



Adaptive Thresholding Function Image Denoising using the CSO Algorithm

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Abstract

The noise signal from the image that occurs from natural including Gaussian, speckle, and salt and pepper noise the image that affects the interpretation of the image. The objectives of this research were to adaptive threshold function method for image denoising in the wavelet domain within the Competitive Swarm Optimizer (CSO) modeling of sub-band coefficients. In this approach, the stochastic global optimization techniques such as Competitive Swarm Optimizer (CSO), Cuckoo Search (CS) algorithm, artificial bee colony (ABC), Genetic algorithm (GA), Jaya Algorithm and particle swarm optimization (PSO) technique were used for learn about the parameters for adaptive thresholding functions that are required for optimizing the performances. The thresholding function is optimized to address such noise through the use of various evolutionary optimization algorithms in terms mean squared error (MSE) to produce better de-noised images. It was found that the CSO algorithm algorithm-based denoising methods give better performance in terms of peak signal-to-noise ratio (PSNR) and Image Quality index (IQI) as compared to Cuckoo-based de-noising approach, which is effective in denoising. Comparative results of the peak signal-to-noise ratio and image quality index demonstrate the robustness of the proposed optimization algorithm.

Keywords: Image de-noising; threshold function; Competitive Swarm Optimizer; salt and pepper noise; speckle noise; Gaussian noise

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1. Introduction

Images are another type of information. That may be affected by interference from the capturing process and data transmission. By noise sources that cover distortions from additive noise (Gaussian noise), multiplicative noise (speckle noise), and salt and pepper noise artefacts within different imaging modalities, which degrade the image quality. The first methods introduced for image denoising were based on statistical filters. The *wavelet thresholding*, has been an active area of interest in the past two decades. Wavelet domain-based noise eliminated techniques require the determination of a threshold value to remove the smaller coefficients of the descriptions sub-bands, while preserving the larger coefficients; as the small coefficients are generally noisy, and the large coefficients contain the main attribute of the image.

As for common properties of wavelet coefficients, such as segmentation [1], the thresholding technique within the wavelet domain has become easy to implement. The *Wavelet shrinkage* methodology, proposed by Donoho et al. [2, 3], classified the wavelet coefficients of real-world noisy data that extended the peak signal-to-noise ratio. Intensive research [4] has been carried out in order to enhance thresholding performance based mostly upon wavelet domain. Various noise models for the distribution of noise wavelet coefficients, such as the hidden Markov model [5] and the Gaussian model were also analyzed. Additionally, perceptual quality of a picture can be improved through

proper shrinkages that uses optimum threshold values for determining various adaptive methods on the basis of Wavelet Transform (WT) [8, 9], discrete cosine transform (DCT), partial differential equations, contourlet transformation, and undecimated Discrete Wavelet Transform DWT [6]. Nasri and Pour [7] introduced the adaptive neural network methodology, which outperformed many existing thresholding methodologies, like soft, hard, garrote, and different varied WT-based approaches. However, limitations exist inside this approach, because of the correct format of threshold values and different thresholding parameters needed during this methodology; a lot of that has been overcome through the employment of the particle swarm optimization (PSO) algorithm as well as the substitution of the sheerest descent gradient-based LMS techniques. This technique gives higher performances with rapid convergence rates. In addition, more time is required to train the new model and build.

The focus of this paper is therefore to improve the effectiveness of the WT methodology, thereby eliminating salt and pepper, speckle, and Gaussian noises, at various intensities. In this paper, based on the discrete wavelet transform and thresholding function, an integrated optimization algorithm is denoising image. The novel idea here is the combination of improved thresholding function algorithm and high efficiency in noise reduction. There are research studies on this method with PSO, CSO, Cuckoo, GA, Jaya and ABC algorithms. Section II outlines the justification for our research and provides background information that supports our work and our proposed methodology. In Section III, we present the qualitative and quantitative results of the proposed method with CSO [10] and modified CSO [supported by peak signal-to-noise ratio (PSNR) and image quality index (IQI)]. Our conclusions are presented in the final section.

