

Research Article

Hydrodynamic Analysis of a Two-Lobe Bearing Lubricated with a non-Newtonian Fluid: Effects of Geometric and Rheological Parameters on Operating Characteristics

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Received 26 July 2025

Revised 25 October 2025

Accepted 2 November 2025

Abstract:

In the context of hydrodynamic lubrication, this paper investigates the contribution of the aspect ratio R/L to the static performance of two-lobe bearings, by incorporating the non-Newtonian rheological response of the lubricant modeled by the power law. The Reynolds equation is reformulated to incorporate fluid rheology and solved numerically using the finite difference method to determine the pressure profile within the lubricant film. The analysis focuses on the cross-impact of the aspect ratio, the power law index n , and the eccentricity ratio ϵ , on key parameters such as film thickness, pressure, load-carrying capacity, attitude angle, frictional force and coefficient, as well as the side leakage. The results indicate that film thickness is only marginally affected by the R/L ratio, but is highly sensitive to the lubricant's rheological properties, especially in high shear zones. A reduction of R/L or an increase of n improves the hydrodynamic support. Furthermore, shear-thickening fluids promote the load-carrying capacity and tribological efficiency, while long bearings exhibit increased sensitivity to leakage effects.

Keywords: Two lobe journal bearings, non-Newtonian lubricant, Power law model, Static characteristics, Aspect ratio

1. Introduction

Compared to conventional circular hydrodynamic bearings, two-lobe hydrodynamic bearings, one of the non-circular configurations, have been designed to strengthen the stability and performance of rotating machinery [1]. Two-lobe hydrodynamic bearings exhibit an asymmetric profile, producing two separate lobes, in contrast to circular bearings, which have uniform clearance around the shaft [2]. Through the creation of two separate pressure zones that better support the shaft, this non-circular structure produces variations in film thickness and pressure profile, increasing load-carrying capacity and lowering the risk of lack of stability in high velocity applications [3]. Furthermore, two-lobe bearings' design enables greater control over their dynamic behavior, which makes them especially appropriate for applications requiring precise rotor control and vibration reduction [2]. This innovative design also reduces frictional power losses, which improves overall system efficiency [4]. As a result, the system life is extended and bearing performance is optimized [1]. Pinkus [3] conducted the preliminary study on two-lobe bearings. The dynamic and static characteristics of four different multi-lobe hydrodynamic bearing configurations were examined by Lund et al. [5].

micropolar fluid lubrication. The impact of aspect ratio on the performance and stability of hydrodynamic journal bearings was investigated by Sharma et al. [33]. Sharma and Awasthi [34] explored the impact of aspect ratio on the

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Sehgal et al. [6] used the method of McCallion et al. [7] for circular bearings, which entails decoupling the Reynolds equation from the thermal transport equation by removing the pressure-dependent term, and focusing instead on heat transfer across the bearing, to perform a theoretical comparison of the thermal response of three bearing configurations: circular, axial groove, and half-offset, in the THD (Thermo-Hydrodynamic) case. The influence of sleeve deformation on the steady response and dynamic behavior of elliptical (two-lobe) bearings operating under micropolar lubrication was presented by Nair et al. [8]. Crosby and Chetti [9] conducted a comparative analysis on the steady and rotor-dynamic characteristics of 2-lobed hydrodynamic bearings, under couple stress liquid model lubrication, and the findings were evaluated against those obtained using classical Newtonian liquids. Using a full 3D TEHD (Thermo-Elastohydrodynamic) model to analyze lubricant performance, Huang et al. [10], presented a comparative study of the various performances of circular, two-lobe, and elliptical bearings. Jain and Sharma [11] investigated the effect of shape irregularities on the dynamic performance of a fluid-lubricated, capillary-compensated, hybrid bearing with four pockets and two lobes using a power-law model. An explicit analytical solution to the Reynolds equation for a finite-length three-lobe bearing was developed by Chasalevris [12]. By altering the eccentricity ratios and bearing speeds, Dang et al. [13] investigated the steady-state thermal performance of an elliptical bearing lubricated with mineral oils based on nanoparticles. Pradhan et al. [14] used the Grey-based Taguchi optimization model [15] which was created with four two-level factors to adjust the bearing constituent material, structural form, and functional settings to study an elliptical bore bearing, including material modeling with isotropic roughness to determine the maximum load-bearing conditions. The characteristics of hydrodynamic bearings with various configurations, including two-lobe bearings operating with non-Newtonian lubricants, were examined by Narwat et al. [16], and Their research shed light on the influence of various bearing geometries on performance metrics such as the thinnest fluid film profile, the fluid film pressure generation, and operational parameters. Kumar et al. [17] compared the stability margin and dynamic performance of circular bearings with multi-lobe bearings (2-4 lobes). Using the molecular gas lubrication (MGL) model, Gharanjik et al. [18] examined the effects of cooling and rotational speed on the non-linearized static dynamic behavior of non-circular three-lobe micro-gas bearings. Using electrorheological (ER) lubricants, Narwat et al. [19] assessed the stability and dynamic performance of a two-lobe hydrodynamic bearing in comparison to a circular bearing.

Numerous studies have investigated hydrodynamic bearings lubricated with power-law fluids to gain deeper insight into their performance across different operating regimes. In circular journal bearings, Dien and Elrod [20] formulated a modified Reynolds equation for lubrication with non-Newtonian fluids based on the power law, characterizing the pressure and velocity field using the regular perturbation technique. A comprehensive analysis of the static and dynamic performance of hydrodynamic bearings lubricated with nanofluids was conducted by Nair et al. [21]. By integrating the influence of lubricant rheology and surface texture, Mongkolwongrojn et al. [22] employed the power law model to assess the stability of thermo-electrohydrodynamic bearings. Taking thermo-hydrodynamic interactions into account, the lubrication in journal bearings using non-Newtonian liquids, described by the power-law rheological model, was examined by Nessil et al. [23]. Mouassa et al. [24] conducted a thorough theoretical and numerical study on the effect of lubricant rheology on the steady-state behavior of flexible and finite plain bearings lubricated with piezoviscous fluids under isothermal conditions. A comparative assessment of hydrodynamic circular and four non-circular bearing configurations, under stationary conditions and lubricated with non-Newtonian liquids described by the power-law rheological model, was carried out by Das and Roy [25]. In a numerical study, Chetti and Zouggar [26] examined the influence of viscoelastic distortion on the steady-state response of a cylindrical journal bearing lubricated with shear-dependent liquids governed by a power-law rheological model. Kumar and Kakoty [27] adopted the Krieger-Dougherty model to evaluate the influence of nanoparticle concentrations on lubricant flow resistance and reformulated the Reynolds formulation to incorporate the couple stress rheological model, thereby studying the stationary parameters of 2-lobed bearings operating with shear-dependent liquids. In a separate theoretical analysis, the impacts of viscosity fluctuations as a function of pressure and viscoelastic deformation on the performance of a circular bearing lubricated with non-Newtonian fluids were examined by Chetti et al. [28]. A recent investigation conducted by Bennaceur and Chetti [29] explored the impact of power law model non-Newtonian lubricants on the static performance of two-lobe hydrodynamic journal bearings.

