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Efficient Modified Estimator for the Mean Estimation Using Auxiliary Information in Sample Surveys

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Several researchers use the auxiliary information to enhance the efficiency of their estimators in estimating the population parameters in sample surveys. Studies aim to find out more efficient estimators than recently proposed estimators and make inferences about the unknown population parameters such as population total, population mean, population proportion, or population variance. And one of the population parameters that are widely studied and used, is the population means. In this paper, the author attempts to develop a new modified estimator for the mean of the population in simple random sampling without replacement (SRSWOR) by utilizing information on four auxiliary variables. Therefore, the new estimators with their properties up to the first degree of approximation such as bias and Mean Squared Error (MSE) have been studied. In addition, the optimum value of the real numbers and the minimum MSE of the proposed estimators have been investigated. A few members were also derived from the proposed estimators by allocating the different suitable values of constants. Furthermore, the efficiency of the proposed estimators has been compared with other relevant existing estimators through theoretical study. While the data of peppermint oil production data in Digha India is used for the empirical study to compare the performance of the new

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estimator with other existing estimators. The results of this paper showed that the new estimators are more efficient under the criteria of MSE and Percent Relative Efficiencies (PRE) as compared to all other consideration estimators for certain natural populations available in the literature.

INTRODUCTION

In the sampling theory literature, many authors have introduced the use of auxiliary information to increase the efficiency of their estimators. For instance, Cochran [1] used the strong positive correlation between study and auxiliary variables to present the ratio estimator. On the other hand, if the correlation between study and auxiliary variables is strongly negative, then the product estimator proposed by Robson [2] and Murthy [3] may be used. Recent developments in both ratio and product estimators, together with their variety of modified forms are clearly described in detail by Sisodia and Dwivedi [4], Pandey and Dubey [5], Upadhyaya and Singh [6], Singh [7], Singh and Tailor [8], Kadilar and Cingi [9, 10], Singh *et al.* [11], Singh and Tailor [12], Al-Omari *et al.* [13], Singh *et al.* [14], Yan and Tian [15], Singh *et al.* [16], Subramani and Kumarpandiyan [17], Jeelani *et al.* [18], and Jerajuddin and Kishun [19].

Many estimators were further extended to cover the proposed previously by Khoshnevisan *et al.* [20]. The estimator of Khoshnevisan *et al.* [20] is given as follows:

$$\bar{y}_1 = \bar{y}\left(\frac{a\bar{X}+c}{\alpha(a\bar{x}+c)+(1-\alpha)(a\bar{X}+c)}\right)^g, \quad (1)$$

Where $\bar{y}$ and $\bar{x}$ are the sample means of the study and auxiliary variables, respectively, whereas $\bar{X}$ is the population mean of the auxiliary variable. The consonants $a(a \neq 0)$ and c are either real numbers or functions of auxiliary variables such as variance, $(S_x^2 = (N-1)^{-1}\sum_{i=1}^{N}(x_i-\bar{X})^2)$, coefficient of variation $(C_x = S_x/\bar{X})$, and correlation coefficient $(\rho_{yx} = S_{yx}/S_yS_x)$. In contrast, the consonants α and g are integer to be determined.

The bias and MSE of this family of Khoshnevisan *et al.* [20] estimators are respectively shown as:

$$Bias(\bar{y}_1) = \frac{(1-f)}{n}\bar{Y}\alpha g\theta C_x^2\left[\frac{(g+1)}{2}\alpha\theta - C\right], \quad (2)$$

$$MSE(\bar{y}_1) = \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + \alpha g\theta C_x^2(\alpha\theta g - 2C)\right], \quad (3)$$

where $f = n/N$, $\theta = a\bar{X}/(a\bar{X}+b)$, $C_y^2 = S_y^2/\bar{Y}^2$, $S_y^2 = (N-1)^{-1}\sum_{i=1}^{N}(y_i-\bar{Y})^2$, $C_{yx} = S_{yx}/\bar{Y}\,\bar{X}$, $C = \rho_{yx}C_y/C_x$, and $S_{yx} = (N-1)^{-1}\sum_{i=1}^{N}(y_i-\bar{Y})(x_i-\bar{X})$.

