MATHEMATICAL MODELLING OF DRYING OF RED DRAGON FRUIT

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Abstract

The solar dryer was constructed in Loei province. The dimension of the dryer is 1.50 m wide, 4.50 m long and 1.20 m high. All sides of the dryer are covered with polycarbonate sheets. Five drying experiments for drying red dragon fruit were conducted. For each experiment, 30 kg of red dragon fruit were dried in the dryer. A system of partial different equations describing heat and moisture transfer during drying the red dragon fruit in the dryer was formulated. This system of partially differential equations was solved numerically using the finite difference method. The simulation results agreed well with the experimental data for solar drying of the red dragon fruit.

Keywords: Solar dryer, Dragon fruit, Mathematical modeling

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Introduction

Dragon fruit (*Hylocereus polyrhizus*) or pitaya originates in Central and South America. Dragon fruit is grown in Asia, Mexico, Central America, South America, and Israel (Benzing, 1990). In Thailand, dragon fruit is known as "kaew mungkorn", which was first introduced in 1997. The pitaya flesh contains small black seeds scattered in white-flesh (*H.undatus*) or red-flesh (*H.polyrhizus*) or yellow-flesh (*H.megalanthus*) (Barbeau, 1990) depending on the cultivar. However, the white- and red- fleshed fruit is very usual in the market. Dragon fruits in Dansai and Phurua Districts, Loei Province are the most famous among consumers and wholesalers because of the fruit's sweet taste. Phurua dragon fruit has become famous and most wanted. The fruit can be harvested from April to October with approximately 1-2 harvests per month. This abundant fruit causes a severe problem for the growers due to its low price. It might be proper for further food processing. Therefore, if farmers can dry fruit processing, it will be an alternative way to solve this problem. Solar drying can be considered as an elaboration of sun drying and is an efficient system of utilizing solar energy (Smitabhindu, 2008; Sharma et al., 2009; Funholi et al., 2010; Janjai & Bala, 2012). Many studies have been reported on natural convection solar drying of agricultural products (Simate, 2001; Vlachos et al., 2002). Thailand is geographically located slightly above the equator so it receives abundant solar radiation. Consequently, the use of solar dryers is a good alternative solution to the problem of dragon fruit drying.

In this work, the objectives of this research were to investigate the performance of the solar dryer for drying red dragon fruit, a simulation model of this dryer for drying red dragon fruit was also developed and the experimental results were used to validate the performance of the model.

Materials and Methods

1. Experimental setup

The new version of the solar dryer was constructed in Loei Province, Thailand. The dryer consists of a polycarbonate sheet on a metal sheet floor. The dryer has a width of 1.50 m, a length of 4.50 m and a height of 1.20 m. To ventilate the dryer, two DC fans operated by 10-W solar cell modules were installed in the wall opposite to the air inlet. The working principle of the direct solar product (red dragon fruit) drying is shown in Figure 1(A) also known as a greenhouse solar dryer. Here the moisture is taken away by the air entering into the dryer from below and sucked from the dryer by the DC fan. In the dryer, of the total solar radiation impinging on the polycarbonate cover, a part is reflected back to the atmosphere and the remaining is transmitted inside the dryer. A part of the transmitted radiation is then reflected back from the product surface and the rest is absorbed by the surface of the product which causes its temperature to increase and thereby emit long wavelength radiations which are not allowed to escape to the atmosphere due to the polycarbonate cover. The overall phenomena cause the temperature above the product inside the cabinet to be higher. The polycarbonate cover in the cabinet dryer thus serves to reduce direct convective losses to the ambient which plays an important role in increasing cabinet temperature. The pictorial view of red dragon fruit being dried in the dryer is shown in Figure 1(B).

