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From Physics to Environmental Policy: Exploring Boltzmann Distribution for Carbon Trading Permit Allocation

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ABSTRACT

Combating climate change and reducing atmospheric CO2 emissions are imperative. While carbon trade, including taxes and trading systems, has been proposed, the allocation of initial permits is challenging. This study addresses this issue, exploring the application of the Boltzmann distribution for carbon emission permit allocation in carbon trading. The Boltzmann distribution method uniquely considers each country's environmental and economic contexts, assigning more responsibility to nations with larger populations and higher emissions, and providing incentives to lower-emission countries. This promotes fairness in global climate change efforts and influences national environmental policies. High-emission countries like China receive a high number of permits, encouraging stronger environmental policies, whereas countries with lower emissions, such as Italy, benefit from additional permits as compared to conventional distribution models, bolstering their existing environmental conservation. This paper applies the Boltzmann distribution to eight countries, considering population, economic size, and CO2 emissions. It effectively balances permits between egalitarianism (population-based) and sovereignty (emission-based) principles, suitable for international carbon trading. This flexible approach provides a practical framework for international emissions rights allocation along with potential applications in broader contexts. Implementation of the Boltzmann distribution in real-world policy faces challenges due to the dynamic nature of international politics and economics. This research offers insights into the process of integrating this method into existing environmental policy frameworks, demonstrating its potential as a tool for enhancing global environmental sustainability. Future research should explore its application in the complex international political and economic environment, furthering its role in global climate policy.

Keywords: Boltzmann distribution, carbon trade, climate change, fair allocation, permit allocation

INTRODUCTION

Scholars around the world have warned that an increase of more than 2°C in average global temperatures will cause hazardous climate change through potential sea-level changes and multispecies extinction; some scientists have argued that as little as a 1°C warming would be significant (Pachauri et al., 2014; Cointe & Guillemot, 2023; Hansen et al., 2006; Lee et al., 2023; Randalls, 2010). In addition, climate change that comes about owing to increasing concentration of global atmospheric carbon dioxide (CO2) is believed to be irremediable for 1,000 years after CO2 emissions cease (Solomon et al., 2009). According to the Stern Review, quick, decisive action is required, and because climate change is not a local issue, it must be addressed globally (Hossain et al., 2023; Stern, 2007).

Numerous ideas and hypotheses, such as the use of carbon taxes and carbon trade, have been suggested for addressing climate change and reducing CO2 emissions into the Earth's atmosphere (Cao et al., 2021; Choi et al., 2022; Gamero et al., 2021; House, 2008; Parhamfar et al., 2024; Tang et al., 2023). Many studies have shown that carbon trade lowers the cost of achieving the targets of the Kyoto Protocol (Oke et al., 2024; Springer, 2003).

The fundamental concept of carbon trade was established over a number of decades (Baumol & Oates, 1971; Coase, 1960; Crocker & Co, 1966; Dales, 1968; Milliman et al., 1989; Montgomery, 1972; Pigou, 2013; Tietenberg, 2006). The flow and value of what is traded in the carbon trading system may depend on initial permit allocation, as well as supply and demand (Victor, 2001). Central to understanding this system is the Coase Theorem, as articulated by Ronald Coase in 1960, which argues that under conditions of well-defined property rights and negligible transaction costs, parties will negotiate to correct externalities and allocate resources efficiently (Ronald H Coase, 1960). This theorem is foundational in the context of carbon trading as it underscores the importance of clearly defined property rights and the role of market mechanisms in addressing environmental externalities, particularly greenhouse gas emissions (Chen et al., 2024; Coase, 1960; Li et al., 2023).

Various methods have been proposed to provide, in the beginning, a given total of emissions permits to participants (Chichilnisky et al., 1993; Zhang et al., 2014; Zhou & Wang, 2016), including auctioning and grandfathering (Cramton & Kerr, 2002; Lai, 2007). Allocating permits is one of the most challenging issues in designing and implementing carbon trade systems. This is because the rule for allocating permits should be simple, established using historical data, and perceived as fair (Sorrell & Skea, 1999). The discussion of how to allocate a determined number of permits fairly is both controversial and important, inasmuch as the tradable emission permits in carbon trade depend on the initial allocation of emissions permits.

Park et al. (2012) introduced the Boltzmann distribution for initial permit allocation in international emissions trading, emphasizing its ability to offer a fair and most probable permit allocation among participating countries. This method was presented as an alternative to existing distribution methods like auctioning and grandfathering, which have been subjects of debate. Park et al. first outlined the basic concept of the Boltzmann distribution and developed its mathematical formula for allocating emissions permits. The novelty of this approach lies in its fairness to the participating countries, as it provides a new perspective on the distribution of a given total amount of emissions permits. To validate this method, the study conducted an empirical data analysis, which included the selection of eight countries - Canada, China, Germany, Italy, Japan, Russia, the U.K., and the U.S. - and analysis of their CO2 emissions for the years 2007 and 2008. The study assumed a global target for these countries to reduce CO2 emissions by 3% compared to 2007. The total amount of emissions permits allowed for 2008 was then allocated to these countries using the Boltzmann distribution.

Park et al. (2012) described the Boltzmann distribution as a simple model for initial allocating emission permits in international carbon trading. Further, they argued that the Boltzmann distribution is a versatile and flexible model because allocating emissions permits can require flexibility in response to different input variables (allocation preferences) and different values of the shape parameter β of the distribution. Park showed only one input variable in the Boltzmann distribution, but its flexibility for different input variables and different values of the shape parameter β still needs to be shown. In this paper, we demonstrate how the Boltzmann distribution offers flexibility and versatility for permit allocation in international carbon trading. Furthermore, we illustrate that this new method has potential for various environmental social science applications.

The rest of this paper is structured as follows. We begin with a literature review of greenhouse gas allocation approaches, and then explain the fundamentals of the Boltzmann distribution and its mathematical formulation for emissions permit allocation. This is followed by an empirical analysis demonstrating the method's practical applications. We also compare the Boltzmann distribution with two other common allocation methods. Finally, we discuss the challenges of and potential for implementing this approach in real-world policy, considering international political and economic dynamics, and suggest directions for future research in global climate policy.

LITERATURE REVIEW

Greenhouse gas allocation approaches

Greenhouse gas emission allocation approaches play a pivotal role in environmental policy and climate change mitigation strategy research. These methods can be categorized into various groups based on different criteria, with each approach distinguished by its characteristics and application methods. Such approaches include the 'Indicator Approach', 'Optimization Approach', 'Game Theoretic Approach', 'Hybrid Approach', and the 'Boltzmann Distribution Approach', which applies physical concepts to allocate initial carbon quotas.

