การคำนวณหาการกระจัดแนวระดับและแนวดิ่งของลูกวอลเลย์บอล ภายใต้แรงเริ่มต้นที่บังคับ

EVALUATION OF HORIZONTAL AND VERTICAL DISPLACEMENT OF A VOLLEYBALL UNDER APPLIED INITIAL FORCE

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บทคัดย่อ

แรงที่กระทำโดยผู้เล่นวอลเลย์บอลเมื่อพวกเขาตีลูกบอลจากดินแดนของตนเองผ่านตาข่ายไปยังขอบเขตของฝ่ายตรงข้ามสามารถคำนวณและสร้างแบบจำลองได้รับการออกแบบโดยมีเงื่อนไขว่าลูกบอลจะต้องไม่ออกจากพื้นที่สนามวอลเลย์บอล ซึ่งงานวิจัยนี้สร้างสื่อการเรียนการสอนนี้ใช้โปรแกรมทางคณิตศาสตร์เพื่ออธิบายการเคลื่อนที่ของวอลเลย์บอล ผลการวิจัยพบว่า แรงของผู้ตีลูกวอลเลย์บอลเป็นฟังก์ชันทางคณิตศาสตร์ของการใช้กำลังเริ่มต้นกับเวลาเกี่ยวข้องกับฟังก์ชันลอการิทึม เลขชี้กำลัง และตรีโกณมิติลูกวอลเลย์บอลลงสนามไม่ออก ซึ่งใช้กฎการเคลื่อนที่ข้อที่สองของนิวตันในการคำนวณหาความเร็วที่ขึ้นกับเวลาและการกระจัดในแนวนอนและแนวตั้งของวอลเลย์บอลภายใต้แรงเริ่มต้นที่ใช้แรงของผู้ตีลูกวอลเลย์บอลสองประเภท แต่ละแรงถูกรวมเข้าด้วยกันเพื่อค้นหาความเร็วเริ่มต้น ผลลัพธ์กำหนดการเคลื่อนที่

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ในแนวตั้งเป็นฟังก์ชันแนวนอน กราฟถูกพล็อตระหว่างการกระจัดในแนวนอน (X) และการกระจัดในแนวตั้ง (Y)

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Abstract

The forces exerted by volleyball players when they hit the ball from their own realm over the net to the realm of the opposing side could be calculated and a model was designed with the condition that the ball must not leave the court area. This research created the instructional material using a mathematical program to describe the movement of volleyball. The research revealed that the volleyball batter force was a mathematical function of applying the initial force to time as it related to the logarithmic, exponential and trigonometric functions. Newton's second law of motion was used to calculate time-dependent velocity and horizontal and vertical displacement of the volleyball under two types of applied initial force. Each force was integrated to find the initial velocity. The results defined the vertical displacement as a horizontal function. A graph was plotted between horizontal displacement (X) and vertical displacement (Y).

Keywords: volleyball, velocity, displacement of the volleyball

Introduction

The trajectory of motion of a volleyball consists of many repetitions of short and forceful exercises, performed throughout the duration of the game. A review of the available literature concluded that the trajectory of motion of a volleyball depended on explosive strength as the applied initial force (Jalilian et al., 2014). However, little information is available regarding volleyball displacement related to apply initial force by volleyball players.

Lidor & Gal (2010) reviewed several studies (n=31) on physical attributes, physiological attributes and on-court performances of female volleyball players. Mohammadi & Malek (2012) researched a design of experiment (DOE) as a procedure of planning controlled experiments to investigate the effect of certain processes on experimental units. Ferrara & Alfredo (2018) designed a training program to evaluate the effectiveness of an instrument to measure the pressure exerted by the palm of the hand on a volleyball during service execution.

Horizontal and vertical displacement of a volleyball were calculated under seven types of applied initial force (Jalilian et al., 2014). In section two of the method of evaluation, we present the calculation of horizontal and vertical displacement. Section three shows numerical calculation methods and graphs for the seven types of applied initial force. Calculations are presented at the end of the document.

Materials and methods

The method of evaluation of vertical and horizontal for volleyball displacement

Experiments were designed which related the fields of physics and mathematics. Newton’s second law of motion, momentum theory and the mathematics of first and second order differential equation were studied by integrating one part at a time using basic calculus, polynomials, trigonometric, logarithmic and exponential functions.