2. Materials and Methods

The basics of de-noising within the wavelet domain, namely speckle, Gaussian and salt and pepper noise, are presented in this section.

Related work

Thersholding function

The selection of the thresholding function is the main proposed of wavelet threshold de-noising. In this method, the thresholding has a considerable effect on the quality of the reconstructed image. Thresholding functions use a patch between soft and hard functions and have advantages over both of them. As they have a fixed feature and rely on the fixed threshold value, Nari and Pour [8] they are not flexible in their usage, as shown in Equation (1).

$$n(x, \lambda, m, k) = \text{real} \begin{cases} x + (k-1)\lambda - \frac{0.5k\lambda m}{x^{m-1}} & x > \lambda \\ 0.5k \frac{k|x|^{m-[(z-k)/k]} \text{sign}(x)}{\lambda^{m+[(z-zk)/k]}} & |x| \leq \lambda \\ x - (k-1)\lambda - \frac{0.5k(-\lambda)m}{x^{m-1}} & x < -\lambda \end{cases} \quad (1)$$

When adapting $k \in (0,1]$ in the given expression, the thresholding function η varies from hard to soft thresholding. The flexibility obtained from adjusting the thresholding from hard to soft results from the variation of k . The parameter m determines the shape which makes the thresholding function more flexible. The λ is threshold value, which plays an important role in the thresholding method. Here λ, m and k are required to obtain the favorable maximum value of thresholded wavelet coefficients.

Discrete wavelet transform

The Discrete Wavelet Transform is an efficient and useful tool for signal and picture processing applications, and is adopted in several rising standards, including one of the newer image compression methods JPEG2000. These options permit the DWT to be tailored to suit a large variety of applications. The Fourier convert of a function $\phi(t)$ for all L2I is $\Psi(\omega)$, as show in the equation below.

$$C_\phi = \int_0^{+\infty} \frac{|\Psi(\omega)|}{\omega} d\omega < \infty \quad (2)$$

The function $\phi(t)$ is the substructure wavelet. A wavelet sequence is obtained by sizing and moving the basis wavelet function:

$$\phi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \phi\left(\frac{t-b}{a}\right) \quad (3)$$

where $x, y \in \mathbb{R}$, $x \neq 0$, x are means as the expand factors and y is determined as the transfer factor.

Evaluation metrics

Evaluate the performance of the de-noising techniques researchers use mean squared error (MSE) and peak signal to noise ratio (PSNR), both of which are fully automated techniques. However, an improved PSNR does not mean that the visual quality of the picture is going to be good. To substitute this shortfall, in this project, Image Quality Index (IQI) is regarded as the second constants for judging the standard of the picture that has been denoised.

Mean Squared Error is the parameter that takes the major notable meaning in term of noise elimination.

$$MSE = \frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n (O(i,j) - D(i,j))^2 \quad (4)$$

O – Perfect image, D – Denoised picture, i – pixel row index, j – pixel Column index

Peak Signal-to-Noise Ratio (PSNR) could be a mathematical measure of picture quality that supports the component variations between two pictures. PSNR in *Daubechies wavelets (db)* is often calculated as follows:

$$PSNR = 10 \log_{10} \left(\frac{MAX_i^2}{MSE} \right) = 20 \log_{10} \left(\frac{MAX_i}{\sqrt{MSE}} \right) \quad (5)$$

Image Quality Index (IQI) (6), which determined Q there are three meanings: wastage of correlation, lightness perversion, and resolution perversion:

$$Q = \frac{\sigma_f \sigma_g}{\sigma_f^2 + \sigma_g^2} \cdot \frac{2f_g}{f^2 + g^2} \cdot \frac{2\sigma_f \sigma_g}{\sigma_f^2 + \sigma_g^2} \quad (6)$$