However, Khoshnevisan et al. [20] estimator is quite difficult to use in practice. Therefore, Yadav *et al.* [21] improved the estimator of Khoshnevisan *et al.* [20] by using the assumption of the values of α and g

equal to one and adding the consonants of b and d in equation (1), as:

$$\bar{y}_2 = \bar{y}\left(\frac{ab\bar{X}+cd}{ab\bar{x}+cd}\right), \tag{4}$$

The bias and MSE Yadav *et al*. [21] estimators are respectively shown as:

$$Bias(\bar{y}_2) = \frac{(1-f)}{n}\bar{Y}\eta C_x^2(\eta - C), \tag{5}$$

$$MSE(\bar{y}_2) = \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + \eta C_x^2(\eta - 2C)\right], \tag{6}$$

where $\eta = ab\bar{X}/(ab\bar{X}+cd)$.

In this present paper, the authors have suggested the modification of Yadav *et al*. [21] by using the concept of power transformation under SRSWOR scheme. The expressions in terms of bias, MSE, and minimum MSE of proposed estimators were obtained. In addition, comparative studies of the proposed estimators with other relevant existing estimators have been considered through the theoretical and empirical studies, which show the efficiency of the proposed estimators was clearly better than the other estimators.

MATERIALS AND METHODS

Modified Estimator

The authors propose to create a new estimator by adjusting the Yadav *et al*. [21] using the concept of power transformation under SRSWOR scheme. When the term of $\left(\frac{ab\bar{X}+cd}{ab\bar{x}+cd}\right)$ from equation (4) is raised to the power of constant g, the new estimator is as follows:

$$\bar{y}_3 = \bar{y}\left(\frac{ab\bar{X}+cd}{ab\bar{x}+cd}\right)^g. \tag{7}$$

After that, if the sample mean $\bar{y}$ in equation (7) is replaced with $\bar{y}\left(\frac{ab\bar{x}+cd}{ab\bar{X}+cd}\right)^{1-g}$, the new modified estimator of population mean is obtained as:

$$\bar{y}_4 = \bar{y}\left(\frac{ab\bar{X}+cd}{ab\bar{x}+cd}\right)^g\left(\frac{ab\bar{x}+cd}{ab\bar{X}+cd}\right)^{1-g},$$

$$= \bar{y}\left(\frac{ab\bar{X}+cd}{ab\bar{x}+cd}\right)^{2g-1}. \tag{8}$$

To obtain the bias and MSE of $\bar{y}_3$ and $\bar{y}_4$ under SRSWOR, let us define

$$\bar{y} = \bar{Y}(1+e_0) \text{ and } \bar{x} = \bar{X}(1+e_1) \tag{9}$$

where e_0 and e_1 are the sampling error on auxiliary and study variables, respectively. Further, one may assume that

$$E(e_0) = E(e_1) = 0. \tag{10}$$

When the population parameter of the auxiliary variable is known, after solving the expectations, the following expression is obtained as

$$\left.\begin{aligned} E(e_0^2) &= \frac{(1-f)}{n}C_y^2, \\ E(e_1^2) &= \frac{(1-f)}{n}C_x^2, \text{ and} \\ E(e_0e_1) &= \frac{(1-f)}{n}CC_x^2. \end{aligned}\right\} \tag{11}$$

The bias of $\bar{y}_3$ can be found as follows:

$$Bias(\bar{y}_3) = E(\bar{y}_3 - \bar{Y})$$

$$= E\left[\bar{y}\left(\frac{ab\bar{X}+cd}{ab\bar{x}+cd}\right)^g - \bar{Y}\right]. \tag{12}$$

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By using the equation (9), one can rewrite the above equation, as

$$Bias(\bar{y}_3) = E\left[\bar{Y}(1+e_0)\left(\frac{ab\bar{X}+cd}{ab\bar{X}(1+e_1)+cd}\right)^g - \bar{Y}\right]$$

$$= E\left[\bar{Y}(1+e_0)\left(1+\eta e_1\right)^{-g} - \bar{Y}\right]. \quad (13)$$

