A Maximum Capture Problem for Public Bicycle Station Allocation and Bicycle Network Design

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Abstract

Bike-sharing systems (BSS) constitute a fundamental component of sustainable transportation development, significantly reducing the dependence on private vehicles, mitigating traffic congestion, and addressing environmental challenges in densely populated urban areas. This study develops a maximum capture problem designed to optimize bike lane network design, with the dual objective of maximizing user adoption and spatial equity by addressing disparities in service accessibility across diverse BSS station locations. The proposed problem employs the Multinomial Logit (MNL) travel choice behavior. The independence of irrelevant alternatives (IIA) property of used to provides a mixed-integer linear programming (MILP) formulation. Through numerical examples of the Chaing Mai transportation network, the proposed MILP can determine the bike lane network and BSS station location to optimize the number of users. Multimodal transport is a key factor in promoting BSS usage, where both the BSS and public transit gain win-win situations. Spatial equity and transportation network characteristics play significant roles in determining the optimum BSS station location. A unit change in the MNL dispersion parameter has a high impact on the bike modal shift, the spatial equity, and bike usage distance travelled.

Keywords: Bike-Sharing Systems, Multinomial Logit, Maximum Capture Problem, Mixed Integer Linear Programming

1. Introduction

1.1 Overview

Transportation pollution is a global issue that accounts for approximately 16.2% of total greenhouse gas emissions worldwide [1]. Most governments have ambitious goals to reach emission levels of net zero by switching to renewable power sources using non-fossil-fuel-based power. Transportation emissions negatively impact public health, where extremely high PM 2.5 levels further reduce bike-sharing uptake. These implications warrant air quality improvement policies in the context of shared-mobility planning [2]. Furthermore, switching to sustainable transportation methods is essential to reduce emissions, improve air quality, and guarantee sustainable cities [3].

Bike-Sharing Systems (BSS) have gained global popularity for reducing pollution, promoting public health, and reducing traffic congestion [4]. Integration of BSS with public transit, including buses, is a sustainable, responsive mobility mode that is supported by GPS tracking, smart locks, and convenient apps to ensure better convenience and operational efficiency [5]. Integration increases operators' revenues and reduces the social costs of urban mobility [6]. BSS efficiency depends on the optimal placement of facilities within networks [7–9], with strategic planning being the key to meeting diverse urban mobility demands. Well-planned inclusive BSS solve urban mobility issues and provide equitable systems [10]. BSS should be strategically combined with comprehensive urban transportation planning to sustain health benefits for cycling [11],[12]. Governments and local authorities have critical responsibilities to address planning for cycling-friendly facilities and coverage of service [13],[14]. The integration of BSS with public transportation makes trips more convenient, reduces private modes of transportation, and increases mode alternatives [15]. Dedicated bike lanes and secure facilities provide user safety and encourage BSS acceptance [16],[17]. Station locations are incorporated into BSS provision and route planning based on geography and population density [18]. Strategically placed station locations enhance multimodal linkages and equity, while maintaining free flow in public rights-of-way to promote an efficient and inclusive urban transportation system [19].

This paper presents a mathematical model to develop a maximum capture problem that optimizes bike-sharing networks in multimodal urban transportation systems. Bike user numbers and spatial equity are maximized to address disparities in the accessibility of services across different BSS station locations. This model applies the Independence of Irrelevant Alternatives (IIA) property, which is

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embedded in the Multinomial Logit (MNL) model to simulate travel choice behavior. Thus, the proposed model can be presented as Mixed Integer Linear Programming (MILP). Through numerical examples, the integration of the BSS infrastructure with the public transport system in the Chiang Mai transportation network improved both systems, specifically for first/last-mile travel. The fare structure plays a significant role in the number of users and connection between the BSS and public transportation systems.

1.2 Literature review

The development of Bike-Sharing Systems (BSS) has demonstrated the potential for urban transportation sustainability. Early BSS models in Amsterdam and Copenhagen initially grappled with theft-related problems, overuse, and operational inefficiencies owing to anonymity in accessing systems and the limited incorporation of technology [20],[21]. Advances in mobile applications, GPS tracking, and integration with public transportation systems have significantly mitigated these problems to an appreciable extent to render increasingly global BSS [22]. Systems such as Velib' in Paris, BIXI in Montreal, and Hangzhou's BSS highlight that equity-focused planning enhances social and environmental benefits. Hangzhou investments in specialized infrastructure have improved accessibility while providing greater equity in society [23]. Hangzhou's targeted investments show improvements in both network effectiveness and equity in accessibility [13]. Key BSS research focuses on optimizing facility placement to achieve optimum placement to match user demand and accessibility [24],[25]. Budget constraints might limit the full consideration of space and environmental aspects, whereas pre-established road infrastructure and capacity constraints will always exist, contributing to bottlenecks that occur frequently and underuse [26]. Mathematical and metaheuristic algorithms have been used to solve problems in optimizing network connections, user cost savings, and continuity problems; however, most disregard equity by not considering demographically heterogeneous urban wards [4]. To improve accessibility and system efficiency, bike-sharing systems require multimodal integration, with spatial equity. Although equity-focused models enhance service distribution and station location [27], their efficacy in first- and last-mile integration is limited, because they frequently do not account for multimodal connectivity. In contrast, location-based optimization, conceptually similar to the Maximum Covering Location Problem (MCLP), is frequently applied to support the best location of bike stations for optimizing the coverage of a bicycle network to financial limits. Nevertheless, the other transport modes provide a moderate challenge for the promotion of cycling in cities, which in turn leads to ineffective integration of bike networks with public transportation systems [28]. These limitations highlight the need for planning approaches that integrate spatial equity with multimodal accessibility to create more efficient and inclusive urban mobility systems. Quantitative tools, including travel pattern evaluation using smart cards, have the potential to optimize routes but face challenges in integrating interplay with transit systems and existing infrastructure constraints [29]. Multimodal transportation in relation to rail and bus systems to realize first-and-last mile travel modes raises travel rates while enabling eco-friendly travel behaviors [15], [30]. Despite these benefits, practical challenges such as station capacity and provision of bikes remain an issue, while spatial equity remains underexplored in depth, with accessibility differences in relation to factors such as income being studied by researchers [18],[31]. Although highlighting the importance of equity in planning, research lacks budget-friendly options for equitable BSS deployment.

Some models integrate multimodal accessibility and spatial equity to enhance network efficiency [32], relying on deterministic frameworks that optimize station distribution based on predefined criteria without capturing stochastic variations in travel behavior. It is necessary to realize user behavior [33] because deterministic models assume rational choices while ignoring user preferences and limitations. Stochastic models that take uncertainty and heterogeneous user behavior into consideration calculate travel choice probabilities based on factors such as distance, convenience, and provision of bikes. This study applied stochastic modeling to optimize station placement, bike allocation, and equitable service access, ensuring system agility and responsiveness to real-world conditions. Several studies have addressed the challenge of the BSS network design problem, as presented in **Table 1**

Table 1 Some cycling network design problems in the literature

Authors (Years)	BSS station location	Cycling network	Multimodal transport	Spatial access equity	Travel choice behavior	
					Deterministic	Stochastic
Mauttone et al. (2017) [4]		✓			✓	
Lin and Yang (2011) [8]		✓	✓		✓	
Li et al. (2013) [11]		✓	✓		✓	
Conrow et al. (2018) [13]	✓	✓		✓	✓	

Table 1 Some cycling network design problems in the literature (cont.)

Authors (Years)	BSS station location	Cycling network	Multimodal transport	Spatial access equity	Travel choice behavior	
					Deterministic	Stochastic
Tavassoli and Tamannaei (2020) [15]	✓	✓	✓		✓	
Padeiro et al (2023) [18]	✓	✓		✓	✓	
Fazio et al. (2021) [25]	✓	✓	✓		✓	
Mix et al. (2022) [26]	✓	✓	✓		✓	
Caggiani et al. (2020b) [27]	✓			✓	✓	
Ospina et al. (2022) [28]	✓	✓		✓	✓	
Akbarzadeh et al. (2018) [29]		✓			✓	
Wei and Zhu (2023) [31]	✓		✓		✓	
Caggiani et al. (2020a) [32]	✓	✓	✓	✓	✓	
This Study	✓	✓	✓	✓		✓

Existing BSS research recognizes key gaps in spatial equity, multimodal transport integration, and intricate user behavior. This study addresses these gaps by proposing an integrated framework that incorporates spatial equity evaluation, multimodal transport optimization, and stochastic modeling to account for diverse user behaviors.