In several research, the impact of various aspect ratios on wear reduction, vibration control, and bearing load carrying capacity has been investigated. Using a variety of aspect ratios, Batra et al. [30] investigated the performance stability of an inverted three-lobe pressure dam bearing. Verma et al. [31] conducted a theoretical study comparing the performance of a multi-recess hydrostatic journal bearing lubricated with micropolar fluids at various aspect ratios with that of a bearing lubricated with Newtonian fluid, using the finite element method for analysis. Sharma et al. [32] analyzed the static and dynamic behavior of a two-lobe pressure-dam bearing at different aspect ratios using

static, dynamic, and stability performances of worn hydrodynamic journal bearings, across a broad range of Sommerfeld numbers. The impact of aspect ratio change on the properties of journal bearings lubricated by non-Newtonian fluids was investigated by Khan et al. [35], varying the power law indices. A linear and nonlinear dynamic analysis was presented by Zare Mehrjardi et al. [36] to investigate the impact of changing the aspect ratio of a non-circular lobe bearing lubricated with micropolar fluids. Ouyang et al. [37] studied the dynamic characteristics of high aspect ratio journal bearings under shaft bending conditions.

Numerous studies have explored non-circular two-lobe hydrodynamic journal bearings, focusing on their film lubrication and the impact of various geometric parameters on their performance [38-41]. Two-lobe bearings are widely employed in high-speed turbomachinery, gas compressors [42], and steam/gas turbines [43], where non-Newtonian or shear-dependent lubricants (such as polymer-enhanced oils, nanoparticle suspensions, or synthetic blends) are increasingly used to improve energy efficiency and thermal stability. This present study holds significant relevance for real bearing design and operation. However, despite these contributions, the combined influence of the aspect ratio (R/L) and the power-law rheological model on the hydrodynamic response of two-lobe journal bearings remains insufficiently explored. Earlier research predominantly focused on either geometric variation within Newtonian or alternative non-Newtonian models, or power-law lubrication under a fixed bearing configuration, neglecting their synergistic impact.

In this work, we propose an original coupled analysis of the influence of the aspect ratio R/L and the rheological behavior according to the power law model on the performance of two-lobe hydrodynamic bearings. The study examines the effect of this interaction on the pressure distribution, load-carrying capacity, tribological behavior, and lateral leakage, for different levels of eccentricity ϵ . The results highlight the understanding of the geometry-rheology interaction, which provides essential support for optimizing bearing selection, improving load capacity, and reducing friction losses under demanding operating conditions. They also provide relevant guidance to designers to determine whether a wider or narrower configuration is best suited for the intended application.

From the findings, it becomes clear that this work lays the groundwork for analyzing the combined influence of the aspect ratio (R/L) and fluid rheology (n). Beyond the theoretical contribution, the results also provide a practical reference for engineers when selecting bearing configurations in modern rotating systems. In general, wider bearings tend to perform better under heavy loads and high speeds where film stability is critical, while narrower ones are often preferred in smaller or moderate-load machines where minimizing friction losses is more important.

2. Mathematical and computational model

2.1 Governing equation

Fig. 1 illustrates the structure of the two-lobe bearing used. Considering the non-Newtonian rheological properties of lubricants characterized by the power-law rheological model, the Reynolds equation governing the behavior of a two-dimensional incompressible lubricating film can be formulated as follows [26]:

$$\frac{\partial}{\partial x} \left[\frac{h^{n+2}}{n} \frac{\partial P}{\partial x} \right] + \frac{\partial}{\partial z} \left[h^{n+2} \frac{\partial P}{\partial z} \right] = 6(\omega R)^n \frac{\partial h}{\partial x} \quad (1)$$

Introducing the following dimensionless variables enables the modified Reynolds equation to be expressed in a dimensionless form:

$$\theta = \frac{x}{R}, \quad \bar{z} = \frac{z}{L}, \quad \bar{P} = \frac{Pc^2}{\mu UR}, \quad \bar{h} = \frac{h}{c}$$

The modified Reynolds equation, therefore, takes the form [26]:

$$\frac{\partial}{\partial \theta} \left[\frac{\bar{h}^{n+2}}{12} \frac{\partial \bar{P}}{\partial \theta} \right] + n \left(\frac{R}{L} \right)^2 \frac{\partial}{\partial \bar{z}} \left[\frac{\bar{h}^{n+2}}{12} \frac{\partial \bar{P}}{\partial \bar{z}} \right] = \frac{1}{2} n \frac{\partial \bar{h}}{\partial \theta} \quad (2)$$

The exponent (n) represents the power-law index; when $n=1$, the fluid exhibits Newtonian behavior. The normalized fluid film thickness associated with the two-lobe journal bearing is defined as follows [9]:

$$\bar{h} = 1 - \varepsilon(\cos \theta \sin \phi - \sin \theta \cos \phi) \pm \delta \sin \theta \quad (3)$$

or

$$\varepsilon = \bar{\varepsilon}(1 - \delta)$$

The symbol (δ) denotes the preload factor, which characterizes the deviation from circularity in the bearing geometry. When this factor is null, the two-lobe configuration reduces to a symmetrical two-groove circular bearing. The film thicknesses in the upper and lower lobes are governed by the positive and negative signs in equation (3), respectively [9].