Rewrite equation (13) in terms of equations (10) and (11); one gets

$$Bias(\bar{y}_3) = E\left[\bar{Y}(1+e_0)(1-g\eta e_1+\frac{g(g+1)}{2}\eta^2 e_1^2) - \bar{Y}\right]$$

$$= \bar{Y}E\left[1-g\eta e_1+\frac{g(g+1)}{2}\eta^2 e_1^2 + e_0 - g\eta e_0 e_1 - \bar{Y}\right]$$

$$= \frac{(1-f)}{n}\bar{Y}g\eta C_x^2\left[\frac{(g+1)}{2}\eta - C\right]. \quad (14)$$

Further, bias of $\bar{y}_4$ is obtained from equation (15) as follows:

$$Bias(\bar{y}_4) = \frac{(1-f)}{n}(2g-1)\eta\bar{Y}C_x^2\left[g\eta - C\right]. \quad (15)$$

In addition, the MSE of $\bar{y}_3$ can be found as follows:

$$MSE(\bar{y}_3) = E(\bar{y}_3 - \bar{Y})^2$$

$$= E\left[\bar{Y}(1+e_0)\left(1+\eta e_1\right)^{-g} - \bar{Y}\right]^2$$

$$= \bar{Y}^2 E\left[-g\eta e_1+\frac{g(g+1)}{2}\eta^2 e_1^2 + e_0 - g\eta e_0 e_1\right]^2$$

$$= \bar{Y}^2 E\left[g^2\eta^2 e_1^2 + e_0^2 - 2g\eta e_0 e_1\right]. \quad (16)$$

Rewrite equation (16) in term of equations (10) and (11), one get

$$MSE(\bar{y}_3) = \bar{Y}^2\left[g^2\eta^2\frac{(1-f)}{n}C_x^2 + \frac{(1-f)}{n}C_y^2 - 2g\eta\frac{(1-f)}{n}CC_x^2\right]$$

$$= \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + g^2\eta^2 C_x^2 - 2g\eta CC_x^2\right]. \quad (17)$$

The MSE of $\bar{y}_4$ can be found from equation (18) as follows:

$$MSE(\bar{y}_4) = \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + (2g-1)\eta C_x^2\left[(2g-1)\eta - 2C\right]\right]. \quad (18)$$

The MSE of $\bar{y}_3$ and $\bar{y}_4$ at (17) and (18) depends on three constants g, η and C. So, keeping the values of g and C fixed. To obtain the value of η that minimizes for $\bar{y}_3$ and $\bar{y}_4$ respectively, one takes the partial derivative of equation (17) for $\bar{y}_3$ and (18) for $\bar{y}_4$ with respect to η and equate in each equation to zero. Therefore, the optimum values of η for $\bar{y}_3$ as

$$\eta = C/g = \eta_{(opt1.)}. \quad (19)$$

and the optimum values of η for $\bar{y}_4$ as

$$\eta = C/(2g-1) = \eta_{(opt2.)}. \quad (20)$$

To replace the optimum values of η for $\bar{y}_3$ equation (19) into equation (17), the minimum MSE of $\bar{y}_3$ is given by:

$$\min.MSE(\bar{y}_3) = \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 - C^2 C_x^2\right]. \quad (21)$$

And the substitution of the optimum values of η for $\bar{y}_4$ of equation (20) in equation (18) yields the minimum MSE of $\bar{y}_4$ as

$$\min.MSE(\bar{y}_4) = \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 - C^2 C_x^2\right], \quad (22)$$

which is the same equation as before.