Applied initial forces for volleyball players as time-dependent functions were determined.

Table 1 Illustration of various the time-dependent applied initial forces.

Function type	Force related to time
1. Trigonometric	$\vec{F}_1(t)=\frac{F_0}{\beta}\cos(\omega t)$
2. Logarithmic	$\vec{F}_2(t)=\frac{F_0}{\beta}\ln(\omega t)$

To calculate vertical displacement as a horizontal function (y(x)) and plot the graph:

1. Calculate the end speed that will become the initial velocity in the projectile motion

Apply the defined forces to be replaced in Newton's second law of motion for integrating to find the speed of the hand (u_1) used to hit the volleyball (Serway & Jewett, 2006). On integration by parts, the method yields

$$\int \frac{\sum \vec{F}}{m} dt = \int du_1 \qquad v = u_1$$

We define the applied initial force as, $\vec{F}_1(t)=\frac{F_0}{\beta}\cos(\omega t)$ we obtain

$$u_1 = \frac{F_0}{\beta m}\left(\frac{\sin(\omega t)}{\omega}\right) \tag{1}$$

We define the applied initial force as, $\vec{F}_2(t)=\frac{F_0}{\beta}\ln(\omega t)$ we obtain

$$u_1 = \frac{F_0 t\left(-1+\log(t)+\log(\omega)\right)}{m} \tag{2}$$

Then, take the speed of the hand (u_1) and apply the laws of conservation of momentum and energy to calculate the end velocity (v_2) of the volleyball that will become the initial velocity of the projectile motion. Related to momentum theory

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \tag{3}$$

Related to the law of conservation of energy

$$m_1\left(u_1^{\,2} - v_1^{\,2}\right) = m_2\left(v_2^{\,2} - u_2^{\,2}\right) \tag{4}$$

Substituting Equation (3) into Equation (4) and rearranging gives

$$v_1 = (u_2 - u_1) + v_2 \tag{5}$$

Substituting the initial velocity v_1 in Equation (3), we get a new equation as the final velocity of a volleyball

$$\frac{2m_1u_1 - m_1u_2 + m_2u_2}{(m_1 + m_2)} = v_2 \tag{6}$$

2. Calculate the velocity of the volleyball under air resistance force by considering a free body diagram (FBD) showing the projectile motion of the volleyball to be used for calculating the displacement under the air resistance force Related to Newton's second law of motion

$$\Sigma F = ma = m\frac{d}{dt}\left(v_x\hat{i} + v_y\hat{j}\right)$$

$$-F_d\cos(\theta)\,\hat{i} - F_d\sin(\theta)\,\hat{j} - mg\hat{j} - F_0\sin^2(\omega t + \phi)\,\hat{j} = m\frac{d}{dt}\left(v_x\hat{i} + v_y\hat{j}\right) \tag{7}$$

From Equation (7), we consider the horizontal motion of a volleyball

$$-F_d\cos(\theta)\hat{i} = m\frac{d}{dt}v_x \tag{8}$$

We define the parameter F_d as αv_x. Let us consider Equation (8) which when integrated can be written as

$$v_x = ce^{-\int \frac{\alpha}{m}\cos(\theta)dt} \tag{9}$$

We use the condition of motion with $v_x(0) = v_0$ and the solution of Equation (8) as

$$v_x(t) = v_0 e^{-\frac{\alpha}{m}\mathrm{t}\cos\theta} \tag{10}$$

We calculate Equation (10) to produce a new equation as the horizontal displacement that is a function of time

$$x(t) = \frac{mv_0 e^{\frac{-\alpha}{m}t\cos\theta}}{-\alpha\cos\theta} + c \tag{11}$$

We use the condition of motion $x(0) = 0$ with to find the value of constant (c), we obtain

$$c = \frac{v_0 m}{\alpha\cos\theta} \tag{12}$$

Substituting Equation (12) into Equation (11) and rearranging gives

$$x(t) = \frac{v_0 m}{\alpha\cos\theta}(1 - e^{\frac{-\alpha}{m}t\cos\theta}) \tag{13}$$