2.2 Boundary conditions

The hydrodynamic pressure at the bearing extremities corresponds to the ambient pressure, i.e.

$$\bar{P}(\theta, 0) = \bar{P}(\theta, 1) = 0 \quad (4)$$

The boundary conditions defined at the inlet of the fluid films in both lobes, excluding the effect of the supply pressure, are given by:

$$\bar{P}(\theta_{s1}, \bar{z}) = 0 \quad (5)$$

$$\bar{P}(\theta_{s2}, \bar{z}) = 0 \quad (6)$$

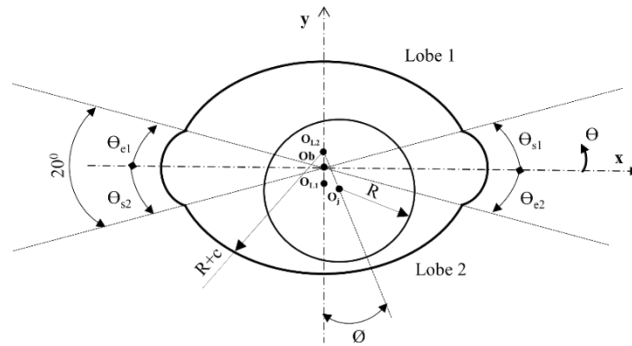


Fig. 1. Schematic representation of the bearing's configuration.

Stieber boundary conditions are implemented at the trailing edges of the bearing. In addition, the pressure within the regions of cavitation is taken to be equal to the ambient pressure. Based on these assumptions, the following boundary conditions are imposed:

$$\frac{\partial \bar{P}(\theta_{t1}, \bar{z})}{\partial \theta} = 0 \quad (7)$$

$$\frac{\partial \bar{P}(\theta_{t2}, \bar{z})}{\partial \theta} = 0 \quad (8)$$

and

$$\bar{P}(\theta, \bar{z}) = 0 \text{ for } \theta_{e1} \leq \theta \leq \theta_{e1} \text{ and } \theta_{e2} \leq \theta \leq \theta_{e2} \quad (9)$$

The pressure distribution over the angular (θ) and axial (z) directions is then integrated to compute the dimensionless load-carrying capacity, expressed as:

$$\bar{W} = (\bar{W}_x^2 + \bar{W}_y^2)^{1/2} \quad (10)$$

Where and are expressed as follows;

$$\bar{W}_x = \int_0^1 \int_0^{2\pi} \bar{P} \cos \theta \, d\theta \, d\bar{z} \quad (11)$$

$$\bar{W}_y = \int_0^1 \int_0^{2\pi} \bar{P} \sin \theta \, d\theta \, d\bar{z} \quad (12)$$

The dimensionless friction force acting on a journal bearing operating under shear-dependent lubrication is evaluated as follows [26]

$$\bar{F}_h = \int_0^{2\pi} \int_0^1 \left(\frac{1}{\bar{h}} + \frac{\bar{h}^n}{2n} \frac{\partial \bar{P}}{\partial \theta} \right)^n \quad (13)$$

As a result, the friction coefficient can be calculated using the following formula:

$$C_f = \frac{\bar{F}_h}{\bar{W}} \quad (14)$$

The dimensionless formulation of the bearing's side leakage is provided in [26];

$$\bar{Q}_s = \int_0^{2\pi} \frac{\bar{h}^{n+2}}{12} \frac{\partial \bar{P}}{\partial \bar{z}} \bigg|_{\bar{z}=0} d\theta \quad (15)$$

2.2 Resolution procedure

To evaluate the film thickness using Eq. (3), an initial estimate of the attitude angle value $\emptyset$ is introduced. The modified Reynolds equation Eq. (2) is then numerically solved using the finite difference method and the Gauss-Seidel iterative method to compute the internal pressure field within the bearing. Iterations continue until the pressure satisfies the convergence criterion, defined as:

$$\left| \frac{\sum (P_{i,j}^{(k)} - P_{i,j}^{(k-1)})}{\sum P_{i,j}^{(k)}} \right| \leq 10^{-3}, \text{ (where k represents the number of iterations).}$$

Once this condition is met, the load-carrying capacity's horizontal and vertical components for each lobe are computed. The journal bearing is considered to be in equilibrium when $|\bar{w}_x/\bar{w}_y| \leq 10^{-3}$, otherwise, the process is repeated using an updated attitude angle. Once the correct attitude angle is obtained, the steady-state performance parameters are then extracted.

3. Results and Discussion

3.1 Validation

In the present study, a wide range of fluids was examined, classified according to their power law index (n), ranging from 0.7 to 1.3. Pseudoplastic fluids, characterized by decreasing viscosity behavior under shear, correspond to $n=0.7$, while Newtonian fluids are represented by $n=1$. At the other end of the spectrum, shear-thickening lubricants, whose

viscosity increases with shear rate, are described by an index of $n=1.3$. For the geometric configuration considered, each groove has an arc length of 20° , and the preload factor of the two-lobe journal bearing is set to 0.5.

The analysis emphasizes the system's steady-state characteristics, in particular the load-carrying capacity, the orientation of the resultant bearing force (attitude angle), the coefficient of hydrodynamic friction, as well as the side leakage. These parameters were evaluated for different aspect ratios $R/L=0.5$ (wide bearing), $R/L=1$, and $R/L=1.5$ (narrow bearings), and for a range of eccentricity ratios from 0.05 to 0.85.

To ensure the accuracy and robustness of the present formulation, the numerical model was systematically validated against well-established benchmark studies from the literature.

3.1.1 Numerical validation

The first comparison was performed with the study by Das et al. [25], which analyzed two-lobe bearings lubricated with power-law non-Newtonian fluids under comparable geometric and operating conditions. All geometric parameters, namely the preload factor (δ), aspect ratio (R/L), and eccentricity ratio (ϵ), as well as the rheological index (n), were kept identical to their configuration to enable a direct validation. In both analyses, the hydrodynamic pressure was set to ambient at the axial extremities and at the inlet of each lobe, while the Stieber boundary condition was imposed in the cavitated (trailing-edge) regions, ensuring that pressure equals the cavitation value and its circumferential gradient is zero. Although expressed in slightly different coordinate systems, these boundary conditions describe the same physical configuration. The present model employed the Gauss–Seidel iterative scheme with a convergence tolerance of 10^{-3} , whereas the relaxation method used by Das et al. [25] applied a stricter tolerance of 10^{-5} . This difference explains the small numerical deviations observed, but the overall trends in attitude angle and Sommerfeld number remain in excellent agreement, as shown in Table 1. This initial comparison confirms the reliability of the implemented numerical approach and builds confidence in its predictive capability.