Therefore, the common minimum MSE of $\bar{y}_3$ and $\bar{y}_4$ is given by:

$$\min.MSE(\bar{y}_3, \bar{y}_4) = \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 - C^2 C_x^2\right]. \quad (23)$$

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To replace the different choices of the constants a, b, c, d, α and g into equations (1), (4), (7), and (8), one can generate a few members of $\bar{y}_1$, $\bar{y}_2$, $\bar{y}_3$, and $\bar{y}_4$ as given in Table 1:

Table 1 A few members of $\bar{y}_1$, $\bar{y}_2$, $\bar{y}_3$, and $\bar{y}_4$

Estimator	Values of constants					
	a	b	c	d	α	g
A few members of $\bar{y}_1$						
$\bar{y}_{1(1)} = \bar{y}\left(\frac{\bar{X}+\beta_1(x)}{\bar{x}+\beta_1(x)}\right)$ Yan and Tian (15)	1	-	$\beta_1(x)$	-	1	1
$\bar{y}_{1(2)} = \bar{y}\left(\frac{n\bar{X}+\rho_{yx}}{n\bar{x}+\rho_{yx}}\right)$ Yadav et al. (21)	n	-	ρ_{yx}	-	1	1
$\bar{y}_{1(3)} = \bar{y}\left(\frac{\beta_1(x)\bar{X}+\rho_{yx}}{\beta_1(x)\bar{x}+\rho_{yx}}\right)$	$\beta_1(x)$	-	ρ_{yx}	-	1	1
$\bar{y}_{1(4)} = \bar{y}\left(\frac{\bar{x}+\beta_1(x)}{\bar{X}+\beta_1(x)}\right)$	1	-	$\beta_1(x)$	-	1	-1
$\bar{y}_{1(5)} = \bar{y}\left(\frac{n\bar{x}+\rho_{yx}}{n\bar{X}+\rho_{yx}}\right)$	n	-	ρ_{yx}	-	1	-1
$\bar{y}_{1(6)} = \bar{y}\left(\frac{\beta_1(x)\bar{x}+\rho_{yx}}{\beta_1(x)\bar{X}+\rho_{yx}}\right)$	$\beta_1(x)$	-	ρ_{yx}	-	1	-1
A few members of $\bar{y}_2$						
$\bar{y}_{2(1)} = \bar{y}\left(\frac{C_x\bar{X}+\beta_1(x)}{C_x\bar{x}+\beta_1(x)}\right)$ Yan and Tian (15)	1	C_x	$\beta_1(x)$	1	-	-
$\bar{y}_{2(2)} = \bar{y}\left(\frac{nC_x\bar{X}+\rho_{yx}}{nC_x\bar{x}+\rho_{yx}}\right)$ Yadav et al. (21)	n	C_x	ρ_{yx}	1	-	-
$\bar{y}_{2(3)} = \bar{y}\left(\frac{\beta_1(x)M_d\bar{X}+\rho_{yx}C_x}{\beta_1(x)M_d\bar{x}+\rho_{yx}C_x}\right)$ Yadav et al. (21)	$\beta_1(x)$	M_d	ρ_{yx}	C_x	-	-
A few members of $\bar{y}_3$						
$\bar{y}_{3(1)} = \bar{y}\left(\frac{C_x\bar{X}+\beta_1(x)}{C_x\bar{x}+\beta_1(x)}\right) = \bar{y}_{2(1)}$	1	C_x	$\beta_1(x)$	1	-	1
$\bar{y}_{3(2)} = \bar{y}\left(\frac{nC_x\bar{X}+\rho_{yx}}{nC_x\bar{x}+\rho_{yx}}\right) = \bar{y}_{2(2)}$	n	C_x	ρ_{yx}	1	-	1
$\bar{y}_{3(3)} = \bar{y}\left(\frac{\beta_1(x)M_d\bar{X}+\rho_{yx}C_x}{\beta_1(x)M_d\bar{x}+\rho_{yx}C_x}\right) = \bar{y}_{2(3)}$	$\beta_1(x)$	M_d	ρ_{yx}	C_x	-	1
$\bar{y}_{3(4)} = \bar{y}\left(\frac{C_x\bar{x}+\beta_1(x)}{C_x\bar{X}+\beta_1(x)}\right)$	1	C_x	$\beta_1(x)$	1	-	-1

