

# The Empirical Equations for Estimating Spanwise Loading Distribution on a Straight-Trapezoidal Wing Planform Formulated by using Data Generated from Lifting-Line Theory and Least Squares Approximation Method

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## Abstract

The empirical equations for estimating a span loading distribution on a straight-trapezoidal wing planform are presented in this paper. The main concept is that the spanwise loading coefficient can be presented as a function of aspect ratio, taper ratio and spanwise position on a wingspan. The lifting-line theory is used to generate data of spanwise loading coefficient at each condition of these three parameters and the curve fitting process is applied to find the suitable equations that can represent the data. These equations can be used to determine the location of center of pressure and spanwise loading distribution on a wingspan. They can be applied in the case of a straight-trapezoidal wing planform that has no geometric and no aerodynamic twist along the spanwise direction. The flow condition is incompressible flow and the range of angle of attack is at the low to moderate which viscosity has no significant impact in the flow field. The results from the empirical equations are compared to those from Anderson's method and experimental data. The comparison shows that the results from these empirical equations agree very well with the data from both theoretical and experimental data which the detail is discussed in this paper.

**Keywords:** Aerodynamics, Straight-Trapezoidal wing, Wingspan load distribution, Lifting-line theory.

## 1. Introduction

This paper is a continuous work from the previous work of Soontornpasatch [1]. In the previous work, the rapid estimation method for calculating spanwise load distribution on a straight wing planform was proposed. The calculation task can be accomplished by using the data of the ratio between local lift coefficient to total wing lift coefficient ( $c_l/C_L$ ) that is generated by lifting-line theory. This data is arranged into a tabular form which can be used to determine spanwise loading distribution along the wingspan. The method is simply, direct forward and can be done manually. The result from this calculation method agrees very well with the results from Anderson's methods and the experimental data. However, by applying this method in some applications, some disadvantages have been found.

First, because the data of  $c_l/C_L$  is arranged in a tabular form, therefore the value of load distribution can be determined only at the selected locations on the wingspan. The value of the intermediate point has to be determined by using approximation methods such as Lagrange polynomial which causes more time and adds steps in the calculation process.

Another disadvantage is that this method is not suitable to be used as a function or subroutine in the computer program. This also comes from the fact that the data is arranged in the tabular form. Therefore, to add this into the computer program, the entire data has to be uploaded and then the approximation method such as linear interpolation has to be applied to

determine the value of load distribution which causes an inconvenience and increases the complexity in the programming process.

One of the solutions to cope with these issues is to represent the spanwise loading distribution into an empirical equation form instead of a tabular form. The first advantage of doing so is that the empirical equation form can be applied to determine load distribution at any point along the wingspan, not just at the selected location. Therefore, the constraint about the location that is mentioned earlier is eliminated.

Another advantage is that it is more easier to apply the empirical formula to create a subroutine in a computer program which largely reduces the complexity in the programming process. In this case, only two wing parameters, aspect ratio and taper, are used as an input data. Then, load distribution can be determined directly by using a set of equations. Therefore, the complex calculation process such as solving system of linear equations process, which is used for lifting-line theory, is not needed. This will be very useful when this subroutine is embedded into the larger program such as the program in optimization process for small UAV design similar to those in the work of Karali et al [2].

Moreover, the empirical equation does not need iterative calculation process. Therefore, it can give results faster than other complex methods such as panel method or computational fluid dynamics program. This is a huge advantage in case of UAV preliminary design and optimization process.

However, it should be noted that the empirical equation also has some limitations because it is based on inviscid incompressible flow assumption. Therefore, it can be applied only in the case that viscosity does not dominate the flow field such as at low to moderate angle of attack.

Finally, the empirical equation can easily be applied in the spreadsheet software and can also be applied for manual calculation process. By considering all of these advantages, this presented work was established.

The main objective of this work is to formulate empirical equations that can represent the spanwise loading distribution of a straight-trapezoidal wing planform that has no geometric and no aerodynamic twist along the spanwise direction. The range of aspect ratio is 4 to 12 and the range of taper ratio is 0.1 to 1.0. The flow condition that is considered in this work is incompressible flow. Because the main assumption of the lifting-line theory that is used to generate loading distribution data is inviscid flow. Therefore, these empirical equations can be used at the low to moderate angle of attack which viscosity has no significant impact in the flow field. An example for the implementation of these equations is provided in this work. The prediction results from this presented work are compared to the results from Anderson's method and also experimental data which the detail is discussed in this work.

## 2. Theoretical estimation of Spanwise loading distribution

Spanwise loading distribution is an important data, especially in the case of wing structural testing process for airframe certification. In this process, the loading distribution along a wingspan and the total lift force must be determined. Then, the lift force at each specific location on the spanwise direction can be found and can be applied in the testing process. The example of this testing process is shown in **Figure 1**. In this case, the port side of the seaplane wing is tested to prove that the wing structure can withstand the maximum load that is expected to occur during flight. The result of this testing process was published in work of Chinvorarat [3].

There are many theoretical methods that can be used to determine spanwise loading distribution. Most of the earlier works were developed based on lifting-line theory. In this case, the distribution of circulation along a wingspan is represented by Fourier series as stated in the work of Glauert [4]. Then, it is combined with induced velocity equation and theoretical lift coefficient equation to construct the equation that is called monoplane equation. When this monoplane equation is solved, the loading distribution can be determined. Many works used this concept to develop calculation methods that are more suitable to be applied for engineering applications such as in the work of Anderson [5] and Sherman [6].



**Figure 1** Shows wing structural testing process

The more sophisticated methods were also developed such as the method in the work of Weissinger [7] and Mutterperl [8] that used modified lifting-line theory to calculate spanwise lift distribution, the work of Falkner [9] and Multhopp [10] that used lifting-surface theory to calculate load distribution on a wingspan. The main advantage of these methods is that they can be applied for a swept-wing planform. The methods from Weissinger, Mutterperl and Falkner were all further evaluated in the work of Dorn and Young [11] by comparing the results from these three methods with the experimental data. The result from this work shows that the Weissinger's method is the best approach of these three methods for spanwise loading calculation and other wing aerodynamic parameters. The method in the work of Weissinger was also used to develop a less complicated calculation process that is more suitable for engineering applications such as in the work of Stevens [12] and the work of Young and Harper [13].

Although these methods were developed many decades ago, they are still being used in aerospace industry in the present day. The methods in the work of Anderson, Sherman, Weissinger, Young and Harper are in the FAA (Federal Aviation Administration, U.S. Department of Transportation) list of the suggestion methods for calculating spanwise loading distribution for airframe certification process [14].

However, the main disadvantage of these methods is that they are difficult to use. Many calculation steps have to be taken and a lot of charts and tables have to be used in the process. In some cases, the iterative calculation process is required to find a solution.

There are some efforts that try to cope with this issue such as in the work of Soontopasatch, which is already mentioned in the previous section. In the work of Schrenk [15] which developed a simple approximation method for spanwise loading distribution on a straight wing planform. This approximation method is easy to apply and can be used in an application that does not require high

accuracy result. The application of Schrenk's method can also be found in recently work of Chinvorarat [16]. Another interesting work is the work of Jones [17] which is also based on Weissinger's method. In this work, the spanwise loading distribution of a spanwise direction can be estimated by using the empirical formulas. They can be applied for both straight and swept wing planform. However, as suggested in the paper, the range of aspect ratio is limited from 1.5 to 8.0 only.

All of the works that are mentioned in this section have both advantages and disadvantages. Therefore, this presented work is a continuous effort that tries to improve the spanwise loading calculation method that is easy to use and can be applied in a practical situation.