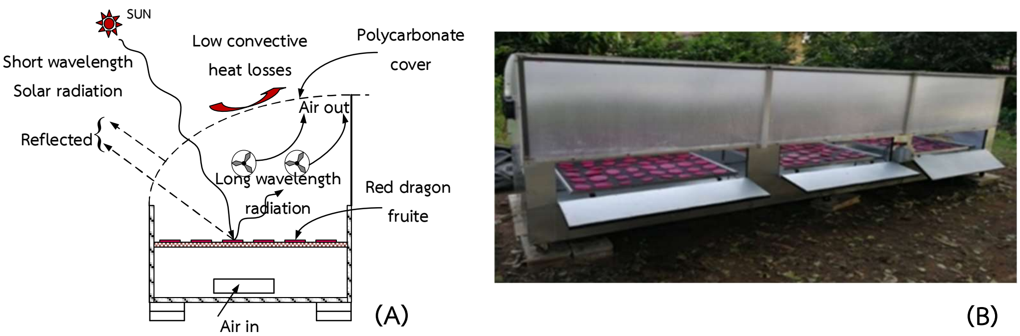

Figure 1 (A) Schematic diagram of energy transfers inside the solar dryer.
(B) Red dragon fruit inside the solar dryer.

2. Experimental procedure

In this study, red dragon fruit was dried in the solar dryer to investigate its potential for drying red dragon fruit. Solar radiation was measured by a pyranometer (Kipp & Zonen model CMP 3, accuracy ± 0.5%). Thermocouples (K type) were used to measure air temperatures in the different positions of the dryer (accuracy ± 2%). A hot wire anemometer (Airflow, model TA5, accuracy ± 2%) was used to monitor the air speed inside the dryer. The relative humidity of ambient air and drying air was periodically measured by hygrometers (Electronnik, model EE23, accuracy ± 2%).

The drying started at 8: am and continued till 6: pm. The positions of all measurements are shown in Figure 2. Five batches of drying tests were carried out. For each batch, 30.0 kg of the red dragon fruit was placed on the trays inside the dryer. The control sample was also dried naturally under the same weather conditions. Samples of products in the dryer were weighed at 1-hour intervals using a digital balance. During the night time, the products were kept in the dryer. The moisture content during drying was estimated from the weight of the product samples and the estimated dried solid mass of the samples. The process was repeated until the desired moisture content was about 8.5% on a wet basis was reached. At the end of the drying process, the exact dry solid mass of the product samples was determined by using the air oven method. The samples were placed in the oven at the temperature of 103°C for 24 hours.

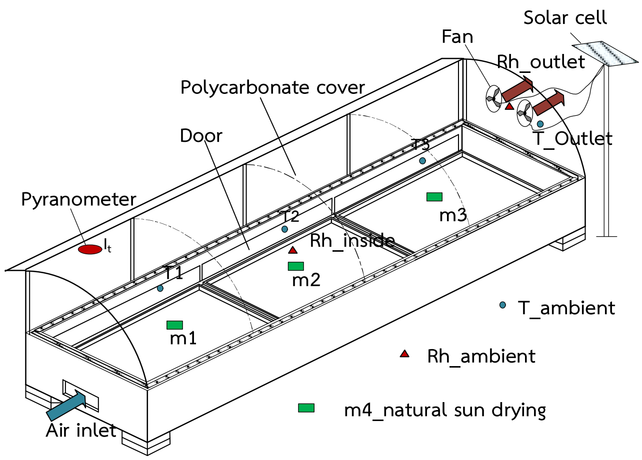

Figure 2 Structure of the dryer and position of thermocouples (T), hygrometer (▲ Rh), solar radiation (● I_t) and product samples for determining moisture content (■ M).

3. Mathematical modeling

For each component of the drying unit, the energy balances and mass balances were written as the followings. There is no stratification of the air inside the dryer. Drying computation is based on a thin layer drying model. Specific heat of air, cover and product are constant. Energy balances in the drying are shown in Figure 3.

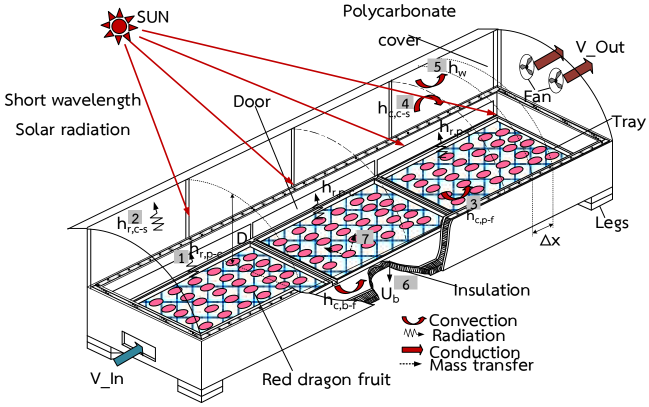

Figure 3 Energy balances in the drying unit: (1) radiative heat transfer between the product and the cover, (2) radiative heat transfer between cover and sky, (3) convective heat transfer between the product and air, (4) convective heat transfer between cover and air, (5) convective heat transfer of cover due to wind, (6) heat loss through back insulator, (7) mass transfer.

