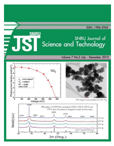
SNRU Journal of Science and Technology 7(2) July – December 2015 90–94

Sakon Nakhon Rajabhat University
Journal of Science and Technology

Journal home page: snrujst.snru.ac.th



Matrices which have similar properties to Padovan Q-Matrix and its generalized relations

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Received: 24 June 2015; **Revised:** 1 September 2015; **Accepted:** 15 October 2015; **Available online:** 15 December 2015

Paper selected from Sakon Nakhon Rajabhat University - International Conference 2015 (SNRU-IC 2015)

Abstract

In this research, we studied and found the new fives matrices of 3×3, which it have similar properties to Padovan Q-Matrix. In the results, we got the new theorems of five matrices that when raised to the n^{th} powers give results matrices whose entries are Padovan numbers and some new relations of Padovan number from these new matrices.

Keywords: Padovan sequences, Padovan Q-Matrix, Padovan numbers

1. Introduction

Nowadays, there are many studies in the literature that concern the sequences such as Fibonacci, Lucas, Pell, Jacobsthal, Padovan, and Perrin [1-5] On the other hand, the matrix sequences have taken so much interest in diffrent types of numbers [6-8]. Therefore, a new matrix sequence related to less known numbers it is worth studying. For Padovan sequence is named after Richard Padovan who attributed its discovery to Dutch architect Hans van der Laan in his 1994 essay *Dom Hans van der Laan: Modern Primitive* [9].

The Padovan sequence is the sequence of integers P_n defined by the initial values $P_0 = 0$, $P_1 = 0$, $P_2 = 1$ and the recurrence relation $P_n = P_{n-2} + P_{n-3}$, for all $n \geq 3$. The first few values of P_n are

0, 0, 1, 0, 1, 1, 1, 2, 2, 3, 4, 5, 7, 9, 12, 16, 21, 28, 37, 49, 65, 86, …

The Padovan Q-Matrix is a 3 x 3 matrix that when raised to the n^{th} power give a matrix whose entries are Padovan numbers [7]. That is

$$Q=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

and the n^{th} powers of Q give

$$Q^n = \begin{bmatrix} P_{n-1} & P_{n+1} & P_n \\ P_n & P_{n+2} & P_{n+1} \\ P_{n+1} & P_{n+3} & P_{n+2} \end{bmatrix}.$$

In this work, we are interested to discover the new matrix which it has similar properties to Padovan Q-matrix and also its generalized relations.

2. Main Results

In main idea of this work, we use the Padovan Q-matrix and consider the initial values of Padovan sequence to construct and rearrange the element of matrix. For the new matrices, we have the following theorems and its relations.

Theorem 1: If $Q_1 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ then $Q_1^n = \begin{bmatrix} P_{n+2} & P_n & P_{n+1} \\ P_{n+1} & P_{n-1} & P_n \\ P_{n+3} & P_{n+1} & P_{n+2} \end{bmatrix}$, for all $n \geq 1$.

Proof:

By the principle of mathematical induction on n. For $n = 1$, it is easy to see that

$$Q_1^1 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} P_3 & P_1 & P_2 \\ P_2 & P_0 & P_1 \\ P_4 & P_2 & P_3 \end{bmatrix} = \begin{bmatrix} P_{1+2} & P_1 & P_{1+1} \\ P_{1+1} & P_{1-1} & P_1 \\ P_{1+3} & P_{1+1} & P_{1+2} \end{bmatrix}$$

So, it's true for $n = 1$. Next, suppose that it is true for $n = k$. That is

$$Q_1^k = \begin{bmatrix} P_{k+2} & P_k & P_{k+1} \\ P_{k+1} & P_{k-1} & P_k \\ P_{k+3} & P_{k+1} & P_{k+2} \end{bmatrix}$$

Consider for $n = k+1$:

$$Q_1^{k+1} = Q_1^k Q_1 = \begin{bmatrix} P_{k+2} & P_k & P_{k+1} \\ P_{k+1} & P_{k-1} & P_k \\ P_{k+3} & P_{k+1} & P_{k+2} \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} P_k + P_{k+1} & P_{k+1} & P_{k+2} \\ P_{k-1} + P_k & P_k & P_{k+1} \\ P_{k+1} + P_{k+2} & P_{k+2} & P_{k+3} \end{bmatrix} = \begin{bmatrix} P_{k+3} & P_{k+1} & P_{k+2} \\ P_{k+2} & P_k & P_{k+1} \\ P_{k+4} & P_{k+2} & P_{k+3} \end{bmatrix}$$

$$= \begin{bmatrix} P_{(k+1)+2} & P_{k+1} & P_{(k+1)+1} \\ P_{(k+1)+1} & P_{(k+1)-1} & P_{k+1} \\ P_{(k+1)+3} & P_{(k+1)+1} & P_{(k+1)+2} \end{bmatrix}$$

Therefore, it is true for every integer $n \geq 1$.