### 3. Theory

#### 3.1 Wing Geometric parameters

A basic geometry of a straight-trapezoidal wing planform can be shown in **Figure 2**. In this figure, the top view of wing right-hand side is shown. The taper ratio ( $\lambda$ ) of this wing can be found by using Eq. (1). The variable  $c_t$  and  $c_r$  are the chord length (m) at the wing tip and wing root, respectively.

$$\lambda = c_t/c_r \quad (1)$$

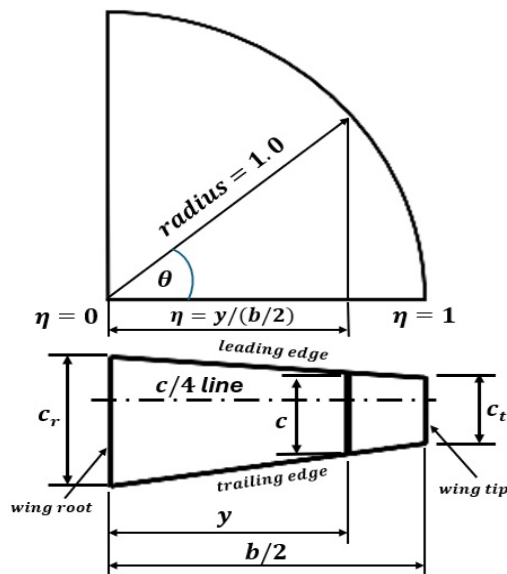


**Figure 2** Basic straight-trapezoidal wing planform

The total wing area ( $S$ , m<sup>2</sup>) can be determined by using Eq. (2), which  $b$  is the length of wingspan (m). The wing aspect ratio (AR) can be calculated by using Eq. (3).

$$S = [(1 + \lambda) c_r b]/2 \quad (2)$$

$$AR = b^2/S \quad (3)$$

The spanwise position ( $y$ , m) is measured from the wing root as shown in **Figure 2**. The non-dimensional

form of this variable can be written in the form as shown in Eq. (4).

$$\eta = \frac{y}{(b/2)} = \cos\theta \quad (4)$$

The local chord length ( $c$ , m) at each position can be determined by using Eq. (5) and the wing average chord ( $\bar{c}$ , m) can be found by using Eq. (6).

$$c = c_r[1 - \eta(1 - \lambda)] \quad (5)$$

$$\bar{c} = S/b \quad (6)$$

#### 3.2 Aerodynamic parameters

The local lift coefficient ( $c_l$ ) distributed along the wingspan can be described as shown in Eq. (7). The variable  $l'$  is lift force per unit length (N/m),  $\rho_\infty$  is air density (kg/m<sup>3</sup>),  $c$  is a local chord length (m) and  $U_\infty$  is free-stream velocity of air (m/s).

$$c_l = \frac{l'}{\frac{1}{2}\rho_\infty U_\infty^2 c} \quad (7)$$

The total lift force coefficient ( $C_L$ ) for the entire wing can be written as shown in Eq. (8) which  $L$  is wing total lift force (N) and  $S$  is total wing area (m<sup>2</sup>).

$$C_L = \frac{L}{\frac{1}{2}\rho_\infty U_\infty^2 S} \quad (8)$$

Another important non-dimensional parameter that is usually used for wingspan loading analysis is spanwise loading coefficient ( $K$ ). This parameter is used in many literatures that are previously mentioned. It can be expressed as shown in Eq. (9).

$$K = \frac{c_l c}{C_L \bar{c}} \quad (9)$$

#### 3.3 Lifting-Line Theory

The lifting-line theory is an important tool for analysis aerodynamic parameters of a straight-wing planform since it was invented in the last century. The main concept is that a straight-wing planform can be represented by using a horse-shoe vortex at a quarter-chord ( $c/4$ ) position. The strength of this horse-shoe vortex or circulation ( $\Gamma$ ) generates lift force. Because the value of vortex strength varies along a wingspan, therefore, an infinite number of horse-shoe vortex is used to represent this effect. For a general lift distribution, the value of  $\Gamma$  can be expressed in the form of Fourier series as stated in the work of Glauert. This can be described as shown in Eq. (10).

$$\Gamma = 2bU_\infty \sum_{n=1}^N A_n \sin n\theta \quad (10)$$

The Eq. (10) can be combined with the induced velocity equation and airfoil lift coefficient equation

[18] to create monoplane equation which can be shown in Eq. (11).

$$\sum_{n=1}^N A_n \sin n\theta (\mu n + \sin \theta) = \mu(\alpha - \alpha_{L=0}) \sin \theta \quad (11)$$

The variable  $\alpha$  and  $\alpha_{L=0}$  are geometric angle of attack and zero-lift angle of attack (rad) of wing cross section. The variable  $\mu$  is a product of local chord length, lift-slop value and the length of wingspan which can be described as shown in Eq. (12). The variable  $a_0$  in Eq. (12) is the lift slope of wing cross section (1/rad) which its theoretical value is  $2\pi$ .

$$\mu = \frac{ca_0}{4b} \quad (12)$$

The monoplane equation can be solved by selecting the positions along a wingspan which value of  $\theta$  at each position can be calculated by using Eq. (4). Then, the geometric and aerodynamic parameters at each position such as local chord length, zero-lift angle of attack are used to create a system of linear equation which can be solved to determine coefficient  $A_n$ . When the value of  $A_n$  is known, it can be used to determine local lift per unit length ( $l'$ ) by using Eq. (13).

$$l' = (2b\rho_\infty U_\infty^2) \sum_{n=1}^N A_n \sin n\theta \quad (13)$$

The local lift coefficient along the wingspan can be calculated by substitute Eq. (13) into Eq. (7). Then, it can be rewritten in the form as shown in Eq. (14). The total wing lift coefficient can be found by using the value of  $A_1$  and AR as described in Eq. (15)

$$c_l = \frac{4b}{c} \sum_{n=1}^N A_n \sin n\theta \quad (14)$$

$$C_L = \pi A_1 AR \quad (15)$$

By using Eq. (5), Eq. (6), Eq. (14) and Eq. (15), the parameters  $c$ ,  $\bar{c}$ ,  $c_l$  and  $C_L$  are known. Then, these parameters are substituted into Eq. (9) and the spanwise loading coefficient distributed along a wingspan can be determined.

### 3.4 Center of Pressure

The general pattern of lift per unit length distribution along a semi-wingspan can be shown in **Figure 3**. The maximum value occurs at the wing root and then decreases toward wing tip area. The value of total lift force for a semi-wingspan ( $L_s$ , N) is half of the total lift force of the entire wing. Therefore, by using Eq. (8),  $L_s$  can be determined by using Eq. (16).

$$L_s = \frac{1}{4} \rho_\infty U_\infty^2 S C_L \quad (16)$$

The center of pressure ( $\bar{\eta}$ ) is the location of  $L_s$  on the semi-wingspan as shown in **Figure 3**. It can be

determined by integrating moment around the wing root axis which can be written as shown in Eq. (17).

$$\bar{y} = \frac{1}{L_s} \int_{y=0}^{y=b/2} y l' dy \quad (17)$$

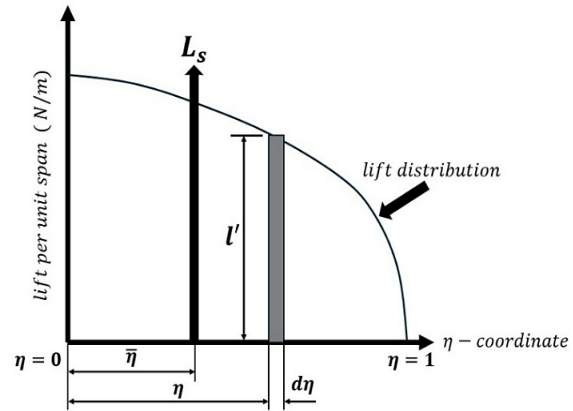


**Figure 3** General pattern of lift per unit length distribution on a semi-wingspan

From Eq. (4), the value of  $y$  can be transformed to  $\eta$  by using the relation in Eq. (18).

$$dy = (b/2) d\eta \quad (18)$$

When substitute Eq. (18) into Eq. (17), it becomes Eq. (19).

$$\bar{\eta} = \int_{\eta=0}^{\eta=1} \eta \left( \frac{b l'}{2 L_s} \right) d\eta \quad (19)$$

To formulate the relation between  $\bar{\eta}$  and  $K$ , Eq. (9) can be rearranged by substituting it with Eq. (6), Eq. (7) and Eq. (8). Then, the Eq. (9) is transformed into Eq. (20).

$$K = \frac{c_l c}{C_L \bar{c}} = \frac{[l' / (\frac{1}{2} \rho_\infty U_\infty^2 c)] c}{[L / (\frac{1}{2} \rho_\infty U_\infty^2 S)] (S/b)} = \frac{b l'}{L} \quad (20)$$

Because the value of total wing lift force  $L$  is twice the total lift force  $L_s$  on a semi-wingspan. Then, Eq. (20) becomes Eq. (21).

$$K = \frac{b l'}{2 L_s} \quad (21)$$

By substituting Eq. (21) into Eq. (19), it can be transformed into Eq. (22).

$$\bar{\eta} = \int_{\eta=0}^{\eta=1} K \eta d\eta \quad (22)$$

The solution of Eq. (22) can be determined by using numerical integration. In this case, Gaussian Quadrature method is chosen. Then, Eq. (22) can be

transformed into Eq. (23) which  $W_j$  is weighting factor.

$$\bar{\eta} = \frac{1}{2} \sum_{j=1}^J W_j (K\eta)_j \quad (23)$$

Because Gaussian Quadrature method performs integration by using a set of collocation points from -1 to 1 on a  $\xi$  space [19]. Therefore, the transformation between  $\xi$  space and  $\eta$  space can be performed by using Eq. (24).

$$\eta = \frac{\xi + 1}{2} \quad (24)$$

The four points in the domain are chosen for this integration process. The value of  $\xi$ ,  $W_j$ ,  $\eta$  and  $\theta$  for all four points are shown in **Table 1**.

