

EVALUATION OF TIME-DEPENDENT PERFORMANCE FOR THE 10,000-METRE MINI MARATHON RUNNER VIA NEWTON'S SECOND LAW OF MOTION

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Abstract

In this paper, the motion models for the 10,000-metre mini marathon runner were proposed. We calculated velocity which depended on time and force parameter for the runner. The velocity from running record data, theoretical mathematics, and physics model were compared. We evaluated the models for the 10,000-metre runner by the method of Newton's second law and differential equations. Data from 10,000-metre men final round of IAAF World Athletics Championships Doha 2019 were used for analysis of velocity and time parameters for runners. The initial velocity, endurance coefficient and coefficient drag force parameters with motion models were defined using Mathematica software. An initial force (f_0) that refined from model 1 was higher than that from model 2 and 3 respectively. The endurance coefficient (β) parameter is lowest in model 1 but highest in model 2 and the coefficient drag force (α) parameter from model 3 is less than model 2 and 1 respectively. Correlation coefficient by the experiment data and motion model 2 is 0.871605 which was more than model 3 (0.870055) and model 1 (0.86942).

Keywords: Velocity, Correlation, Endurance

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Introduction

With “Kawkonlakaw” trend that running for donation for hospital makes running is famous sport for health. The 10,000-metre run or mini marathon is a famous running event that has registered in run event more than the 21,000-metre run or half marathon and 42,000-metre or marathon (Bruce & Tim, 1981). The 10,000-metre run is suitable for both new runners and old runners because of its moderate distance (Begizew et al., 2018).

Therefore, this study developed mathematics and physics model for the 10,000-metre runner. The scheme of the paper is as follows. We constructed force models for the 10,000-metre runner. We calculated velocity depended on time for the runner. We compared the results of velocity from running record data and theoretical mathematics and physics model.

Helene & Yamashita (2010) proposed that a sprint's time-dependent force decreases with the according to $f(t) = F - \beta t$, where F is initial force (N), β is the endurance coefficient (s^{-1}). Tiago et al. (2016) studied the drive force, the maintenance time-dependent force ($f_m = f_i \exp(-ct)$) for runner when c is a constant value (s^{-1}). Begizew et al. (2018) studied about lower-extremity running-related injuries among 10,000-meter long distance runners in Ethiopia. They established the incidence and the risk factors associated with lower-extremity running-related injuries amongst 10,000-meter long distance runners in Ethiopia. Pietro et al. (2014) estimated the energetics and biomechanics of accelerated/decelerated running on flat terrain base on its biomechanical similarity to constant speed running up/down an ‘equivalent slope’ dictated by forward acceleration (a_f).

Keller (1974) developed a mathematics and physics model of competitive running that predicts the form of the time-dependent velocity curve for a runner using his resources in an optimal way. According to Keller's theory, the time-dependent force $f(t)$ per unit mass at time t , applied by a sprinter in the direction of motion, may be written as $f(t) = \frac{dv(t)}{dt} + \frac{v(t)}{\tau}$ where $v(t)$ is the time-dependent velocity and τ is a damping coefficient (N) (Joseph, 1973).

Materials and Method

Linear equation of motion models

We are interested in studying the motion of male 10,000-metre runner when the applied force is time-dependent decreasing function; that is $f = f(t)$. The mass of 10,000-metre runner m starting (run) with an initial velocity $v(0)=0$ is exerted on by the applied force time-dependent decreasing function:

$f_{r1}(t) = \frac{f_0}{\beta^2}(t^2 + 2t)e^{-\beta^2(t+2)} + \frac{f_0}{\beta^2}$ (model 1), $f_{r2}(t) = \frac{f_0}{\beta^2}(t^2 + 4t + 4)e^{-\beta^2(t+2)} + \frac{f_0}{\beta^2}$ (model 2) and $f_{r3}(t) = \frac{f_0}{\beta^2}(t^2 + 6t + 9)e^{-\beta^2(t+2)} + \frac{f_0}{\beta^2}$ (model 3) where t is the time of 10,000-metre runner, f_0 is the initial force for runner (N) and the performance parameter β (s^{-1}) is the positive constant (Tiago et al., 2016).