Each approach is further detailed based on its underlying theoretical basis and application, allocation principles, and key features (Table 1). The Indicator Approach, for instance, is based on specific indicators, and aligns with fairness principles (Pan et al., 2014; Zhou et al., 2013), whereas the Optimization Approach focuses on economic efficiency, and minimizes total abatement costs (Chiu et al., 2015; Gomes & Lins, 2008; Sun et al., 2014; Zhou et al., 2014). The Game Theoretic Approach involves complex negotiations and equilibrium solutions, considering both fairness and efficiency (Filar et al., 1997; Liao et al., 2015; Zhang et al., 2014). Meanwhile, the Hybrid Approach offers flexibility, but may sometimes lack transparency (Berk & den Elzen, 2001; Yu et al., 2014), while the Boltzmann Distribution Approach determines carbon allocations through a probabilistic approach (Park, 2020; Park et al., 2012; Tan et al., 2017). This type of classification plays a crucial role in helping researchers evaluate and understand the characteristics, strengths, and applicability of each approach, and aids in making informed decisions with respect to environmental policy-making and climate change strategies.

In this paper, we base our research on the Boltzmann Distribution Approach, which offers a new perspective on the carbon allocation issue, and which has unique advantages and applicability compared to other existing approaches. This research will explore the theoretical basis and practical applications in cases of carbon allocation based on the Boltzmann distribution, thereby making a significant contribution to environmental policy and climate change strategies.

Table 1

Overview of Approaches for Greenhouse Gas Emission Allocation

Approach	Description	Allocation Principles	Key Features
Indicator Approach	Allocation based on specific indicators	Fairness (sovereignty, egalitarianism, horizontal equity, vertical equity, polluter pays)	Commonly used, simpler approach based on individual or composite indicators
Optimization Approach	Involves linear or nonlinear programming models for efficient allocation	Efficiency (minimization of total abatement cost)	Focuses on economic efficiency, can be complex in application
Game Theoretic Approach	Treats emissions allocation as a strategic game among participants	Fairness and Efficiency	Involves negotiations and equilibrium solutions, often complex and less transparent
Hybrid Approach	Combines various methods and principles	Fairness and Efficiency	Flexible but can lack transparency, incorporates multiple fairness and efficiency criteria
Boltzmann Distribution Approach	Permit allocation in emissions trading using the Boltzmann distribution	Fairness and Probability	Utilizes the concept of the Boltzmann distribution from physics to allocate initial carbon quotas

Note. Adapted from “Permit allocation in emissions trading using the Boltzmann distribution,” by J. W. Park, C. U. Kim, & W. Isard, 2012, *Physica A: Statistical Mechanics and its Applications, 391*(20), p. 4883–4890. https://doi.org/10.1016/j.physa.2012.05.052. Copyright 2012 by Elsevier. From “Regional allocation of carbon emission quotas in China: Evidence from the Shapley value method,” by Y. J. Zhang, A.-D. Wang, & Y.-B. Da, 2014*, Energy Policy, 74*, p. 454–464. https://doi.org/10.1016/j.enpol.2014.08.006. Copyright 2014 by Elsevier. From “Carbon dioxide emissions allocation: A review,” by P. Zhou & M. Wang, 2016, *Ecological Economics, 125*, p. 47–59. https://doi.org/10.1016/j.ecolecon.2016.03.001. Copyright 2016 by Elsevier.

METHODOLOGY

The Boltzmann distribution, which has been broadly used in other fields, including mathematics and economics, is derived from the physical sciences, and is based on maximum entropy (Park & Kim, 2021; Park et al., 2022; Yakovenko & Rosser Jr, 2009). The principle of maximum entropy, through the Boltzmann distribution, is introduced to international carbon trade, providing helpful guidelines for allocation of permits among multiple countries (Park et al., 2012). In physics, the Boltzmann probability (P_i), that a particle remains in the i^{th} substate, is inversely proportional to the exponential function of the substate energy E_i.

$$P_i \propto C_i e^{-\beta E_i} \tag{1}$$

where $\beta = 1/kT$ (k: Boltzmann constant, T: absolute temperature).

The Boltzmann distribution delivers the most probable, natural, and unbiased distribution of a physical system at thermal equilibrium. As part of this study, we substitute a carbon trading system composed of all involved countries for the concept of a physical system. Furthermore, we substitute people of the involved countries for physical substates, and emissions permits for physical particles. We assume that all individuals in country *i* contribute uniformly to its total CO2 emissions, and substitute the potential energy of a physical substate *i* with the "allocation potential energy per unit of population" ($\tilde{E}_i$) of country *i.* With this substitution, the probability that a unit emissions permit is distributed to a country *i* is proportional to its total population, and is inversely proportional to the exponential function of the allocation potential energy per unit of population ($\tilde{E}_i$) (Park et al., 2012).

$$P_i \propto C_i e^{-\beta \tilde{E}_i} \tag{2}$$

where β = constant (≥0)

$\tilde{E}_i$ = allocation potential energy per unit of population of a country *i*

C_i = total population of a country *i*

For instance, $\hat{E}_i$ in Eq. (2) can be defined as the negative value of CO2 emissions per unit of population. However, if the negative allocation potential energy per unit of population ($-\hat{E}_i$) is replaced with the allocation preference per unit of population ($\hat{E}_i$), interpreting Eq. (2) becomes more intuitive. $\hat{E}_i$ can be defined in multi-faceted ways, and can also take account of various political and economic parameters, e.g., it is proportional to the current CO2 emissions per unit of population of country *i*, energy per unit of population of country *i*, or GDP per unit of population of country *i*. To illustrate, if $\hat{E}_i$ is defined as the positive value of CO2 emissions per unit of population, then emissions permits are more likely to be allocated to the countries with higher CO2 emissions per unit of population:

$$P_i \propto C_i e^{\beta \hat{E}_i} \tag{3}$$

where β = constant (≥0)

$\hat{E}_i$ = allocation preference per unit of population of country *i*

C_i = total population of country *i*.