The Newtonian case ($n=1$) was then compared with the classical semi-analytical data of Lund and Thomsen [5], which remain a benchmark reference for the static characteristics of oil-lubricated journal bearings. The comparison was performed using the same geometry ($\delta=0.5$, $L/D=1$) defined in their study (Table 2). Both analyses applied the same boundary conditions, where the pressure was set to zero at the axial edges and along the leading and trailing film boundaries, ensuring complete consistency between the two models. The load capacity and attitude angle obtained from the present formulation show excellent agreement with Lund and Thomsen's results, with discrepancies remaining below 1%, as summarized in Table 2. This strong correspondence confirms that the developed model accurately reproduces the established analytical trends for two-lobe configurations and validates the physical soundness of the hydrodynamic formulation.

3.1.2 Experimental validation

For experimental validation, the proposed model was compared to the results obtained by Bouyer et al. [44] on a two-lobe hydrodynamic bearing. The geometric and operational parameters were defined in such a way as to faithfully reproduce the experimental configuration: preload factor $\delta=0.524$, aspect ratio $R/L=0.73$, and supply pressure $p_s=0.17$ MPa. The lubricant used in both studies is an ISO VG 46 oil, behaving as a Newtonian fluid within the operational temperature range between 40 – 60°C . For the numerical simulations, a viscosity of $\mu=0.0416$ Pa·s at 40°C was considered. The dimensionless pressure was determined from the classical hydrodynamic scale factor, $P = \frac{\mu UR}{c^2}$, leading to a reference pressure equal to 1.86 MPa.

The results presented in Figure 2 highlight a very good agreement between the predicted pressure distribution and Bouyer's experimental measurements for a rotational speed of 3500 rpm and an applied load of 6000 N. This concordance is particularly notable for eccentricity ratios ϵ between 0.35 and 0.40, where both the amplitude and the angular position of the pressure peak are correctly reproduced. The pressure is almost identical for an eccentricity of 0.4. The minor discrepancies observed between the two sets of results can be attributed to thermal effects, the present modeling being conducted under isothermal conditions, whereas the experimental tests take into account temperature variations.

Moreover, the excellent agreement obtained with the results of Das et al. [25], Lund and Thomsen [5], as well as Bouyer et al. [44], confirms the robustness of the mathematical formulation used. This inter-study consistency provides a solid basis for the analysis of the coupled effects of the aspect ratio and non-Newtonian rheology in two-lobe hydrodynamic bearings. All the validations conducted, numerical, analytical, and experimental, attest to the reliability of the proposed model and its predictive capacity for studying the tribological behavior of this type of bearing.

Table 1: Comparison of static characteristics of two-lobe journal bearing lubricated with non-Newtonian fluids $\delta=0.5$, $R/L=0.5$ [26]

ϵ	n	ϕ		S	
		Present	Das et al. [25]	Present	Das et al. [25]
0.15	0.8	94.5	94.04	0.568	0.591
	1.0	91.9	92.31	0.446	0.452
	1.1	90.6	90.5	0.398	0.391
0.25	0.8	90.7	90.46	0.289	0.295
	1.0	88.2	88.45	0.225	0.227
	1.1	87.1	86.84	0.201	0.196
0.35	0.8	84.4	84.05	0.161	0.159
	1.0	81.7	81.94	0.121	0.122
	1.1	80.5	80.34	0.106	0.107

Table 2: Comparison of static characteristics of two-lobe journal bearing lubricated with Newtonian fluid $\delta=0.5$, $R/L=0.5$

Parameter	ϵ	R/L	Lund et al. [5]	Present Study	Deviation (%)
Load Carrying Capacity (W)	0.5	0.5	1.4210	1.4119	0.64
Load Carrying Capacity (W)	0.7	0.5	2.6306	2.6246	0.23
Attitude Angle (θ)	0.5	0.5	88.28	88.2	0.09
Attitude Angle (θ)	0.7	0.5	81.8	81.7	0.12

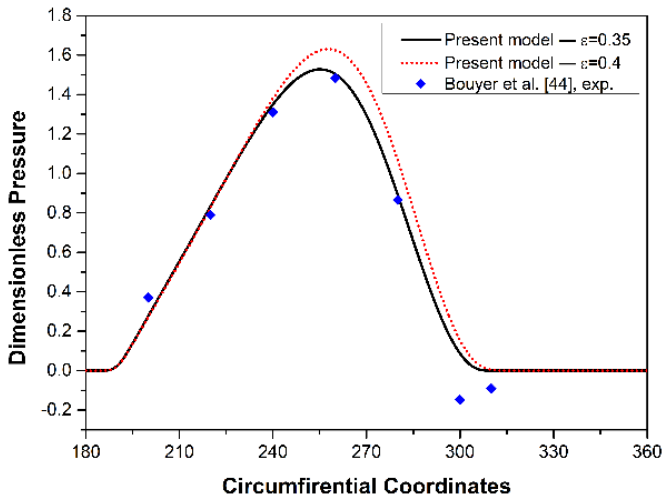


Fig. 2. Comparison of predicted and experimental dimensionless pressure distributions for a two-lobe journal bearing at 3500 rpm and 6000 N.

3.2 Evaluation of Lubricant Film Thickness in the Circumferential Direction

Fig. 3 illustrates the circumferential profile of fluid thickness in a two-lobe hydrodynamic journal bearing, operating with non-Newtonian lubricant using the power law model, for an eccentricity fixed at $\epsilon=0.5$ and an ellipticity $\delta=0.5$. The rheological index (n) ranges from 0.7 (pseudoplastic fluid) to 1.3 (shear-thickening fluid), whereas the aspect ratio R/L corresponds to one wide bearing and two narrow bearings.