**Table 1** Parameters for Gaussian integration

| $j$ | $\xi$    | $W_j$   | $\eta$  | $\theta$<br>(rad) |
|-----|----------|---------|---------|-------------------|
| 1   | -0.86114 | 0.34785 | 0.06943 | 1.50131           |
| 2   | -0.33998 | 0.65215 | 0.33001 | 1.23448           |
| 3   | 0.33998  | 0.65214 | 0.66999 | 0.83660           |
| 4   | 0.86114  | 0.34785 | 0.93057 | 0.37483           |

The calculation process for Eq. (23) is straight forward. When the monoplane equation or Eq. (11) is solved, the value of  $A_n$  is known. Then, the value of  $c_l$  for each  $\theta$  in the **Table 1** can be determined by Eq. (14). With the Eq. (5), Eq. (6) and Eq. (15), the value of  $K$  at each  $\theta$  can be calculated. Then, by using  $\eta$  and  $W_j$  with  $K$ , the value of  $\bar{\eta}$  can be determined.

## 4. Empirical equations from data generated by using lifting-line theory

### 4.1 Concept

The data in many literatures that are previously mentioned in the section 2 shows that the spanwise loading coefficient ( $K$ ) depends on the spanwise position, taper ratio, aspect ratio and sweep angle at  $c/4$  line ( $\Lambda_{c/4}$ ) only. By using this fact, it can be assumed that the value of  $K$  can be represented as a function of  $\eta$ ,  $\lambda$ , AR and  $\Lambda_{c/4}$ . Because this presented work considers only the case of straight wing planform. Then,  $\Lambda_{c/4}$  is zero and can be neglected. Therefore, the relation of  $K$  and other variables can be written as shown in Eq. (25).

$$K = f(\eta, \lambda, AR) \quad (25)$$

However, the data from the work of Jones also shows that the center of pressure location  $\bar{\eta}$  on a swept-wing planform can be presented as a function of  $\lambda$  and AR. Therefore, this also be assumed to be the same for a straight-wing planform. This relation can be represented as shown in Eq. (26).

$$\bar{\eta} = f(\lambda, AR) \quad (26)$$

Therefore, Eq. (25) can be rewritten in the form as shown in Eq. (27).

$$K = f(\eta, \bar{\eta}) \quad (27)$$

By considering Eq. (26) and Eq. (27), this indicates that the calculation process to determine spanwise loading coefficient can be divided into two steps. First, the value of  $\bar{\eta}$  must be determined by using  $\lambda$  and AR as the input parameters. Then, it is used to calculate loading coefficient  $K$  at each location of  $\eta$  along a wingspan. Therefore, the empirical equations for Eq. (26) and Eq. (27) are formulated based on this calculation process. This can be achieved by using a monoplane equation to calculate  $\bar{\eta}$  and distribution of  $K$  for each  $\lambda$  and AR condition. Then, the results are arranged into the sets of discrete data and the curve fitting method is applied to find the suitable approximating equations that can represent them. In this presented work, the least squares approximation is chosen for curve fitting process.

Although the empirical equation for calculation of  $\bar{\eta}$  from the work of Jones can be applied for a straight wing planform by setting  $\Lambda_{c/4} = 0$ . This equation is not used in this presented work. This comes from the limitation of this equation which can be used for aspect ratio up to 8 as suggested in the paper. However, in this presented work, the highest aspect ratio is 12. Therefore, the new equation for  $\bar{\eta}$  is formulated instead.

### 4.2 Equations for Estimating Center of Pressure Location

The data of  $\bar{\eta}$  for a straight-trapezoidal wing planform at  $\lambda$  from 0.1 to 1.0 and AR from 4 to 12 is shown in **Table 2**. These values are generated by solving a monoplane equation or Eq. (11). The data in the **Table 2** is used to create a graph in **Figure 4** that shows the relation between  $\bar{\eta}$ ,  $\lambda$  and AR. However, to avoid congestion in this graph, only  $\lambda = 0.1, 0.2, 0.3, 0.5, 0.7, 0.9$  and 1.0 are plotted.

**Table 2** Shows the value of  $\bar{\eta}$  at each  $\lambda$  and AR from lifting-line

| AR \ $\lambda$ | 4      | 6      | 8      | 10     | 12     |
|----------------|--------|--------|--------|--------|--------|
| 0.1            | 0.4010 | 0.3949 | 0.3906 | 0.3873 | 0.3848 |
| 0.2            | 0.4127 | 0.4092 | 0.4066 | 0.4047 | 0.4031 |
| 0.3            | 0.4209 | 0.4195 | 0.4185 | 0.4177 | 0.4171 |
| 0.4            | 0.4273 | 0.4277 | 0.4280 | 0.4282 | 0.4285 |
| 0.5            | 0.4307 | 0.4345 | 0.4359 | 0.4370 | 0.4380 |
| 0.6            | 0.4372 | 0.4403 | 0.4427 | 0.4446 | 0.4461 |
| 0.7            | 0.4412 | 0.4454 | 0.4486 | 0.4512 | 0.4533 |
| 0.8            | 0.4448 | 0.4500 | 0.4539 | 0.4571 | 0.4597 |
| 0.9            | 0.4480 | 0.4541 | 0.4587 | 0.4624 | 0.4654 |
| 1.0            | 0.4510 | 0.4579 | 0.4631 | 0.4672 | 0.4706 |

Instead of using direct value of AR, it was found that using  $\ln(AR)$  in the x-axis can make the graph in a linear pattern as shown in **Figure 4**. Based on this pattern, the relation between  $\bar{\eta}$ ,  $\lambda$  and AR can be represented by using linear equation form as shown in Eq. (28) which  $a_0$  is y-axis intercept and  $a_1$  is slope.

$$\bar{\eta} = a_0 + a_1 \ln(AR) \quad (28)$$

To find the coefficient  $a_0$  and  $a_1$ , the least squares approximation method is applied to the data in **Table 2**. The value of  $a_0$  and  $a_1$  for each  $\lambda$  condition can be found as shown in **Table 3**.

From **Table 3**, it was found that the value of  $a_0$  varies in a short range between 0.4215 and 0.4275 as shown in **Figure 5**. The value of  $a_0$  changes rapidly at  $\lambda$  from 0.1 to 0.4 and this value is nearly at constant level from  $\lambda = 0.4$  to 0.9. Then, the value decreases at  $\lambda$  from 0.9 to 1.0. By considering this pattern, it was found that it is very difficult to represent the data with an algebraic polynomial equation. Moreover, the deviation between minimum and maximum value through the entire range of considered  $\lambda$  is only 1.423 percent. Therefore, it was judged that it is better to define  $a_0$  by using the average value in the **Table 3** and its value is shown in Eq. (29).

$$a_0 \approx 0.4253 \quad (29)$$

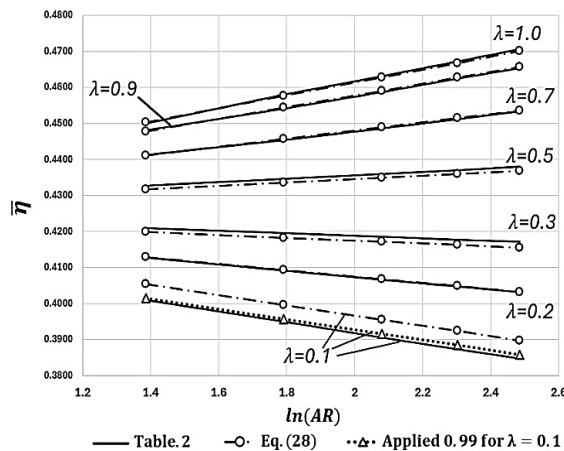


**Figure 4** Shows the relation between  $\bar{\eta}$ ,  $\lambda$  and AR

The data in **Table 3** also shows that at  $\lambda$  from 0.1 to 0.3, the slope  $a_1$  is negative which indicates that the center of pressure location moves toward wing root position when aspect ratio increases. In contrast, at  $\lambda$  from 0.4 to 1.0, the slope  $a_1$  is positive which indicates that the center of pressure moves toward the wingtip when aspect ratio increases. These trends also can be seen in **Figure 4**.