4. Uncertainty analysis

Uncertainty analysis refers to the uncertainty or error in experimental data. A systematic error in the experimental data is a repeated error of constant value and the random error is due to imprecision. Systematic error can be removed by calibration but random error cannot be removed. The imprecision due to the random error can be defined numerically by a calibration. Statistical analysis (analysis of variance) was carried out to assess whether there exists any significant difference in drying (moisture removed) in the solar dryer in comparison to drying in the traditional sun drying system with a level of significance, and a similar analysis was also conducted for the temperature inside the dryer and the ambient temperature as well as the humidity inside the dryer and the ambient relative humidity.

The suitability of the model was evaluated using the value of the root mean square difference (RMSD) was calculated using the following equation:

$$RMSD = \frac{\sqrt{\frac{\sum_{i=1}^{N}(y_{pre,i}-y_{meas,i})^2}{N}}}{\frac{\sum_{i=1}^{N}y_{meas,i}}{N}} \times 100 \quad (1)$$

where $y_{pre,i}$ and $y_{meas,i}$ are the predicted and measured values of variable y, respectively, and N is total number of data points.

5. Mathematical modeling

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Energy balance for the cover of the drying unit

The energy balance for the polycarbonate plates cover is considered. There are five terms to find the rate of thermal energy accumulation in the cover. These are rate of the radiative heat transfer between the cover and the product, rate of the convective heat transfer between the moist air steam in the drying unit and the cover, rate of heat loss from the cover due to wind, rate of radiative heat loss from the cover to the sky, and rate of solar energy absorption by the cover. From these five terms, the energy balance of the polycarbonate plates cover can be written as:

$$\rho_c \delta_c C_c \frac{dT_c}{dt} = h_{r,p-c}(T_p - T_c) + h_{c,c-f}(T_f - T_c) + h_w(T_a - T_c) + h_{r,c-s}(T_s - T_c) + \alpha_c I_t \quad (2)$$

Where ρ_c is density of the cover material (kg/m^3), δ_c is thickness of the cover (m), C_c is specific heat of cover material (J/kg-K), T_c is temperature of the cover of the drying unit (K), $h_{r,p-c}$ is radiative heat transfer coefficient between the cover and the product (W/m^2-K), T_p is the product temperature (K), $h_{c,c-f}$ is convective heat transfer coefficient between the cover and the air (W/m^2-K), T_f is temperature of the air steam in the collector (K), h_w is convective heat transfer coefficient between the cover and the ambient due to wind (W/m^2-K), T_a is ambient temperature (K), $h_{r,c-s}$ is radiative heat transfer coefficient between the cover and the sky (W/m^2-K), T_s is the sky temperature (K), α_c is the absorptance of the cover material (-), I_t is incident solar radiation (W/m^2).

Energy balance for the moist air steam in the drying unit

The rate of enthalpy change of the moist air steam inside the dryer can be calculated from two terms. The first terms are the rate of the convective heat transfer between the product and the moist air steam. Next terms are the rate of convective heat transfer between the moist air steam and the cover. This energy balance can be written as:

$$DG(C_f + C_v H)\frac{dT_{fl}}{dx} = h_{c,p-f}(T_p - T_{fl}) + h_{c,f-c}(T_c - T_{fl}) \tag{3}$$

Where D is diffusivity of red dragon fruit (m^2/s), G is mass flow rate of air ($kg/s\text{-}m^2$), C_f is specific heat of air (J/kg-K), C_v is specific heat of water vapour (J/kg-K), H is humidity ratio (kg/kg), T_{fl} is temperature of the moist air steam in the drying unit (K), $h_{c,p-f}$ is convective heat transfer coefficient between the product and the air ($W/m^2\text{-}K$).