The main goal of this study is to develop an efficient framework for BSS network design that prioritizes multimodal integration and spatial equity. To address this issue, we develop a Maximum Capture Problem (MCP) model to optimally place bike stations and design a network. Strategies aimed at mitigating accessibility disparities support inclusive transportation planning, while ensuring spatial equity.

This paper is divided into five sections. Next section describes the MNL model. Section 3 presents the framework and describes the proposed mathematical model. Numerical examples are presented in section 4. Section 5 concludes the study with key insights and highlights potential directions for future research.

1.3 Multinomial Logit (MNL) model

The Multinomial Logit (MNL) model is a widely adopted discrete choice model. A key advantage of this model lies in its closed-form probability expression. It can be presented as Eq. (1).

$$\hat{P}_{r(t,v)}^{ijm} = \frac{exp\left(-\theta g_{r(t,v)}^{ijm}\right)}{\sum_{n\in M^{ij}}\sum_{s\in T}\sum_{\substack{w\neq s\\ w\in T}}\sum_{k\in R_{(s,w)}^{ijn}} exp\left(-\theta g_{k(s,w)}^{ijn}\right)} \quad (1)$$

where:

$\hat{P}_{r(t,v)}^{ijm}$	Probability of choosing route r of mode m passing through bike-sharing parking facilities t between OD pair ij
$g_{r(t,v)}^{ijm}$	Generalized travel cost on route r of mode m passing through bike-sharing parking facilities t between OD pair ij
θ	Dispersion parameter related to travelers' perception variance

The MNL model exhibits the Independence of Irrelevant Alternatives (IIA) property. The probability ratio between options *r* and *k* remains unchanged even when new options are added or removed [34], as shown in Eq. (2).

$$\frac{\hat{P}_{r(t,v)}^{ijm}}{\hat{P}_{k(s,w)}^{ijn}} = \frac{exp\left(-\theta g_{r(t,v)}^{ijm}\right)}{exp\left(-\theta g_{k(s,w)}^{ijn)}\right)} \quad (2)$$

2. Mathematical model

In this section, a mathematical model based on the Maximum Capture Problem addresses the BSS allocation and network design challenges, with a focus on spatial equity. It starts with key assumptions and advances to the formulation of a Mixed Integer Linear Programming (MILP) model. The following notation is used throughout this study, as shown in **Table 2**

Table 2 The following notation is used throughout this study

Indices	Definition
	Sets
M	Set of all modes
A	Set of potential links to install bike lane
IJ	Set of origin-destination (OD) pairs
T	Set of potential bike-sharing parking facility locations
M^{ij}	Set of modes between OD pair $ij \in IJ$
$M^{ij[b]}$	Set of modes using shared bike between OD pair $ij \in IJ$
$R_{(t,v)}^{ijm}$	Set of routes in mode $m \in M^{ij}$ passing through bike-sharing parking facility $t \neq v \in T$ between OD pair $ij \in IJ$

Table 2 The following notation is used throughout this study (cont.)

Indices	Definition
Sets	
A_r	Set of potential links to install bike lane on route $r \in R_{(t,v)}^{ijm}$
Parameters	
q_{ij}	Travel demand between OD pair $ij \in IJ$
$g_{r(t,v)}^{ijm}$	Generalized travel cost on route $r \in R_{(t,v)}^{ijm}$ of mode $m \in M^{ij}$ passing through bike-sharing parking facilities $t \neq v \in T$ between OD pair $ij \in IJ$
θ	Dispersion parameter
$\check{c}_t$	Cost of installing bike-sharing parking facility at site $t \in T$
$\tilde{c}_a$	Cost of installing bike lane on link $a \in A$
C_t	Capacity of bike-sharing parking facility at site $t \in T$
B	Available budget
ψ	Allowable volume capacity ratio
Λ	A large number
Variables	
x_t	1 if shared bike parking facility site $t \in T$ is installed, or 0 otherwise
h_a	1 if bike lane is installed on link a$\in A$, or 0 otherwise
$\hat{P}_{r(t,v)}^{ijm}$	Probability of choosing route $r \in R_{(t,v)}^{ijm}$ of mode $m \in M^{ij}$ passing through bike-sharing parking facilities $t \neq v \in T$ between OD pair $ij \in IJ$
$f_t^{[b]}$	Bike-sharing travel demand at parking facilities $t \in T$
α	Tolerance between bike-sharing parking facility locations regarding the volume-capacity ratio

2.1 Assumptions

Assumption 1: Shared bike riders travel solely in bike lanes. The implementation of bicycle lanes has a negligible effect on the generalized travel cost of the other transportation modes.

Assumption 2: Users start or continue to ride the shared bike only at the BSS parking facility. Travelers are both accessible and egress eligible at the transit station.

Assumption 3: Equity is measured based on the difference in the ratio between the number of BSS users at the facility and the capacity of the facility.

Assumption 4: All potential bike lanes satisfy the safety standard, which is a fundamental requirement in transportation.

These assumptions are utilized with the aim of streamlining the travel patterns of multimodal transportation. It is assumed that all travelers use shared bicycle ride vehicles within the installed bike lanes. The operators of the BSS position bicycle exclusively at the selected bike-sharing parking facilities. The objective is to optimize the use of the right of way among various modes of transportation within a given locality. It is assumed that the installed bike lane has a negligible effect on the overall travel expenses of other modes of transportation. This assumption may hold significant weight when considering that the implementation of a bike lane would result in a reduction in road width. This assumption is made to simplify a generalized travel cost function that would otherwise be complex. For equity, the ratio between the number of BSS users at the parking facility and the facility capacity is adopted. The greater the equity, the less the difference between the ratios at each location.

Without loss of generality, this study considers four modes of transport in a hypernetwork, where multimodal transport can be considered through the travel route, as presented in **Figure 1** Modes of transportation and general patterns. The four modes of transport include 1) auto (A), 2) transit (T), 3) shared-bike (B), and 4) shared-bike-transit (BT). The present study categorizes the BSS under investigation as belonging to the dockless BSS and hybrid BSS classes. Specifically, the BSS requires a parking facility for docking stations or dockless systems at the onset of BSS utilization. Both the A and T modes do not utilize BSS. In mode B, travelers walk to the BSS parking facility before using the shared bike. BT mode refers to a form of transportation that utilizes multiple modes of transportation. BSS have been integrated into public transit systems. In the BT mode, commuters utilize public transportation either by walking or using a shared-bike. Similarly, when exiting the transit system, they also walked or used a shared-bike.