The algebraic equation for  $a_1$  can also be found by using least squares method. However, by plotting the graph between  $\lambda$  and  $a_1$ , it was found that the pattern becomes a curved line pattern as shown in **Figure 6**. Therefore, the quadratic polynomial form is chosen in this case. The equation for  $a_1$  can be described as shown in Eq. (30).

$$a_1 = -0.0202 + 0.0611\lambda - 0.0229\lambda^2 \quad (30)$$

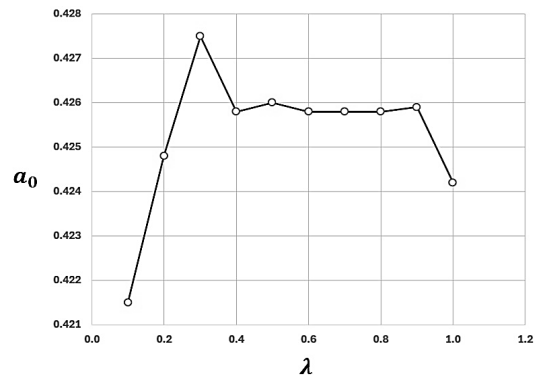


**Figure 5** Shows the relation between  $\lambda$  and  $a_0$

**Table 3** Shows the value of  $a_0$  and  $a_1$  for each  $\lambda$

| $\lambda$ | $a_0$  | $a_1$   |
|-----------|--------|---------|
| 0.1       | 0.4215 | -0.0148 |
| 0.2       | 0.4248 | -0.0088 |
| 0.3       | 0.4275 | -0.0035 |
| 0.4       | 0.4258 | 0.0011  |
| 0.5       | 0.4260 | 0.0048  |
| 0.6       | 0.4258 | 0.0081  |
| 0.7       | 0.4258 | 0.0110  |
| 0.8       | 0.4258 | 0.0136  |
| 0.9       | 0.4259 | 0.0159  |
| 1.0       | 0.4242 | 0.0187  |

The result of Eq. (30) agrees very well with the data in the **Table 3** as shown in **Figure 6**. The prediction from Eq. (28) with includes both Eq. (29) and Eq. (30) is also shown in the **Figure 4**. The results at every  $\lambda$  from 0.2 to 1.0 agree very well with the data in **Table 2** and the error from the prediction is less than 1 percent in every case.

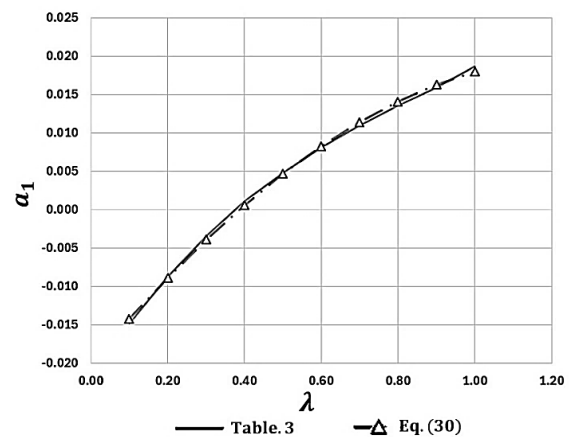


**Figure 6** The relation between  $\lambda$  and  $a_1$

However, at  $\lambda = 0.1$ , the Eq. (28) yields the overestimate value at all range of  $\ln(AR)$  which is noticeable in the graph. This is caused by using average value of  $a_0$  which is higher than exact value of  $a_0$  at condition  $\lambda = 0.1$ . In this case, it was found that by multiplying 0.99 to the result from Eq. (28), the

error can be reduced to less than 1 percent and the data is more aligned with the data from **Table 2** as shown at the bottom of the graph. However, it should be emphasized that this approach is applied for the condition of  $\lambda = 0.1$  only.

### 4.3 Equations for Spanwise Loading Coefficient

#### 4.3.1 Relation between $K$ , $\eta$ and $\bar{\eta}$

When the value of  $K$  and  $\bar{\eta}$  for each  $\lambda$  and AR are calculated, the graph that shows the relation between  $K$ , and  $\bar{\eta}$  at each  $\eta$  can be created. The trend of this data can be shown in **Figure 7**.

The graph in **Figure 7** can be divided into two parts. In the first part, the value of  $K$  varies with  $\bar{\eta}$  in a linear pattern which occurs in the range of  $\eta$  from 0 to 0.9. In this case, the slope of each line varies with  $\eta$ . At each  $\eta$ , there is a small shift in y-axis direction of the line which is caused by the changing of AR. In the **Figure 7**, the solid line represents the condition at  $AR=12$  and the dot line represents the condition at  $AR=4$ . For all cases, the solid line always be the longest line and the dot line always be the shortest. The chain line represents the other AR conditions that are placed between 4 to 12.

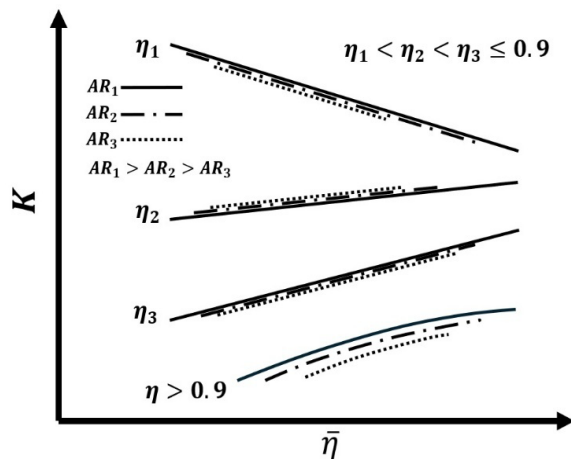


**Figure 7** Shows the overall trend of the relation between  $K$ ,  $\eta$  and  $\bar{\eta}$

By evaluating through the entire data, it was found that the magnitude of the shift by AR is small, which is in the range of about 3 to 5 percent.

By considering this trend in the graph, it is judged that the relation between  $K$  and  $\bar{\eta}$  in the range of  $\eta$  from 0 to 0.9 can be represented by using linear equation form as shown in Eq. (31). In this case,  $b_0$  is y-intercept and  $b_1$  is slope of the graph.

$$K = b_0 + b_1 \bar{\eta} \quad (31)$$

Because both coefficient  $b_0$  and  $b_1$  varies with  $\eta$ , therefore they can be found by arranging the data in a form similar to those in **Table 3**. Then, the least squares approximation method is applied to formulate the relation between these three variables. However, because the value of  $b_0$  and  $b_1$  for each  $\eta$  has a small shift caused by AR. Therefore, the average value of  $b_0$

and  $b_1$  for each  $\eta$  are used in the curved fitting process.

For second part of the graph, the relation between  $K$  and  $\bar{\eta}$  can be represented by a curved line as shown at the bottom of **Figure 7**. This occurs when the value of  $\eta$  is more than 0.9. The shift caused by AR also occurs in this range. However, the magnitude of the shift in this case is higher when compared to those at  $\eta < 0.9$ . It was found that this can be as high as 12.711 percent.

By using Eq. (31) to represent the data in this range, it was found that this approach can lead to average error about 15 percent. Therefore, it was judged that the relation between  $K$  and  $\bar{\eta}$  at  $\eta > 0.9$  should be represented in the quadratic equation form instead, as shown in Eq. (32). Because the shift caused by AR is more severe in this range, using average value for  $b_2$ ,  $b_3$  and  $b_4$  can lead to higher error in prediction result. Therefore, the weighing function ( $W$ ) that represents the effect of the shifting from AR is used in this case, which is also shown in Eq. (32).

$$K = W(b_2 + b_3 \bar{\eta} + b_4 \bar{\eta}^2) \quad (32)$$

#### 4.3.2 Equations for $0.0 \leq \eta \leq 0.9$

At each  $\eta$ , the value of y-intercept ( $b_0$ ) and slope ( $b_1$ ) for each AR condition can be found by applying least squares approximation method to the data of  $K$  and  $\bar{\eta}$ . When  $b_0$  and  $b_1$  at each AR are known, the average value of both variables can be found. This process is applied to the every  $\eta$  from 0.0 to 0.9. The average value of  $b_0$  and  $b_1$  for each  $\eta$  can be shown in **Table 4**.