Allocating permits through the Boltzmann distribution

Assume that *n* countries participate in the international carbon market. Suppose the total available emissions permits ($\tilde{N}$) are assigned to the countries, and that country *i* has a population C_i, and assume as well that the allocation preference per unit of population of $\hat{E}_i$. $\tilde{N}$ is a big number so that the permit unit can be triflingly small. The normalization condition ($\sum_{i=1}^{n} P_i$) is given, so the probability that $\tilde{N}$ is assigned to country *i* can be shown in Eq. (4):

$$P_i = \frac{C_i e^{\beta \hat{E}_i}}{\sum_{i=1}^{n} C_i e^{\beta \hat{E}_i}}, \quad for\ i = 1,2,\dots,n \tag{4}$$

Next, the number of emissions permits that are assigned to country *i* is

$$N_i = \tilde{N} \times P_i = \tilde{N} \times \frac{C_i e^{\beta \hat{E}_i}}{\sum_{i=1}^{n} C_i e^{\beta \hat{E}_i}}, \quad for\ i = 1,2,\dots,n \tag{5}$$

The allocation preference per unit of population ($\hat{E}_i$) and the value of β in the Boltzmann distribution are two key factors in permit allocation, and are related to the responsibility for future reduction. If $\hat{E}_i$ is defined as energy use per unit of population, then a country with the highest energy uses per unit of population will receive the most available emissions permits. This country might be less accountable for future reductions, and receive more benefits through the permit allocation. The value of β in the Boltzmann distribution is related to the range of permit assignments. $\hat{E}_i$ can involve political and economic factors that need to be considered meticulously when allocating limited permits, and the β-value provides a "weight" to the allocation preference per unit of population. These two key factors in the Boltzmann distribution provide permit-allocation flexibility and versatility not easily achieved using conventional allocation methods. If a policymaker chooses one of the conventional fairness notions to allocate limited permits, the decision cannot be easily adjusted. For example, if initial permits are allocated to countries according to egalitarian principles, then permits are simply distributed in proportion to population. Under this system, China, having the largest population globally, would receive most of the available permits. Countries with the smallest populations would be left unsatisfied, and adjustments to the allocation would be difficult. Allocating permits using the Boltzmann distribution, on the other hand, can be adjusted via the allocation preference per unit of population ($\hat{E}_i$) and the β-value.

The optimal β-value for determining the optimal allocation in the Boltzmann distribution

This section presents a methodology for selecting the optimal β-value in permit allocation using the Boltzmann distribution. Suppose $\hat{E}_i$ is defined as the actual CO2 emissions per unit of population of country *i*.

To observe diverse outcomes from different values of β for allocating emissions permits, two obvious conditions need to be meticulously considered where the β-value approaches the limiting value: 0 or ∞. In the Boltzmann probability of Eq. (4), when β is 0, only one factor, the population of the participating country, is considered. In this case, every person in each of the involved countries acquires a uniform and identical number of emissions permits, which is consequently assigned in proportion to the total population of country *i*. We can thus understand that this situation represents egalitarianism, one of the fairness notions for allocating emissions permits (Ringius et al., 2002; Rose et al., 1998). If β is near 0, then those countries having lower CO2 emissions per unit of population and a huge population can meet their demand for CO2 emissions fairly easily. But this situation disadvantages countries with higher CO2 emissions per unit of population; thus, they might prefer bigger β-values. As β increases and approaches ∞, the Boltzmann probability yields non-zero values for only the few countries having the highest CO2 emissions per unit of population, and it represents a tremendously biased and weighted permit allocation. Otherwise, the Boltzmann probability is 0 for all other countries. In this case, countries having the highest CO2 emissions per unit of population derive the greatest benefit insofar as they receive all the obtainable emissions permits. By contrast, all other countries would remain unsatisfied, receiving just a few permits. Hence, these countries will prefer smaller β-values.

Based on the above cases, countries with large populations and commensurably small CO2 emissions per unit of population will prefer smaller β-values. Conversely, countries with relatively high CO2 emissions per unit of population will desire larger values of β. Because no singular value fulfills the needs of all countries, it may be difficult to reach conflict-free consensus on an appropriate β-value. In other words, allocating permits using the Boltzmann distribution can accommodate a broad range of permit allocations. Determining the β-value actually creates a conflict between countries with comparatively large populations and countries with comparatively high CO2 emissions per unit of population.

To resolve the conflict, in this paper, we try to determine the β-value for optimal permit allocation using the least squares (Y) calculation between allocated permits and actual CO2 emissions $Y = \sum_{i=0}^{n}(permit_i - demand_i)^2$ (Park et al., 2012). Here, it is assumed that the actual CO2 emissions of country *i* refers to the demand for CO2 of each country. The least square minimizes the sum of the squares of the differences between allocated permits (N_i) and actual CO2 emissions (CO_{2i}). Determining the optimal allocation involves finding the right number of allocated permits to achieve the least squares. Therefore, we want to find the optimal β-value (β*) that results in the least squares; that is,

$$\begin{aligned}\beta^* &= \arg\min_{\beta}\ Y\ (N_1, N_2, \ldots, N_n) \\ &= \arg\min_{\beta} \sum_{i=1}^{n}(N_i(\beta) - CO_{2i})^2\end{aligned} \tag{6}$$

subject to

$$N_i(\beta) = \widetilde{N} \times \frac{c_i e^{\beta \hat{E}_i}}{\sum_{i=1}^{n} c_i e^{\beta \hat{E}_i}} \tag{7}$$

Practically, the numerical value of the optimal β-value can be determined using the extremum condition:

$$\frac{\partial Y}{\partial \beta} = \sum_{i=1}^{n} \frac{\partial Y}{\partial N_i} \cdot \frac{\partial N_i}{\partial \beta} = 0 \qquad (8)$$

Next, the obtained permit allocation $\{N_1, N_2, \cdots, N_n\}$ with the optimal value of β becomes the optimal permit allocation given by the Boltzmann distribution.

DATA ANALYSIS

Eight countries—Canada, China, Germany, Italy, Japan, Russia, the U.K., and the U.S—were chosen for an example analysis to show how permits can be allocated in practice using a Boltzmann distribution. The U.S. refused to ratify the Kyoto Protocol for several reasons. One of its main concerns was that China, a developing country, was not among those required to reduce emissions. GDP growth rate in China averaged 15.36% from 1990 until 2010, much higher than the average annual growth rate of developed countries. Its annual growth of emissions has increased as well. Since 2000, China's CO2 emissions have nearly tripled, to approximately 10 billion tonnes (Schiermeier, 2012; Xu et al., 2018). Unlike developed countries, developing countries such as India are China are concerned about "historical responsibility" for the accumulated stock of carbon constantly emitted by developed economies. Therefore, whereas developed countries generally favor permit allocation based on historical or current CO2 emissions, developing countries, including China and India, are more likely to favor permit allocation in proportion to population.

