

Research Article

Thermo-Mechanical Stress Analysis of a Linearly Functionally Graded Rotating Disk with Uniform and Tapered Cross Sections

R. Madan^{1,*}

S. Bhowmick²

P.P. Khobragade³

¹ Department of Mechanical Engineering, Graphic Era (Deemed to be University), Dehradun 248002, India

² Department of Mechanical Engineering, National Institute of Technology Raipur, Raipur 492010, India

³ Department of Civil Engineering, Dev Bhoomi Uttarakhand University, Dehradun 248007, India

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Abstract:

Functional gradation of materials in a component, whether metal-metal, metal-ceramic, or ceramic-ceramic, offers benefits such as high toughness and strength at elevated temperatures, which are desirable traits in rotating disks. With the advent of FGMs, isotropic material components began to be replaced. In the present study, Thermo-mechanical stress analysis on rotating disks was performed for isotropic materials (both metal and ceramic), and their results were compared with those of a linearly varying functionally graded (FG) disk. The approach utilized is the Variational principle, deriving the solution through Galerkin's error minimization method. Comparison plots of displacement, radial and tangential strains, as well as radial, tangential, and von Mises stresses, were presented to illustrate the advantages of advanced materials such as FGMs (functionally graded materials) over isotropic materials. The study reveals the benefits of functional grading of disks over those made of isotropic materials. Moreover, the analysis was performed for uniform as well as tapered disk profiles.

Keywords: Linear gradation, functionally graded, rotating disk, material modelling, variable cross sections

1. Introduction

Mostly, metal–ceramic combinations are used, and due to the integration of two different materials, metals and ceramics, the graded material attains the properties of both. As a result, the final component possesses characteristics like a high strength-to-weight ratio and wear resistance, which have applications in various sectors such as aerospace, nuclear energy, electronics, power plants, biomaterials, commodities, in cutting tools, and data storage devices [1], [2], [3], [4]. Stresses in rotating structures like a disk can be altered by changing either the profile or by changing the material with different gradations [5]. A study of different disk profiles, such as concave, convex, and linear variation, demonstrates that a concave-thickness disk is the lightest in weight [6]. Also, there are several methods to formulate and solve the problem, such as the Fredholm integral equation [7], a semi-analytical method [8], the complementary functions method [9], Galerkin's, and Runge-Kutta methods [10].

A tangential stress in a disk can be kept constant by maintaining an arbitrary variation of shear modulus [11]. A nonlinear constrained optimization problem has been suggested to mitigate the stresses linked with functionally graded structures [12]. Modeling of FGMs can be formulated by two approaches: In the first case, the variation in the material property is considered directly, which could be a power law [13], [14], trigonometric law [15], exponential law [16], etc. In the second case, a variation in material composition, i.e., volume fraction, is varied. Due to the different thermo-mechanical properties of the two mixed materials, which vary along a particular direction, there is a possibility of delamination. However, with recent advancements in fabrication techniques, it has become possible

* Corresponding author: R. Madan
E-mail address: royalmadan6293@gmail.com



to achieve gradation according to power law, sigmoid law, exponential law, or other models. Despite these options, linear gradation is generally preferred [17]. The material properties of graded material can be formulated with several approaches, such as the rule of mixture (ROM)[18], the Mori-Tanaka [19], and others, modified rule of mixture (MROM) [20], etc. In one of the studies, various modeling methods, such as ROM, Mori-Tanaka, Halpin-Tsai, and others, were tested for different combinations of materials covering a wide range of material types. The results illustrate that the Halpin-Tsai model matches well with the experimental data published in the literature [21],[22]. In [23], presented an exact analytical solution for FG thick-walled cylinders under combined thermal and mechanical loads, validated by FEM. The benefits of the dynamic response of FG structures, such as shells, beams, plates, and sandwich structures, have been studied [24], [25], [26], [27]. The material gradation in these structures can be unidirectional [28], [29], [30], [31], bidirectional [32], or multidirectional [33], [34]. During the fabrication of FGM, porosity develops. To analyze the behavior of the structure for various porosity distributions, extensive research has been conducted [24], [35], [36], [37]. To avoid the development of porosities, the process parameters, such as sintering temperature, holding time and temperature, compaction load, scan rate, and melting temperature, need to be optimized [38], [39]. Apart from the discussed material modeling methods of composites and FGMs, there are several other approaches that can also be utilized [40], [41], [42].