**Table 4** The average value of  $b_0$  and  $b_1$  for each  $\eta$

| $\eta$ | $b_0$   | $b_1$   |
|--------|---------|---------|
| 0.0    | 3.6042  | -5.3368 |
| 0.2    | 2.9199  | -3.8877 |
| 0.4    | 1.7128  | -1.3397 |
| 0.6    | 0.2340  | 1.7407  |
| 0.8    | -1.1149 | 4.3893  |
| 0.9    | -1.4113 | 4.6994  |

From **Table 4**, the relation of  $b_0$  and  $b_1$  to  $\eta$  can be formulated by applying least square method. In this case, it was found that the quartic polynomial equation is suitable for representing the data. The equation for  $b_0$  and  $b_1$  are shown in Eq. (33) and Eq. (34).

$$b_0 = 3.6057 - 2.4539\eta - 3.7837\eta^2 - 8.4094\eta^3 + 9.7256\eta^4 \quad (33)$$

$$b_1 = -5.3410 + 5.6002\eta + 4.9351\eta^2 + 24.9249\eta^3 - 26.1423\eta^4 \quad (34)$$

From **Table 4**, from position  $\eta = 0.0$  to 0.4, the slope of the graph  $b_1$  is negative which indicates that loading coefficient decreases when  $\bar{\eta}$  increases. On the other hand, at position  $\eta \geq 0.6$ , the slope of the graph is positive which shows the increasing of

loading coefficient when  $\bar{\eta}$  increases. This trend can also be seen in **Figure 7**.

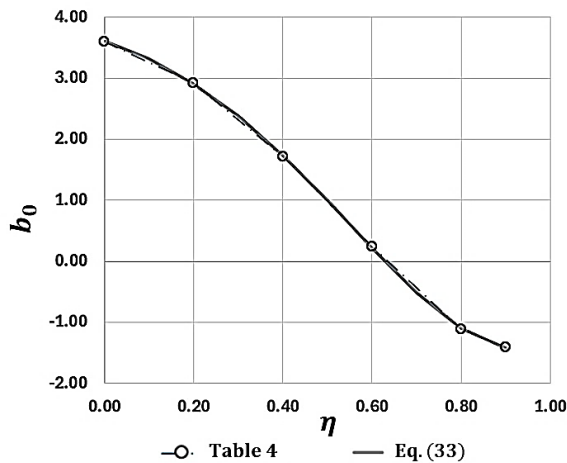


**Figure 8** Comparison of  $b_0$  from Eq. (33) and data from **Table 4**.

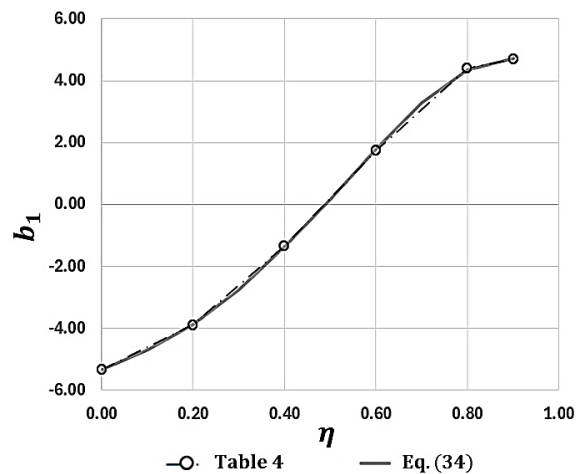


**Figure 9** Comparison of  $b_1$  from Eq. (34) and data from **Table 4**.

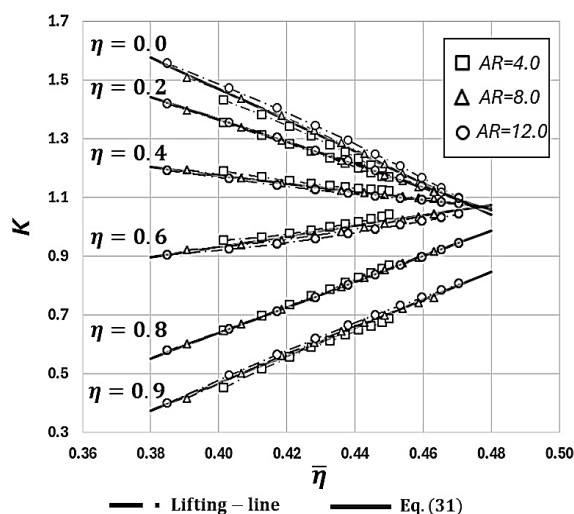


**Figure 10** Comparison of  $K$  from empirical equations and data from lifting-line theory.

The comparison between the results of Eq. (33) and Eq. (34) and the data in **Table 4** are shown in **Figure 8** and **Figure 9**, respectively. These two figures show that the results from both equations agree very well with the data. In this case, the average error for  $b_0$  and  $b_1$  from prediction are 1.828 and 1.359 percent, respectively.

The prediction of loading coefficient  $K$  by using Eq. (31) that includes Eq. (33) and Eq. (34) is shown in **Figure 10**. When compared with the data generated from lifting-line theory, the result from the Eq. (31) agrees very well. Through the entire range of the data, the average error is 0.9158 percent.

#### 4.3.3 Equations for $\eta \geq 0.9$ at condition AR=12

The trend of the graph for this range is a curved pattern as mentioned earlier. Therefore, the quadratic form is used in this case. The least square approximation method is also used to determine  $b_2$ ,  $b_3$  and  $b_4$  for each  $\eta$ . The value of  $\eta$  are 0.9, 0.925, 0.95 and 0.975. Then, these coefficients are arranged into a tabular form similar to those in the **Table 4**. Then, the relation of  $b_0$ ,  $b_1$ ,  $b_2$  to  $\eta$  can be formulated.

However, as previously mentioned, the shift of the graph caused by changing of AR is more severe than those in the case that  $\eta < 0.9$ . Therefore, only the equation for the condition that AR=12 is formulated in this way. To determine the value of  $K$  for other AR condition, the weighing function  $W$  is used for this purpose. The process to determine  $W$  will be separately described in the next section.

By using least squares method, the value of  $b_2$ ,  $b_3$  and  $b_4$  at each  $\eta$  can be shown in **Table 5**. Then, by applying the least square method again to the data in the **Table 5**, the relation of  $b_2$ ,  $b_3$ ,  $b_4$  to  $\eta$  can be formulated as shown in Eq. (35), Eq. (36) and Eq. (37), respectively. In this case, it was found that the quadratic form is suitable to represent the data.

**Table 5** Shows the value of  $b_2$ ,  $b_3$  and  $b_4$  for each  $\eta$

| $\eta$       | $b_2$   | $b_3$   | $b_4$    |
|--------------|---------|---------|----------|
| <b>0.900</b> | -2.7040 | 10.7892 | -7.0776  |
| <b>0.925</b> | -3.1738 | 12.9106 | -9.7514  |
| <b>0.950</b> | -3.4295 | 14.2164 | -11.7799 |
| <b>0.975</b> | -3.1667 | 13.4367 | -11.9780 |

$$b_2 = 260.35 - 555.98\eta + 293.02\eta^2 \quad (35)$$

$$b_3 = -1040.87 + 2212.84\eta - 1160.45\eta^2 \quad (36)$$

$$b_4 = 922.17 - 1923.68\eta + 990.27\eta^2 \quad (37)$$

By using Eq. (32), Eq. (35), Eq. (36) and Eq. (37), the value of loading coefficient  $K$  at  $\eta \geq 0.9$  for AR=12 condition can be determined. In this case, the weighing function  $W$  is set to be 1.0.

The comparison between the results from Eq. (32) for AR=12 and the data generated from lifting-line is shown in **Figure 11**. This graph shows that the result from prediction agrees very well with the data from lifting-line theory at all position for  $\eta \geq 0.9$ . The average error in this case is 0.4721 percent.

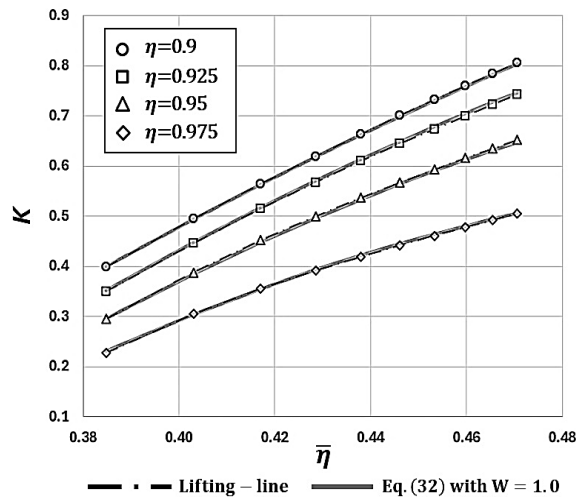


**Figure 11** Comparison of  $K$  from equations and data generated from lifting-line theory at wing tip area for  $AR = 12$ .

