A Survey of Approximate Methods for the Traveling Salesman Problem

Impacts of Wind Energy on Generation Adequacy

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Abstract

The traveling salesman problem (TSP) is a famous problem in the class of combinatorial optimization problems. Its special case is a symmetric TSP in which the matrix of the distances between each pair of cities is symmetric. Several approaches have been proposed up to date for solving the symmetric TSP, especially the approximate methods, which have made remarkable advances. This article aims to survey these techniques including the approximation algorithms, the neural network methods, and the metaheuristics. The hybridization schemes based on these manners are also reviewed.

Keywords

approximation algorithms, neural network, metaheuristics, symmetric TSP, review

1. Introduction

The traveling salesman problem (TSP) is one of the well-known *NP*-hard problems in the field of the combinatorial optimization. The objective of the TSP is to find the shortest Hamiltonian cycle among n cities, where the salesman visits each of the n cities exactly once and then returns to the starting city. The TSP can be classified into two types: symmetric and asymmetric cases. In the symmetric case, the distance between each pair of cities is the same in each opposite direction, i.e., $d_{ij} = d_{ji}$ for all cities i, j in $N = \{1, 2, 3, \ldots, n\}$. Otherwise, the distance between two cities is different. However, this article focuses on the symmetric TSP.

There are several algorithms designed to solve the symmetric TSP [1]. The brute-force exact method would be to try all possible permutations. This problem is traditionally solved by an exact algorithms based on an integer linear programming, i.e., the cutting plane method [2], the branch and bound method [3], and other exact algorithms appeared in [1,4]. However, this paper aims to survey the approximate methods that are proposed for solving the symmetric TSP in the last decade.

The rest of paper is organized as follows. A survey of some classical approximation algorithms is provided in Section 2. Section 3 provides a review of methods based on the neural network techniques, and Section 4 gives a survey of metaheuristics. Finally, Section 5 concludes this paper.

2. Conventional Approximate Methods

Mathematicians propose many approximation algorithms (or heuristics) for solving a near-optimal solution. One of traditionally approximate approaches is the nearest neighbor heuristic, which is used to construct a good initial tour [5]. In addition, some heuristics are still implemented for solving the TSP in some recent literatures, including the Lin-Kernighan heuristic, the genetic algorithm, and the simulated annealing algorithm, as described below.

2.1 Lin-Kernighan heuristic

The Lin-Kernighan (LK) heuristic [6] is designed to generate *k*-opt moves in the neighborhood by deleting *k* edges and inserting *k* new edges that yield an improvement of these moves. In addition, Helsgaun [7] improves the design and implementation of LK heuristic and shows that his modified Lin-Kernighan-Helsgaun (LKH) version is highly effective. Later, he modifies the k-exchange neighborhood strategy in his LKH version, and it is highly effective and efficient for solving extremely large problems [8].

The LK heuristic is a notably original version of the k-exchange neighborhoods which it decomposes the sequence of cities into the 2- or 3-opt move [6,8]. The 2-opt algorithm is the well-known simple local search that executes on edges recombination [4,9], and it is a special case of the k-exchange neighborhoods [7]. Moreover, the LK heuristic and the 2-opt algorithm are embedded as the improvement procedure in other approaches for solving the TSP [9 - 12].

2.2 Genetic algorithm

The genetic algorithm (GA) is a well-known heuristic based on a randomized search strategy inspired by natural evolution [13]. It consists of the evaluation, crossover, and mutation steps. The evaluation step reproduces a population of chromosomes, and better chromosomes (based on their fitness value) are selected for the next generation of population with some probability. The crossover step randomly selects pairs of survival chromosomes to the next generation and mates them for producing new chromosomes. The mutation step randomly chooses a chromosome completed by the crossover step and mutates it at a particular point for a new population. All steps are repeated until some stopping criterion is reached.

In order to execute on GA operators, many coding and representations are presented for crossover and mutation operators. Larrañaga et al. [14] provide a review of these different GA representations and operators for the TSP. Many variations of GA applied for the TSP are proposed throughout the past three decades, for example:

Marinakis *et al*. [15] utilize the GPRSP technique in [10] to generate an initial population of GA, and employ the Lagrangean relaxation method to compute the upper and lower bounds of tour length that are utilized to calculate the threshold value in the termination process. Yang et al. [12] propose the generalized chromosome genetic algorithm (GCGA) to solve the symmetric TSP.