The $f_{r1}(t) = \frac{f_0}{\beta^2}(t^2 + 2t)e^{-\beta^2(t+2)} + \frac{f_0}{\beta^2}$, $f_{r2}(t) = \frac{f_0}{\beta^2}(t^2 + 4t + 4)e^{-\beta^2(t+2)} + \frac{f_0}{\beta^2}$ and

$f_{r3}(t) = \frac{f_0}{\beta^2}(t^2 + 6t + 9)e^{-\beta^2(t+2)} + \frac{f_0}{\beta^2}$ are the force of ground field act to the runner

due to force and weight (gravitation force) of the runner that press to the ground field.

The β is speed endurance of runner. We assume that the air resistance is proportional to velocity. Thus, the Newton's second law of equation of motion is:

$$\sum F = m \frac{dv}{dt},$$

$$f(t) - \alpha v = \frac{mdv}{dt} \quad (1)$$

$$\frac{dv_{r1}}{dt} + \frac{\alpha}{m} v_{r1} = \frac{f_0}{m\beta^2}(t^2 + 2t)e^{-\beta^2(t+2)} + \frac{f_0}{m\beta^2} \quad (2)$$

$$\frac{dv_{r2}}{dt} + \frac{\alpha}{m} v_{r2} = \frac{f_0}{m\beta^2}(t^2 + 4t + 4)e^{-\beta^2(t+2)} + \frac{f_0}{m\beta^2} \quad (3)$$

$$\frac{dv_{r3}}{dt} + \frac{\alpha}{m} v_{r3} = \frac{f_0}{m\beta^2}(t^2 + 6t + 9)e^{-\beta^2(t+2)} + \frac{f_0}{m\beta^2} \quad (4)$$

where $v_{r1}(t)$, $v_{r2}(t)$ and $v_{r3}(t)$ (Hutem, 2019; Hutem et al., 2017) are the velocity time-dependent of the 10,000-metre runner (ms^{-1}) and α is the coefficient drag force (Jalilian et al., 2014; Keokhumcheng, 2020). The equation (2), (3) and (4) are called the

formula for in-homogeneous linear differential equation. The solution of equation (2), (3) and (4) is:

$$v_{r1}(t) = e^{-\frac{\alpha}{m}t} \left[\frac{f_0}{m\beta^2} \int (t^2 + 2t) e^{-\beta^2(t+2)} e^{\frac{\alpha}{m}t} dt + \frac{f_0}{m\beta^2} \int e^{\frac{\alpha}{m}t} dt + c_1 \right] \quad (5)$$

$$v_{r2}(t) = e^{-\frac{\alpha}{m}t} \left[\frac{f_0}{m\beta^2} \int (t^2 + 4t + 4) e^{-\beta^2 t - 2\beta^2 + \frac{\alpha}{m}t} dt + \frac{f_0}{m\beta^2} \int e^{\frac{\alpha}{m}t} dt + c_2 \right] \quad (6)$$

$$v_{r3}(t) = e^{-\frac{\alpha}{m}t} \left[\frac{f_0 e^{-2\beta^2}}{m\beta^2} \int (t^2 + 6t + 9) e^{\left(\frac{\alpha}{m} - \beta^2\right)t} dt + \frac{f_0}{m\beta^2} \cdot \frac{m}{\alpha} e^{\frac{\alpha}{m}t} + c_3 \right] \quad (7)$$