Where

$$S \quad f = \frac{1}{M} \sum_{i=1}^M f_i, g = \frac{1}{M} \sum_{i=1}^M g_i, \sigma_f^2 = \frac{1}{M-1} \sum_{i=1}^{M-1} (f_i - \bar{f})^2$$

$$\sigma_g^2 = \frac{1}{M-1} \sum_{i=1}^{M-1} (g_i - \bar{g})^2, \sigma_{fg} = \frac{1}{M-1} \sum_{i=1}^{M-1} (f_i - \bar{f})(g_i - \bar{g})$$

In this section, the equation (6) is the relation coefficient with these two values g and f ; Then, the level of linear relation between g and f measured. The dynamic variance is between – 1 and 1. The next component is a value rang of [0, 1]. The mean lightness is between g and f , σ_g and σ_f is considered because of the approximation of distinction of g and f , The final parameter with a value range of [0,1] measures the homologous and difference of the image. The dynamic range of Q is [–1, 1]. The most effective value 1 is achieved, where $g_i = f_i$ for all $i = 1, 2, \dots, M$. the minimum value -1 when the condition $g_i = 2f_i - f_i$ for all $i = 1, 2, \dots, M$.

Gaussian noise can be from natural sources like thermal vibration of atoms, similarly because the separate nature of radiation found in heat materials. Gaussian noise usually infests the gray color values in an exceedingly pixel model image, which is why the Gaussian noise layout is intended to normalize the various gray values.

Speckle noise, which in an increasing noise, happens inside coherent imaging systems, like in lasers, radar, and acoustics. Generally, in terms of multiplicative nature of speckle noise, it should initially be transformed to an additive noise source through the application of a power operator.

Images corrupted by salt and pepper noise, which is often found in data transmission, represent the incorrect values of pixels that have degraded or changed. While some neighboring pixels may remain unchanged, the corrupted image pixel values of eight bit transmissions occur in maximum or minimum pixel values of 255 or 0, respectively.

Proposed method

The objective of this research is to analyze and improve the de-noising CSO algorithm and wavelet transform methods. Optimization would occur through the use of the CSO with PSNR as its fitness function. CSO provides outstanding performance over the genetic algorithm (GA), particle swarm optimization (PSO), artificial bee colony (ABC), cuckoo search algorithm (CSA), and Jaya algorithm. Furthermore, we employed a soft computing technique to compare our proposed method with the other existing methods listed above.

The minimum value of MSE is measured through calculation of Equation (4). We used thresholding function parameters for the minimum value of MSE in order to obtain the maximum evaluation quality of the thresholding function. The fitness conditions for the CSO are show in (7).

$$f = MSE(p, \hat{p}) = \frac{1}{N} \sum_{n=1}^N (p(n) - \hat{p}(n))^2 \quad (7)$$

Where N is the size of the sub-band, $p(n)$ is the WT coefficients of original signal picture, and $\hat{p}(n)$ is the thresholded WT coefficients of a noisy picture.

Accordingly, a fitness worth depends upon the threshold wavelet coefficients $\hat{p}(n)$, that are obsessed on the thresholding function η , where η is the function of λ , k , and m . The parameters λ , k and m are thus chosen as search particles within the CSO-based method. In optimizing the thresholding function using (1), numerous types of optimization techniques are exploited the aim of which is to obtain the minimum value of Equation (7) which may be enforced in each monitored and unattended way. The algorithmic steps for de-noising the image-based CSO are explained as follows:

Step 1: Start with the various thresholding parameters (λ , k , m) where $\lambda = (1-120)$, $k = (0.1-1)$ and $m = (1-3)$ represent the range of the thresholding parameters.

Step 2: Find answers Fitness F for every particle to prepare for a repetitive process in search of the most effective resolution, where F_{pd} is the initial fitness.

Step 3: Initialize the constants and different variables, as follows:

1 $V_{l,k}(t)$ is that the positive acceleration fixed rate, that is the coefficients of the self-memorize element value and the other values around in order; where $V_{l,k}(t)$ is taken as 4.