With increasing interest in the fabrication of FGMs, new methods are being investigated. The well-known methods are centrifugal casting [43], [44], powder metallurgy [45], [46] and additive manufacturing [47], [48], [49], [50], [51]. The selection of the right method depends on various factors like the geometry and grading requirements. For example, the gradation can be continuous in continuous casting, when the operating parameters like mold rotational speed, mold pouring temperature, melting temperature, etc [43], [44], [52]. On the other hand, the powder metallurgy method allows layer-wise development of structure where each layer is assigned with different composition. The fabrication of a disk graded with was performed using the powder metallurgy method. A three-layered disk made of a combination of Al and Al_2O_3 material was developed, and characterization was performed. The optical images depict and confirm the successful fabrication of the FG disk [53]. Furthermore, the analysis was extended to develop a five-layered disk made of Al as the matrix and SiC as reinforcement. Stress analysis was also performed using finite element studies confirms stress jump reduction with an increase in the number of layers [54]. The greater the number of layers, the stronger the interfacial bonding and the lower the chance of disk delamination [48], [55]. In one of the studies, a wire arc additive manufacturing method was used to develop low alloy and austenite steel-based FGM, and a 46 % increase in tensile strength [56]. Moreover, a functionally graded Al7075T6 suspension plate reinforced with graphene and B_4C using stir casting was fabricated. Mechanical testing and microscopy confirmed improved hardness and fatigue strength, with only 5.1% deviation between experimental and computational material gradation estimates [57]. A Cu/SiC axially graded FGM beam was fabricated using powder metallurgy. Microstructural and hardness analysis confirmed uniform SiC distribution, reduced porosity, and increased hardness with higher SiC content, supporting applications in rotor blades, gears, and implants [58]. Four-layered W-Cu and diamond/W-Cu FGMs were fabricated via microwave sintering. The addition of tungsten-coated diamond enhances thermal conductivity and shock resistance. Microstructural analysis confirmed improved properties, with optimal results at 1200 °C for 10–30 minutes [59]. To enhance the performance of rotating structures, future research can focus on tailoring material properties to minimize induced stresses, as well as employing optimization approaches like differential evolution, particle swarm optimization, and genetic algorithms [60], [61], [62], [63].

As discussed, there are many modelling techniques available, but to effectively estimate the properties of functionally graded materials, the Halpin-Tsai model is employed. The disk profile changes from uniform to tapered, ensuring the mass of the disk remains constant. To date, considerable research has been conducted on the analysis of FG structures; however, their comparative study with homogeneous materials has not been performed previously. For the first time, stress and strain plots of metal, ceramic, and graded material have been performed for a uniform and tapered disk. A comparison of stresses in pure metal, pure ceramic, and graded material is studied to realize the benefits of functionally graded materials in rotating disks. The material under consideration, Al- Al_2O_3 , holds promise for high-temperature structural applications. An energy method was employed to study the rotating disk problem. The presented results offer valuable insights for industries seeking to establish application-specific material gradation guidelines.

1.1 Research objectives

- (a) To apply the variational principle and Galerkin's error minimization method for evaluating stress and strain in rotating disks.

- (b) To conduct a comparative analysis of isotropic metal, ceramic, and linearly graded materials under identical thermal and mechanical loading conditions.
- (c) To assess the impact of disk geometry, particularly uniform and tapered profiles, while maintaining constant disk mass across configurations.
- (d) To quantify and visualize displacement fields, radial and tangential strain, and stress distributions (radial, tangential, and von Mises) for different material compositions.
- (e) To provide insights that aid the selection of material gradation strategies for rotating components in structural and thermal environments.