From Equation (7), we consider the vertical motion of a volleyball

$$-F_d\sin\theta - mg - F_0\sin^2(\omega t - \phi) = \frac{mdv_y}{dt} \tag{14}$$

After rearranging

$$\frac{dv_y}{dt} + \frac{\alpha}{m}v_y\sin\theta = -\left(g + \frac{F_0}{m}\sin^2(\omega t - \phi)\right) \tag{15}$$

Substituting Equation (15) into the completed solution of the non-homogeneous first order linear differential equation (Tikjha et al., 2018), we get

$$\int e^{\frac{\alpha}{m}t\sin\theta}\frac{F_0}{m}\sin^2(\omega t+\phi)\,dt=\frac{F_0}{2m}\left[\int e^{\frac{\alpha}{m}t\sin\theta}dt-\int e^{\frac{\alpha}{m}t\sin\theta}\cos\left(2(\omega t+\phi)\right)dt\right] \tag{16}$$

Which on second right term side integration by parts method yields. After rearranging

$$\int e^{\frac{\alpha}{m}\sin\theta t}\cos\left(2\omega t+2\phi\right)dt=\frac{e^{\frac{\alpha}{m}t\sin\theta}}{2\omega}\sin\left(2\omega t+2\phi\right)-\frac{\alpha\sin\theta}{m2\omega}\int e^{\frac{\alpha}{m}\sin\theta t}\sin\left(2\omega t+2\phi t\right)dt \tag{17}$$

Which on second right term side integration by parts method yields

$$\begin{aligned}&\int e^{\frac{\alpha}{m}\sin\theta t}\sin\left(2\omega t+2\phi t\right)dt\\&=\frac{-e^{\frac{\alpha}{m}\sin\theta}\cos\left(2\omega t+2\phi\right)}{2\omega}+\int\frac{\alpha\sin\theta}{2m}e^{\frac{\alpha}{m}\sin\theta t}\cos\left(2\omega t+2\phi\right)dt\end{aligned} \tag{18}$$

Substituting Equation (18) into Equation (17) and rearranging gives

$$\int e^{\frac{\alpha}{m}\sin\theta t}\cos\left(2\omega t+2\phi\right)dt=\frac{\left(2m^2\omega e^{\frac{\alpha}{m}\sin\theta t}\sin\left(2\omega t+2\phi\right)+\alpha m\sin\theta e^{\frac{\alpha}{m}\sin\theta t}\cos\left(2\omega t+2\phi\right)\right)}{\left(4m^2\omega^2+\alpha^2\sin^2\theta\right)} \tag{19}$$

Substituting Equation (19) into Equation (16) and rearranging gives velocity as a function of time $v(t)$ as

$$v(t)=\frac{-gm}{\alpha\sin\theta}-\left(\frac{F_0}{2m}\left(\frac{m}{\alpha\sin\theta}-\frac{\left(2m^2\omega\sin\left(2\omega t+2\phi\right)+\alpha m\sin\theta\cos\left(2\omega t+2\phi\right)\right)}{\left(4m^2\omega^2+\alpha^2\sin^2\theta\right)}\right)\right)+ce^{\frac{-\alpha}{m}\sin\theta t} \tag{20}$$

We use the condition of motion with $t=0, v_y(0)=0$ to find the value of constant (c), we obtain

$$c=v_0+\frac{gm}{\alpha\sin\theta}+\frac{F_0}{2\alpha\sin\theta}-\frac{F_0\left(2m^2\omega\sin\left(2\phi\right)+\alpha m\sin\left(\theta\right)\cos\left(2\phi\right)\right)}{2m\left(4m^2\omega^2+\alpha^2\sin^2\theta\right)} \tag{21}$$

Substituting Equation (21) into Equation (20) and rearranging gives

$$v_y(t) = \left(v_0 + \frac{gm}{\alpha \sin\theta} + \frac{F_0}{2\alpha \sin\theta} - \frac{F_0\left(2\omega m^2 \sin(2\phi) + \alpha m \sin\theta \cos(2\phi)\right)}{2m\left(4m^2\omega^2 + \alpha^2 \sin^2\theta\right)} \right) e^{\frac{-\alpha}{m}\sin\theta t}$$
$$- \frac{gm}{\alpha \sin\theta} - \frac{F_0}{2\alpha \sin\theta} + \frac{F_0\left(2m^2\omega \sin(2\omega t + 2\phi) + \alpha m \sin\theta \cos(2\omega t + 2\phi)\right)}{2m\left(4m^2\omega^2 + \alpha^2 \sin^2\theta\right)} \quad (22)$$