Proposition 2: For all integers m, n such that $0<m<n$, we have the following relations :

$$P_n = P_{m+2} \cdot P_{n-m} + P_m \cdot P_{n-m-1} + P_{m+1} \cdot P_{n-m+1} \quad (1)$$

and

$$P_n = P_{m+1} \cdot P_{n-m+1} + P_{m-1} \cdot P_{n-m} + P_m \cdot P_{n-m+2} \quad (2)$$

Proof: From Theorem 1 and the laws of exponent for the square matrix, we have the following:

$$Q_1^n = Q_1^m Q_1^{n-m}$$

$$\begin{bmatrix} P_{n+2} & P_n & P_{n+1} \\ P_{n+1} & P_{n-1} & P_n \\ P_{n+3} & P_{n+1} & P_{n+2} \end{bmatrix} = \begin{bmatrix} P_{m+2} & P_m & P_{m+1} \\ P_{m+1} & P_{m-1} & P_m \\ P_{m+3} & P_{m+1} & P_{m+2} \end{bmatrix} \begin{bmatrix} P_{n-m+2} & P_{n-m} & P_{n-m+1} \\ P_{n-m+1} & P_{n-m-1} & P_{n-m} \\ P_{n-m+3} & P_{n-m+1} & P_{n-m+2} \end{bmatrix}$$

By consider the corresponding elements on left and right-hand of above equation. We have,

$$P_n = P_{m+2} \cdot P_{n-m} + P_m \cdot P_{n-m-1} + P_{m+1} \cdot P_{n-m+1}$$

and

$$P_n = P_{m+1} \cdot P_{n-m+1} + P_{m-1} \cdot P_{n-m} + P_m \cdot P_{n-m+2}$$

Remark 1: In Proposition 2(1), If m = 3, then we have

$$\begin{aligned} P_n &= P_5 \cdot P_{n-3} + P_3 \cdot P_{n-4} + P_4 \cdot P_{n-2} \\ &= 1 \cdot P_{n-3} + 0 \cdot P_{n-4} + 1 \cdot P_{n-2} \qquad (\because P_4 = P_5 = 1, P_3 = 0) \\ &= P_{n-3} + P_{n-2} \end{aligned}$$

In Proposition 2(2), If m = 2, then the result is similarly to the Proposition 2(1). This results are the generalized relation of Padovan sequences. Moreover, in our research, we have the following theorems and propositions for the new four matrices which it have similar properties to Padovan *Q*-matrix. For proving of these theorems and propositions are omitted, since it similar to the proving of Theorem 1 and Proposition 2.

Theorem 3: If $Q_2 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ then $Q_2^n = \begin{bmatrix} P_{n-1} & P_n & P_{n+1} \\ P_{n+1} & P_{n+2} & P_{n+3} \\ P_n & P_{n+1} & P_{n+2} \end{bmatrix}$, for all $n \geq 1$.

Proposition 4: For all integers m, n such that $0<m<n$, we have the following relations :

$$P_n = P_{m-1} \cdot P_{n-m} + P_m \cdot P_{n-m+2} + P_{m+1} \cdot P_{n-m+1} \quad (3)$$

and

$$P_n = P_m \cdot P_{n-m-1} + P_{m+1} \cdot P_{n-m+1} + P_{m+2} \cdot P_{n-m} \quad (4)$$

Theorem 5: If $Q_3 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ then $Q_3^n = \begin{bmatrix} P_{n+2} & P_{n+3} & P_{n+1} \\ P_{n+1} & P_{n+2} & P_n \\ P_n & P_{n+1} & P_{n-1} \end{bmatrix}$, for all $n \geq 1$.

Proposition 6: For all integers m, n such that $0<m<n$, we have the following relations :

$$P_n = P_{m+1} \cdot P_{n-m+1} + P_{m+2} \cdot P_{n-m} + P_m \cdot P_{n-m-1} \quad (5)$$

and

$$P_n = P_m \cdot P_{n-m+2} + P_{m+1} \cdot P_{n-m+1} + P_{m-1} \cdot P_{n-m} \tag{6}$$

Theorem 7: If $Q_4 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ then $Q_4^n = \begin{bmatrix} P_{n+2} & P_{n+1} & P_n \\ P_{n+3} & P_{n+2} & P_{n+1} \\ P_{n+1} & P_n & P_{n-1} \end{bmatrix}$, for all $n \geq 1$.

Proposition 8: For all integers m, n such that $0 < m < n$, we have the following relations :

$$P_n = P_{m+2} \cdot P_{n-m} + P_{m+1} \cdot P_{n-m+1} + P_m \cdot P_{n-m-1} \tag{7}$$

and

$$P_n = P_{m+1} \cdot P_{n-m+1} + P_m \cdot P_{n-m+2} + P_{m-1} \cdot P_{n-m} \tag{8}$$

Theorem 9: If $Q_5 = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ then $Q_5^n = \begin{bmatrix} P_{n+2} & P_{n+1} & P_{n+3} \\ P_n & P_{n-1} & P_{n+1} \\ P_{n+1} & P_n & P_{n+2} \end{bmatrix}$, for all $n \geq 1$.

Proposition 10: For all integers m, n such that $0 < m < n$, we have the following relations :

$$P_n = P_m \cdot P_{n-m+2} + P_{m-1} \cdot P_{n-m} + P_{m+1} \cdot P_{n-m+1} \tag{9}$$

and

$$P_n = P_{m+1} \cdot P_{n-m+1} + P_m \cdot P_{n-m-1} + P_{m+2} \cdot P_{n-m} \tag{10}$$

3. Conclusion

From results of the research, we have five matrices which have similar properties to Padovan Q-Matrix and its generalized relations. In future, we may extend this work by extending the size of matrix and find the matrix which raised to the n^{th} powers give results matrices whose entries are Padovan numbers and some new relations or its applications.

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