With this definition, the parameter becomes $\delta = \frac{\alpha}{m} - \beta^2$. Therefore the velocity time-dependent model 1,2 and 3 of the 10,000-metre runner can be written as:

$$v_{r1}(t) = \frac{f_0 e^{-\beta^2(t+2)}}{m\beta^2} \left(\frac{t^2}{\delta} - \frac{2t}{\delta^2} + \frac{2}{\delta^3} + \frac{2t}{\delta} - \frac{2}{\delta^2} \right) + \frac{f_0}{\alpha\beta^2} + c_1 e^{-\frac{\alpha}{m}t} \quad (8)$$

$$v_{r2}(t) = \frac{f_0 e^{-\beta^2(t+2)}}{m\beta^2} \left(\frac{2}{\delta^3} - \frac{(2t+4)}{\delta^2} + \frac{(t^2+4t+4)}{\delta} \right) + \frac{f_0}{\alpha\beta^2} + c_2 e^{-\frac{\alpha}{m}t} \quad (9)$$

$$v_{r3}(t) = \frac{f_0 e^{-\beta^2(t+2)}}{m\beta^2} \left(\frac{2}{\delta^3} - \frac{(2t+6)}{\delta^2} + \frac{(t^2+6t+9)}{\delta} \right) + \frac{f_0}{\alpha\beta^2} + c_3 e^{-\frac{\alpha}{m}t} \quad (10)$$

It should be noted that if the initial velocity time-dependent model 1,2 and 3 is zero, that is, if at $t = 0$, $v(0) = 0$ we obtain:

$$\therefore c_1 = - \left[\frac{f_0}{\alpha\beta^2} + \frac{f_0 e^{-2\beta^2}}{m\beta^2} \left(\frac{2}{\left(\frac{\alpha}{m} - \beta^2\right)^3} - \frac{2}{\left(\frac{\alpha}{m} - \beta^2\right)^2} \right) \right] \quad (11)$$

$$\therefore c_2 = - \left[\frac{f_0}{\alpha\beta^2} + \frac{f_0 e^{-2\beta^2}}{m\beta^2} \left(\frac{2}{\delta^3} - \frac{4}{\delta^2} + \frac{4}{\delta} \right) \right] \quad (12)$$

$$\therefore c_3 = - \left[\frac{f_0}{\alpha\beta^2} + \frac{f_0 e^{-2\beta^2}}{m\beta^2} \left(\frac{2}{\delta^3} - \frac{6}{\delta^2} + \frac{9}{\delta} \right) \right] \quad (13)$$

Substitution equation (11), (12) and (13) into equation (8), (9) and 10, we get therefore the velocity time-dependent model 1 complete solutions of the 10,000-metre runner as the followings:

$$v_{r1}(t) = \frac{f_0 e^{-\beta^2(t+2)}}{m\beta^2} \left(\frac{2}{\left(\frac{\alpha}{m} - \beta^2\right)^3} - \frac{(2t+2)}{\left(\frac{\alpha}{m} - \beta^2\right)^2} + \frac{t^2 + 2t}{\frac{\alpha}{m} - \beta^2} \right) + \frac{f_0}{\alpha\beta^2} - \left(\frac{f_0}{\alpha\beta^2} + \frac{f_0 e^{-2\beta^2}}{m\beta^2} \left(\frac{2}{\left(\frac{\alpha}{m} - \beta^2\right)^3} - \frac{2t}{\left(\frac{\alpha}{m} - \beta^2\right)^2} \right) \right) e^{\frac{\alpha}{m}t} \quad (14)$$

$$v_{r2}(t) = \frac{f_0 e^{-\beta^2(t+2)}}{m\beta^2} \left(\frac{2}{\left(\frac{\alpha}{m} - \beta^2\right)^3} - \frac{(2t+4)}{\left(\frac{\alpha}{m} - \beta^2\right)^2} + \frac{(t^2 + 4t + 4)}{\frac{\alpha}{m} - \beta^2} \right) + \frac{f_0}{\alpha\beta^2} - \left[\frac{f_0}{\alpha\beta^2} + \frac{f_0 e^{-2\beta^2}}{m\beta^2} \left(\frac{2}{\left(\frac{\alpha}{m} - \beta^2\right)^3} - \frac{4}{\left(\frac{\alpha}{m} - \beta^2\right)^2} + \frac{4}{\frac{\alpha}{m} - \beta^2} \right) \right] e^{\frac{\alpha}{m}t} \quad (15)$$