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Estimator	Values of constants					
	a	b	c	d	α	g
$\bar{y}_{3(5)}=\bar{y}\left(\frac{nC_x\bar{x}+\rho_{yx}}{nC_x\bar{X}+\rho_{yx}}\right)$	n	C_x	ρ_{yx}	1	-	-1
$\bar{y}_{3(6)}=\bar{y}\left(\frac{\beta_1(x)M_d\bar{x}+\rho_{yx}C_x}{\beta_1(x)M_d\bar{X}+\rho_{yx}C_x}\right)$	$\beta_1(x)$	M_d	ρ_{yx}	C_x	-	-1
A few members of $\bar{y}_4$						
$\bar{y}_{4(1)}=\bar{y}\left(\frac{C_x\bar{X}+\beta_1(x)}{C_x\bar{x}+\beta_1(x)}\right)=\bar{y}_{3(1)}=\bar{y}_{2(1)}$	1	C_x	$\beta_1(x)$	1	-	1
$\bar{y}_{4(2)}=\bar{y}\left(\frac{nC_x\bar{X}+\rho_{yx}}{nC_x\bar{x}+\rho_{yx}}\right)=\bar{y}_{3(2)}=\bar{y}_{2(2)}$	n	C_x	ρ_{yx}	1	-	1
$\bar{y}_{4(3)}=\bar{y}\left(\frac{\beta_1(x)M_d\bar{X}+\rho_{yx}C_x}{\beta_1(x)M_d\bar{x}+\rho_{yx}C_x}\right)=\bar{y}_{3(3)}=\bar{y}_{2(3)}$	$\beta_1(x)$	M_d	ρ_{yx}	C_x	-	1
$\bar{y}_{4(4)}=\bar{y}\left(\frac{C_x\bar{x}+\beta_1(x)}{C_x\bar{X}+\beta_1(x)}\right)=\bar{y}_{3(4)}$	1	C_x	$\beta_1(x)$	1	-	0
$\bar{y}_{4(5)}=\bar{y}\left(\frac{nC_x\bar{x}+\rho_{yx}}{nC_x\bar{X}+\rho_{yx}}\right)=\bar{y}_{3(5)}$	n	C_x	ρ_{yx}	1	-	0
$\bar{y}_{4(6)}=\bar{y}\left(\frac{\beta_1(x)M_d\bar{x}+\rho_{yx}C_x}{\beta_1(x)M_d\bar{X}+\rho_{yx}C_x}\right)=\bar{y}_{3(6)}$	$\beta_1(x)$	M_d	ρ_{yx}	C_x	-	0

Efficiency Comparisons

In this section, the authors intend to compare the efficiency of $\bar{y}_3$ and $\bar{y}_4$ with other existing estimators, as shown in Table 1. It is well known that the MSE of the unbiased estimator $\bar{y}$ under SRSWOR is given by

$$MSE(\bar{y})=\frac{(1-f)}{n}\bar{Y}^2C_y^2. \quad (24)$$

The expressions for the MSE of a few members of $\bar{y}_1$, $\bar{y}_2$, $\bar{y}_3$, and $\bar{y}_4$ are derived as

$$MSE(\bar{y}_{1(1)})=\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2+\theta_1C_x^2(\theta_1-2C)\right] \quad (25)$$

$$MSE(\bar{y}_{1(2)})=\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2+\theta_2C_x^2(\theta_2-2C)\right] \quad (26)$$

$$MSE(\bar{y}_{1(3)})=\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2+\theta_3C_x^2(\theta_3-2C)\right] \quad (27)$$

$$MSE(\bar{y}_{1(4)})=\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2+\theta_1C_x^2(\theta_1+2C)\right] \quad (28)$$

$$MSE(\bar{y}_{1(5)})=\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2+\theta_2C_x^2(\theta_2+2C)\right] \quad (29)$$

$$MSE(\bar{y}_{1(6)})=\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2+\theta_3C_x^2(\theta_3+2C)\right] \quad (30)$$

$$MSE(\bar{y}_{2(1)},\bar{y}_{3(1)},\bar{y}_{4(1)})=\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2+\eta_1C_x^2(\eta_1-2C)\right] \quad (31)$$

$$MSE(\bar{y}_{2(2)},\bar{y}_{3(2)},\bar{y}_{4(2)})=\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2+\eta_2C_x^2(\eta_2-2C)\right] \quad (32)$$