2.3 Simulated annealing algorithm

The simulated annealing (SA) algorithm is a method that imitates the material annealing process in metallurgy [4]. For solving the TSP, SA starts from an initial tour (at high temperature), and the temperature is gradually decreased to achieve a minimum tour length. It generates a new tour and calculates the difference in tour length between the current tour and a new tour. If the new tour is shorter, it becomes the current tour; otherwise, the new tour is accepted with some probability. These processes are repeated until some stopping criterion is reached.

Tian *et al.* [16] suggest that the perturbation scheme with a reverse and/or move a partial sequence of cities in a tour is the most efficient mechanism to design the neighborhood structure in the SA algorithm for solving the symmetric TSP. Later, Liu et al. [17] apply this perturbation scheme in their proposed algorithm to generate the neighboring solution. An adaptive simulated annealing algorithm with greedy search [18] combines the vertex-insert mutation, block-insert mutation, and block-reverse mutation strategies to generate the candidate solution and applies the greedy search to speed up the convergence rate.

This section provides a review of conventional approximation algorithms and their variants for the symmetric TSP. The next section discusses the methods-based neural network techniques applied to solve the symmetric TSP.

3. Neural Network Techniques

Over the past three decades, an enormous amount of literature involves the design of algorithms based on the neural networks derived from a variety of learning principles [19]. In the TSP context, the nearest neuron is mapped to a data point (city) presented in the neural network, and then this neuron and its related data points are initiated into a set of neurons. Next, the rest of neurons (outside the set) are moved toward the first data point into the target location. These processes are repeated until all data points are in the target location.

Cochrane and Beasley [19] provide a review of other neural network approach for solving the TSP published between 1985 and 2000; in addition, some algorithms proposed in the past decade can be seen in [20]. This section provides a review of some algorithms that are widely used for the performance comparison, as follows:

Cochrane and Beasley [19] introduce the co-adaptive net algorithm based on self-organizing map that combines lateral interactions between neurons or co-operation between them. Later, the constructive-optimizer neural network [20] utilizes a feedback structure (similar to the Hopfield-type neural networks) and a competitive training algorithm (similar to the Kohonen-type self-organizing maps). Moreover, the improved elastic net algorithm [21] applies several time-dependent parameters into the original elastic net algorithm as the penalty.

This section provides a review of the algorithms-based neural network schemes for solving the TSP. The next section provides a survey of metaheuristics that are recently proposed to solve the symmetric TSP.

4. Metaheuristics

In the last decade, metaheuristics provide a generalized frameworks to build heuristics for solving the combinatorial optimization problems by efficiently exploring the search space in order to produce high-quality solutions [22]. The techniques range from simple local search procedures to complex learning processes, and they may incorporate with mechanisms to avoid getting the local optima traps. Moreover, the genetic algorithm and the simulated annealing algorithm are sometimes classified as metaheuristics [22]. The metaheuristics for solving the TSP are discussed below.

4.1 Ant colony optimization

The ant colony system (ACS) [23] is based on the nature-inspired evolution process, where a set of ants is generated for searching for a good solution in parallel and for exploiting via pheromone information. Later, Qi [24] applies the randomized algorithm into the tour construction procedure of ACS for decreasing the time to compute the transition probability that the ant moves from the current city to a new city. Moreover, ACS can incorporate other metaheuristics, such as the cooperative genetic ant systems [25].

4.2 Iterated local search

The iterated local search (ILS) is a simple and widely applicable metaheuristic which iteratively applies the local search to improve a candidate solution [26]. ILS applies local search by moving from one solution to another in the search space with the improving change in the value of objective function until some stopping criterion is satisfied. In addition, the acceptance criterion is applied for accepting the new solution during two loops of the ILS.

Several algorithms share the ILS characteristics, but they are under different names: chained local optimization [27], iterated tabu search [28], and variable neighborhood search [29]. Furthermore, the ILS with the Lin-Kernighan local search procedure is reported as the most powerful algorithm for TSP [9,27].