In this paper, the allocation of permits is considered using the Boltzmann distribution with different definitions of the allocation preference per unit of population ($\hat{E}_i$) of a country *i*. Here, $\hat{E}_i$ of a country *i* provides the standard for allocation of permits. Determining which country might be more obligated to future reductions depends on how $\hat{E}_i$ is defined. In the next empirical data analysis, to evaluate and compare different definitions of $\hat{E}_i$ of country *i*, it is defined as one of three specific criteria of country *i* in 2010: CO2 emissions per unit of population, energy use per unit of population, or GDP per unit of population.

The primary reason for selecting data from the year 2010 in this study was to effectively demonstrate the application results of the Boltzmann distribution method. The core objective of this research is to illustrate how the Boltzmann distribution can be applied to the allocation of carbon emission permits, and the data from a specific year is utilized to validate the efficacy of this method. The year 2010 was chosen not only because it provides sufficient data availability and quality to achieve this purpose but also because it represents a suitable time point for capturing significant changes or trends relevant to the application of the Boltzmann distribution. In other words, this year (2010) was not the main focus of the research; rather, it was selected to demonstrate the results of applying the Boltzmann distribution. Therefore, this study concentrates not on the data of a particular year, but on the effectiveness and practicality of the Boltzmann distribution method itself, offering a new approach that can enhance the fairness and efficiency of carbon allocation.

Assume that the global target for eight countries in 2010 is to reduce global CO2 emissions by 3% compared with 2009. Strictly speaking, the total number of available permits for 2010 is 17,268 (1000 kt), allocated to the chosen eight countries. First, the permits are allocated in proportion to seven criteria: population, emissions, energy use, GDP, emissions per unit of population, energy use per unit of population, and GDP per unit of population (Table 2). Specifically, population and emissions correspond to existing notions of fairness for egalitarianism and sovereignty. For example, sovereignty implies that "all nations have equal rights to pollute and to be protected from pollution" (Ringius et al**.**, 2002; Rose & Stevens, 1993; Rose et al**.**, 1998; Soltau, 2009). As shown in Figure 1, China receives the largest allocation when permits are distributed proportionally based on population, energy use, or emissions; the U.S. receives the second-most, and Russia the third-most. While these three countries receive the majority of the permits, other countries remain relatively unfulfilled, obtaining only a small fraction of the available permits. When GDP is used as the criterion, Japan replaces Russia in the top three permit recipients; China is seventh, and Russia is last**.**

Table 2

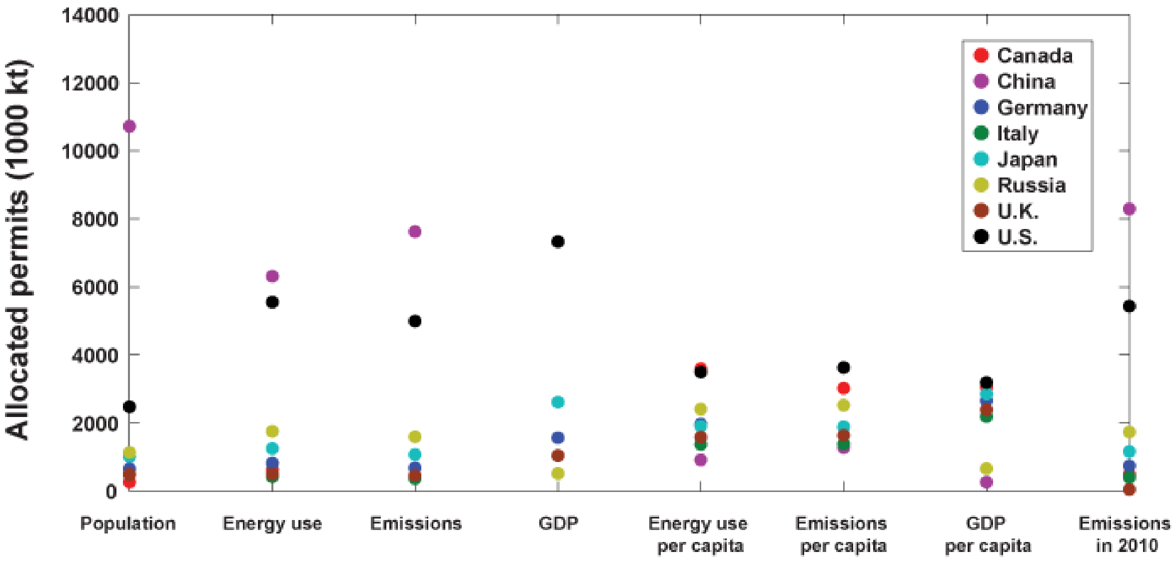
Seven Simple Criteria for Allocating Emissions Permits Between Eight Countries for 2010.

Country	Population (million)	Emissions (1000 kt)	Energy use (1000 kt of oil equivalent)	GDP (billion $)	Emissions per capita* (1000 kt)	Energy use per capita* (1000 kt of oil equivalent)	GDP per capita* (billion $)
Canada	34	499	251	1,577	14.62	7.35	46.21
China	1,338	8,287	2,517	5,931	6.19	1.88	4.43
Germany	82	745	330	3,304	9.11	4.03	40.41
Italy	60	406	170	2,055	6.72	2.81	33.98
Japan	127	1,171	499	5,495	9.19	3.92	43.12
Russia	142	1,741	702	1,525	12.23	4.93	10.71
U.K.	62	494	202	2,286	7.93	3.24	36.70
U.S.	309	5,433	2,216	14,958	17.56	7.16	48.36

Note. * The allocation preference per unit of population ($\hat{E}_i$) of country *i* in the Boltzmann distribution. Adapted from "Country Data," by World Bank, World Bank website (https://data.worldbank.org/country). Copyright by World Bank.

Figure 1

Permit Allocation According to Seven Criteria: Population, Energy Use, Emissions, GDP, Energy Use per Unit of Population, Emission per Unit of Population, and GDP per Unit of Population.



Note. The permits are allocated to eight countries, in proportion to their values for each criterion; actual emissions data for the eight countries in 2010 are added as a reference.