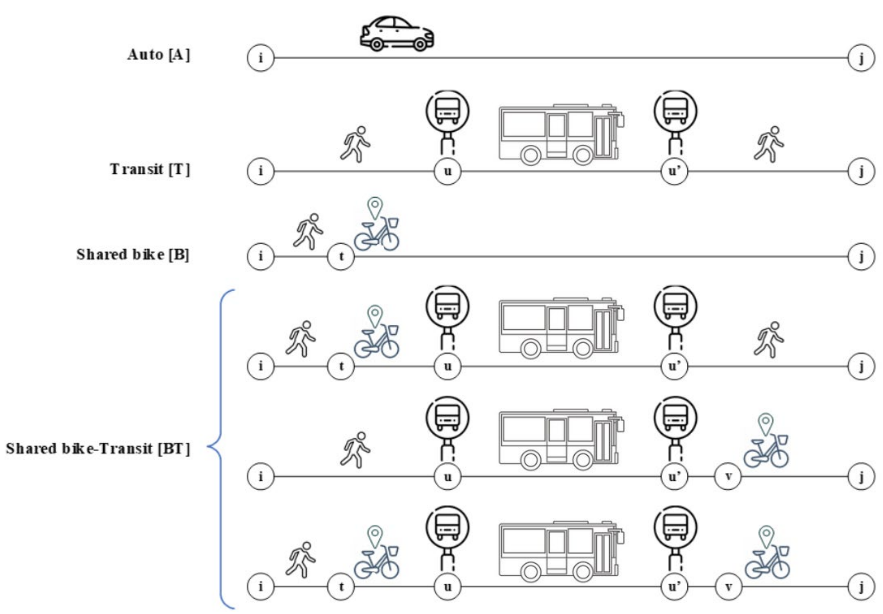

Figure 1 Modes of transportation and general patterns

2.2 Mathematical programming

Based on the assumptions mentioned above, the spatial equity-based maximum capture problem for bike-sharing parking facility allocation and path network design can be formulated as the following MILP:

$$\max W_1 \sum_{ij\in IJ} q_{ij}\left(\sum_{m\in M^{ij[b]}}\sum_{t,v\in T}\sum_{r\in R^{ijm}_{(t,v)}} \hat{P}^{ijm}_{r(t,v)}\right) - W_2\alpha \tag{3}$$

s.t.

$$\sum_{a\in A} h_a \tilde{c}_a + \sum_{t\in T} x_t \check{c}_t \le B, \tag{4}$$

$$\sum_{m\in M^{ij}}\sum_{t,v\in T}\sum_{r\in R^{ijm}_{(t,v)}} \hat{P}^{ijm}_{r(t,v)} = 1, \tag{5}$$

$$\hat{P}^{ijm}_{r(t,v)} \le h_a, \quad \forall a\in A_r, r\in R^{ijm}_{(t,v)}, t,v \in T, m\in M^{ij[b]}, ij\in IJ, \tag{6}$$

$$\hat{P}^{ijm}_{r(t,v)} \le x_t, \quad \forall r\in R^{ijm}_{(t,v)}, t,v\in T, m \in M^{ij[b]}, ij\in IJ, \tag{7}$$

$$\hat{P}^{ijm}_{r(t,v)} \le x_v, \quad \forall r\in R^{ijm}_{(t,v)}, t,v\in T, m \in M^{ij[b]}, ij\in IJ, \tag{8}$$

$$\hat{P}^{ijm}_{r(t,v)} \le \frac{exp\left(-\theta g^{ijm}_{r(t,v)}\right)}{exp\left(-\theta g^{ijn}_{k(s,w)}\right)} \hat{P}^{ijm}_{k(s,w)} + (2 - x_s - x_w) + \sum_{a\in A_k}(1-h_a), \quad \forall r\in R^{ijm}_{(t,v)}, k\in R^{ijn}_{(s,w)}, t,v,s,w\in T, m,n\in M^{ij}, ij\in IJ, \tag{9}$$

$$\left|\frac{\sum_{ij\in IJ} q_{ij}\left(\sum_{m\in M^{ij[b]}}\sum_{t\in T}\sum_{r\in R^{ijm}_{(t,v)}} \hat{P}^{ijm}_{r(t,v)}\right)}{C_v} - \frac{\sum_{ij\in IJ} q_{ij}\left(\sum_{m\in M^{ij[b]}}\sum_{w\in T}\sum_{k\in R^{ijm}_{(s,w)}} \hat{P}^{ijm}_{k(s,w)}\right)}{C_s}\right| \le \alpha + \Lambda(2 - x_v - x_s), \quad \forall v,w\in T, \tag{10}$$

$$\sum_{ij\in IJ} q_{ij}\left(\sum_{m\in M^{ij[b]}}\sum_{t\in T}\sum_{r\in R^{ijm}_{(t,v)}} \hat{P}^{ijm}_{r(t,v)}\right) \le \psi C_v, \ \forall v\in T, \tag{11}$$

$$\hat{P}^{ijm}_{r(t,v)} \in [0,1], \quad \forall r\in R^{ijm}_{(t,v)}, t,v\in T, m\in M^{ij}, ij\in IJ, \tag{12}$$

$$h_a \in \{0,1\}, \ \forall a\in A, \tag{13}$$

$$x_t \in \{0,1\}, \ \forall t\in T, \tag{14}$$

$$\alpha \ge 0. \tag{15}$$

Eq. (3) defines the objective function to maximize the usage of BSS and spatial equity. The weight parameters W_1 and W_2 balance the trade-off between users and equity levels. In addition to the budget constraints in Eq. (4), which guarantees that the costs of constructing bike lanes and stations remain within the budget limits (B). Eq. (5) provides the flow conservation to $\hat{P}^{ijm}_{r(t,v)}$ as representing the probability of selecting other modes of travel. The logical constraints in Eqs. (7)–(8) are incorporated based on *Assumptions 1 and 2* such that for $\hat{P}^{ijm}_{r(t,v)}$ ranges from 0 to 1, depending on the presence of relevant bike lanes and stations. Combined with the flow-conservation constraint in Eq. (5) and Eq. (9) helps to define the choice probabilities in the MNL model used in the objective function. The second and third terms on the right-hand side of Eq. (9) represents logical statements related to $\hat{P}^{ijm}_{k(s,w)}$. The MNL model maintains the Independence of Irrelevant Alternatives (IIA) for the probability ratio between $\hat{P}^{ijm}_{r(t,v)}$ and $\hat{P}^{ijm}_{k(s,w)}$. Eq. (10) expresses equity, where α represents the absolute difference in the ratio of BSS users to station capacity, as defined in *Assumption 3*. This Eq. is linearized for computational efficiency in Eqs. (16)–(17).

$$\left(\frac{\sum_{ij\in IJ} q_{ij}\left(\sum_{m\in M^{ij[b]}}\sum_{t\in T}\sum_{r\in R^{ijm}_{(t,v)}}\hat{P}^{ijm}_{r(t,v)}\right)}{\underline{C_v}} \frac{\sum_{ij\in IJ} q_{ij}\left(\sum_{m\in M^{ij[b]}}\sum_{w\in T}\sum_{k\in R^{ijm}_{(s,w)}}\hat{P}^{ijm}_{k(s,w)}\right)}{C_s}\right) \le \alpha + \Lambda(2 - x_v - x_s), \forall v, w \in T, \tag{16}$$

$$\left(\frac{\sum_{ij\in IJ} q_{ij}\left(\sum_{m\in M^{ij[b]}}\sum_{w\in T}\sum_{r\in R^{ijm}_{(t,v)}}\hat{P}^{ijm}_{r(s,w)}\right)}{\underline{C_v}} \frac{\sum_{ij\in IJ} q_{ij}\left(\sum_{m\in M^{ij[b]}}\sum_{t\in T}\sum_{r\in R^{ijm}_{(t,v)}}\hat{P}^{ijm}_{r(t,v)}\right)}{C_v}\right) \le \alpha + \Lambda(2 - x_v - x_s), \forall v, w \in T, \tag{17}$$

Eq. (1 1) sets the allowable ratio between the number of BSS users at the parking facility and the capacity at each installed facility. Eqs. (1 2)–(1 5) describe the decision variables.

2.3 Proposition

The MILP in Eqs. (3)–(9) and (11)–(17) generate the maximum number of BSS users and equity under MNL travel choice behavior.

Proof. assume that there are at least two routes connecting each OD pair. In addition, the A and/or T modes are available for all OD pairs. The proof focuses on Eq. (9). We separate the proof into two cases.

Case 1: When $(2\text{-x}_\text{s}\text{-x}_\text{w})+\sum_{a\in A_k}(1\text{-h}_\text{a}) \ge 1$, x_s and/or x_w and/or h_a correspond to $\hat{P}^{ijm}_{k(s,\,w)}$ equals 0. According to Eqs. (6)–(8), $\hat{P}^{ijm}_{k(s,\,w)}=0$. From Eq. (5) and Eq. (12) $\hat{P}^{ijm}_{r(t,\,v)} \in [0,\,1]$.