$$MSE(\bar{y}_{2(3)},\bar{y}_{3(3)},\bar{y}_{4(3)})=\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2+\eta_3C_x^2(\eta_3-2C)\right] \quad (33)$$

$$MSE(\bar{y}_{3(4)},\bar{y}_{4(4)})=\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2+\eta_1C_x^2(\eta_1+2C)\right] \tag{34}$$

$$MSE(\bar{y}_{3(5)},\bar{y}_{4(5)})=\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2+\eta_2C_x^2(\eta_2+2C)\right] \tag{35}$$

$$MSE(\bar{y}_{3(6)},\bar{y}_{4(6)})=\frac{(1-f)}{n}\bar{Y}^2\left[C_y^2+\eta_3C_x^2(\eta_3+2C)\right] \tag{36}$$

where $\theta_1=\bar{X}/(\bar{X}+\beta_1(x))$,
$\theta_2=n\bar{X}/(n\bar{X}+\rho_{yx})$,
$\theta_3=\beta_1(x)\bar{X}/(\beta_1(x)\bar{X}+\rho_{yx})$,
$\eta_1=C_x\bar{X}/(C_x\bar{X}+\beta_1(x))$,
$\eta_2=nC_x\bar{X}/(nC_x\bar{X}+\rho_{yx})$,
$\eta_3=\beta_1(x)M_d\bar{X}/(\beta_1(x)M_d\bar{X}+\rho_{yx}C_x)$.

It is observed from equations (23) to (36) that the estimators $\bar{y}_3$ and $\bar{y}_4$ are more efficient than

(i) the unbiased estimator $\bar{y}$ if

$$MSE(\bar{y})-\min.MSE(\bar{y}_3,\bar{y}_4)=C^2C_x^2>0 \tag{37}$$

(ii) the estimator $\bar{y}_{1(1)}$ if

$$MSE(\bar{y}_{1(1)})-\min.MSE(\bar{y}_3,\bar{y}_4)=\theta_1(\theta_1-2C)+C^2>0 \tag{38}$$

(iii) the estimator $\bar{y}_{1(2)}$ if

$$MSE(\bar{y}_{1(2)})-\min.MSE(\bar{y}_3,\bar{y}_4)=\theta_2(\theta_2-2C)+C^2>0 \tag{39}$$

(iv) the estimator $\bar{y}_{1(3)}$ if

$$MSE(\bar{y}_{1(3)})-\min.MSE(\bar{y}_3,\bar{y}_4)=\theta_3(\theta_3-2C)+C^2>0 \tag{40}$$

(v) the estimator $\bar{y}_{1(4)}$ if

$$MSE(\bar{y}_{1(4)})-\min.MSE(\bar{y}_3,\bar{y}_4)=\theta_1(\theta_1+2C)+C^2>0 \tag{41}$$

(vi) the estimator $\bar{y}_{1(5)}$ if

$$MSE(\bar{y}_{1(5)})-\min.MSE(\bar{y}_3,\bar{y}_4)=\theta_2(\theta_2+2C)+C^2>0 \tag{42}$$

(vii) the estimator $\bar{y}_{1(6)}$ if

$$MSE(\bar{y}_{1(6)})-\min.MSE(\bar{y}_3,\bar{y}_4)=\theta_3(\theta_3+2C)+C^2>0 \tag{43}$$

(viii) the estimator $\bar{y}_{2(1)},\bar{y}_{3(1)},\bar{y}_{4(1)}$ if

$$MSE(\bar{y}_{2(1)},\bar{y}_{3(1)},\bar{y}_{4(1)})-\min.MSE(\bar{y}_3,\bar{y}_4)=\eta_1(\eta_1-2C)+C^2>0 \tag{44}$$

(ix) the estimator $\bar{y}_{2(2)},\bar{y}_{3(2)},\bar{y}_{4(2)}$ if

$$MSE(\bar{y}_{2(2)},\bar{y}_{3(2)},\bar{y}_{4(2)})-\min.MSE(\bar{y}_3,\bar{y}_4)=\eta_2(\eta_2-2C)+C^2>0 \tag{45}$$