2. R_1 , R_2 , and R_3 square measure indiscriminately selects a fixed rate that maintains the range of the population. These are square measures regularly spread at the range $[0, 1]$.
3. The iteration number (i) is decided which contains a spread of the most range of iterations (I_{\max}).
4. w is the inertia issue which begins at 0.8, and reduces linearly to 0.5, inside every iteration i .
5. In the position and the speed of the particle are $X_{l,k}$ and $V_{l,k}$, which $X_{l,k}$ is the parameter prepared randomly and $V_{l,k}$ is the parameter prepared to zero.
6. $R_2(k, t)$ and $[X_{w,k}(t) - X_{l,k}(t)]$ square measure the personal best position and global best position.
7. $X_{w,k}(t)$ is the personal best position and $X_{l,k}(t)$ is the global best position, severally. The position of $X_{l,k}(t)$ is held at $X_{w,k}(t)$, and also the position of $X_{l,k}(t)$ is held at $X_{w,k}(t)$.

Step 4: Update the velocities and positions as in (8) and (9), as follows:

$$\begin{aligned} v_{l,k}(t+1) &= R_1(k, t)v_{l,k}(t) + R_2(k, t)(X_{w,k}(t) - X_{l,k}(t)) \\ &\quad + \varphi R_3(k, t)(\bar{X}_k(t) - X_{l,k}(t)) \end{aligned} \quad (8)$$

$$X_{l,k}(t+1) = X_{l,k}(t) + V_{l,k}(t+1) \quad (9)$$

Step 5: Update the position and speed, which are limited by the scope value. The scope value is 85% of the most and lowest value. steps 3 – 5, until the stoppi

Step 6: Repeat s ng criteria are met or the utmost variety of iterations are reached.

The CSO methodology is further illustrated in the following flowchart, (Fig. 1).

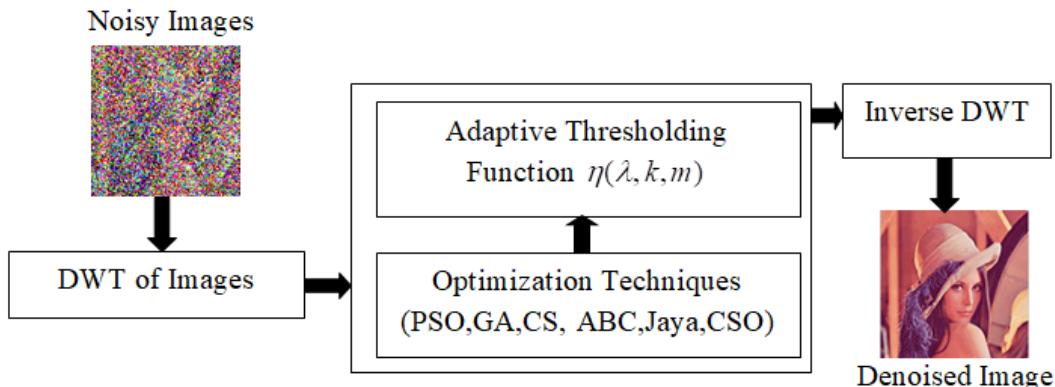


Fig. 1 The Methodology research model.

3. Results and Discussion

In this section, we shall discuss performance results of the proposed approach on the standard 512×512 quality image *Lena* (Fig. 2), which is impure with salt and pepper, Gaussian, and speckle noise of different degradation levels. The results from the use of the proposed filter for reduction were estimated with wavelets through PSNR and IQI visual quality metrics. Below are the implementation details for the various methods and data about the noise sources and their intensities.

A. Implementation details and parameters

A computer simulation was utilized to present the test results in order to create visual images similar to the original 512×512 resolution. Some implementation constraints imposed upon the wavelet are as follows: db4 wavelet name was used in the wavelet thresholding methods containing Level 3 decomposition. Comparative analysis approaches included the particle swarm optimization (PSO), cuckoo search algorithm (CSA), genetic algorithm(GA), Artificial Bee Colony algorithm (ABC), and

the Jaya algorithm (Jaya). The parameters of the CSO optimization algorithm were: *The experiments were proceeded to estimate the performance of the existing de-noising methods with noises at completely different levels.*



Fig. 2 Original *Lena* image, showing an almost clear background.