2. Material grading and mathematical formulation

2.1. Volume fraction

The gradation variation because of change in composition of metal and ceramic (V_m) and (V_c) is given by Eq. (1) for grading index (n)=1, and plotted in Figure 1(a).

$$V_c = \left(\frac{r-a}{b-a} \right)^n ; \quad V_m = 1 - V_c \quad (1)$$

2.2 Mass density of FGM

For mass density, rule of mixture is used, given by Eq. (2), and plotted in Figure 1(b); the density values of Al and Al_2O_3 are taken as 2700 kg/m^3 and 3960 kg/m^3 .

$$\rho_f = \rho_m V_m + \rho_c V_c \quad (2)$$

2.3 Effective Young's modulus of FGM

The Halpin-Tsai method was utilized to effectively estimate the Young's modulus of the material. The term ξ is considered as 2, as the particle shape can be a square/ circular, which has an aspect ratio of unity. The Young's modulus and thermal expansion coefficient of Al and Al_2O_3 are taken as 67 GPa, $25 \times 10^{-6}/\text{K}$ and 393 GPa, $8 \times 10^{-6}/\text{K}$, respectively. Figure 1(a-c) illustrates the variation of the material properties along the disk radius. Eq. 3 is a Halpin-Tsai model used to calculate Young's modulus and thermal expansion for FGM.

$$\frac{P}{P_m} = \frac{1 + \xi \eta V_f}{1 - \eta V_f} ; \text{ where } \eta = \frac{P_f / P_m - 1}{P_f / P_m + \xi} \text{ and } \xi = 2 \frac{a}{b} \quad (3)$$

Figure 2 shows a disk of depth h_0 with radius values at the inner and outer as a and b . The disk rotates at an angular speed, ω . Considering the disk to be thin, a plane stress state is employed. The energy method is utilized, involving the computation of strain energy and work potential. Identification of variables is achieved through stress-strain and strain-displacement relationships. The displacement obtained from the displacement formulation is used to get the strains, and from the strains, stresses are identified. With the variation principle, the calculation becomes simpler. A detailed derivation of the same, along with the solution algorithm, can be seen in [64].

By employing energy method and normalizing (ξ)=($r-a$)/($b-a$), and $\bar{r} = (b-a)$, thus obtained:

$$\delta \left[\frac{\pi}{1-\mu^2} \int_0^1 E(\xi) \bar{r} \left\{ \frac{u^2}{\bar{r}\xi + a} + \frac{2\mu u}{\bar{r}} \frac{du}{d\xi} + \frac{\bar{r}\xi + a}{\bar{r}^2} \left(\frac{du}{d\xi} \right)^2 \right\} h d\xi \right. \\ \left. - 2\pi \omega^2 \bar{r} \int_0^1 \rho(\xi) (\bar{r}\xi + a)^2 u h d\xi - \frac{2\pi}{(1-\mu)} \int_0^1 E(\xi) \alpha(\xi) (\bar{r}\xi + a) \bar{r} \left(\frac{u}{(\bar{r}\xi + a)} + \frac{du}{\bar{r} d\xi} \right) T(\xi) h d\xi \right] = 0 \quad (4)$$