It is observed that the film thickness exhibits a typical periodic behavior of two-lobe journal bearings, with two zones of minimum film located around $\theta=65^\circ$ and 295° , and two zones of maximum film at $\theta=0^\circ$ and 180° , reflecting the symmetrical and the double undulation induced by the elliptical shape of the bearing structure. The influence of the index (n) is significant, for the same geometry, pseudoplastic fluids ($n=0.7$) exhibit a lower film thickness, especially in the first minimum zone, due to a lower apparent viscosity under high shear, which favors flow and reduces surface separation. Conversely, shear-thickening fluids ($n=1.3$) generate an increase in film thickness attributed to the increase in local viscosity under the effect of shear, leading to an improved load support. An inverse behavior is noted in the second minimum zone at $\theta=295^\circ$. It is noted that the increase of the R/L ratio has a limited influence on changes in the lubricant thickness. The film thickness reaches a maximum value for the three R/L ratios, particularly marked at $\theta=180^\circ$, reflecting an improvement in the capacity to maintain the lubricating film in the restricted space.

This combined observation offers novel insights, indicating that the rheological properties of the lubricant predominantly influence the film profile, rather than alterations in the geometrical aspect ratio. This comprehensive understanding enhances previous interpretations derived from single-parameter analyses and underscores the necessity of concurrently evaluating fluid rheology and geometry in the design of compact or high-speed bearing systems.

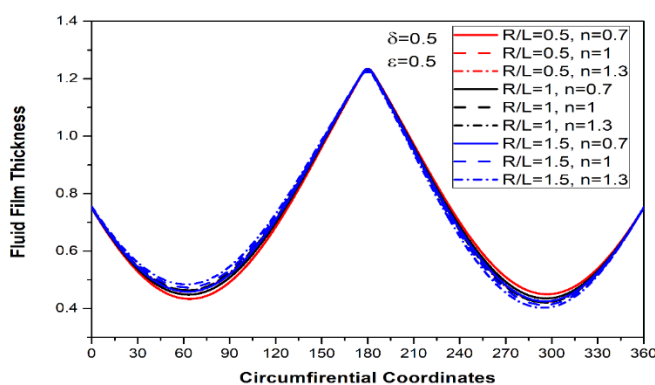


Fig. 2. Circumferential distribution of the lubricant film thickness at the midplane of the bearing, shown for various power-law indices and aspect ratios.

3.3 Evaluation of Circumferential Pressure Profiles

Fig. 4 presents the circumferential variation of dimensionless pressure at the bearing's midplane, for various power-law indices (n) and aspect ratios R/L , with an eccentricity fixed at $\epsilon=0.5$ and an ellipticity of $\delta=0.5$. The lubricant film behavior is modeled using a power-law non-Newtonian fluid, which highlights the combined influence of fluid rheology and bearing geometry on the generation of hydrodynamic pressure.

It is noted that the decrease in the aspect ratio (wider bearing) favors a significant increase in the pressure developed in the lubricating film, due to the larger contact surface between the journal and the bearing. Furthermore, an increase in the power index also leads to a higher pressure, a direct consequence of the reduction in viscosity under shear, which accentuates the hydrodynamic effect in the convergent zone.

It is also noted that two distinct pressure peaks are located respectively around $\theta \approx 40^\circ$ and $\theta \approx 265^\circ$, characteristic of the two-lobe geometry. These pressure maxima correspond to the areas of thinner film thickness, where the velocity gradient is highest, thus favoring the generation of bearing loads. The height and location of the pressure peaks vary significantly depending on the values of (n) and R/L . For a ratio $R/L=0.5$, the maximum pressure developed is significantly higher than in the configurations at $R/L=1$ and $R/L=1.5$. This improvement is explained by the ability of the wide bearing to maintain a wider and more uniform pressure distribution, due to its larger lubricating surface. Regarding the effect of fluid rheology, the findings reveal that shear-thickening liquids ($n=1.3$) produce the peak pressure values, followed by Newtonian fluids ($n=1$) and finally pseudoplastic fluids ($n=0.7$). This behavior is explained by the fact that, in high shear zones, the effective viscosity of shear-thickening fluids increases, which increases the flow resistance and allows to maintain higher pressure gradients in the lubricant film. Conversely,

pseudoplastic fluids, whose viscosity decreases under shear, favor a faster flow, thus reducing the pressure rise in the convergent zone.

The wide bearings (low R/L) promote a stronger and more uniform pressure field, an effect further enhanced when the lubricant follows a power-law behavior. This coupled influence between geometry and rheology provides a more realistic description of pressure generation in non-Newtonian two-lobe bearings. This is the first study to quantify the combined effect of R/L and (n) on the circumferential pressure field, showing geometry as the primary organizer of the pressure pattern and rheology as the amplifier of its magnitude, especially in wide configurations.

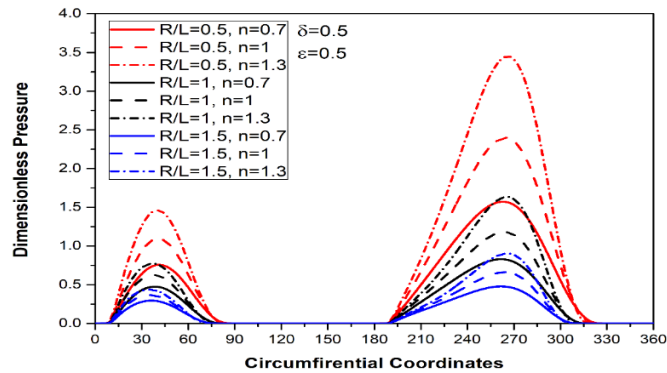


Fig. 3. Pressure distribution along the circumference at the midplane of the bearing for different power law indices and aspect ratios values.

3.4 Influence of Fluid Rheology and Aspect Ratio on the Dimensionless Load Carrying Capacity

This paragraph examines the overall influence of lubricant rheology, eccentricity, and bearing geometry on the load capacity of two-lobe journal bearings. Hence, the present study provides the first quantitative framework linking aspect ratio and non-Newtonian rheology to load-supporting performance in two-lobe bearings, bridging the gap between theoretical modeling and real-design optimization. Fig. 5 highlights a monotonic and non-linear increase of the dimensionless load carrying capacity with the eccentricity ratio ϵ , a direct consequence of the amplified hydrodynamic effect in the converging zone, where the thinning of the lubricant film within the bearing leads to a more pronounced increase in hydrodynamic pressure.