B. Experiment: Gaussian, salt and pepper, and speckle noise

In the first results, the performance of the proposed method is compared with state of the art algorithms on the standard *Lena* image which has been tested for adulterate with Gaussian noise at an intensity of 0.1; speckle noise, at an intensity of 0.1; and salt and pepper, at an intensity of 0.05. PSNR and IQI values for the highest noise intensity are shown in the Tables 1 – 3, in which the best values are highlighted. Image comparisons were also attained in order to thoroughly investigate the de-noising behavior all of the approaches contained within our study.

The proposed approach exceeded the study criterion, as the CSO optimization proved to be superior for proficient noise removal. The resulting PSNR and IQI values of the de-noising methods after the addition of the Gaussian noise are presented in Table 1. The PSNR values observed through each iteration appeared to be stable.

Table 1 The comparison of parameter values after the addition of Gaussian noise variance 0.1 to the test image.

No. of iteration	PSO		Cuckoo		ABC		GA		Jaya		CSO (proposed)	
	PSNR	IQI	PSNR	IQI	PSNR	IQI	PSNR	IQI	PSNR	IQI	PSNR	IQI
10	30.1134	0.09082	29.0950	0.09135	27.2615	0.091766	34.71734	0.090975	27.64687	0.090094	34.8113	0.091485
50	30.1185	0.09135	30.0377	0.08941	27.2619	0.090131	34.60134	0.089817	27.64926	0.089921	34.8113	0.091230
100	30.1257	0.09038	30.0607	0.09047	27.3291	0.098480	34.71134	0.080309	27.64270	0.090813	34.8113	0.090803
150	30.1168	0.09129	30.1134	0.09005	27.3762	0.090281	34.71134	0.080415	27.63932	0.090128	34.8113	0.091118
200	30.1462	0.09052	30.1235	0.09058	27.4349	0.090508	34.64256	0.070165	27.63891	0.089718	34.8113	0.090131
250	30.1169	0.09028	30.1348	0.08983	27.4734	0.090441	34.71124	0.089888	27.64096	0.091318	34.8113	0.090555
300	30.1009	0.09119	30.1009	0.09003	27.5323	0.090192	34.71134	0.080611	27.63333	0.089301	34.8113	0.089610
350	30.1046	0.09119	30.0929	0.09038	27.5154	0.0906100	34.71134	0.080337	27.63453	0.091072	34.8113	0.090412
400	30.1015	0.09043	30.1140	0.09049	27.5985	0.089937	34.53034	0.060529	27.64092	0.090743	34.8113	0.091270
450	30.1236	0.09000	30.1344	0.08981	27.6636	0.090859	34.61234	0.070082	27.64070	0.090361	34.8113	0.090414
500	30.1047	0.09037	30.1149	0.08946	27.6641	0.089798	34.71134	0.080214	27.63309	0.090361	34.8113	0.090591

We may conclude from Table 1, *Lena* image, that the proposed de-noising algorithm outperforms other approaches in terms of both PSNR and IQI values. Visual comparisons also show that the CSO algorithm, which is specifically designed for Gaussian noise, resulted in higher PSNR values.

The performance comparison (PSNR and IQI) of the state-of-the-art de-noising algorithms applied to the *Lena* image damaged with Gaussian noise at 0.1 is shown in (Fig. 3).

A visual comparison of all de-noising approaches demonstrates that the proposed CSO algorithm applied to the salt and pepper noise at an intensity of 0.05 of the *Lena* image yielded exceptional results. Neither wavelet methods was capable of satisfactorily removing the salt and pepper noise, as compared to the CSO algorithm results, which are specifically tasked to eliminate salt and pepper noise.