(x) the estimator $\bar{y}_{2(3)},\bar{y}_{3(3)},\bar{y}_{4(3)}$ if

$$MSE(\bar{y}_{2(3)},\bar{y}_{3(3)},\bar{y}_{4(3)})-\min.MSE(\bar{y}_3,\bar{y}_4)=\eta_3(\eta_3-2C)+C^2>0 \tag{46}$$

(xi) the estimator $\bar{y}_{3(4)},\bar{y}_{4(4)}$ if

$$MSE(\bar{y}_{3(4)},\bar{y}_{4(4)})-\min.MSE(\bar{y}_3,\bar{y}_4)=\eta_1(\eta_1+2C)+C^2>0 \tag{47}$$

(xii) the estimator $\bar{y}_{3(5)},\bar{y}_{4(5)}$ if

$$MSE(\bar{y}_{3(5)},\bar{y}_{4(5)})-\min.MSE(\bar{y}_3,\bar{y}_4)=\eta_2(\eta_2+2C)+C^2>0 \tag{48}$$

Empirical Study

To examine the efficiency of the estimators discussed in the present paper, the authors considered the data provided in Yadav *et al*. [21], which details are as follows:

Yadav *et al*. [21] have used the peppermint oil production data in Digha India, and assumed that the yield of peppermint oil (in kilogram) and the area of the field in Bigha were taken as the study and auxiliary variables, respectively. The description of population parameters is summarized in Table 2.

Table 2 Parameters and constants of the population under study

$N=150$	$\bar{X}=4.20,$	$C_x=0.73$
$n=40$	$\bar{Y}=33.46$	$C_y=0.76$
$M_d=3$	$\rho_{yx}=0.91$	$\beta_1(x)=2.80$

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The criteria for comparing the efficiency of estimators in this study, the percent relative efficiencies (PRE) of all existing estimators with respect to unbiased estimator $\bar{y}$ was used and presented in Table 3 as follows:

The PRE of all estimators with respect to $\bar{y}$ is calculated by

$$PRE(\bar{y},\,\bar{y}_i)=\frac{MSE(\bar{y})}{MSE(\bar{y}_i)}\times 100 \tag{49}$$

where $\bar{y}_i$ denote the estimator that we are interested in comparing with $\bar{y}$ such as $\bar{y}_{1(1)}$, $\bar{y}_{1(2)}$, …., and so on.

Table 3 MSE and PRE of all existing estimators

Estimator	MSE	PRE
$\bar{y}$	12.9333	100.0000
A few members of $\bar{y}_1$		
$\bar{y}_{1(1)}$	3.6633	353.0511
$\bar{y}_{1(2)}$	2.2498	574.8555
$\bar{y}_{1(3)}$	2.2276	580.5824
$\bar{y}_{1(4)}$	30.7946	41.9985
$\bar{y}_{1(5)}$	47.2251	27.3865
$\bar{y}_{1(6)}$	44.1987	29.2617
A few members of $\bar{y}_2$		
$\bar{y}_{2(1)}=\bar{y}_{3(1)}=\bar{y}_{4(1)}$	4.3757	295.5691
$\bar{y}_{2(2)}=\bar{y}_{3(2)}=\bar{y}_{4(2)}$	2.2477	575.4136
$\bar{y}_{2(3)}=\bar{y}_{3(3)}=\bar{y}_{4(3)}$	2.2371	578.1210
A few members of $\bar{y}_3$		
$\bar{y}_{3(4)}=\bar{y}_{4(4)}$	28.0104	46.1732
$\bar{y}_{3(5)}=\bar{y}_{4(5)}$	47.1335	27.4397
$\bar{y}_{3(6)}=\bar{y}_{4(6)}$	46.6203	27.7418
$\bar{y}_3,\bar{y}_4$ (Proposed estimator)	2.2232	581.7336

From Table 3, one can derive two preliminary results as follows:

(i) Among all estimators considered in this population group, it is envisaged that the estimators $\bar{y}_{1(1)},\bar{y}_{1(2)},\bar{y}_{1(3)},(\bar{y}_{2(1)}=\bar{y}_{3(1)}=\bar{y}_{4(1)})$, $(\bar{y}_{2(2)}=\bar{y}_{3(2)}=\bar{y}_{4(2)})$, and $(\bar{y}_{2(3)}=\bar{y}_{3(3)}=\bar{y}_{4(3)})$ are the parts of the ratio estimator that will be effective when the data are positively correlated. Furthermore, the data of this section are positively correlated so that the values of MSE of these estimators have small when compared with other existing estimators.