$$v_{r3}(t) = \frac{f_0 e^{-\beta^2(t+2)}}{m\beta^2} \left(\frac{2}{\left(\frac{\alpha}{m} - \beta^2\right)^3} - \frac{(2t+6)}{\left(\frac{\alpha}{m} - \beta^2\right)^2} + \frac{(t^2 + 6t + 9)}{\frac{\alpha}{m} - \beta^2} \right) + \frac{f_0}{\alpha\beta^2} - \left[\frac{f_0 e^{-2\beta^2}}{m\beta^2} \left(\frac{2}{\left(\frac{\alpha}{m} - \beta^2\right)^3} - \frac{6}{\left(\frac{\alpha}{m} - \beta^2\right)^2} + \frac{9}{\frac{\alpha}{m} - \beta^2} \right) \right] e^{\frac{\alpha}{m}t} \quad (16)$$

A graph of the velocity time-dependent motion is given by equation (14), (15) and (16) for the 10,000-metre runner at the 2019 IAAF World Athletics Championships which is shown by the velocity function of time curve in section 4 as numerical results. The velocity of equation (14) as a function of time is presented in Figure 1. The velocity of equation (15) as a function of time is presented in Figure 2. The velocity of equation (16) as a function of time is presented in Figure 3.

Calculation procedure the runner velocity from experiment.

1. The IAAF World Athletics Championships Doha 2019 10,000-metre men results record has been downloaded from the world athletics website.
2. Distance and times from record have been input to the data table.
3. Time information form from minute: second: millisecond has been changed into the second form.

4. The velocity every 100 metres has been calculated by using formula

$$velocity = \frac{distance}{time}$$

5. We can create ordered pairs between times and velocity of the 10,000-metre runner.

Calculation procedure on Mathematica.

1. We can use “Clear” command to clear parameter value of model1, data1, f_0 , β , α and m

2. The value of m has been inputted, we can find the value from the average mass value of first four fastest 10,000 metres men runner at the IAAF World Athletics Championships DOHA 2019.

3. Data1, 101 ordered pairs between times and velocity from the IAAF World Athletics Championships DOHA 2019 record, has been inputted.

4. The velocity time-dependent model equation has been inputted.

5. We can use “FindFit” command to find f_0 , β and α parameter value.

6. We can use “Clear” command to clear parameter value of model1, data1, f_0 , β , α and m

7. The value of f_0 , β , α and m value has been inputted.

8. We can use “Plot” command to plot a model1 graph between time (s) and velocity (m/s).

9. Data2, 101 ordered pairs between times and velocity from the IAAF World Athletics Championships DOHA 2019 record, has been inputted.

10. We can use “Plot” command to plot a graph between time (s) and velocity (m/s) from ordered pairs in data2.

11. We can use “Show” command to compare the model 1 graph and data 2 graph in same frame.

12. We can use “Clear” command to clear parameter value of f_0 , β , α , t and m

13. The value of f_0 , β , α and m has been inputted.

14. The value of t has been inputted and use “SetPrecision” command to find velocity from $t=15.54$ to $t=1608.36$ correspondingly.

15. We can be using “Correlation” command to find correlation value of velocity from experiment and velocity from the motion model.

Results

According to experiment data, 10,000-metre runners usually use a highest velocity at the start and relatively stable speed for long distance run before accelerating at the end as shown in the Table 1 and Figure 1-3.