Case 2: When $(2\text{-x}_\text{s}\text{-x}_\text{w})+\sum_{a\in A_k}(1\text{-h}_\text{a}) = 0$, all x_s, x_w, and h_a corresponding to $\hat{P}^{ijm}_{k(s,\,w)}$ equal 1. Then, we have

$$\hat{P}^{ijm}_{r(t,v)} \le \frac{exp\left(-\theta g^{ijm}_{r(t,v)}\right)}{exp\left(-\theta g^{ijn}_{k(s,w)}\right)} \hat{P}^{ijm}_{k(s,w)}, \quad \forall r \in R^{ijm}_{(t,v)}, k \in R^{ijn}_{(s,w)}, t, v, s, w \in T, m, n \in M^{ij}, ij \in IJ.$$

From the objective function and Eq. (5),

$$\frac{exp\left(-\theta g^{ijm}_{r(t,v)}\right)}{exp\left(-\theta g^{ijn}_{k(s,w)}\right)} \hat{P}^{ijm}_{k(s,w)} + \frac{exp\left(-\theta g^{ijm'}_{r'(t',v')}\right)}{exp\left(-\theta g^{ijn}_{k(s,w)}\right)} \hat{P}^{ijm}_{k(s,w)} + \cdots + \frac{exp\left(-\theta g^{ijm''}_{r''(t'',v'')}\right)}{exp\left(-\theta g^{ijn}_{k(s,w)}\right)} \hat{P}^{ijm}_{k(s,w)} = 1$$

Rearranging,

$$\hat{P}^{ijm}_{k(s,w)} = \frac{exp\left(-\theta g^{ijm}_{k(s,w)}\right)}{\sum_{n\in M^{ij}}\sum_{t\in T}\sum_{\substack{v\neq t\\ v\in T}}\sum_{r\in R^{ijn}_{(t,v)}} exp\left(-\theta g^{ijn}_{r(t,v)}\right)},$$

which corresponds to Eq. (1).

The first term in the objective function $\sum_{ij\in IJ} q_{ij}\left(\sum_{m\in M^{ij[b]}}\sum_{t,\,v\in T}\sum_{r\in R^{ijm}_{(t,\,v)}}\hat{P}^{ijm}_{r(t,\,v)}\right)$ provides the MNL choice behavior. This completes this proof.

Proof. We assume that there are at least two routes connecting an origin-destination (OD) pair across different travel modes. This proof focuses on Eq. (9), emphasizing the Independence of Irrelevant Alternatives (IIA), where the probability of selecting route r with mode m is defined in Eq. (1) and derived from the IIA property in Eq. (6). We consider two cases:

Case 1: When there is an installation of bike stations and lanes, $(2\text{-x}_\text{s}\text{-x}_\text{w})+\sum_{a\in A_k}(1\text{-h}_\text{a}) = 0$, thus both origin and destination stations exist ($x_s = x_w = 1$) and all routes include bike lanes ($h_a = 1$ for all a ∈ A_k), resulting in zero additional terms in Eq. (9).

Case 2: If no stations or lanes exist, $(2\text{-x}_\text{s}\text{-x}_\text{w})+\sum_{a\in A_k}(1\text{-h}_\text{a}) \ge 1$, thus at least one infrastructure is missing (x_s and/or $x_w = 0$) or some routes lack bike lanes ($h_a = 0$), yielding a positive value that affects the route choice probability. The relationship between the different travel modes is expressed as follows:

Auto /Transit:

$$\hat{P}^{ij}_{r_{auto}} \le \frac{exp\left(-\theta g^{ij(auto)}_{r_{auto}}\right)}{exp\left(-\theta g^{ij}_{r_{transit}}\right)} \hat{P}^{ij}_{r_{transit}}$$
$$\hat{P}^{ij}_{r\_transit} \le \frac{exp\left(-\theta g^{ij}_{r\_transit}\right)}{exp\left(-\theta g^{ij}_{r\_auto}\right)} \hat{P}^{ij}_{r\_auto}$$

Thus,

$$\hat{P}^{ij}_{r_{auto}} = \frac{exp\left(-\theta g^{ij(auto)}_{r_{auto}}\right)}{exp\left(-\theta g^{ij}_{r_{transit}}\right)} \hat{P}^{ij}_{r_{transit}}$$
$$\hat{P}^{ij}_{r\_transit} = \frac{exp\left(-\theta g^{ij}_{r\_transit}\right)}{exp\left(-\theta g^{ij}_{r\_auto}\right)} \hat{P}^{ij}_{r\_auto}$$

Auto /Bike:

$$\hat{P}^{ij}_{r_{auto}} \le \frac{exp\left(-\theta g^{ij}_{r_{auto}}\right)}{exp\left(-\theta g^{ij}_{r_{bike}}\right)} \hat{P}^{ij}_{r_{bike}} + (2 - x_t - x_v) + \sum_{a\in A_{r_{bike}}}(1 - h_a)$$

Installed:

$$\hat{P}^{ij}_{r_{auto}} \le \frac{exp\left(-\theta g^{ij}_{r_{auto}}\right)}{exp\left(-\theta g^{ij}_{r_{bike}}\right)} \hat{P}^{ij}_{r_{bike}}$$
$$\hat{P}^{ij}_{r\_bike} \le \frac{exp\left(-\theta g^{ij}_{r\_bike}\right)}{exp\left(-\theta g^{ij}_{r\_auto}\right)} \hat{P}^{ij}_{r\_auto}$$

Thus,

$$\hat{P}^{ij}_{r\_auto} = \frac{exp\left(-\theta g^{ij}_{r\_auto}\right)}{exp\left(-\theta g^{ij}_{r\_bike}\right)} \hat{P}^{ij}_{r\_bike}$$

$$\hat{P}^{ij}_{r_{bike}} = \frac{exp\left(-\theta g^{ij}_{r_{bike}}\right)}{exp\left(-\theta g^{ij}_{r_{auto}}\right)} \hat{P}^{ij}_{r_{auto}}$$

Absent:

$$\hat{P}^{ij}_{r\_auto} \leq \frac{exp\left(-\theta g^{ij}_{r\_auto}\right)}{exp\left(-\theta g^{ij}_{r\_bike}\right)} \hat{P}^{ij}_{r\_bike} + Positive\ Value$$

Transit /Bike:

$$\hat{P}^{ij}_{r_{\text{transit}}} \leq \frac{exp\left(-\theta g^{ij}_{r_{\text{transit}}}\right)}{exp\left(-\theta g^{ij}_{r_{bike}}\right)} \hat{P}^{ij}_{r_{bike}} + (2 - x_t - x_v) + \textstyle\sum_{a \in A_{r\_bike}} (1 - h_a)$$

Installed:

$$\hat{P}^{ij}_{r\_\text{transit}} \leq \frac{exp\left(-\theta g^{ij}_{r\_\text{transit}}\right)}{exp\left(-\theta g^{ij}_{r\_bike}\right)} \hat{P}^{ij}_{r\_bike}$$
$$\hat{P}^{ij}_{r\_bike} \leq \frac{exp\left(-\theta g^{ij}_{r\_bike}\right)}{exp\left(-\theta g^{ij}_{r\_\text{transit}}\right)} \hat{P}^{ij}_{r\_\text{transit}}$$

Thus,

$$\hat{P}^{ij}_{r\_bike} = \frac{exp\left(-\theta g^{ij}_{r\_bike}\right)}{exp\left(-\theta g^{ij}_{r\_\text{transit}}\right)} \hat{P}^{ij}_{r\_\text{transit}}$$
$$\hat{P}^{ij}_{r\_\text{transit}} = \frac{exp\left(-\theta g^{ij}_{r\_\text{transit}}\right)}{exp\left(-\theta g^{ij}_{r\_bike}\right)} \hat{P}^{ij}_{r\_bike}$$

Absent:

$$\hat{P}^{ij}_{r\_\text{transit}} \leq \frac{exp\left(-\theta g^{ij}_{r\_\text{transit}}\right)}{exp\left(-\theta g^{ij}_{r\_bike}\right)} \hat{P}^{ij}_{r\_bike} + Positive\ Value$$