On the contrary, the estimators $\bar{y}_{1(4)},\bar{y}_{1(5)},\bar{y}_{1(6)},(\bar{y}_{3(4)}=\bar{y}_{4(4)})$, $(\bar{y}_{3(5)}=\bar{y}_{4(5)})$, and $(\bar{y}_{3(6)}=\bar{y}_{4(6)})$ are the parts of the product estimator. Due to this, they have large MSE values compared to other estimators. For proposed estimators $\bar{y}_3,\bar{y}_4$ (at its optimum), once observed that they have the smallest MSE compared to all estimators in this study. Therefore, one can infer from the values of MSE that among all estimators, the proposed estimators $\bar{y}_3,\bar{y}_4$ (at its optimum) are the best in the sense of having the smallest MSE.

(ii) When considering the values of PRE of all estimators, one can find that the PRE of estimators $\bar{y}_{1(1)},\bar{y}_{1(2)},\bar{y}_{1(3)},(\bar{y}_{2(1)}=\bar{y}_{3(1)}=\bar{y}_{4(1)})$, $(\bar{y}_{2(2)}=\bar{y}_{3(2)}=\bar{y}_{4(2)})$, and $(\bar{y}_{2(3)}=\bar{y}_{3(3)}=\bar{y}_{4(3)})$ have larger PRE values than the estimators $\bar{y}_{1(4)},\bar{y}_{1(5)},\bar{y}_{1(6)},(\bar{y}_{3(4)}=\bar{y}_{4(4)})$, $(\bar{y}_{3(5)}=\bar{y}_{4(5)})$, and $(\bar{y}_{3(6)}=\bar{y}_{4(6)})$. Moreover, the proposed estimator $\bar{y}_3,\bar{y}_4$ (at its optimum) has the largest PRE value compared to all estimators.

From empirical study, one can conclude that the proposed estimators are more desirable overall than those under optimum conditions for this population data.

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RESULTS AND DISCUSSIONS

It is well known in sample surveys that when the correlation between the study and auxiliary variables is positive, the performance of the ratio estimator is generally more efficient than the product estimator, which corresponds to the result of Table 3. Table 3 shows that the MSE of the proposed estimator $\bar{y}_3, \bar{y}_4$ (at its optimum) are consistently less than the other existing estimators under consideration for this population. While the value of PRE of its are persistently higher than other ones. Therefore, one can summarize that the proposed estimator $\bar{y}_3, \bar{y}_4$ (at its optimum) performs well compared to other existing estimators in terms of MSE and PRE. However, among all estimators, besides the proposed estimator $\bar{y}_3, \bar{y}_4$ (at its optimum), one recommends using a few members of the estimator $\bar{y}_{1(3)}$ as an inferior alternative to the proposed estimators $\bar{y}_3, \bar{y}_4$ for estimating the population mean. Because the values of MSE and PRE are relatively close to the proposed estimators $\bar{y}_3, \bar{y}_4$.

CONCLUSION

This paper proposed new modified estimators for estimating the population means by adapting the Yadav et al. [21] estimator and using the information on auxiliary variables. The expressions for bias, MSE, the optimum value of the real numbers and the minimum MSE of the proposed estimators are investigated and compared with relevant existing estimators. In addition, the efficiency of all estimators was evaluated from theoretical and empirical studies, which found that the proposed estimators are always better than the mentioned existing estimators in literature, having the smallest and highest values of MSE and PRE, respectively. Therefore, the proposed estimators are recommended for their practical use in estimating the population means when the auxiliary information is available.

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