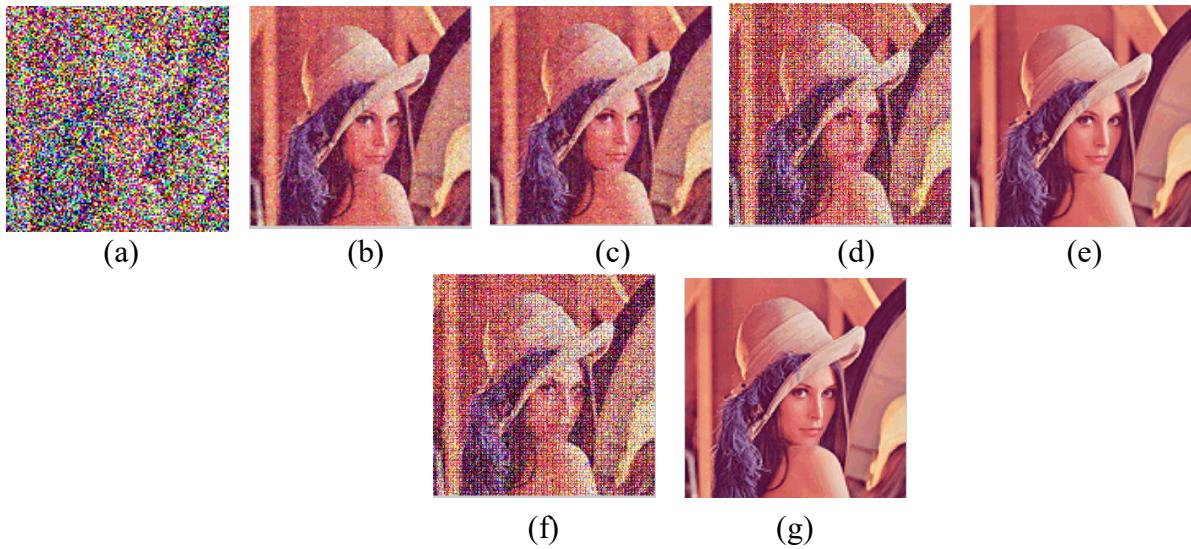


Fig. 3 (a) noisy image; the (b) PSO, (c) Cuckoo, (d) ABC, (e) GA, (f) Jaya, and (g) the proposed CSO algorithms.

Table 2 The comparison of parameter values after the addition of salt and pepper noise variance 0.05 to the test image.

No. of iteration	PSO		Cuckoo		ABC		GA		Jaya		CSO (proposed)	
	PSNR	IQI	PSNR	IQI								
10	31.9390	0.20556	34.6066	0.20702	27.8812	0.20489	34.6105	0.20484	31.2832	0.20320	34.8113	0.20555
50	31.8869	0.20611	34.7139	0.20490	28.0513	0.20512	34.5301	0.20628	31.0824	0.20450	34.8113	0.20214
100	31.9514	0.20823	34.6176	0.20454	28.0950	0.20096	34.6023	0.20462	30.9757	0.20569	34.8113	0.20410
150	31.9078	0.20663	34.7251	0.20687	28.1350	0.20548	34.5072	0.20537	30.9602	0.20730	34.8113	0.20777
200	32.9273	0.20429	34.7057	0.20312	28.8447	0.20258	34.6103	0.20556	30.9704	0.20634	34.8113	0.20525
250	31.9054	0.20418	34.7993	0.20583	28.7030	0.20529	34.4803	0.20353	30.9856	0.20339	34.8113	0.20284
300	32.8785	0.20283	34.7975	0.20513	29.2834	0.20701	34.7015	0.20454	30.9802	0.20384	34.8113	0.20678
350	31.8996	0.20445	34.7234	0.20368	29.6027	0.20532	34.6025	0.20353	30.9887	0.20378	34.8113	0.20576
400	32.9303	0.20595	34.7900	0.20573	30.4760	0.20707	34.6106	0.20794	30.9649	0.20673	34.8113	0.20423
450	32.9113	0.20367	34.7010	0.20426	31.0777	0.20438	34.7104	0.20346	30.9854	0.20489	34.8113	0.20797
500	32.9557	0.20725	34.7108	0.20785	31.2399	0.20717	34.7254	0.20478	30.9495	0.20622	34.8113	0.20405

Table 2 outlines the results of the de-noising methods after removing salt and pepper noise in terms of both PSNR and IQI values. It can be seen that the performance of the proposed filter algorithm, through the behavior of the CSO algorithm, is far superior to that of all other de-noising techniques in terms of PSNR and IQI values. Visual comparison confirms that all of the comparative filter algorithms are ineffective for both Gaussian and salt and pepper noise removal.