Related to the derivative of position with respect to time

$$v_y(t) = \frac{dy}{dt} \qquad dy = v_y(t)\,dt \quad (23)$$

Substituting Equation (22) into Equation (23) and integrating by part, we generate a new Equation as the vertical displacement as a function of time $y(t)$

$$y(t) = \left(\frac{m}{\alpha \sin\theta}\left(1 - e^{\frac{-\alpha}{m}\sin\theta}\right)\left(v_0 + \frac{gm}{\alpha \sin\theta} + \frac{F_0}{2\alpha \sin\theta} - \frac{F_0\left(2\omega m^2 \sin(2\phi) + \alpha m \sin\theta \cos(2\phi)\right)}{2m\left(4m^2\omega^2 + \alpha^2 \sin^2\theta\right)} \right)\right)$$
$$- \frac{gmt}{\alpha \sin\theta} - \frac{F_0 t}{2\alpha \sin\theta} + \left(\frac{F_0 2\omega m^2\left(\cos(2\phi) - \cos(2\omega t + 2\phi)\right)}{4m\omega^2\left(4m^2\omega^2 + \alpha^2 \sin^2\theta\right)} \right) + \left(\frac{F_0 \alpha m \sin\theta\left(\sin(2\omega t + 2\phi) - \sin\phi\right)}{4m\omega\left(4m^2\omega^2 + \alpha^2 \sin^2\theta\right)} \right) + y_0 \quad (24)$$

3. Calculate the time that a volleyball takes to move in two dimensions. After rearranging we obtain

$$t = \frac{m}{\cos\theta \ln\left(\frac{mu_0 - \alpha \cos\theta x}{mu_0}\right)} \quad (25)$$

4. Take the time in the x-axis to be replaced in the y-axis to find the y-axis displacement as a function of the x-axis displacement. After rearranging we obtain

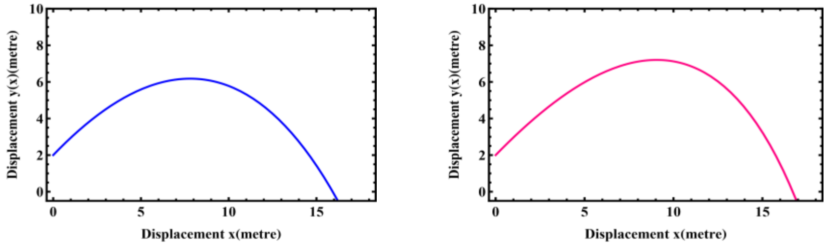
$$y(x) = \left(\frac{m}{\alpha \sin\theta} \left(1 - e^{\frac{-\alpha}{m}\sin\theta t} \right) \left(v_0 + \frac{gm}{\alpha\sin\theta} + \frac{F_0}{2\alpha\sin\theta} - \frac{F_0 \left(2\omega m^2 \sin(2\phi) + \alpha m \sin\theta\cos(2\phi) \right)}{2m\left(4m^2\omega^2 + \alpha^2\sin^2\theta\right)} \right) \right) - \frac{gmt}{\alpha\sin\theta} - \frac{F_0 t}{2\alpha\sin\theta} + \left(\frac{F_0\, 2\omega m^2 \left(\cos(2\phi) - \cos(2\omega t + 2\phi) \right)}{4m\omega\left(4m^2\omega^2 + \alpha^2\sin^2\theta\right)} \right) + \left(\frac{F_0 \alpha m \sin\theta \left(\sin(2\omega t + 2\phi) - \sin\phi \right)}{4m\omega\left(4m^2\omega^2 + \alpha^2\sin^2\theta\right)} \right) + y_0 \qquad (26)$$

5. Plot the graph between horizontal displacement (x) and vertical displacement (y) in program mathematica From Equation (26) is the vertical displacement for volleyball of the horizontal displacement show graph in program. Putting Equation (26) into program mathematica (Zimmerman & Olness, 2002) for plot graph. Next, we can be using the applied initial force $F_1(t)$ and $F_2(t)$ (Ricardo, 2014) evaluation of the vertical displacement $y(x)$ via the first-order linear differential equation. The initial force parameter (F_0) of the applied initial force is the independent variable. The initial velocity parameter (v_0) of the volleyball is the independent variable.