Energy balance for the product

The rate of enthalpy change of the product can be calculated from five terms. These are the rate of the sensible and latent heat for the evaporization of the moisture from the product, the rate of the convective heat transfer between the air steam and the product, the rate of the radiative heat transfer between the cover and the product, the rate of heat loss through the back insulator from the product to ambient air, and the rate of solar energy absorbed by the product. The energy balance on the product can be given as:

$$\rho_s p(C_p + C_w M)\frac{dT_p}{dt} = \left[h_{fg} + C_V(T_p - T_f)\right] DG\frac{dH}{dx} + h_{c,p-f}(T_f - T_p) + h_{r,p-c}(T_c - T_p) + U_b(T_a - T_p) + \tau_c \alpha_p I_t \tag{4}$$

Where ρ_s is density of the red dragon fruit (kg/m^2), C_w is specific heat of water (J/kg -K), M is moisture content of red dragon fruit (%, db), h_{fg} is latent heat of vaporization of moisture from red dragon fruit (J/kg), U_b is heat loss coefficient of the absorber through the back insulator ($W/m^2\text{-}K$), α_p is absorbtance of peeled longan (-).

Mass balance equation

The rate of moisture transfer between the product and the air steam can be written as:

$$DG\frac{dH}{dx} = -\rho_s p\frac{dM}{dt} \tag{5}$$

Heat transfer and heat loss coefficients

The radiative heat transfer coefficient ($h_{r,c-s}$) from the cover with temperature of T_c to the sky with the equivalent temperature T_s was computed as (Duffie & Beckman, 1991):

$$h_{r,c\text{-}s} = \varepsilon_c \sigma (T_c^2 + T_s^2)(T_c + T_s) \tag{6}$$

where ε_c is the emissivity of the cover, σ is Stefen-Boltzmann constant. The convective heat transfer coefficient from the cover to the ambient air due to wind (h_w) was calculated from, where V_a is the wind speed in m/s.

$$h_w = 5.7 + 3.8\,V_a \tag{7}$$

6. Thin layer drying and equilibrium moisture content

To obtain the thin layer drying equation, thin layer drying experiments of red dragon fruit were conducted under controlled conditions of the drying air in a laboratory dryer. The following thin layer drying model based on Modified Page's equation was used to fit the drying data obtained from these experiments:

$$\frac{M-M_e}{M_0-M_e} = \exp-(Kt)^n \tag{8}$$

where M (decimal, d.b.) is the product moisture content at time t (hour), M_0 (decimal, d.b.) is initial moisture content, M_e (decimal, d.b.) is the equilibrium moisture content. The drying parameters K, and n are given as:

$$K = -1.72520+0.08627T+0.05243rh \tag{9}$$

$$n = 0.33843+0.02731T-0.07335rh \tag{10}$$

The following empirical equation developed for equilibrium moisture content (M_e, % db) of red dragon fruit. Where T is temperature (°C) rh is relative humidity (%). We also conducted experiments to determine the equilibrium moisture content under controlled conditions of temperature and relative humidity. The result is written as:

$$a_w = \frac{1}{1+\left[\dfrac{29.167-0.875T}{M_e}\right]^{5.532}} \tag{11}$$

where a_w is water activity. The water activity is equal to the relative humidity in percent divided by 100.

7. Method of solution for the drying model

The system of Eq. (2-4) was solved numerically using the finite difference technique. On the basis of the drying air temperature and relative humidity for all the sections of the drying unit, the constants k and n were computed from Eq. (9-10) and equilibrium moisture content (M_e) of the red dragon fruit was calculated by using the Modified Page's equation. Using the K, n and M_e values, the change in moisture content of red dragon fruit ΔM for all the sections for a time interval Δt were calculated using Eq. (5). Next the system of finite difference equation derived from Eq. (2-4) for the interval Δt for the entire length of drying unit was formulated shown in eq. (12).

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{vmatrix} T_c \\ T_a \\ T_p \\ T_f \end{vmatrix} = \begin{vmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{vmatrix} \qquad (12)$$

This system of equations is a set of implicit equations and was solved using the Gauss–Jordan elimination method. Using the recent value of drying air temperature and relative humidity for the different sections of the entire length of drying unit, the change in moisture content of red dragon fruit, ΔM for next time interval for the different sections of the entire length of drying unit were calculated, and the process was repeated until the final time was reached. The numerical solution was programmed in Compaq Visual Fortran version 6.6 and flow chart for Fortran are shown in Figure 4.