Bike/ Bike:
Between bike routes r and k:

$$\hat{P}^{ij}_{r_{bike}} \leq \frac{exp\left(-\theta g^{ij}_{r_{bike}}\right)}{exp\left(-\theta g^{ij}_{k_{bike}}\right)} \hat{P}^{ij}_{k_{bike}} + (2 - x_t - x_v) + \textstyle\sum_{a \in A_{k\_bike}} (1 - h_a)$$

Installed:

$$\hat{P}^{ij}_{r\_bike} \leq \frac{exp\left(-\theta g^{ij}_{r\_bike}\right)}{exp\left(-\theta g^{ij}_{k\_bike}\right)} \hat{P}^{ij}_{k\_bike}$$
$$\hat{P}^{ij}_{k\_bike} \leq \frac{exp\left(-\theta g^{ij}_{k\_bike}\right)}{exp\left(-\theta g^{ij}_{r\_bike}\right)} \hat{P}^{ij}_{r\_bike}$$

Thus,

$$\hat{P}^{ij}_{r\_bike} = \frac{exp\left(-\theta g^{ij}_{r\_bike}\right)}{exp\left(-\theta g^{ij}_{k\_bike}\right)} \hat{P}^{ij}_{k\_bike}$$
$$\hat{P}^{ij}_{k\_bike} = \frac{exp\left(-\theta g^{ij}_{k\_bike}\right)}{exp\left(-\theta g^{ij}_{r\_bike}\right)} \hat{P}^{ij}_{r\_bike}$$

Absent:

$$\hat{P}^{ij}_{r\_bike} \leq \frac{exp\left(-\theta g^{ij}_{r\_bike}\right)}{exp\left(-\theta g^{ij}_{k\_bike}\right)} \hat{P}^{ij}_{k\_bike} + Positive\ Value$$
$$\hat{P}^{ij}_{k\_bike} \leq \frac{exp\left(-\theta g^{ij}_{k\_bike}\right)}{exp\left(-\theta g^{ij}_{r\_bike}\right)} \hat{P}^{ij}_{r\_bike} + Positive\ Value$$

The relationship between the two travel modes shows that when bike stations and lanes are installed, $(2\text{-}x_s\text{-}x_w)+\sum_{a\in A_k}(1\text{-}h_a)=0$, resulting in the probability of choosing route $\hat{P}^{ijm}_{r_{t,v}}$ depending solely on the relative generalized costs, consistent with the IIA property. In contrast, when bike stations and lanes are not installed, Eq. (6)–(8) yield $(2\text{-}x_s\text{-}x_w)+\sum_{a\in A_k}(1\text{-}h_a)\geq 1$, making the probability $\hat{P}^{ijm}_{r\_(t,v)}$ dependent on more than just the relative generalized costs, because of violating the IIA property. This completes this proof.

3. Numerical example

An analysis of the characteristics of the MILP model was demonstrated through a simulation of the transportation network in Chiang Mai (**Figure 2**). Located in northern Thailand, the city is endowed with a rich cultural heritage, beautiful natural scenes, and lively urban life, all of which are key tourist attractions for both locals and foreigners. The growth in the city is greatly dependent on the tourism sector, and many activities are organized around its historical sites, outdoor activities, festivals celebrating colors, and markets. As part of the overall public transport master plan for the city, Chiang Mai is also developing a Light Rail Transit (LRT) system of three lines (blue, red, and green), aimed at linking some of the most important places and promoting sustainable mobility. Additionally, the government initiated a Non-Motorized Mobility program by integrating the BSS as a crucial component to complement the LRT, thereby providing an eco-friendly way of getting around in Chiang Mai. Computational experiments were conducted on a workstation with an Intel 11th Gen Core i5-11400H processing unit (2.70 GHz, 6 cores, 12 threads) and 16GB of RAM. The MILP was run in Python 3.10.11 using PuLP 2.7.0, run via the CBC solver from PyCharm 2024.3.1.1.

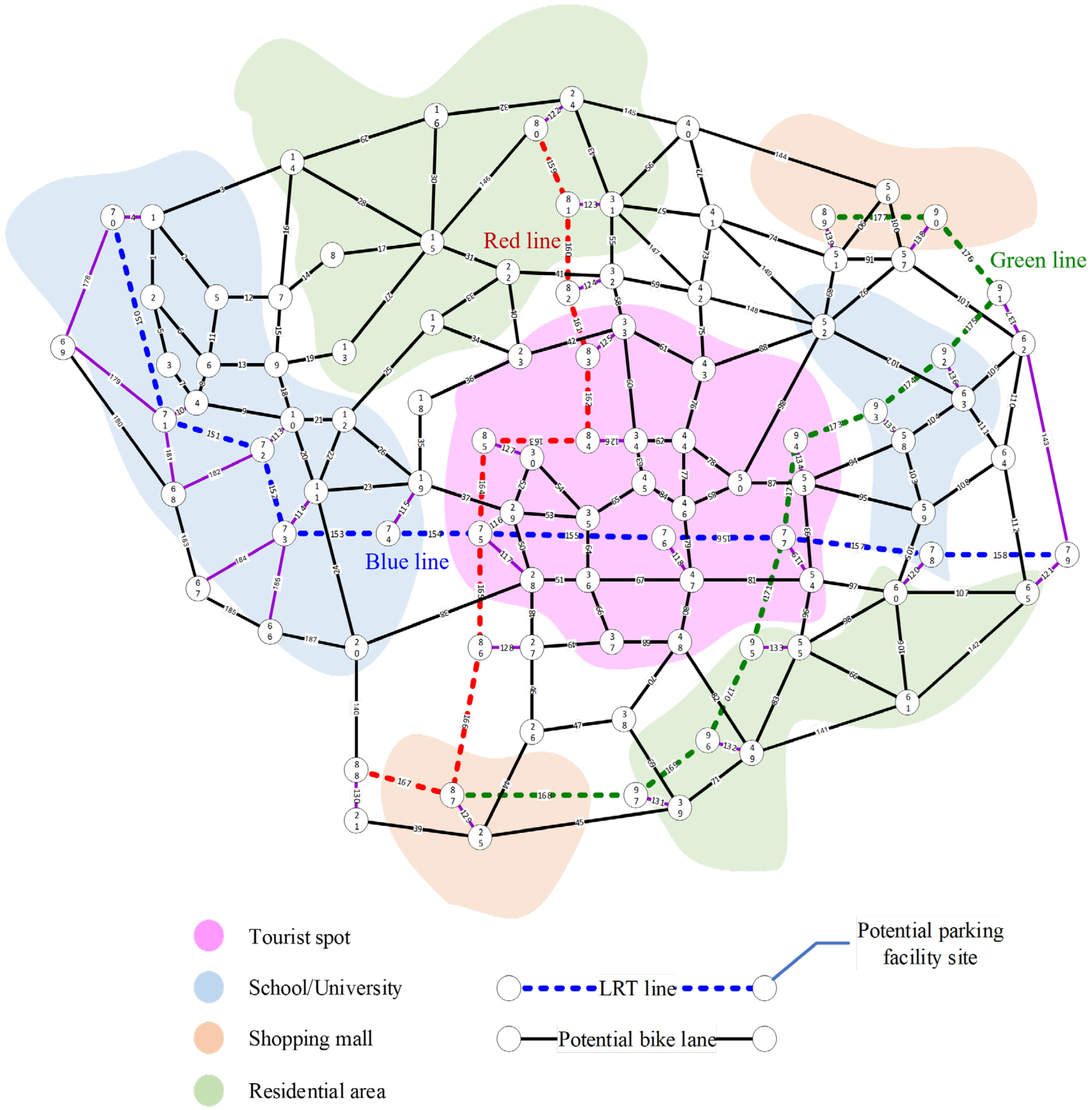

Figure 2 Potential bike lane and parking facility site in city of Chiang Mai

We set W_1 = 0.7 and W_2= 30 to balance the difference in scale between bike demand (hundreds) and equity (i.e., α typically 0.1–0.3). In scenario 1 without LRT integration (**Figure 3**), the model activates nodes of bike stations (x_t = *1*) and installs bike lanes (h_a = *1*) according to Eqs. (6)–(8) into a connected network that users can navigate through many possible routes in an urban area. The network still features widely distributed bike parking locations; however, in certain corridors not reached by these bike lanes, private cars may still be necessary.