Thus, China and Russia obtain just a tiny fraction of the available permits compared with their actual emissions**.** In this situation, they might be unsatisfied**.** When permits are assigned in proportion to emissions per unit of population, the U.S. receives the largest number of permits; Canada ranks second, and China is last. In this situation, China is unsatisfied. When permits are instead assigned in proportion to energy use per unit of population, Canada receives the largest number of permits, and the U.S. receives a slightly smaller number. Here, Canada and the

U.S. might be satisfied, but China, with its relatively low energy use per unit of population, is unsatisfied, obtaining just a few permits. Using GDP per unit of population as the criterion, the U.S. receives the largest number of permits, Canada receives the second-most, and Japan the third-most. Under this criterion, China is last, and the number of emissions permits provided to Russia only slightly exceeds the number provided to China. This is true even if China has the highest CO2 emissions and Russia has the third-largest emissions of the eight countries**.** In this situation, China and Russia might be unsatisfied. If a policymaker chooses any one of the seven criteria to allocate the limited permits proportionally, the procedure is strict and straightforward. On the other hand, decisions made using a single criterion cannot be adjusted regardless of dissatisfaction among countries.

In this study, permit allocation was analyzed using the Boltzmann distribution with energy use, emissions use, and GDP per unit of population as allocation preferences, as depicted in Figure 2. Figure 2-a1 illustrates the allocation of emission permits based on emissions per unit of population, showing that as the β-value increases, the U.S. receives more permits while China receives fewer. At certain β-values, the U.S. nearly acquires all available permits, indicating a preference for higher β-values, whereas China and other countries prefer lower β-values, due to the inverse relationship with permits received. As shown in Figure 2-b1, with energy use as the allocation criterion, the distribution changes, with the U.S. and Canada receiving more permits at higher β-values, and China receiving fewer. Canada, being the largest energy user per capita, benefits from higher β-values, while other countries, including China, are left with fewer permits. Similarly, as shown in Figure 2-c1, where GDP per unit of population is the criterion, the U.S., with the highest GDP per capita, benefits most from higher β-values, receiving almost all available permits, while China and other countries prefer lower β-values to meet their demands. These findings illustrate the competitive dynamics in permit allocation between countries like the U.S., China, and Canada under different β-value scenarios. The graphs in Figure 2 help to illustrate and emphasize the need for careful consideration of β-values for equitable permit distribution.

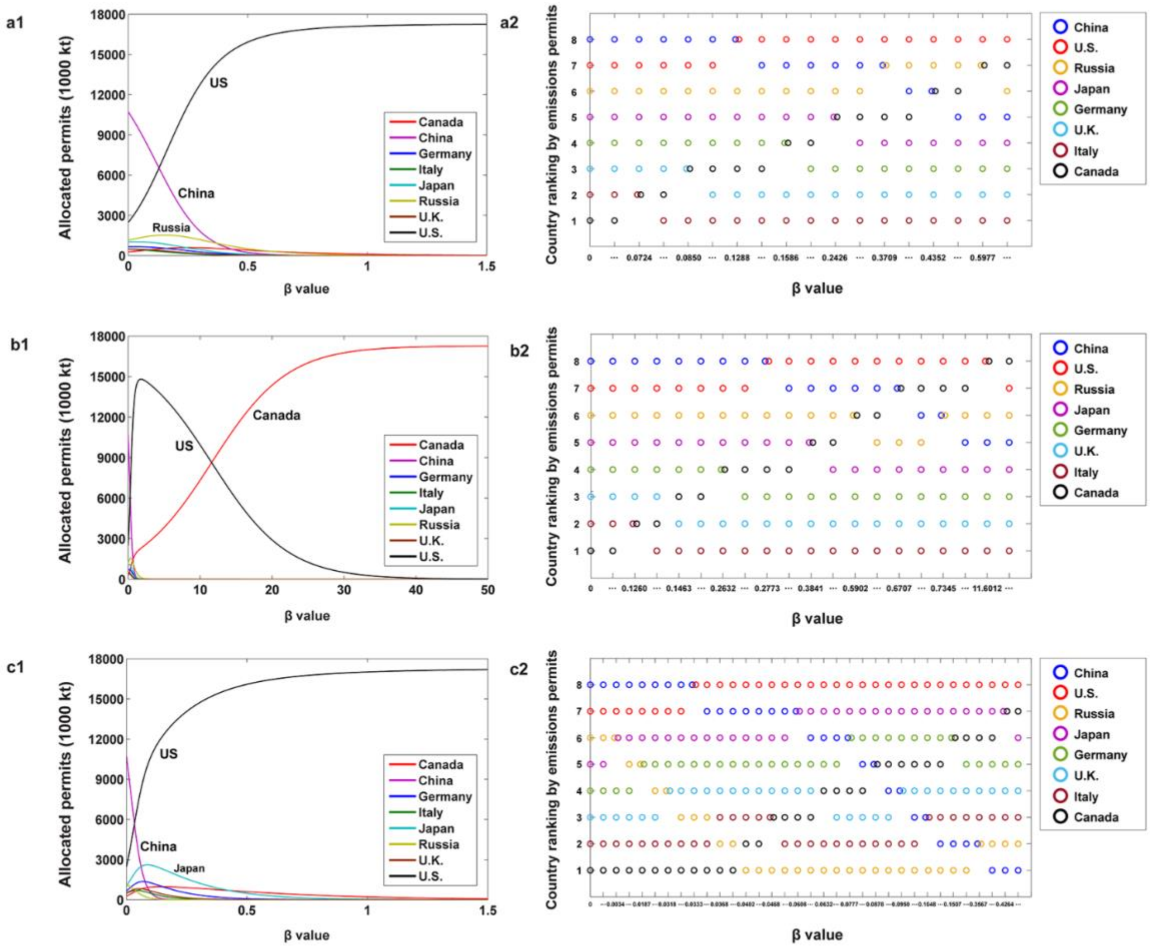
The empirical data analysis displayed in Figure 2 demonstrates that permit allocation using the Boltzmann distribution is highly flexible, with the nature of competition among countries varying based on the input variable. When the β-value represents energy use per capita, its range is broader (0 to ~50) compared to emissions per capita (0 to ~1.5) or GDP per capita (0 to ~1.5) due to smaller disparities in energy use. With GDP per capita, the ranking changes among the eight countries are more pronounced since more countries (Canada, Germany, Italy, Japan, the U.K., and the U.S.) have above-average GDP per capita than those with above-average emissions or energy use. Figures 2-a1, b1, and c1 illustrate how permit allocation shifts with varying β-values, while Figures 2-a2, b2, c2 show the changing country rankings for emissions permits. The frequency of ranking changes is notably higher when GDP is measured on a per capita basis, reflecting smaller differences in GDP among countries with larger economies.