Table 3 The comparison of parameter values after the addition of speckle noise variance 0.1 to the test image.

No. of iteration	PSO		Cuckoo		ABC		GA		Jaya		CSO (proposed)	
	PSNR	IQI	PSNR	IQI								
10	33.9830	0.24130	33.9350	0.24100	27.8812	0.20489	33.5130	0.24070	29.1150	0.24190	34.8113	0.24049
50	33.9110	0.24130	33.8840	0.24080	28.0513	0.20512	32.1304	0.24080	29.0880	0.24070	34.8113	0.24008
100	33.9160	0.24070	33.9260	0.24090	28.0950	0.20096	32.4709	0.24150	29.0510	0.24140	34.8113	0.24032
150	33.9160	0.24040	33.9330	0.24120	28.1350	0.20548	33.7102	0.24170	29.0640	0.24130	34.8113	0.24123
200	33.9170	0.24110	33.9180	0.24090	28.8447	0.20258	32.5710	0.24120	29.0460	0.24120	34.8113	0.24155
250	33.9280	0.24080	33.9260	0.24160	28.7030	0.20529	33.3021	0.24120	29.0430	0.24120	34.8113	0.24196
300	33.9150	0.24110	33.9280	0.24070	29.2834	0.20701	32.6015	0.24130	29.0470	0.24160	34.8113	0.24115
350	33.9310	0.24160	33.9240	0.24150	29.6027	0.20532	33.7023	0.24120	29.0410	0.24120	34.8113	0.24063
400	33.9310	0.24110	33.9440	0.24180	30.4760	0.20707	33.7106	0.24130	29.0510	0.24240	34.8113	0.24128
450	33.9230	0.24090	33.9260	0.24200	31.0777	0.20438	33.6304	0.24180	29.0560	0.24140	34.8113	0.24185
500	33.9210	0.24110	33.9180	0.24150	31.2399	0.20717	33.7054	0.24160	29.0490	0.24190	34.8113	0.24142

Table 3 shows the comparative performances of all algorithms on the standard *Lena* image damaged with multiplicative (speckle noise) at 0.1 intensity. Again, the proposed CSO de-noising algorithm outperformed all others, as evidenced in both PSNR and IQI values. Their IQI values

averaged 0.20513, or approximately 3.41 %, which is double that of the best case. The research found iteration tested 10 de-noising observed that were PSNR values higher and the same constant compared to other methods. Since the CSO can work without using neighborhood controls, which this means that each particle will participate in completion only once, as shown these PSNR values are shown in table 1-3.

4. Conclusion

We propose herein the optimization algorithm referred to as the Competitive Swarm Optimization algorithm. Research comparisons were made among different de-noising algorithms and measures in terms of PSNR and IQI values, in which the proposed method outperformed the other algorithm tested. In this paper, we explored the CSO as an optimization tool for different de-noising algorithms, on different kinds of noise and at a variety of noise intensities contaminated. The proposed approach proved to be superior to the other methods in de-noising speckle, salt and pepper, and Gaussian noise. On average, a 5-10% rise in PSNR was achieved for each type of noise at varying intensities. Compared with GA, the average enhancement of six PSNR was double that of the other methods. The successful PSNR values remained constant throughout each round of the test. We can, therefore, may conclude that the proposed is more effective de-noising all kinds of noise at a variety of noise intensities contaminated. Furthermore, CSO, being an increase performance approach, obtained the best results in this study. However, due to its computational inefficiency, different variants of the CSO are the current focus of several research studies. This suggests that a comparison of the Artificial intelligence techniques used in image processing is required. In future works, we intend to investigate more thoroughly the behavioral characteristics of the CSO algorithm.

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