The acceptable tolerance in bike usage to capacity ratio (α = 0.15) remains the highest at 0.27 and the lowest at 0.12, defining the maximum tolerance (α) value across all other stations, defining BSS equity aligning with Eq. (10). The bike usage to capacity ratio $\left(\frac{f_t^{[b]}}{C_t}\right)$ analysis, considering both pickup and drop-off stations, reveals that the maximum difference in bike usages to capacity defines α. A smaller α indicates higher equity across stations, whereas a larger α signifies lower equity (**Figure 4**). The similar pickup and drop-off ratios across stations indicate a balanced utilization of bicycles throughout the network.

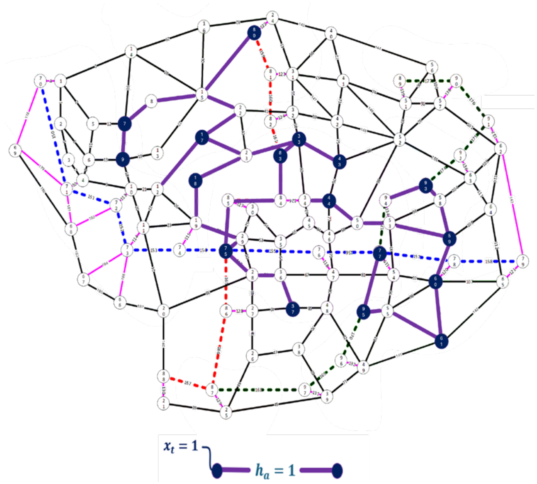

Figure 3 Bike Station and Lane Locations (Scenario 1)

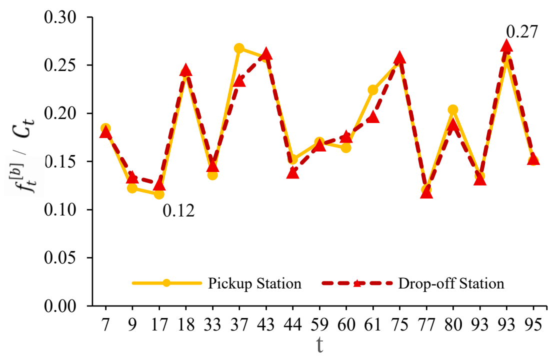

Figure 4 Station Usage Ratios $f_t^{[b]}/C_t$ at Pickup and Drop-off Station

In the BSS with LRT scenario (**Figure 5**), stations adjacent or on LRT stops would be in the modality to enhance accessibility for bike-transit choice even from short-distance cycling and long-distance BT. The system indicates that total travel distance (TTD) is 324.44 km to be dedicated to the bike routes: specifically, 95.12 km is set apart for the first and last mile trips, demonstrating increased bicycle use, specifically for short connections. LRT also changes BSS usage patterns dramatically, enhances first- and last-mile connections, and adds to the total travel distance to bicycle segments. However, equity concerns become more pronounced as demand accumulates at LRT stations.

The equity variable (α) is 0.28, indicating an increase in the overall usage of BT. This has mainly been attributed to higher bike replenishment at stations adjacent to LRT stops, which causes demand fluctuations throughout the network. The system maintains a reasonable level of equity among the increasing number of bike users, which implies the development of the bike-transit system.

Additionally, the TTD accounts for 100.4 km of walking, reflecting foot-based segments from one's origin to a bike or LRT station or from the station to the destination. Despite the availability of bike-transit (BT) travel, walking remains essential for short links, reinforcing the model's multimodal integration.

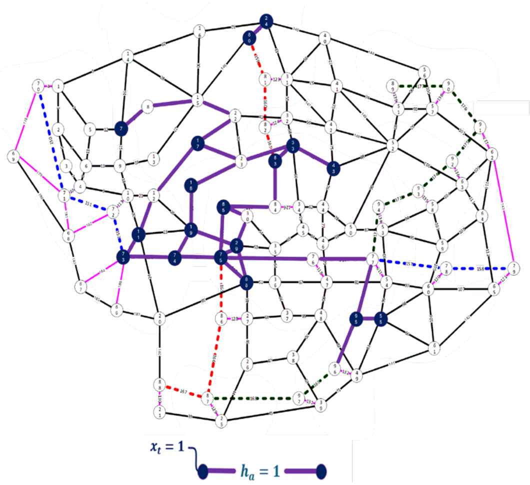

Figure 5 Example of Bike Station and Lane Locations (Scenario 2)

Comparison of the station usage ratios between the without-LRT mode (Scenario 1) and the LRT mode (Scenario 2) (**Figure 6**). In Scenario 2, most bike stations are installed near or at the LRT station (e.g., node 28) to accommodate bike-transit transportation, resulting in higher bike-user demand, reflecting the increased propensity for BT travel. LRT stations show significantly increased usage once a station is introduced there (e.g., node 75), whereas some stations (e.g., nodes 11 and 19) are no longer installed because demand has shifted toward BT alternatives, leading to station relocations closer to BT corridors. Bike utilization at each station stays within its capacity, guaranteeing that no station exceeds its allotted amount. To prevent overcrowding, the usage ratios remained within ψ (e.g., 0.8). (Eq.(1 1)). Optimizing ψ levels helps manage station usage by preventing overcrowding and ensuring a spatial equity distribution.

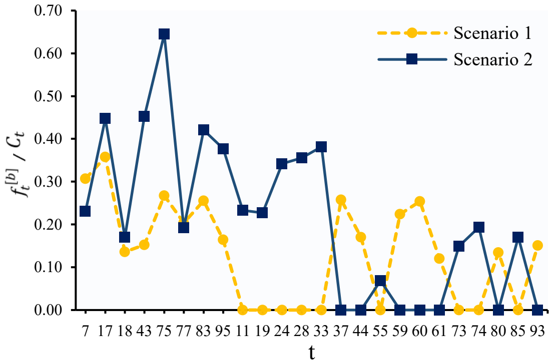

Figure 6 Comparison of Station Usage Ratios in Scenario 1 and Scenario 2

When varying the weights in the objective function, it was observed that increasing W_2 significantly reduces α (**Figure 7**), as the model places more emphasis on minimizing disparity across stations. Conversely, increasing W_1 leads to higher total bike demand (**Figure 8**), as the model prioritizes maximizing user adoption. This trade-off confirms the model's ability to respond to policy preferences between maximizing usage and promoting spatial equity.

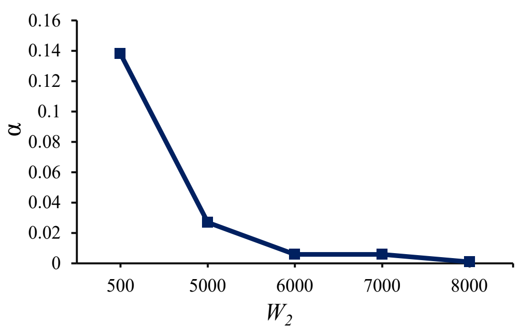

Figure 7 Effect of W_2 on α

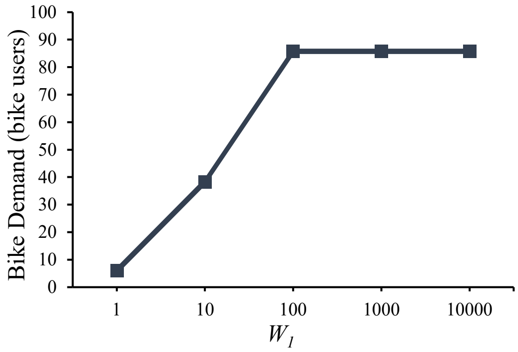

Figure 8 Effect of W_1 on Bike Demand

In a Multinomial Logit (MNL) model, the dispersion parameter (θ) governs the degree of stochasticity in choice behavior by scaling the generalized cost. In this study, we adopt the negative of the dispersion parameter, denoted as –θ, to represent the reversed scaling effect. This allows us to examine the relationship between the degree of stochasticity in choice behavior and the relative influence of generalized costs on decision-making. When θ is higher, stochasticity decreases, leading to more deterministic choices where the influence of lower $g_{r(t,v)}^{ijm}$ options dominate.