Table 1 Representation time, velocity and percentage difference velocity value parameter of running for the 10,000-metre runner: IAAF World Athletics Championships Doha 2019 and motion model

Time	Velocity experiment (IAAF World Athletics Championships Doha 2019)	Velocity			% Difference Velocity experiment and Velocity model		
		Model	Model	Model	Model	Model	Model
		1	2	3	1	2	3
15.54	6.44	6.43	6.43	6.45	0.08	0.08	0.23
31.31	6.39	6.42	6.40	6.38	0.50	0.19	0.12
47.72	6.29	6.28	6.29	6.29	0.11	0.05	0.05
64.13	6.24	6.21	6.23	6.23	0.44	0.12	0.12
80.43	6.22	6.18	6.19	6.20	0.59	0.43	0.27
97.21	6.17	6.16	6.17	6.17	0.20	0.04	0.04
113.98	6.14	6.15	6.16	6.16	0.14	0.30	0.30
130.47	6.13	6.15	6.15	6.15	0.30	0.30	0.30
147.07	6.12	6.15	6.15	6.15	0.50	0.50	0.50
163.67	6.11	6.15	6.15	6.15	0.65	0.65	0.65
180.20	6.10	6.14	6.15	6.15	0.58	0.75	0.75
196.52	6.11	6.14	6.14	6.14	0.55	0.55	0.55
213.02	6.10	6.14	6.14	6.14	0.61	0.61	0.61
229.36	6.10	6.14	6.14	6.14	0.59	0.59	0.59
245.81	6.10	6.14	6.14	6.14	0.62	0.62	0.62
261.92	6.11	6.14	6.14	6.14	0.51	0.51	0.51
278.11	6.11	6.14	6.14	6.14	0.45	0.45	0.45
294.56	6.11	6.14	6.14	6.14	0.48	0.48	0.48
311.10	6.11	6.14	6.14	6.14	0.53	0.53	0.53
327.24	6.11	6.14	6.14	6.14	0.46	0.46	0.46
343.43	6.11	6.14	6.14	6.14	0.41	0.41	0.41
359.90	6.11	6.14	6.14	6.14	0.44	0.44	0.44
376.31	6.11	6.14	6.14	6.14	0.46	0.46	0.46

Table 1 Representation time, velocity and percentage difference velocity value parameter of running for the 10,000-metre runner: IAAF World Athletics Championships Doha 2019 and motion model (cont.)

Time	Velocity experiment	Velocity			% Difference Velocity		
	(IAAF World Athletics				experiment and Velocity model		
	Championships Doha	Model	Model	Model	Model	Model	Model
	2019)	1	2	3	1	2	3
392.91	6.11	6.14	6.14	6.14	0.52	0.52	0.52
409.43	6.11	6.14	6.14	6.14	0.55	0.55	0.55
425.53	6.11	6.14	6.14	6.14	0.49	0.49	0.49
440.64	6.13	6.14	6.14	6.14	0.20	0.20	0.20
456.28	6.14	6.14	6.14	6.14	0.06	0.06	0.06
472.25	6.14	6.14	6.14	6.14	0.01	0.01	0.01
488.23	6.14	6.14	6.14	6.14	0.08	0.08	0.08
505.38	6.13	6.14	6.14	6.14	0.10	0.10	0.10
521.23	6.14	6.14	6.14	6.14	0.01	0.01	0.01
537.64	6.14	6.14	6.14	6.14	0.03	0.03	0.03
554.20	6.13	6.14	6.14	6.14	0.08	0.08	0.08
570.73	6.13	6.14	6.14	6.14	0.12	0.12	0.12
587.33	6.13	6.14	6.14	6.14	0.17	0.17	0.17
603.80	6.13	6.14	6.14	6.14	0.20	0.20	0.20
620.24	6.13	6.14	6.14	6.14	0.22	0.22	0.22
636.30	6.13	6.14	6.14	6.14	0.18	0.18	0.18
652.38	6.13	6.14	6.14	6.14	0.14	0.14	0.14
668.66	6.13	6.14	6.14	6.14	0.14	0.14	0.14
684.51	6.14	6.14	6.14	6.14	0.07	0.07	0.07
700.40	6.14	6.14	6.14	6.14	0.01	0.01	0.01
716.51	6.14	6.14	6.14	6.14	0.01	0.01	0.01
732.58	6.14	6.14	6.14	6.14	0.04	0.04	0.04
748.55	6.15	6.14	6.14	6.14	0.08	0.08	0.08
765.12	6.14	6.14	6.14	6.14	0.05	0.05	0.05
781.61	6.14	6.14	6.14	6.14	0.02	0.02	0.02
797.86	6.14	6.14	6.14	6.14	0.02	0.02	0.02
813.20	6.15	6.14	6.14	6.14	0.14	0.14	0.14