The power law index (n) , reflecting the fluid rheology, also plays a determining role. Unlike classical cylindrical configurations, the results obtained indicate that the load carrying capacity increases with (n) , with shear-thickening lubricants ($n=1.3$) generating a higher load capacity than Newtonian or pseudoplastic fluids. This behavior is attributed to the increase in effective viscosity in high shear stress regions, which reinforces the flow resistance and favors the formation of more intense pressure gradients. Moreover, in terms of geometry, wide bearings ($R/L=0.5$) exhibit significantly higher load-carrying capacity than narrow bearings, regardless of the values of ϵ and n . This performance is attributed to the larger available lubrication surface, which enables a more uniform pressure distribution and more effective load transmission.

These results highlight the complex interaction between fluid rheology and system geometry in the generation of the supporting film. They suggest that, in high-eccentricity conditions, the use of shear-thickening lubricants combined with wide bearings is an effective strategy to maximize the load-carrying capacity.

The current results demonstrate that geometry and rheology act in a coupled manner: a wider bearing enhances the shear field distribution, allowing shear-thickening fluids to develop greater load capacity than predicted by classical single-parameter models. Such coupling had not been quantitatively characterized in prior studies. From an engineering standpoint, this outcome offers practical design guidance. For high-speed or heavily loaded systems, combining a moderate shear-thickening lubricant with a wide two-lobe bearing can increase load capacity and film stability without excessive frictional penalty. Conversely, for low-load, low-speed applications where minimizing drag is critical, a narrower bearing with a nearly Newtonian or mildly pseudoplastic lubricant may be preferable.

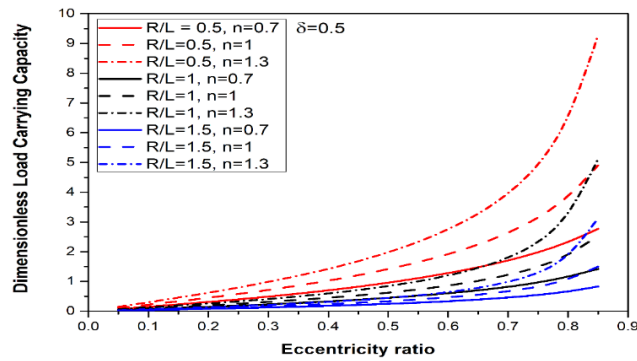


Fig. 4. Dimensionless Load Carrying Capacity response to eccentricity ratio under various power-law indices and aspect ratios.

3.5 Influence of Fluid Rheology and Aspect Ratio on the Attitude Angle

Fig. 6 presents the attitude angle response of a two-lobe journal bearing under varying fluid rheology (index n) and aspect ratio R/L , with constant ellipticity $\delta=0.5$. The attitude angle is observed to decrease monotonically as the eccentricity ratio increases, regardless of the configuration considered. This decrease reflects that an eccentric journal tends to align more rapidly toward the direction of the applied load, thereby reducing the angle between the applied load direction and the line joining the centers of the rotor and bearing shell. Under constant eccentricity, a rise in the power-law index (n), reflecting a shear-thickening behavior of the lubricating fluid, also induces a reduction in the attitude angle. This results from the increase in the effective viscosity under shear, which limits the fluid's ability to flow quickly and creates a more intense local pressure field. In turn, this increases the hydrodynamic moment applied to the journal, inducing a more inclined orientation (smaller attitude angle).

Furthermore, at a fixed R/L ratio, the attitude angle decreases with increasing power law index (n), indicating that the rheological behavior of the fluid significantly influences the stability of the lubricating film. In particular, a fluid with shear thickening behavior ($n>1$) favors a more marked reduction of the attitude angle, which is explained by better bearing capacity and increased stiffness of the film under such conditions. Thus, the effect of the geometric ratio R/L is also marked, lower values of R/L lead to higher attitude angles for the same combination of ε and n . This is explained by the lower capacity of narrow bearings to develop a stabilizing pressure in the lubricating film, making the system more sensitive to eccentricity.

The results demonstrate that the aspect ratio (R/L) controls the overall equilibrium orientation by influencing the axial pressure distribution, whereas the rheological properties of the lubricant, through its shear-dependent viscosity, adjust the intensity of this orientation. The analysis thus offers a cohesive comprehension of the interplay between film geometry and non-Newtonian behavior in establishing the equilibrium configuration and load alignment of two-lobe journal bearings. From a practical perspective, these findings suggest that selecting an appropriate combination of bearing aspect ratio and lubricant rheology can enhance load alignment and improve the operational efficiency of real engineering systems.

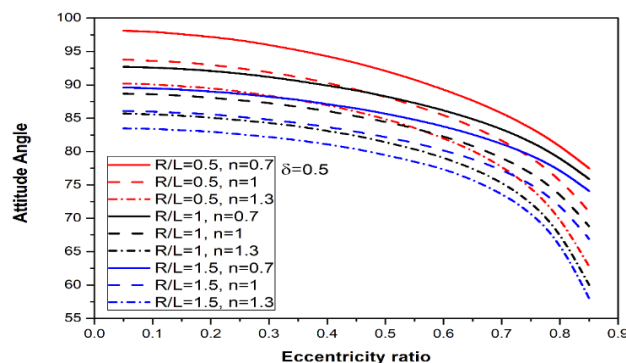


Fig. 5. Attitude angle response to eccentricity ratio under various power-law indices and aspect ratios.