#### 4.3.4 Weighting function

The value of weighting function  $W$  in Eq. (32) is determined by dividing the value of loading coefficient, calculated from the lifting-line ( $K_{LF}$ ), with the value of loading coefficient at  $AR=12$  that is calculated by using Eq. (32). This can be written in the form of Eq. (38). When all the values of  $W$  are determined, they are used in the curved fitting process. However, it was found that the value of  $W$  also slightly varies with  $\bar{\eta}$ . Therefore, the value of  $W$  at each  $\eta$  and  $AR$  that is used in this curved fitting process is the average value. The result of  $W$  from Eq. (38) is shown in **Table 6**.

$$W = \frac{K_{LF}}{(b_2 + b_3\bar{\eta} + b_4\bar{\eta}^2)_{AR=12}} \quad (38)$$

From **Table 6**, the relation between  $W$  and  $AR$  at each  $\eta$  can be shown in **Figure 12**. In this case, it was found that, by using the value of  $\ln(AR)$  and  $\ln(W)$  instead of using the direct value of these variables, this can make the graph in a slightly linear pattern. Therefore, the equation for  $W$  is formulated by using  $\ln(AR)$  and  $\ln(W)$  as the main variables which can be shown in Eq. (39).

$$\ln(W) = c_0 + c_1 \ln(AR) \quad (38)$$

**Table 6** The average  $W$  from Eq. (38) at each  $\eta$  and  $AR$

| $\eta$<br>AR | 0.900  | 0.925  | 0.950  | 0.975  |
|--------------|--------|--------|--------|--------|
| 4            | 0.9544 | 0.9217 | 0.8987 | 0.8432 |
| 6            | 0.9698 | 0.9446 | 0.9317 | 0.8877 |
| 8            | 0.9825 | 0.9639 | 0.9699 | 0.9278 |
| 10           | 0.9929 | 0.9800 | 0.9841 | 0.9633 |
| 12           | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

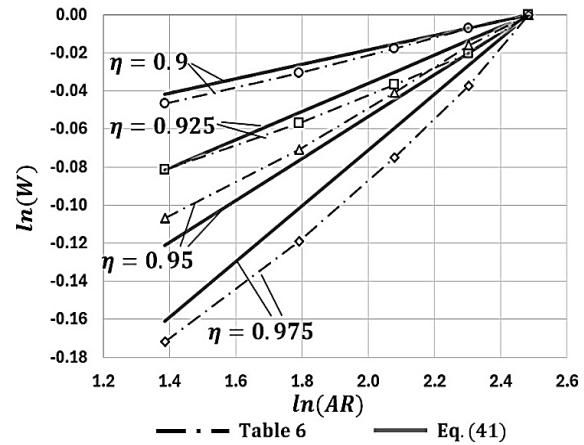


**Figure 12** Shows relation between  $\ln(AR)$  and  $\ln(W)$

From **Figure 12**, it clearly shows that every line of data intersects the x-axis when the value of  $AR=12$  which  $\ln(AR) = 2.4849$ . Therefore, the terms in Eq. (39) can be reduced by replacing  $\ln(AR)$  with  $\ln(AR) - 2.4849$  instead. By doing so, the y-intercept ( $c_0$ ) becomes zero and Eq. (39) can be rewritten as shown in Eq. (40).

$$\ln(W) = c_1 [\ln(AR) - 2.4849] \quad (40)$$

In this form, only the slope of the graph ( $c_1$ ) is needed to be determined. Then, the weighting function can be written as shown in Eq. (41).

$$W = e^{c_1 [\ln(AR) - 2.4849]} \quad (41)$$

The relation of  $c_1$  to  $\eta$  is found by applying least square method to the data from **Table 6**. This equation can be shown in Eq. (42). The comparison between the data from **Table 6** and the prediction result from Eq. (41), which using  $c_1$  from Eq. (42), can also be shown in **Figure 12**.

$$c_1 = -1.2633 + 1.446\eta \quad (42)$$

The comparison of loading coefficient  $K$  from Eq. (32), which applies weighting function from Eq. (41), with the data from lifting-line for  $\eta$  at 0.925, 0.95 and 0.975 is shown in **Figure 13**. The average error between them is 0.7697 percent.

Although the prediction of weighting function is slightly difference from the desired value which can be noticeable in **Figure 12**, this has no significant impact for the calculation for value of  $K$ . This comes from the fact that this deviation is only in the second digits of number such as the one at  $\eta = 0.975$  as shown in the **Figure 12**. Therefore, this has a little effect on the prediction value of  $K$ .

It should be noted that the quadratic and cubic forms of Eq. (39) were also tried in this formulating process. However, it was found that this makes only little improvement on the accuracy of predicting of  $W$

and  $K$  but adds a lot of complexity into the Eq. (39). Therefore, it is judged that Eq. (41) and Eq. (42) are enough for the prediction of  $W$ .

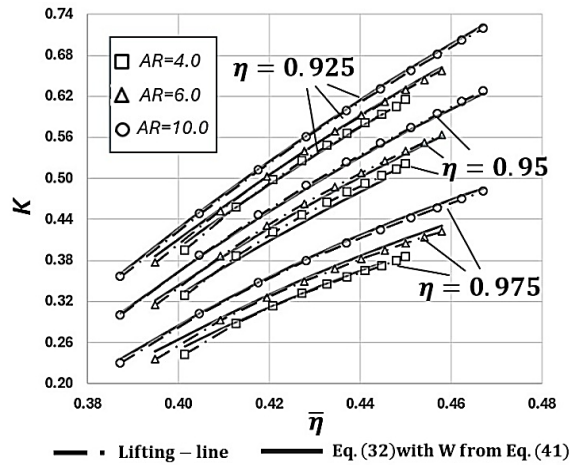


**Figure 13** Comparison of  $K$  from empirical equations and data from lifting-line theory.

#### 4.3.5 Equation for $\eta = 0.9$

For the position  $\eta=0.9$ , the value of loading coefficient can be calculated by using both Eq. (31) and Eq. (32). This comes from the fact that the data from lifting-line at this position is used in the curved-fitting process for both equations. To find the best option, the results from both equations are compared with the data from lifting-line. It was found that the average error of the prediction from Eq. (31) is 1.4159

percent. In the case of Eq. (32), the average error is 0.2748 percent. Therefore, it was judged that the Eq. (32) is more suitable for calculating the value of  $K$  at  $\eta=0.9$ .

## 5. Implementation

All equations for calculating spanwise loading distribution  $K$  are summarized in **Table 7**. The calculating procedure is straight forward. First, aspect ratio and taper ratio are calculated by using Eq. (1) and Eq. (3), respectively. These two variables are used to determine the location for center of pressure by using Eq. (28), (29) and (30). Then, the loading coefficient for  $0.0 \leq \eta < 0.9$  can be found by Eq. (31), (33) and (34).

For the wing tip area or at  $0.9 \leq \eta \leq 0.975$ , the Eq. (32) and (35) to (37) are used. The weighting function can be found by using Eq. (41) and (42). After the value of loading coefficient is known, the lift distribution along the wingspan can be determined.

For a demonstration purpose, an example is provided in this section. The geometric data of a straight-wing planform and flight condition are given as shown in **Table 8** and the calculation result of lift distribution is shown in **Table 9**. By using Eq. (1) and Eq. (3), the value of taper ratio and aspect ratio in this case are 0.45 and 11.08, respectively. The value of wing average chord ( $\bar{c}$ ) is calculated by using Eq. (6) which is 2.211 m. Then, from Eq. (28) to (30), the location of center of pressure  $\bar{\eta}$  is 0.4317.