Figure 3 presents the preferred β-value ranges for eight countries under different definitions, where each country achieves more (or less) than the share it deems equitable. In Figure 3-a (emissions per capita), Germany, Japan, and Russia cannot fulfill their demand, thus preferring β-values where they receive the maximum permits. Canada, Russia, and the U.S. favor higher β-values, while other countries opt for lower ones. Figure 3-b (energy use per capita) shows a similar pattern, with Germany, Japan, and Russia again unable to meet their demand. Canada and the U.S. prefer higher β-values, while China, Italy, and the U.K. lean towards lower ones. These scenarios indicate challenges in reaching a consensus on a single β-value that satisfies all countries. However, in Figure 3-c (GDP per capita), all countries tend to prefer lower β-values, suggesting that reaching consensus might be easier in this case, likely at lower β-value ranges.

In Figure 3, it's shown that no single β-value can satisfy all countries, making consensus challenging. Park et al. (2012) suggested using a least-squares calculation to find an optimal β-value for permit allocation. Our analysis, as detailed in Table 3, reveals that when CO2 emissions per capita is used, the least-squares value is the lowest, suggesting this as the best allocation preference. When comparing egalitarianism, sovereignty, and the Boltzmann distribution, four countries (Japan, Germany, the U.K., and Italy) receive similar permit numbers across all criteria; however, Canada, China, Russia, and the U.S. show varied results. For instance, China receives the most permits under egalitarianism, while Canada, the U.S., and Russia fare better under sovereignty. The Boltzmann distribution yielding the lowest least-squares value most closely represents actual CO2 emissions, suggesting its potential as an optimal method in international carbon trading.

Figure 2

Allocating Permits in International Carbon Trade Using the Boltzmann Distribution.

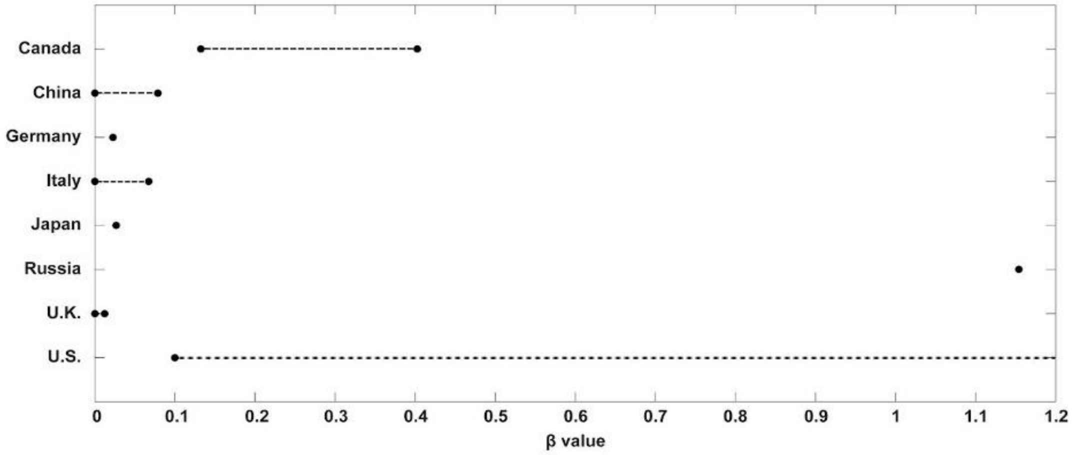


Note. (a) Allocation preference ($\hat{E}_i$) is emissions per unit of population in 2010. (b) Allocation preference ($\hat{E}_i$) is energy use per unit of population in 2010. (c) Allocation preference ($\hat{E}_i$) is GDP per unit of population in 2010. For emissions per unit of population and GDP per unit of population, the competition is derived for China and the U.S. For energy use per unit of population, the competition to allocate emissions permits is mainly derived for three countries: Canada, China, and the U.S.

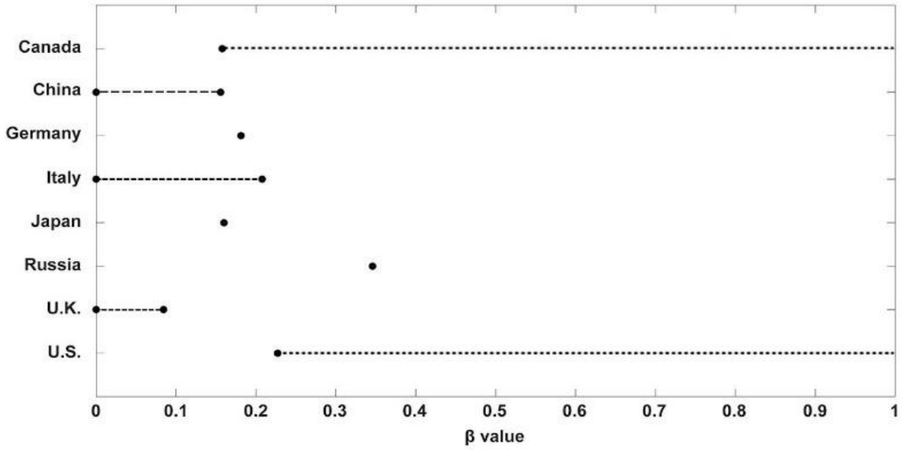
Figure 3

The Preferred β-Value Range for Eight Countries Using the Three Definitions of $\hat{E}_i$.

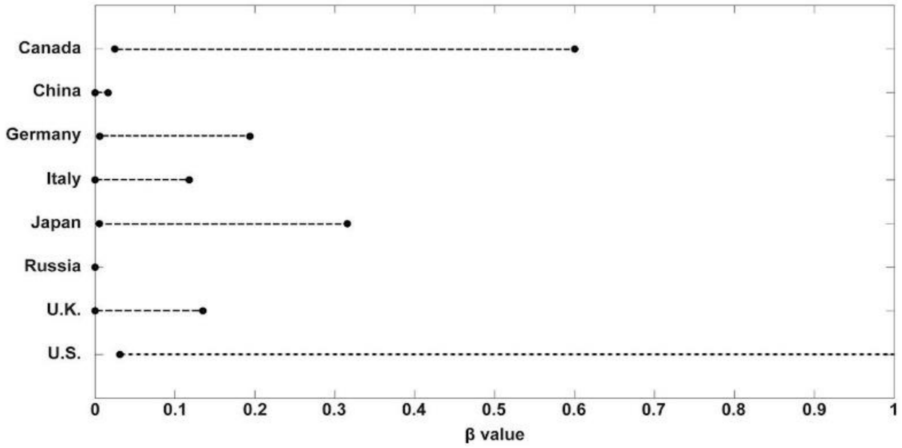
a. Emissions per unit of population



b. Energy use per unit of population



c. GDP per unit of population



Note. Although the results show that each country obtains more than its demand at the preferred β-value, consensus on acceptable β-values cannot be easily reached because a single value fails to satisfy all countries.