3.6 Influence of Fluid Rheology and Aspect Ratio on Dimensionless Frictional Force

In Fig. 7, the combined effect of the geometric ratio R/L and the index (n) on the normalized friction force in a two-lobe hydrodynamic journal bearing is illustrated, under varying eccentricity ratios and a fixed ellipticity of $\delta=0.5$. It is clearly observed that the friction force increases monotonically with the eccentricity, a direct consequence of the thinning of the lubricant film, which intensifies the shear stresses in the fluid. Furthermore, a rise in the power law index (n) leads to an increase in the friction force, due to the shear-thickening behavior of the lubricant; the apparent viscosity increases under the effect of shear, which increases the sliding resistance between the journal and the bearing. Furthermore, the impact of the geometric ratio R/L is notable. Wide bearings systematically generate higher friction forces than narrow bearings, for the same value of $n>1$. This behavior is explained by a more uniform and widespread distribution of hydrodynamic pressure along the bearing, which enlarges the shear zone and intensifies the viscous interactions in the fluid film. Conversely, in narrow bearings, the capacity to develop a stabilizing pressure remains more localized, which limits the amplitude of the friction forces.

From a practical standpoint, this means that while wide bearings operating with shear-thickening fluids can deliver superior load capacity, they do so at the cost of increased frictional losses. Designers can therefore use the present results to balance performance and efficiency, selecting narrower bearings or shear-thinning lubricants when energy conservation and reduced drag are prioritized.

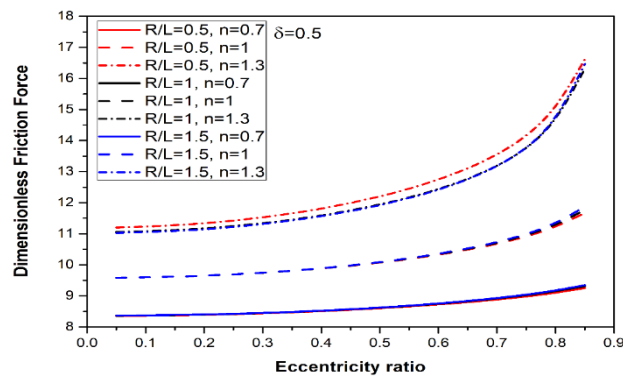


Fig. 6. Friction force response to eccentricity ratio under various power-law indices and aspect ratios.

3.7 Influence of Fluid Rheology and Aspect Ratio on Friction Coefficient

Fig. 8 illustrates the change in normalized friction coefficient with eccentricity ratio ϵ considering various power-law indices (n) and geometric ratios R/L , in a two-lobe hydrodynamic journal bearing operating with both Newtonian and non-Newtonian lubricants. The ellipticity is held constant at $\delta=0.5$.

The findings indicate that the friction coefficient diminishes as the eccentricity rises. Such behavior results from the improved load-carrying capacity of the lubricant film, which in turn lowers the proportion between the tangential frictional force and the radial load supported by the bearing. As the load capacity generated increases, the relative contribution of the frictional force to the normal force diminishes, resulting in a lower friction coefficient. The influence of the rheological behavior of the fluid is also significant. For a given geometric configuration, an increase in the index (n), reflecting a more shear-thickening behavior, leads to a further drop in the friction coefficient due to the increased apparent viscosity under shear.

Bearing geometry, defined by the ratio R/L , exerts a considerable impact on the tribological behavior of the system. It appears that the so-called wide bearings have the lowest friction coefficients, while the narrow bearings generate significantly higher values. This trend can be attributed to a more effective distribution of hydrodynamic pressure across the entire contact surface in the case of wide bearings, which promotes more effective separation between the moving components. In contrast, for narrow bearing configurations, the load is concentrated over a reduced contact area, leading to increased local pressure and, consequently, a higher friction coefficient. This observation highlights the synergistic effect between geometric and rheological parameters and demonstrates the importance of jointly optimizing these factors to minimize energy losses in lubricated systems.

The interplay of geometry and lubricant rheology affects friction in two-lobe bearings: wide bearings with shear-thickening fluids improve load capacity and film stability but also raise frictional losses, while narrow bearings or pseudoplastic lubricants lower friction, which is better for energy efficiency in delicate or low-power systems.

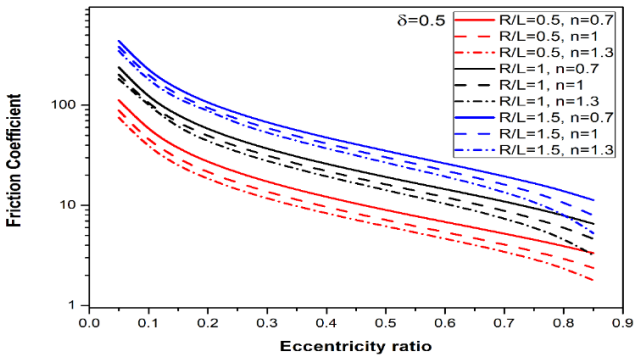


Fig. 7. Friction coefficient response to eccentricity ratio under various power-law indices and aspect ratios.

3.8 Influence of Fluid Rheology and Aspect Ratio on Dimensionless Side Leakage

Fig.9 illustrates the change in dimensionless side leakage under different eccentricity ratios, based on multiple combinations of geometric ratios (R/L) and power-law indices (n), with the ellipticity fixed at $\delta = 0.5$. It can be observed that the side leakage tends to increase with rising eccentricity ratio across all configurations. This trend is particularly pronounced for wide bearings (R/L=0.5), which exhibit significantly greater side leakage values compared to other bearings. This increase is explained by the reduction in available lateral surface area for fluid dissipation, leading to intensified transverse flow.

The power-law index effect (n) on side leakage is also noteworthy. For a given geometry, increasing (n) contributes to a rise in the dimensionless side leakage. At R/L=0.5, the effect of the index (n) is clearly noticeable, as the three curves corresponding to n=0.7, n=1, and n=1.3 are distinctly separated and arranged in increasing order. In fact, the side leakage increases with the index (n), the lowest value being recorded for n=0.7. This is explained by the fact that a pseudoplastic fluid (n<1) has a higher apparent viscosity at low shear rates, which slows down side flow, while a shear-thickening fluid (n>1) tends to flow more easily under stress, hence an increase in the side leakage. On the other hand, for R/L=1 ratios, the difference between the curves becomes very small, even negligible, which reflects a lower sensitivity to (n) in the narrow configurations. This diminution can be attributed to the stabilization of the pressure field and the reduced influence of the fluid’s rheological properties on side leakage in narrower bearings.