**Table 7** Summary of equations

| Variable                                                   | Equations                                                                                                                                                                                                                                                            | Equation No.                                                         |
|------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| Center of pressure ( $\bar{\eta}$ )                        | $\bar{\eta} = a_0 + a_1 \ln(AR)$<br>$a_0 = 0.4253$<br>$a_1 = -0.0202 + 0.0611\lambda - 0.0229\lambda^2$                                                                                                                                                              | Eq. (28)<br>Eq. (29)<br>Eq. (30)                                     |
| Loading coefficient ( $K$ ) for $0.0 \leq \eta < 0.9$      | $K = b_0 + b_1 \bar{\eta}$<br>$b_0 = 3.6507 - 2.4539\eta - 3.7837\eta^2 - 8.4094\eta^3 + 9.7256\eta^4$<br>$b_1 = -5.3410 + 5.6002\eta + 4.9351\eta^2 + 24.9249\eta^3 - 26.1423\eta^4$                                                                                | Eq. (31)<br>Eq. (33)<br>Eq. (34)                                     |
| Loading coefficient ( $K$ ) for $0.9 \leq \eta \leq 0.975$ | $K = W(b_2 + b_3 \bar{\eta} + b_4 \bar{\eta}^2)$<br>$b_2 = 260.35 - 555.98\eta + 293.02\eta^2$<br>$b_3 = -1040.87 + 2212.84\eta - 1160.45\eta^2$<br>$b_4 = 922.17 - 1923.68\eta + 990.27\eta^2$<br>$W = e^{c_1 [l \ln(AR) - 2.4849]}$<br>$c_1 = -1.2633 + 1.446\eta$ | Eq. (32)<br>Eq. (35)<br>Eq. (36)<br>Eq. (37)<br>Eq. (41)<br>Eq. (42) |

**Table 8** Wing geometric parameters and flight conditions

| Parameters                                        | Value                   |
|---------------------------------------------------|-------------------------|
| Wingspan ( $b$ )                                  | 24.5 m                  |
| Chord length at wing root ( $c_r$ )               | 3.05 m                  |
| Chord length at wing tip ( $c_t$ )                | 1.37 m                  |
| Wing angle of attack ( $\alpha$ )                 | 3.0°                    |
| Wing zero-lift angle of attack ( $\alpha_{L=0}$ ) | -2.5°                   |
| Air density at altitude ( $\rho_\infty$ )         | 0.046 kg/m <sup>3</sup> |
| Air velocity ( $U_\infty$ )                       | 318 km/hr               |

The total wing lift coefficient can be determined by using Eq. (43). In this case, the theoretical value of wing lift-slope ( $a$ ) is estimated by using Eq. (44) which  $\alpha$  and  $\alpha_{L=0}$  are angle of attack and zero-lift angle of attack (deg), respectively. The value of  $a$  in this case is 0.0926 (1/deg).

$$C_L = a(\alpha - \alpha_{L=0}) \quad (43)$$

$$a = f \frac{2\pi}{1+(2/AR)} \times 0.0174 \quad (44)$$

**Table 9** Calculation result for the example

| $\eta$ | $b_0$  | $b_1$  | $b_2$  | $b_3$  | $b_4$   | $c_1$ | $W$   | $K$   | $c$<br>(m) | $c_l$ | $l'$<br>(N/m) |
|--------|--------|--------|--------|--------|---------|-------|-------|-------|------------|-------|---------------|
| 0.00   | 3.606  | -5.341 | -      | -      | -       | -     | -     | 1.300 | 3.050      | 0.480 | 262.59        |
| 0.10   | 3.315  | -4.709 | -      | -      | -       | -     | -     | 1.282 | 2.882      | 0.501 | 258.97        |
| 0.20   | 2.912  | -3.866 | -      | -      | -       | -     | -     | 1.243 | 2.715      | 0.515 | 251.06        |
| 0.30   | 2.381  | -2.756 | -      | -      | -       | -     | -     | 1.191 | 2.547      | 0.526 | 240.61        |
| 0.40   | 1.730  | -1.385 | -      | -      | -       | -     | -     | 1.131 | 2.379      | 0.535 | 228.55        |
| 0.50   | 0.990  | 0.175  | -      | -      | -       | -     | -     | 1.065 | 2.211      | 0.542 | 215.10        |
| 0.60   | 0.215  | 1.791  | -      | -      | -       | -     | -     | 0.989 | 2.044      | 0.545 | 199.69        |
| 0.70   | -0.515 | 3.270  | -      | -      | -       | -     | -     | 0.896 | 1.876      | 0.538 | 181.03        |
| 0.80   | -1.101 | 4.351  | -      | -      | -       | -     | -     | 0.777 | 1.708      | 0.512 | 157.03        |
| 0.90   | -      | -      | -2.686 | 10.722 | -7.023  | 0.038 | 0.997 | 0.632 | 1.540      | 0.462 | 127.62        |
| 0.92   | -      | -      | -3.139 | 12.738 | -9.451  | 0.067 | 0.995 | 0.595 | 1.507      | 0.444 | 120.17        |
| 0.94   | -      | -      | -3.359 | 13.826 | -11.087 | 0.096 | 0.992 | 0.540 | 1.473      | 0.412 | 109.00        |
| 0.96   | -      | -      | -3.344 | 13.986 | -11.930 | 0.125 | 0.990 | 0.466 | 1.440      | 0.364 | 94.13         |
| 0.98   | -      | -      | -3.094 | 13.217 | -11.981 | 0.154 | 0.988 | 0.374 | 1.406      | 0.300 | 75.61         |
| 1.00   | -      | -      | -      | -      | -       | -     | -     | 0.000 | 1.370      | 0.000 | 0.00          |

The variable  $f$  in Eq. (44) is the coefficient for the effect from taper ratio and aspect ratio that can be found by using the graph provided in the work of Anderson, which, in this case, is 0.99. Then, from Eq. (43), the value of  $C_L$  is 0.509. The value of loading coefficient  $K$  is calculated by using the equations in **Table 7** and the result is shown in **Table 9** which is generated by using spread sheet program. When distribution of  $K$  along the wingspan is known, the local lift coefficient  $c_l$  can be found by using Eq. (9) which  $\bar{c}$  and  $C_L$  are used in this equation.

The variation of local chord length can be found by using Eq. (5) which is shown in the tenth column of **Table 9**. Then, the lift per unit length  $l'$  at each position can be determined by using Eq. (7). The distribution of lift per unit span is shown in **Figure 14**.

It should be noted that the calculation process to determine  $C_L$  in this example is for demonstration purpose only. In practical, the value of  $C_L$  that comes from other sources such as an experimental data can also be applied.

## 6. Comparison

### 6.1 Comparison with selected theoretical method

The presented method and Anderson's method are used to calculate the distribution of loading coefficient  $K$  of a straight-trapezoidal wing planform. The computer code was created to perform this task by using Python 3.12.1. The numerical library Numpy 1.26.0 is used in this computational process. The equations in **Table 7** are used to create a subroutine in the program which uses taper ratio and aspect ratio as the input variables.

The subroutine for Anderson's methods was also created. Because this method is a manual calculation method that includes a lot of tables and graphs. Therefore, all the data in the paper is transformed into a numerical form and stored in the program. The linear interpolation scheme is also applied in this subroutine.

Anderson's method is one of the suggested methods for calculating spanwise loading distribution for airframe certification process by the FAA. It is considered as one of the accepted methods that is used in aerospace industry. Therefore, it is chosen as a reference method for this comparison.

The basic concept of Anderson's Method can be briefly described as follows. This method considers that the lift distribution along wingspan can be divided into two components. First, the basic loading distribution that occurs at wing zero-lift condition. This parameter mainly depends on the wing twisted angle. Another component is addition loading distribution which depends only on the wing geometric planform. This parameter is independent from the angle of attack. The formular for these two components were derived in Anderson's work. The coefficients and parameters for each aspect ratio and taper ratio that are used in the calculation process are arranged in the charts and tables.

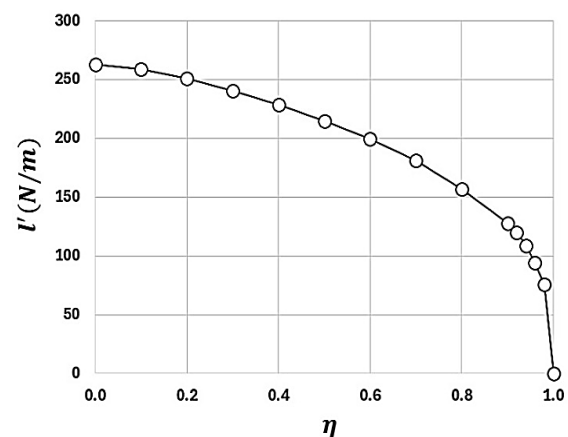


**Figure 14** Shows distribution of lift per unit span from the example.

In this computational process, the value of aspect ratio varies from 4 to 12 with step 2. The taper ratio

varies from 0.1 to 1.0 with step 0.1. The positions along the wingspan  $\eta$  are 0.0 to 0.2 with step 0.2 and the positions at the wing tip area are 0.9, 0.95 and 0.975. At  $\eta = 1$ , the value of  $K$  is set to be zero. The results in each case from both methods are compared to each other.

The deviation ( $\delta$ ) of the result between these two methods is calculated by using Eq. (45).

$$\delta = \left| \frac{K_{\text{present}} - K_{\text{Anderson}}}{K_{\text{Anderson}}} \right| \times 100\% \quad (45)$$

The result from this computational process shows that the average deviation between the presented method and Anderson's method through the entire consideration range is 7.268 percent, which the presented method tends to yield overprediction value compared to the result from Anderson's method.