4.3 Other metaheuristics

The memetic algorithm (MA) proposed by Merz and Freisleben [30] utilizes the generic greedy recombination operator to generate the offspring tour from the parents, and applies the edge exchange strategy of the Lin-Kernighan local search into the mutation operator. The hybridization manners of the MA with other methods can be seen in [31]. Moreover, the adaptive tabu search [32] utilizes the backtracking method to perturb the solution and uses the adaptive radius strategy to intensify the search in the near-optimal region. The particle swarm optimization for solving the symmetric TSP is presented in [33], and the honey bees mating optimization algorithm is proposed by Marinakis *et al*. [34].

The summary of some approximate approaches published in the last decade is shown in Table 1. The classification of these methods arranges on the population-based approaches, the non-population-based approaches or the single solution search methods, and the hybrid schemes. Moreover, the assessments on the strengths and weaknesses of these methods are provided.

Table 1 The summary of some approximation methods published in the last decade

Method	Year	Evaluation	
		Strengths	Weaknesses
Non-population-based approaches (single solution search)			
Greedy randomized adaptive search procedure (GRASP) [10]	2005	• Generate an initial tour by randomly selecting the candidate edges from list of the best promising candidate edges.	• The neighborhood search space is limited and the solution may back to the visited solutions during a run.
Lin-Kernighan-Helsgaun heuristic (version 2) [8]	2009	• Allow moving with any *k*-submove and searching for both sequential moves and non-sequential moves.	• The value of k during program execution is constant.
Adaptive simulated annealing algorithm with greedy search [18]	2011	• Combine three different mutation strategies to generate an initial solution.	• The solution may back to the visited solutions during a run.
		• Control parameter is adaptable and dynamic during the algorithm execution.	
Adaptive tabu search [32]	2012	• Diversify the search by the backtracking method and intensify the search for reaching a global optimum.	• An initial tour is constructed by using a random scheme, which may not generate a good initial tour.
Population-based approaches			
Generalized chromosome genetic algorithm (GCGA) [12]	2008	• Remove the limitation of triangle inequality of the cost matrix.	• A more effective local search method is needed.
Multiple phase neighborhood search-GRASP algorithm [11]	2009	• Utilize flexible scheme of the neighborhood search.	• The running time for solving large instances should be further diminished.
Genetic-based particle swarm optimization algorithm [33]	2012	• Reduce the complexity of searching process with large number of cities before using the PSO algorithm.	• For large instance size, the search performance depends on the specified number of clusters.

Table 1 The summary of some approximation methods published in the last decade (Continued)

Method	Year	Evaluation	
		Strengths	Weaknesses
Hybrid approaches			
Ant colony system hybridized with randomized algorithm [24]	2007	• Reduce the computational time for computing the transition probability that the ant moves between solutions.	• The vast running time still occurs in large TSP instance that its size is larger than 500 cities.
Hybridization of tabu search and simulated annealing algorithms based on adaptive cooling schedule [17]	2009	• Expand the neighborhood search and eliminate the backtracking solutions.	• The search process still chances to encounter worse solutions.
		• Allow upward and down move of temperature in the cooling schedule.	• The algorithm should be executed on larger TSP problem.
Honey bees mating optimization algorithm [34]	2011	• Utilize several neighborhood search strategy and apply adaptive memory into crossover operator to store the queen.	• The running time should be further diminished.
Cooperative genetic ant systems (CGAS) [25]	2012	• Combine and execute concurrently between GA and ACS.	• The vast runtime occurs in large TSP instance.

5. Conclusion

This paper provides an overview of recent advances in the approximate methods for solving TSP, including the approximation algorithms, the neural network techniques, the metaheuristics, and the hybridization of metaheuristics. To develop the approach further for the TSP, the hybridization of metaheuristics is a competitive and effective method, and it becomes a recent direction of the approximation manner. This technique should consist of the diversification and intensification strategies, the adaptable scheme of searching in several neighborhood spaces, the elimination of turning back to the visited solutions, the restart manner for escaping from local optima regions, and the parameter setting with problem constraints. Finally, the integration with Lin-Kernighan local search is the most effective and efficient technique to improve the search performance of metaheuristic in both terms of the solution quality and the computational time.

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