Table 3

Permit Allocation Using the Boltzmann Distribution and Two Conventional Allocation Criteria (Egalitarianism and Sovereignty).

Country	Egalitarianism	Def	Sovereignty	Def	Boltz1	Def	Boltz2	Def	Boltz3	Def
China	10,716	2,429	7,621	-666	7,906	-381	7,743	-544	7,729	-558
U.S.	2,478	-2,955	4,997	-436	5,069	-364	4,835	-598	4,284	-1,149
Russia	1,141	-600	1,601	-140	1,445	-295	1,463	-278	932	-809
Japan	1,021	-150	1,077	-94	985	-186	1,082	-89	1,590	419
Germany	655	-90	686	-60	628	-117	709	-36	967	221
Canada	273	-226	459	-40	430	-69	553	54	453	-46
U.K.	499	5	454	-40	430	-64	466	-28	684	190
Italy	485	78	374	-33	375	-32	417	11	629	223
Least square* $Y = \sum_{i=1}^{n} (permits_i - demand_i)^2$	15,082,107		669,512		422,793 (*β*=0.0897)		743,777 (*β*=0.1881)		2,599,009 (*β*=0.0199)	

Note. Def (Deficiency) = Allocated permits – Actual emissions.
Egalitarianism: countries receive emissions permit units proportional to their population.
Sovereignty: countries receive emissions permit units proportional to their emissions.
Boltz1: allocation preference per capita ($\hat{E}_i$) of a country *i* is defined as emissions per unit of population of country *i* in 2010.
Boltz2: allocation preference per capita ($\hat{E}_i$) of a country *i* is defined as energy use per unit of population of country *i* in 2010.
Boltz3: allocation preference per capita ($\hat{E}_i$) of a country *i* is defined as GDP per unit of population of country *i* in 2010.
*When the least square value **(***Y***)** has its minimum at a *β*-value, it can be considered a useful reference *β*-value**.**

Figure 4 shows that under egalitarianism, China receives the most permits due to its large population, making it China's preferred criterion. However, the U.S., with high emissions but a smaller population, may find egalitarianism the least favorable as it doesn't fully meet the country's emissions needs. The least squares calculation, as illustrated in Table 3 and the spider chart in Figure 4, confirms egalitarianism as the most biased method among the three examined. When comparing the Boltzmann distribution to sovereignty, the results are similar, primarily because both methods use emissions as the key input for permit allocation.

However, as shown in Table 3 and Figure 4, there is dissimilarity between sovereignty and the Boltzmann distribution. Using sovereignty, emissions permits assigned to five countries having the middle amount of CO2 emissions are slightly more numerous than the number allocated to them using the Boltzmann distribution. By contrast, when using the Boltzmann distribution, emissions permits assigned to China and the United States, which have the largest CO2 emissions, and Italy, which has the smallest CO2 emissions, are larger than those allocated to these countries under sovereignty. Comparing the Boltzmann distribution and sovereignty, when emissions permits are distributed, Italy, with the smallest CO2 emissions, can attain more benefits, i.e., more emissions permits, using the Boltzmann distribution than under sovereignty. Furthermore, the least-squares value using the Boltzmann distribution is lower than under sovereignty. The study's ultimate aim is to minimize the least squares value, and the Boltzmann distribution yields the lowest least-squares value of the three criteria. If "unbiased" means minimizing least-squares calculations and finding the optimal allocation in the carbon trading system, then the Boltzmann distribution can provide the least biased standard for assigning limited emission permits of the three allocation methods.

Utilizing the Boltzmann distribution for carbon emission permit allocation provides a tailored strategy that considers the unique environmental and economic circumstances of each country. This method assigns more responsibility to countries with larger populations and higher CO2 emissions, while offering appropriate incentives for those with lower emissions, thereby promoting fairness in international climate change efforts.

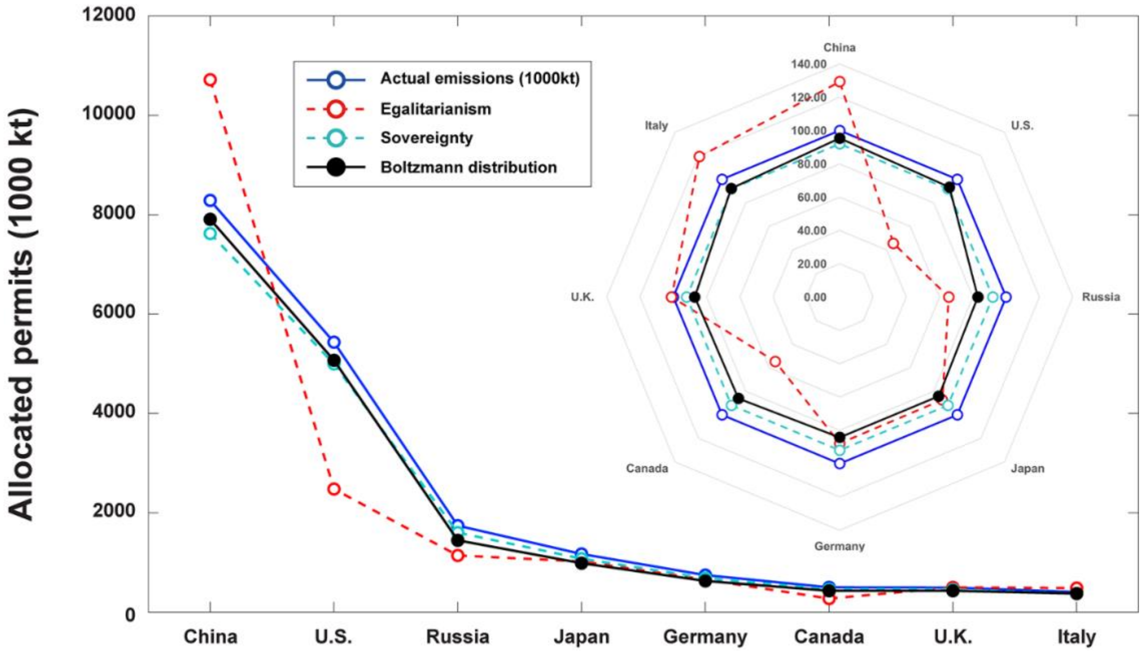
The application of the Boltzmann distribution significantly influences national environmental policies and planning. For instance, countries like China, with large populations and high emissions, receive more permits, pushing them to enhance their environmental policies and actively engage in global climate initiatives. Conversely, countries with lower emissions such as Italy can benefit from more permits under this method, enabling them to strengthen their environmental conservaton efforts.