The level of deviation comes from the fact that Anderson's method used only four terms of Fourier series in the calculation process. However, the presented work used eight terms of Fourier series in the calculation process. Therefore, this causes the overprediction of presented work compared to Anderson's method.

However, the level of  $\delta$  can be reduced by introducing correction factor ( $f_c$ ) in the calculation process. This correction factor is used to multiply with the value of  $K$  after it is calculated by Eq. (31) and (32). To find the appropriate value of a correction factor. The further investigation is performed by using the same computer code which includes a correction factor in the computational process. By varying the value of  $f_c$  from 0.9 to 0.99, it was found that the value of  $f_c$  that can yield the minimum average  $\delta$  is 0.9537 which is shown in **Figure 15**.

This correction factor can reduce the average  $\delta$  from 7.268 to 4.059 percent and give a better prediction result for loading coefficient. The examples of the comparison between the results from Anderson's method and presented method, both with and without applying  $f_c$ , are shown in **Figure 16** and **Figure 17**.

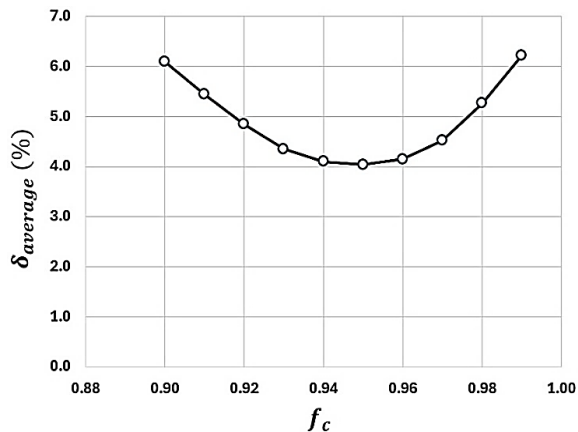


**Figure 15** Shows correction factor  $f_c$  and average  $\delta$

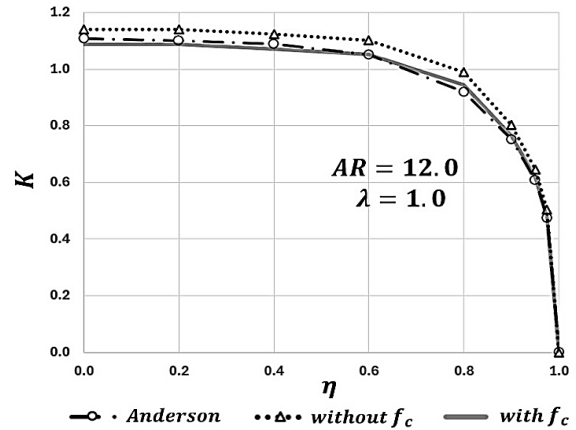


**Figure 16** Shows the comparison of the result from Anderson's method and presented method with and without  $f_c$  at  $AR = 6.0$  and  $\lambda = 0.4$ .

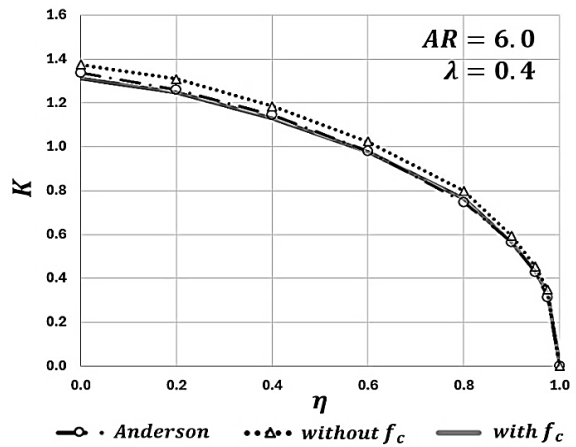


**Figure 17** Shows the comparison of the result from Anderson's method and presented method with and without  $f_c$  at  $AR = 12.0$  and  $\lambda = 1.0$ .

## 6.2 Comparison with experimental data

The presented method is used to calculate the distribution of lift force per unit span on the straight wings and then compares with the experimental data. In this case, the correction factor  $f_c$  is also applied in calculation process.

The first case is the experimental data of a high aspect ratio straight wing from Whitcomb [20]. This straight wing planform has taper ratio 0.4 and aspect ratio 9.18. The wing angle of attack are  $2.0^\circ$  and  $7.0^\circ$ . The wing lift coefficient for both conditions are 0.34 and 0.745, respectively. The Reynolds number in this experiment is 0.9 million. The comparison of the distribution of lift per unit span is shown in **Figure 18**. This figure shows that the results from the calculation agree very well with the experimental data in both cases. The average deviation between calculation results and the experimental data are 4.830 and 4.108 percent for  $2.0^\circ$  and  $7.0^\circ$ , respectively.

Another case is the experimental data from Theil and Weissinger [21]. In this case, the value of taper ratio and aspect ratio are 0.5 and 5, respectively. The angles of attack are  $5.7^\circ$  and  $11.4^\circ$  and lift coefficient

related to each angle of attack are 0.42 and 0.83, respectively. The Reynolds number in this test is one million. The comparison for this case is shown in **Figure 19**. This graph also shows a very good agreement between the calculation results and experimental data. The average deviation for angle of attack  $5.7^\circ$  and  $11.4^\circ$  are 3.886 and 4.654 percent, respectively.

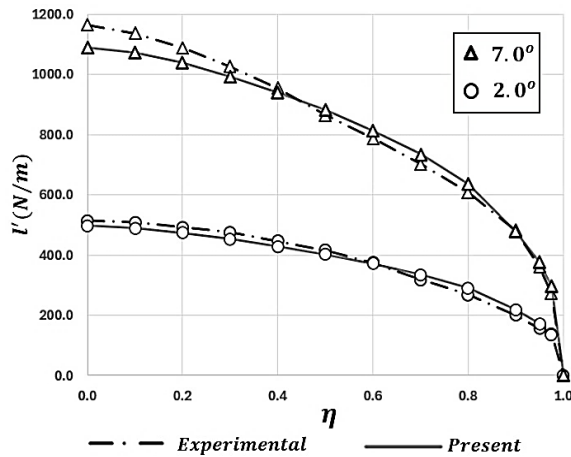


**Figure 18** Shows comparison between the present method and experimental data for  $\lambda = 0.4$  and  $AR = 9.18$

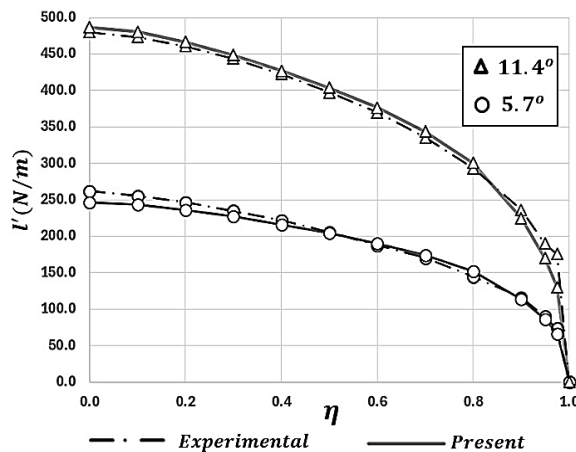


**Figure 19** Shows comparison between the present method and experimental data for  $\lambda = 0.5$  and  $AR = 5$

## 7. Conclusion

The empirical equations for estimating spanwise loading coefficient for a straight-trapezoidal wing planform are formulated by using data generated from lifting-line theory. The least squares approximation method is used to formulate these equations. These empirical equations can be used to determine center of pressure location and spanwise loading distribution on a wingspan. These equations are easy to apply. The calculation process can be accomplished by using only taper ratio and aspect ratio as input data and other complex calculation processes such as matrix solving or an iterative calculation process is not required.

The result from the empirical equations agrees very well when compared with the Anderson's method. The average deviation of the results between these two methods is 4.059 percent. However, it should be noted that the correction factor is also required in the presented calculation process.

The result from the empirical equations also agrees very well when compared with the experimental data. They performed very well in the range at low to moderate lift coefficients, which is the range that this method is expected to be used. The average deviation between calculation results and experimental data is in the range about 3.886 to 4.830 percent.

However, it should be mentioned that some coefficients in this work are formulated by using average value of the data. Therefore, the explicit relations between some parameters may disappear and correction factors are needed in some situations. Another point is about the change of loading coefficient behavior at the wing tip area. Although the empirical equations formulated can represent the data very well. The fundamental understanding about this changing behavior is still unknown and this issue should be further investigated in the future.

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