As θ decreases, travelers exhibit more stochastic behavior, making choices more dispersed to other modes rather than strictly favoring the lowest-cost cycling option (**Figure 9**). On the other hand, as θ increases, travelers become more sensitive to cost, increasing their preference for bicycles because of lower generalized costs. This shift is reflected in the total bike travel distance (TTD). In addition, walking still appears where station placement (t, v) does not align perfectly with origins (i) and destinations (j), thus supporting a foot-based link for shorter connections.

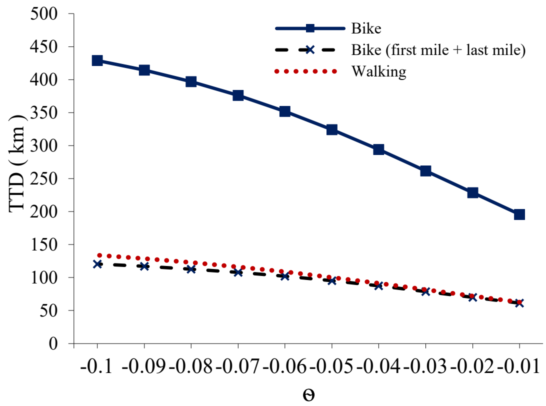

Figure 9 Effect of θ on TTD

While α increases alongside total bike demand (**Figure 10**), this change reflects a redistribution of demand rather than fundamental inequity (**Figure 11**). The model amplifies the cost differences, naturally guiding users toward more cost-effective stations and routes. However, this also ensures that bike availability remains sufficient across the network. Given a significant increase in users, the observed increase in α remains within a reasonable range meaning that this difference is still manageable and reflects cost-sensitivity in user preferences and station attractiveness. It does not cause severe imbalances or service failures but rather represents a practical trade-off between promoting bike adoption and maintaining spatial equity. indicating that the model contributes to the trade-off between adoption and equity.

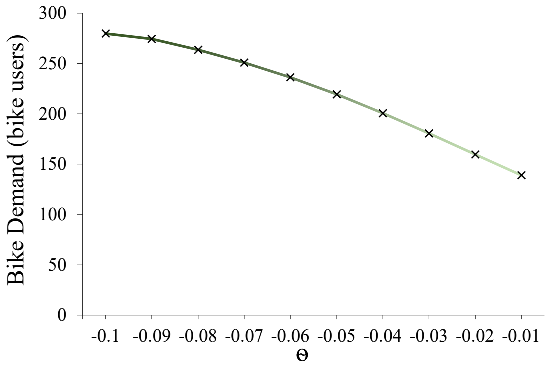

Figure 10 Effect of θ on Bike Demand

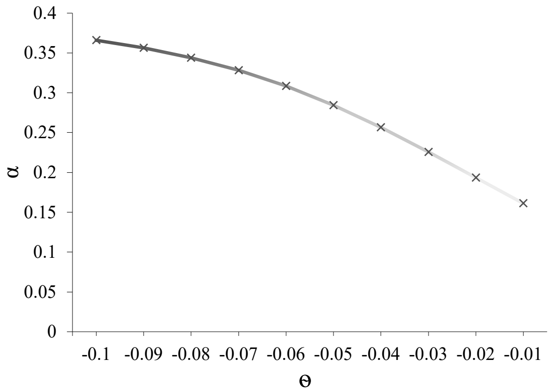

Figure 11 Effect of θ on Equity Distribution (α)

This effect is expected when travelers respond to financial incentives, even though increasing θ shifts demand toward more cost-effective corridors. These findings demonstrate the model's ability to change travel behavior while maintaining equity. Future adjustments to station locations and pricing strategies could further support the system's main goal of promoting bike utilization and spatial equity by further optimizing demand distribution.

4. Conclusions and remarks

This paper presented a framework for developing a bike-sharing system to address complicated urban travel demand through the Multinomial Logit (MNL) model and Maximum Capture Problem (MCP) to enhance spatial equity in bicycle access. Numerical analysis, applied to a simulated transportation network in Chiang Mai, explored the characteristics of a Mixed-Integer Linear Programming (MILP) model under different scenarios, particularly examining the impact of the dispersion parameter (θ), which governed stochasticity in travelers' decision-making. Findings revealed that as θ increased, travelers exhibited higher cost sensitivity, leading to an increased reliance on bicycles due to their lower generalized cost. This aligned with the goal of promoting sustainable mobility, as demonstrated by a rise in total bike travel distance (TTD) as well as increasing overall bike demand.

However, the natural consequence of heightened cost sensitivity is a more concentrated demand pattern, reflected in an increase in the equity index α. This illustrates how economic incentives influence user behavior in a cost-driven system, rather than a basic equity problem. The observed redistribution of demand followed a predictable trend: travelers preferred minimizing costs that led to density within corridors, while higher usage at certain stations corresponded to reduced growth at others. This indicates an inherent trade-off in maximizing efficiency with a perfectly evenly distributed demand. Notably, although clustered demand might have occurred, bicycle access remained sufficient across the system, ensuring that no areas faced critical shortages. The increased value of α is simply a natural outcome of cost optimization under efficiency among users in the system.

These findings reinforce the model's ability to maintain network accessibility in addition to stimulating total bicycle adoption. Further refinements could explore additional strategies to enhance the demand balance, while the present model could benefit from the incorporation of these adaptive pricing strategies, allowing for the maintenance of spatial equity with greater flexibility despite its existing customs of optimizing cost efficiency.

Examining the utility of bicycle users in relation to interactions with other modes of transportation and the non-exclusive use of bike lanes presents an intriguing avenue for future research. Incorporating factors such as inconvenience and discomfort associated with bicycle usage could provide a more accurate reflection of real-world scenarios.

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6. References

[1] J. Bukhari, A. G. Somanagoudar, L. Hou, O. E. Herrera and W. Mérida, "Zero-Emission Delivery for Logistics and Transportation: Challenges, Research Issues, and Opportunities," *ArXiv*, vol. 2205.15606, pp. 1–20 2022, doi: 10.48550/arXiv.2205.15606.

[2] Y. Ou, Z. Bao, S. Thomas Ng, and W. Song, "Estimating the effect of air quality on Bike-Sharing usage in Shanghai, China: An instrumental variable approach," *Travel Behaviour and Society*, vol. 33, 2023, doi: 10.1016/j.tbs.2023.100626.

[3] F. U. Rehman, M. M. Islam and Q. Miao, "Environmental sustainability via green transportation: A case of the top 10 energy transition nations," *Transportation Policy*, vol. 137, pp. 32–44, 2023, doi: 10.1016/j.tranpol.2023.04.013.

[4] A. Mauttone, G. Mercadante, M. Rabaza and F. Toledo, "Bicycle network design: Model and solution algorithm," *Transportation Research Procedia*, vol. 27, pp. 969–976, 2017, doi: 10.1016/j.trpro.2017.12.119.

[5] Z. Yin, Y. Guo, M. Zhou, Y. Wang and F. Tang, "Integration between Dockless Bike-Sharing and Buses: The Effect of Urban Road Network Characteristics," *land*, vol. 13, no. 8, 2024, Art. no. 1209, doi: 10.3390/land13081209.

[6] S. Liu, Z. -J. M. Shen and X. Ji, "Urban Bike Lane Planning with Bike Trajectories: Models, Algorithms, and a Real-World Case Study," *Manufacturing & Service Operations Management*, vol. 24, no. 5, pp. 2500–2515, 2022, doi: 10.1287/msom.2021.1023.