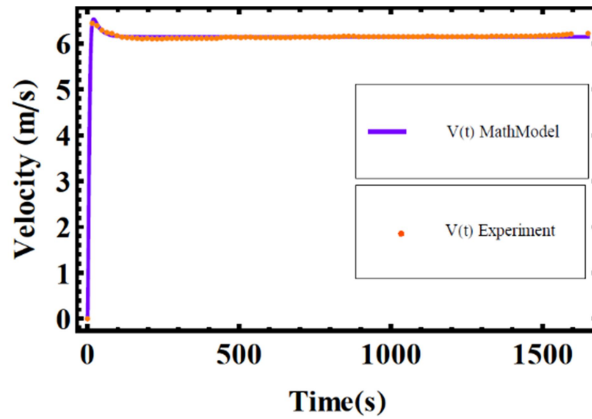
Table 1 Representation time, velocity and percentage difference velocity value parameter of running for the 10,000-metre runner: IAAF World Athletics Championships Doha 2019 and motion model (cont.)

Time	Velocity experiment	Velocity			% Difference Velocity		
	(IAAF World Athletics				experiment and Velocity model		
	Championships Doha	Model	Model	Model	Model	Model	Model
	2019)	1	2	3	1	2	3
828.86	6.15	6.14	6.14	6.14	0.21	0.21	0.21
844.59	6.16	6.14	6.14	6.14	0.27	0.27	0.27
860.75	6.16	6.14	6.14	6.14	0.28	0.28	0.28
877.29	6.16	6.14	6.14	6.14	0.25	0.25	0.25
893.91	6.15	6.14	6.14	6.14	0.21	0.21	0.21
910.52	6.15	6.14	6.14	6.14	0.17	0.17	0.17
926.91	6.15	6.14	6.14	6.14	0.15	0.15	0.15
943.14	6.15	6.14	6.14	6.14	0.16	0.16	0.16
959.53	6.15	6.14	6.14	6.14	0.14	0.14	0.14
976.13	6.15	6.14	6.14	6.14	0.11	0.11	0.11
992.51	6.15	6.14	6.14	6.14	0.10	0.10	0.10
1,008.76	6.15	6.14	6.14	6.14	0.10	0.10	0.10
1,024.81	6.15	6.14	6.14	6.14	0.12	0.12	0.12
1,041.07	6.15	6.14	6.14	6.14	0.12	0.12	0.12
1,057.42	6.15	6.14	6.14	6.14	0.11	0.11	0.11
1,073.36	6.15	6.14	6.14	6.14	0.15	0.15	0.15
1,089.15	6.15	6.14	6.14	6.14	0.19	0.19	0.19
1,105.19	6.15	6.14	6.14	6.14	0.21	0.21	0.21
1,120.96	6.16	6.14	6.14	6.14	0.25	0.25	0.25
1,136.85	6.16	6.14	6.14	6.14	0.28	0.28	0.28
1,153.64	6.15	6.14	6.14	6.14	0.23	0.23	0.23
1,169.80	6.15	6.14	6.14	6.14	0.24	0.24	0.24
1,186.12	6.15	6.14	6.14	6.14	0.24	0.24	0.24
1,202.28	6.15	6.14	6.14	6.14	0.24	0.24	0.24
1,218.90	6.15	6.14	6.14	6.14	0.21	0.21	0.21
1,234.91	6.15	6.14	6.14	6.14	0.23	0.23	0.23

Table 1 Representation time, velocity and percentage difference velocity value parameter of running for the 10,000-metre runner: IAAF World Athletics Championships Doha 2019 and motion model (cont.)