The parameter of ω is the angular frequency parameter. The parameter of β is the damping coefficient. The parameter of g is the acceleration due to gravity. The parameter of α is the air-resistance coefficient. The parameter $\omega = 1\ rad/s$, $\beta = 2.5$, $\alpha = 1$, $g = 10\ m/s^2$, $\theta = \frac{\pi}{4}\ rad$, $\phi = \frac{\pi}{3}\ rad$ and $y_0 = 2\ m$ of the vertical displacement $y(x)$ equation is the control variable for all the apply initial force ($F_1(t)$, and $F_2(t)$).

Results and discussion

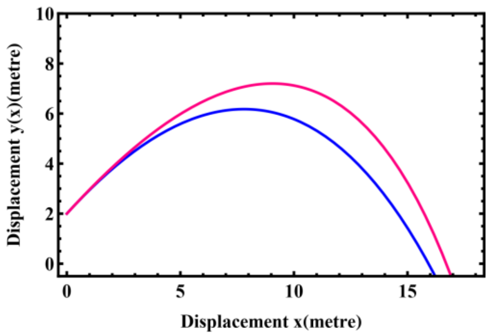
Analyze the graph of the volleyball motion. Consider whether the graph of the volleyball motion that is served with various kinds of forces meets the conditions of moving in a complete projectile motion, having vertical displacement over the height of the net. The vertical displacement $y(x)$ for case the applied initial forces $F_1(t)$ and $F_2(t)$ are plotted in Figure 1.



(a) (b)

Figure 1 Plots of the vertical displacement $y(x)$, which illustrate the right-hand sides of Equation (26), as a function of the horizontal displacement (a) The vertical displacement is a function of the x-axis in the boundary of the volleyball field of the applied initial forces $F_1(t)=\frac{F_0}{\beta}\cos(\omega t)$ (the blue hard thing line) (b) The vertical displacement is a function of the x-axis in the boundary of the volleyball field of the applied initial forces $F_2(t)=\frac{F_0}{\beta}\ln(\omega t)$ (the pink hard thing line).

The vertical displacement $y(x)$ for case the comparison of the applied initial forces are plotted in Figure 2.



(a)

Figure 2 Plots of the vertical displacement $y(x)$, which reveal the right-hand sides of Equation (26), as a function of the horizontal displacement (a) We must be the comparison of the vertical displacement $y(x)$ evaluated from different the applied initial forces ($F_1(t)$ and $F_2(t)$).

These can be used to model the velocity of the curved movement of a volleyball as follows: Replace the force and initial velocity in the vertical displacement of the horizontal function $y(x)$. Plot the graph between the vertical displacement (y) and horizontal displacement (x) and compare a suitable graph with the curved movement of a volleyball.

From the Figure 1(a), the initial force used to hit the volleyball is 650.00 N. The initial velocity of volleyball is 133.79 m/s. The displacement in the x-axis is 16.00±0.01 meters. The y-axis displacement which is a function of the x-axis is 6.50±0.01 meters and almost out the boundary of the volleyball court (= 18 meters). From the Figure 1(b), the initial force used to hit the volleyball is 140.00 N. The initial velocity of volleyball is 76.27 m/s. The displacement in the x-axis is 17.00±0.01 meters. The y-axis displacement which is a function of the x-axis is 7.00±0.01 meters and almost out the boundary of the volleyball court (= 18 meters). Figure 2 (a), we must be comparison the vertical displacement $y(x)$ produced from different types of the applied initial force. These can be summary the applied initial force $F_1(t)$, $F_2(t)$ have similar the initial force and the initial velocity.

Conclusions

Results showed that the apply initial force $F_1(t)$ and $F_2(t)$ as a pure and mixed time function dependent the parameter α, ω, β, θ, ϕ, F_0 and the initial velocity of the projectile of motion for volleyball and in accordance with the conditions of horizontal displacement over 9 meters not exceeding 18 meters and vertical displacement not less than 2.24 meters. From Figure 2 (b), if higher the initial force used to hit the volleyball and the initial velocity affect decreasing value of the horizontal and the vertical displacement of $y(x)$.

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