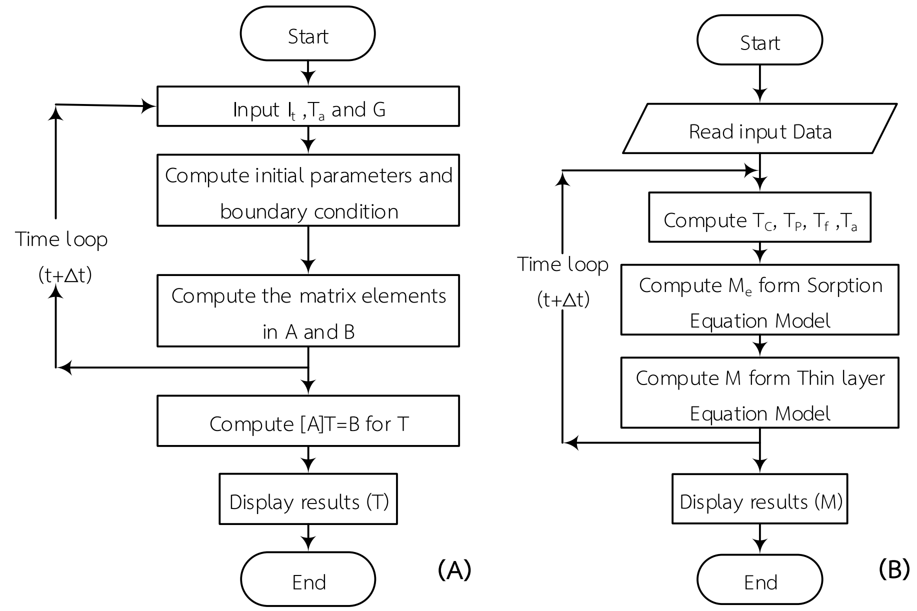

Figure 4 Schematic diagram of temperature (A) and moisture content (B) calculation.

Results

1. Experimental results

Figure 5(A) shows the variations of solar radiation during a typical experimental run of solar drying of red dragon fruit in a side loading type solar dryer. The solar radiation varied between 0-1,018 W/m^2 during the drying period. There were some fluctuations in solar radiation due to clouds and rainfall. Figure 5(B) shows the comparison of the temperature changes at different positions inside the collector for a typical experimental run of solar drying of red dragon fruit in the side loading dryer. The temperatures in the different positions increase till noon and then again decrease in the afternoon.

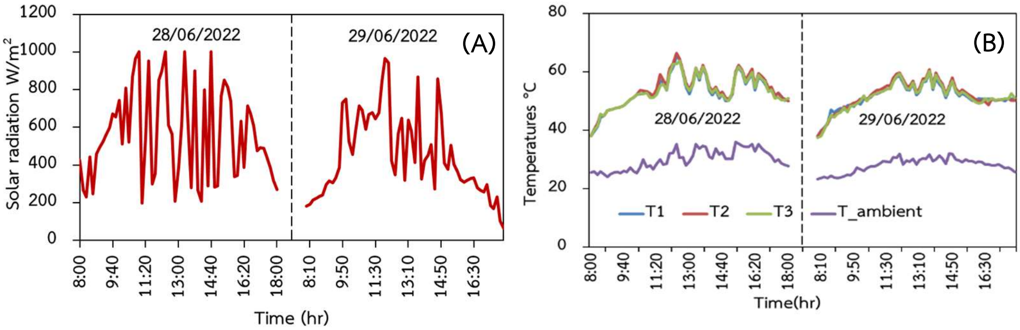

Figure 5 (A) Variation of solar radiation and (B) variation of temperature inside the dryer.