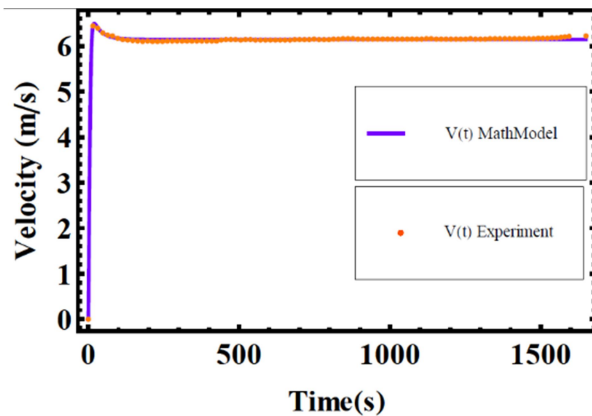
Time	Velocity experiment	Velocity			% Difference Velocity		
	(IAAF World Athletics				experiment and Velocity model		
	Championships Doha	Model	Model	Model	Model	Model	Model
	2019)	1	2	3	1	2	3
1,251.05	6.15	6.14	6.14	6.14	0.24	0.24	0.24
1,267.09	6.16	6.14	6.14	6.14	0.26	0.26	0.26
1,283.72	6.15	6.14	6.14	6.14	0.23	0.23	0.23
1,300.16	6.15	6.14	6.14	6.14	0.21	0.21	0.21
1,316.10	6.15	6.14	6.14	6.14	0.24	0.24	0.24
1,331.94	6.16	6.14	6.14	6.14	0.27	0.27	0.27
1,348.25	6.16	6.14	6.14	6.14	0.26	0.26	0.26
1,364.50	6.16	6.14	6.14	6.14	0.26	0.26	0.26
1,380.96	6.16	6.14	6.14	6.14	0.25	0.25	0.25
1,397.01	6.16	6.14	6.14	6.14	0.26	0.26	0.26
1,413.13	6.16	6.14	6.14	6.14	0.27	0.27	0.27
1,429.35	6.16	6.14	6.14	6.14	0.27	0.27	0.27
1,445.02	6.16	6.14	6.14	6.14	0.31	0.31	0.31
1,460.79	6.16	6.14	6.14	6.14	0.34	0.34	0.34
1,476.29	6.16	6.14	6.14	6.14	0.39	0.39	0.39
1,491.73	6.17	6.14	6.14	6.14	0.44	0.44	0.44
1,507.11	6.17	6.14	6.14	6.14	0.50	0.50	0.50
1,522.46	6.17	6.14	6.14	6.14	0.56	0.56	0.56
1,537.82	6.18	6.14	6.14	6.14	0.61	0.61	0.61
1,552.98	6.18	6.14	6.14	6.14	0.68	0.68	0.68
1,567.27	6.19	6.14	6.14	6.14	0.80	0.80	0.80
1,580.48	6.20	6.14	6.14	6.14	0.98	0.98	0.98
1,594.24	6.21	6.14	6.14	6.14	1.13	1.13	1.13
1,608.36	6.22	6.14	6.14	6.14	1.25	1.25	1.25
Correlation Coefficient					0.86942	0.871605	0.870055

We show plot of velocity for 10,000-metre runner from the experimental data and motion models.