This dual dependence of side leakage on both R/L and n demonstrates that controlling leakage in two-lobe bearings can't be achieved just by changing the shape or the fluid's properties alone. The interaction between geometry and rheology revealed in this study yields a new quantitative perspective on cross-flow behavior in non-Newtonian lubrication.

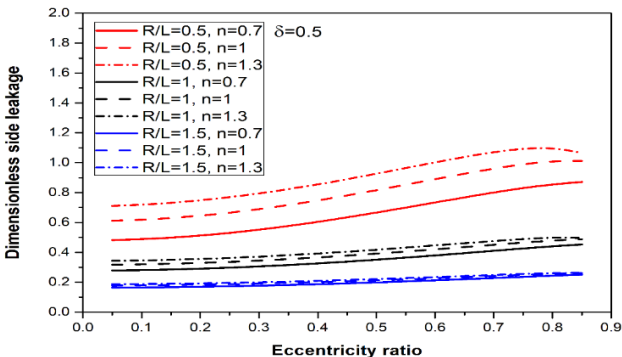


Fig. 8. Dimensionless Side Leakage response to eccentricity ratio under various power-law indices and aspect ratios.

The observed effects of the power-law index (n), aspect ratio (R/L), and eccentricity ratio (ϵ) on key steady-state performance parameters are compiled in Table 3 to summarize the main findings. The trends reinforce the combined impact of rheology and geometry found in the earlier analyses.

Table 2: Summary of the observed impacts of the power-law index (n), aspect ratio (R/L), and eccentricity ratio (ϵ) on key steady-state performance parameters of two-lobe journal bearings.

Parameter	Increasing n (shear-thinning → pseudoplastic)	Increasing R/L (wide → narrow bearing)	Increasing ϵ	Increasing ϵ
Pressure distribution $\bar{P}$	Increases with n	Decreases with higher R/L	Not explicitly analyzed, inferred from load trends indicating that a higher ϵ increases pressure accumulation in converging regions.	Increases with n and decreases with R/L , shear-thickening fluids, and wider bearings enhance pressure buildup.
Load-Carrying Capacity $\bar{W}$	Increases with n	Decreases with higher R/L	Increases non-linearly with ϵ	Rises with higher n and lower R/L due to stronger pressure fields and larger supporting area.
Frictional force $\bar{F}_h$	Increases with n	Decreases with higher R/L , notable only for $n > 1$	Increases with ϵ	Increases with n ; slightly higher for wider bearings owing to larger shear area.
Friction coefficient C_f	Decreases with n	Increases with higher R/L	Slightly decreases with ϵ	Decreases with n , wide bearings show lower C_f due to improved load efficiency.

4. Conclusions

This work examines the behavior of a hydrodynamic two-lobe journal bearing system subjected to shear-dependent lubrication (non-Newtonian) described by the power-law rheological model. The analysis emphasizes the combined effect of the aspect ratio R/L , the power law index (n), and the eccentricity ratio ϵ on the film behavior and tribological performance of the system. The main conclusions can be summarized as follows:

- The lubricating film thickness shows a low sensitivity to the R/L ratio but remains strongly influenced by the rheology of the fluid; it decreases for pseudoplastic fluids and increases for shear-thickening lubricants, especially in high shear zones.
- Pressure distribution is significantly affected by the geometry and nature of the lubricant. A decrease in the R/L ratio or an increase in the (n) index leads to higher and more evenly distributed pressure peaks, thereby enhancing the hydrodynamic effect.
- The load-carrying capacity improves as both eccentricity and the power-law index (n) increase, reflecting the film’s improved resistance to thinning in the converging regions. Conversely, this capacity diminishes as the R/L ratio increases, since wide bearings provide a more favorable load-supporting surface.
- The attitude angle progressively reduces as the values of ϵ , (n), and R/L increase, reflecting a faster alignment of the journal in the direction of the load due to a more intense and symmetrical pressure field.
- The friction force increases with ϵ and (n), due to intensified shear effects. However, wide bearings generate overall higher friction forces than narrow bearings because of their larger contact area. Conversely, the friction coefficient decreases with increasing load capacity, indicating improved tribological efficiency in configurations with high load support and low tangential resistance.

- The side leakage rate is strongly dependent on the R/L ratio and the power law index (n). It increases with eccentricity, especially in wide bearings lubricated with shear-thickening fluids. This trend is less pronounced in narrow bearings, where sensitivity to the fluid's rheology becomes negligible.
- Wide two-lobe hydrodynamic bearings combined with moderately shear-thickening lubricants are recommended for high-speed or heavily loaded applications such as turbines, compressors, and pumps, where high load-carrying capacity and good film stability are required.
- Narrow two-lobe bearings, combined with pseudoplastic lubricants, are better suited for precision instruments or low-power rotary devices, where the reduction of friction and orifice losses is advantageous.

Nomenclature

c	Major clearance
c_m	Minor clearance
C_f	Friction coefficient
e	Eccentricity
e_p	Ellipticity, $(c-c_m)$
F_h	Friction force
$\bar{F}_h$	Dimensionless friction force
h	Oil film thickness
$\bar{h}$	Dimensionless oil film thickness
L	Bearing length
O	Center of the bearing
O_j	Center of Journal
P	Pressure
$\bar{P}$	Dimensionless pressure
Q_s	Side leakage
$\bar{Q}_s$	Dimensionless Side leakage
R	Bearing radius
W	Bearing load
W_x, W_y	The horizontal and vertical bearing load respectively
$\bar{W}$	Dimensionless bearing load, $W(c/R)^2 / \mu_i \omega RL$
x, y, z	Circumferential, radial and axial coordinates respectively
S	Non-dimensional Sommerfeld number = $1/(2\pi W)$

Greek letters

δ	e_p/c , Preload factor
$\mathcal{E}$	e/c_m , Eccentricity ratio based on minor clearance
$\bar{\epsilon}$	e/c , Eccentricity ratio based on major clearance
θ	Angular coordinate
θ_{e1}, θ_{e2}	Angular coordinates at the end of the bearing pads
θ_{s1}, θ_{s2}	Angular coordinates at the start of the bearing pads
θ_{t1}, θ_{t2}	Angular coordinates at the trailing edges
μ	Viscosity
μ_i	Oil inlet viscosity
ω	Angular velocity of the journal
ϕ	Attitude angle

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