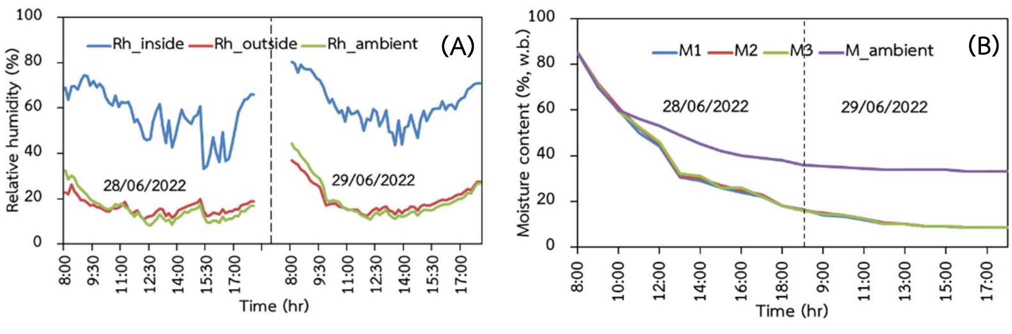

Figure 6 (A) Relative humidity and (B) variation of the moisture content.

Figure 6 (A) shows relative humidities at the different positions inside the dryer for a typical experimental run during the solar drying of red dragon fruit in the side loading type solar dryer. The relative humidity inside the drying unit depends on the moisture released from the product and the air temperature. In most cases, the relative humidity at the outlet of the dryer was higher than that of the middle. The moisture content of red dragon fruit in the dryer was reduced from an initial value of 85% (w.b.) to a final value of 8.45% (w.b.) in 2 days (Figure 6(B)).

2. Modeling result

Figure 7(A) shows typical comparisons between the predicted and experimental temperature values for solar drying of red dragon fruit. The predicted temperature shows plausible behaviour and the agreement between the predicted and observed values at the middle of the dryer is good. Figure 7(B) shows comparisons of the predicted and observed moisture contents of red dragon fruit inside the dryer. The model predictions for drying red dragon fruit were evaluated based on root mean square difference (RMSD). RMSD of the prediction of the temperatures inside the dryer were 4.52% and RMSD of the predictions of moisture contents was 5.81%.

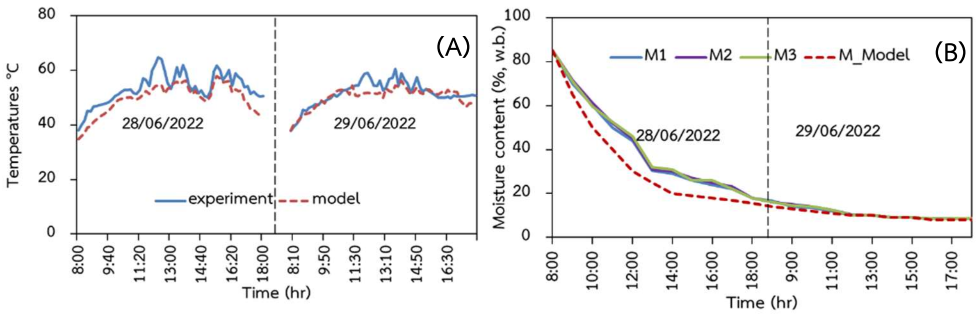

Figure 7 (A) Comparison between the simulated and observed temperatures.
(B) Comparison between the simulated and observed moisture content.

Discussions

As expected, a high drying temperature caused a lower air relative humidity in the solar dryer, resulting in a greater driving force for the heat to transfer to the samples and consequently a shorter drying time (Akter et al., 2010). Moreover, moisture diffusivity is higher at high temperature (Chantaro et al., 2008). Statistical analysis shows that there is no significant difference in solar drying of red dragon fruit in the different locations of the solar dryer at a significance level of 10%. However, there is a significant difference in drying after hours of initial drying between drying inside the solar dryer and sun drying of red dragon fruit.

Thus, the drying in the solar dryer resulted in a reduction in drying time and the production of better-quality dried products. The colour of the dried red dragon fruit was comparable to that of high quality dried red dragon fruit in markets when the colour was tested (Nor et al., 2021).

Conclusions

Solar radiation has high variation throughout the experiment days. Sinusoidal, like around the peak at noon, the solar radiation influences other ambient parameters of the red dragon fruit. Inside the dryer, air temperature variation follows the variation of the solar radiation. The ambient relative humidity varies almost the inverse pattern with the solar radiance.

The system of partial differential equations for heat and moisture transfer has been used for the simulation drying of red dragon fruit in the solar dryer. From the validation, the simulated air temperature inside the dryer reasonably agreed with the measured temperature. Good agreement was found between the experimental and simulated moisture contents. This model can be used to provide design data for side loading solar dryers and optimize the solar dryer.

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