[7] C. Karolemeas, A. Vassi, S. Tsigdinos and E. Bakogiannis, "Measure the ability of cities to be biked via weighted parameters, using GIS tools. The case study of Zografou in Greece," *Transportation Research Procedia*, vol. 62, pp. 59–66, 2022, doi: 10.1016/j.trpro.2022.02.008.

[8] J. -R. Lin and T. -H. Yang, "Strategic design of public bicycle sharing systems with service level constraints," *Transportation Research Part E: Logistics and Transportation Review*, vol. 47, no. 2, pp. 284–294, 2011, doi: 10.1016/j.tre.2010.09.004.

[9] Y. Yao, Y. Zhang, L. Tian, N. Zhou, Z. Li and M. Wang, "Analysis of Network Structure of Urban Bike-Sharing System: A Case Study Based on Real-Time Data of a Public Bicycle System," *sustainability*, vol. 11, no. 19, 2019, doi: 10.3390/su11195425.

[10] T. Ahmed, A. Pirdavani, G. Wets and D. Janssens, "Bicycle Infrastructure Design Principles in Urban Bikeability Indices: A Systematic Review," *Sustainability*, vol. 16, no. 6, 2024, doi: 10.3390/su16062545.

[11] J. -R. Lin, T. -H. Yang, and Y. -C. Chang, "A hub location inventory model for bicycle sharing system design: Formulation and solution," *Computers & Industrial Engineering*, vol. 65, no. 1, pp. 77–86, 2013, doi: 10.1016/j.cie.2011.12.006.

[12] Y. Lyu, M. Cao, Y. Zhang, T. Yang and C. Shi, "Investigating users' perspectives on the development of bike-sharing in Shanghai," *Research in Transportation Business & Management*, vol. 40, 2021, Art. no. 100543 doi: 10.1016/j.rtbm.2020.100543.

[13] L. Conrow, A. T. Murray and H. A. Fischer, "An optimization approach for equitable bicycle share

station siting," *Journal of Transport Geography*, vol. 69, pp. 163–170, 2018, doi: 10.1016/j.jtrangeo.2018.04.023.

[14] D. Duran-Rodas, B. Wright, F. C. Pereira and G. Wulfhorst, "Demand And/oR Equity (DARE) method for planning bike-sharing," *Transportation Research Part D: Transport and Environment*, vol. 97, 2021, Art. no. 102914, doi: 10.1016/j.trd.2021.102914.

[15] K. Tavassoli and M. Tamannaei, "Hub network design for integrated Bike-and-Ride services: A competitive approach to reducing automobile dependence," *Journal of Cleaner Production*, vol. 248, 2020, Art. no. 119247, doi: 10.1016/j.jclepro.2019.119247.

[16] J. Bao, T. He, S. Ruan, Y. Li and Y. Zheng, "Planning Bike Lanes based on Sharing-Bikes' Trajectories," in *Proc. 23rd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, Halifax, Canada, 2017, pp. 1377–1386, doi: 10.1145/3097983.3098056.

[17] K. Ketikidis, A. Papagiannakis and S. Basbas, "Identifying and Modeling the Factors That Affect Bicycle Users' Satisfaction," *sustainability*, vol. 15, no. 18, 2023, Art. no. 13666, doi: 10.3390/su151813666.

[18] M. Padeiro, "Cycling infrastructures and equity: an examination of bike lanes and bike sharing system in Lisbon, Portugal," *Cities & Health*, vol. 7, no. 5, pp. 729–743, 2022, doi: 10.1080/23748834.2022.2084589.

[19] S. Shaheen and A. Cohen. *Shared Micromobility Policy Toolkit Docked and Dockless Bike and Scooter Sharing*, Institute of Transportation Studies, Berkeley, 2019. Accessed: Feb. 4, 2025. [Online]. Available: https://doi.org/10.7922/g2th8jw7.

[20] P. DeMaio, "Bike-sharing: History, Impacts, Models of Provision, and Future," *Journal of Public Transportation*, vol. 12, no. 4, pp. 41–56, 2009, doi: 10.5038/2375-0901.12.4.3.

[21] S. A. Shaheen, S. Guzman and H. Zhang, "Bikesharing in Europe, the Americas, and Asia," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 2143, no. 1, pp. 159–167, 2010, doi: 10.3141/2143-20.

[22] E. Fishman, "Introduction," in *Bike Share*, 1st ed. New York, NY, USA: Routledge, 2019, ch. 1, sec. 1.4, pp. 8–9.

[23] Z. Chen, D. van Lierop and D. Ettema, "Dockless bike-sharing systems: what are the implications?," *Transport Reviews*, vol. 40, no. 3, pp. 333–353, 2020, doi: 10.1080/01441647.2019.1710306.

[24] M. Castiglione, R. De Vincentis, M. Nigro and V. Rega, "Bike Network Design: an approach based on micro-mobility geo-referenced data," *Transportation Research Procedia*, vol. 62, pp. 51–58, 2022, doi: 10.1016/j.trpro.2022.02.007.

[25] M. Fazio, N. Giuffrida, M. Le Pira, G. Inturri and M. Ignaccolo, "Bike oriented development: Selecting locations for cycle stations through a spatial approach," *Research in Transportation Business & Management*, vol. 40, 2021, Art. no. 100576, doi: 10.1016/j.rtbm.2020.100576.

[26] R. Mix, R. Hurtubia and S. Raveau, "Optimal location of bike-sharing stations: A built environment and accessibility approach," *Transportation Research Part A: Policy and Practice*, vol. 160, pp. 126–142, 2022, doi: 10.1016/j.tra.2022.03.022.

[27] L. Caggiani, R. Camporeale, B. Dimitrijević and M. Vidović, "An approach to modeling bike-sharing systems based on spatial equity concept," *Transportation Research Procedia, vol. 45*, pp. 185–192, 2020, doi: 10.1016/j.trpro.2020.03.006.

[28] J. P. Ospina, J. C. Duque, V. Botero-Fernández and A. Montoya, "The maximal covering bicycle network design problem," *Transportation Research Part A: Policy and Practice*, vol. 159, pp. 222–236, 2022, doi: 10.1016/j.tra.2022.02.004.

[29] M. Akbarzadeh, S. S. Mohri and E. Yazdian, "Designing bike networks using the concept of network clusters," *Applied Network Science*, vol. 3, no. 1, 2018, Art. no. 12, doi: 10.1007/s41109-018-0069-0.

[30] D. Guo, E. Yao, S. Liu, R. Chen, J. Hong and J. Zhang, "Exploring the role of passengers' attitude in the integration of dockless bike-sharing and public transit: A hybrid choice modeling approach," *Journal of Cleaner Production.*, vol. 384, 2023, Art. no. 135627, doi: 10.1016/j.jclepro.2022.135627.

[31] B. Wei and L. Zhu, "Exploring the Impact of Built Environment Factors on the Relationships between Bike Sharing and Public Transportation: A Case Study of New York," *ISPRS International Journal of Geo-Information*, vol. 12, no. 7, 2023, Art. no. 12, doi: 10.3390/ijgi12070293.

[32] L. Caggiani, A. Colovic and M. Ottomanelli, "An equality-based model for bike-sharing stations location in bicycle-public transport multimodal mobility," *Transportation Research Part A: Policy and Practice*, vol. 140, pp. 251–265, 2020, doi: 10.1016/j.tra.2020.08.015.

[33] S. R. Gehrke, A. Akhavan, P. G. Furth, Q. Wang and T. G. Reardon, "A cycling-focused accessibility tool to support regional bike network connectivity," *Transportation Research Part D: Transport and Environment*, vol. 85, 2020, Art. no. 102388, doi: 10.1016/j.trd.2020.102388.

[34] Y. Sheffi, "Discrete Choice Models and Traffic Assignment," in *Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods*, 1st ed. Englewood Cliffs, NJ, USA: Prentice-Hall, 1985, ch. 10, pp. 262–284.

[35] Office of Transport and Traffic Policy and Planning, "Chiang Mai Public Transportation Master Plan," OTP, Bangkok, Thailand, Final Rep., 2017. Accessed: Sept. 23, 2025. [Online]. Available: https://www.otp.go.th/edureport/view?id=129.