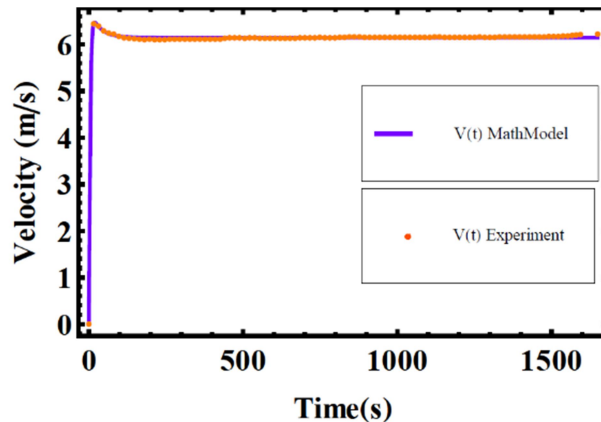
$$m = 60 \text{ kg}, f_0 = 6.35835 \text{ N}, \beta = 0.631508 \text{ s}^{-1}, \alpha = 2.59491$$

Figure 1 Comparison of the velocity-time obtained from the experimental data and motion model 1 of the applied force time-dependent decreasing function for 10,000-metre runner in IAAF World Athletics Championships Doha 2019.



$$m = 60 \text{ kg}, f_0 = 5.40964 \text{ N}, \beta = 0.645901 \text{ s}^{-1}, \alpha = 2.1106$$

Figure 2 Comparison of the velocity-time obtained from the experimental data and motion model 2 of the applied force time-dependent decreasing function for 10,000-metre runner in IAAF World Athletics Championships Doha 2019.



$$m = 60 \text{ kg}, f_0 = 5.01715 \text{ N}, \beta = -0.664321 \text{ s}^{-1}, \alpha = 1.85051$$

Figure 3 Comparison of the velocity-time obtained from the experimental data and motion model 3 of the applied force time-dependent decreasing function for 10,000-metre runner in IAAF World Athletics Championships Doha 2019.

Discussions

From Table 1, it is the comparison of the quantity velocity of the 10,000-metre runner in IAAF World Athletics Championships Doha 2019. The applied force is time-dependent decreasing function of mathematical and physics motion model 1, 2 and 3 with velocity data obtained from the experiment. The velocity of 10,000-metre runner from applied force is time-dependent decreasing function of mathematical, which is in agreement with previous studies in velocity of long distance runner (Bruce & Tim, 1981), and physics motion model 1 has a more different percentage from experiment. This is consistent with correlation coefficient model = 0.86942 but the velocity of 10,000-metre runner from the applied force is time-dependent decreasing function of mathematical and physics motion model 2 has less different percentage from the experiment. This is consistent with correlation coefficient model = 0.871605. The velocity of 10,000-metre runner from the applied force is time-dependent decreasing function of mathematical and physics motion model 3 has less different percentage from the experiment. This is consistent with correlation coefficient model = 0.870055.

Apart from time, coefficient drag force (α), speed endurance of runner (β) and initial force (f_0) are the factors that affect running speed. Compared to motion model of the 100-metre runner's by Helene & Yamashita (2010); Tiago et al. (2016) and

Keller (1974), and the 10,000-metre runner's motion model, it was found that β of the 10,000-metre runner's value was greater than β of the 100-metre runner's motion model due to mini marathon is longer distance than 100-metre sprint (Thongsit et al., 2019). The 10,000-metre runner's running time is more than 160 times of 100-metre runners. The value of f_0 in a 100-metre sprint is higher than that in a 10,000-metre runner because of short sprints, the time required for sprints is much less, so it takes effort to reach the initial speed as much as possible. Finally, the variable (α) is the wind force that hits the runner's body, which can vary according to the position and weather conditions in each location.

Conclusions

An initial force (f_0) that refined from model 1 was higher than that from model 2 and model 3 respectively. The endurance coefficient (β) was lowest in model 1 and the coefficient drag force (α) from model 3 is less than that in model 2 and model 1 respectively. Correlation coefficient by the experiment data and motion model 2 is more than that in model 1 and 3. The motion model 2 is the most suitable model to explain the 10,000-metre mini marathon runner factors.

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