Moreover, this method provides crucial criteria for policy-making in sustainable urban and regional planning. For example, in setting carbon emission reduction targets at city and regional levels, the Boltzmann distribution-based permit allocation can be applied. This facilitates the setting of carbon reduction goals tailored to each area's characteristics and needs, seeking effective solutions for their achievement.

This study applied the Boltzmann distribution method for allocating carbon emission rights to eight countries: Canada, China, Germany, Italy, Japan, Russia, the United Kingdom, and the United States. This method considers factors like population, economic size, and each country's CO2 emissions, reflecting the country’s current status and future carbon reduction needs. Each

Figure 4

Allocated Permits Using the Boltzmann Distribution and Two Conventional Allocation Criteria: Egalitarianism and Sovereignty.



Note. The Boltzmann distribution shows a well-proportioned permit allocation between egalitarianism and sovereignty. The spider chart shows that Canada, the U.S., and Russia receive fewer permits under egalitarianism because of their small populations, leading to a large permit deficiency between the three criteria. It is clear that allocating permits under egalitarianism is the most biased of the three studied criteria.

country can redistribute the allocated carbon emission rights as per the needs of its cities, regions, and industries. This approach aids in balancing economic growth with environmental protection, essential for both regional economic development and environmental preservation.

Advanced nations, with high industrialization and significant historical responsibility for carbon emissions, may have higher reduction targets and comparatively fewer rights. Developing nations, prioritizing economic growth and poverty reduction, can receive more per capita emission rights to balance development with environmental protection. This demonstrates the effectiveness of the Boltzmann distribution in establishing a fair and efficient global carbon market, enabling precise carbon management and reduction strategies, and playing a crucial role in achieving carbon reduction targets while balancing sustainable economic growth with environmental protection.

DISCUSSION AND CONCLUSIONS

This paper highlights the importance of CO2 emissions permit allocation in determining carbon emission reduction responsibilities at national and corporate levels. Permit allocation varies across countries, regions, and industries, with egalitarianism (allocation based on population) and sovereignty (allocation based on emissions) being the predominant methods. While countries like China, the UK, and Italy might benefit from egalitarianism due to their large populations or lower emissions, nations such as Canada, Germany, Japan, Russia, and the U.S. favor sovereignty for permit allocation.

The Kyoto Protocol states that Russia has no responsibility for reducing carbon emissions. After adopting the Paris Agreement, Russia officially agreed to a target reduction of GHG emissions to a range of 70 to 75% of 1990 levels by 2030 (Korppoo, 2022). However, Russia will need power for purposes such as home heating; therefore, some carbon will be required and consumed. A greater demand for energy may also emerge in the U.S. steel industry. Canadian steel plants could use carbon to produce a still more profitable kind of steel. Japan has emerged as a major carbon demander because of its control of the world automobile production. In addition, Germany is a major carbon demander because of its dominance of world steel production combined with automobile sales. These carbon demanders want to receive more permits; but under egalitarianism, they receive fewer permits than under sovereignty. If a permit allocation rule is too rigid, neither countries with large populations nor those with relatively high emissions can negotiate peacefully. However, if the allocation rules are simple yet versatile and flexible, it becomes easy to sit at the negotiating table and reach a unified solution to reduce CO2 emissions among multiple countries.

This paper introduces a simplified permit allocation method using the Boltzmann distribution, adaptable to varying population sizes and CO2 emissions levels across countries. The method effectively balances permits between egalitarian and sovereignty principles, suitable for international carbon trading involving multiple countries. It provides a flexible approach for international emissions rights allocation that is also potentially applicable to broader economic and environmental issues.

This study presents a theoretically robust application of the Boltzmann distribution for carbon trading permit allocation, acknowledging complexities in real-world policy implementation due to international political and economic diversity. It initiates a discussion on simplifying its implementation and integration into international environmental policies. This research highlights the Boltzmann distribution's potential as a practical policy tool for enhancing global environmental sustainability, suggesting the need for further research on its application in the complex realm of international politics and economics.

Although this study applies the Boltzmann distribution method focused on eight countries, it acknowledges the limitation of not fully capturing the entire complexity of global economic and environmental impacts. In particular, it will be crucial to include countries with varying economic standings and different levels of emissions in order to enhance the universality and adaptability of the research. Such an expanded empirical analysis will not only provide a deeper understanding of how the Boltzmann distribution method can be applied in diverse international environments, but also assist in making a substantial contribution to global environmental policy. This approach also lays the groundwork for future researchers to conduct more comprehensive studies in this area, playing a vital role in strengthening the policy implications of the research findings.

The Boltzmann distribution method for carbon allocation, while simple and fair, risks oversimplifying the complexities of international environmental politics and economics. Deeper analysis of international dynamics is required, particularly with respect to the inherent tension between developed and developing countries. Introducing variable allocation standards to reflect economic disparities, technology transfer, and financial aid for developing countries' eco-friendly transitions are essential considerations. Additionally, temporal flexibility in allocation, aligned with each country's development and environmental goals, and adjustments based on economic, technological, and policy changes are crucial for effectively integrating this method into international environmental policy.

The Boltzmann distribution method in our study is crucial for adapting to the dynamic nature of carbon trading markets, influenced by economic factors and technological advancements. It supports the main goals of climate policy, including behavioral change and technological innovation. Recognizing market fluidity, our method is designed for adaptability, allowing recalibrations in response to market and technological changes. This approach marks a shift from traditional methods to more flexible strategies, accommodating the diversity of carbon markets, and it is vital to effective climate policy outcomes. Future research should focus on further aligning the Boltzmann distribution with the evolving carbon trading landscape and climate policy goals.

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