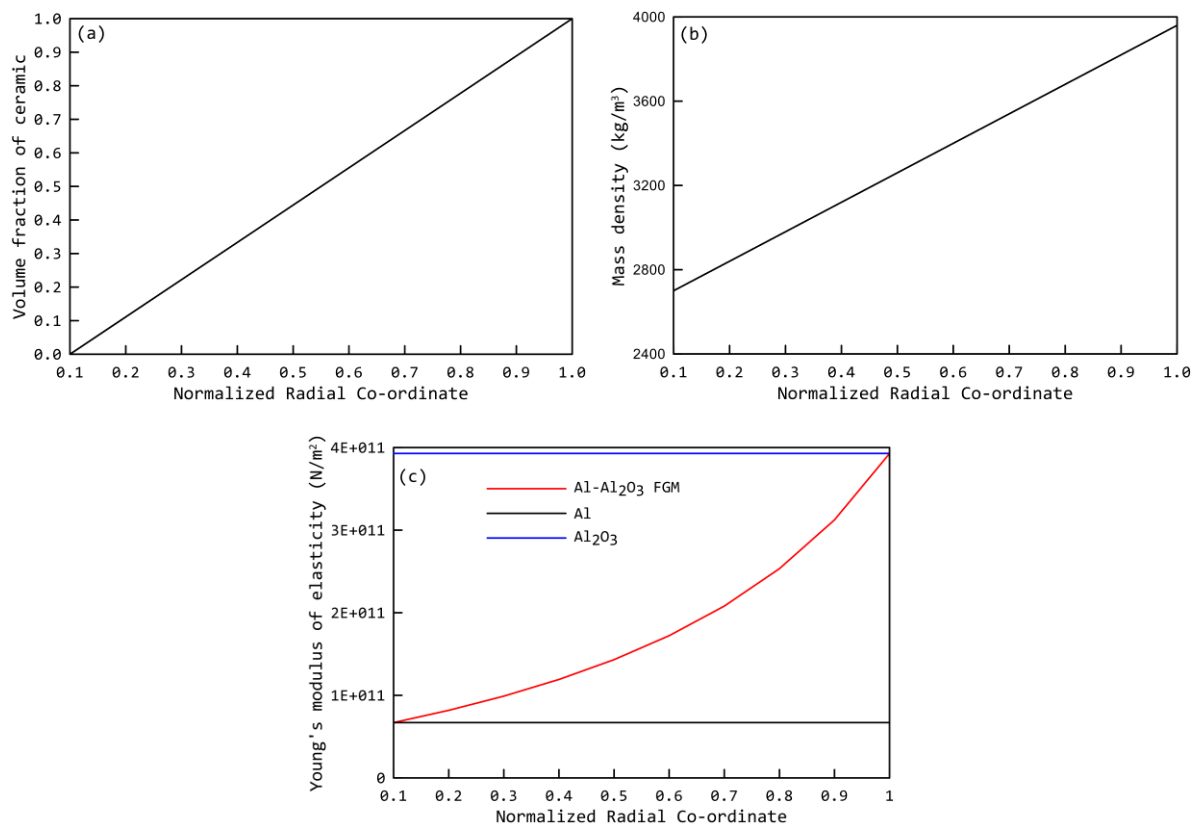


Fig. 1. (a) Volume fraction of ceramic (b) Mass density of FGM (c) Young's modulus of elasticity

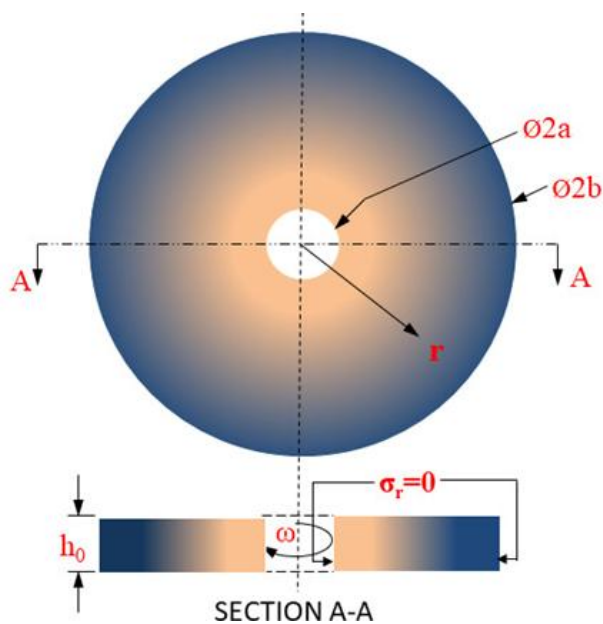


Fig. 2. Linearly graded rotating disk

The polynomial form of displacement is assumed as:

$$u(\xi) = \sum c_i \varphi_i, \quad i = 1, 2, 3, \dots, n_f \quad (5)$$

Here, φ_i are the trial functions considered using the Gram–Schmidt scheme. A free-free boundary condition, $(\sigma_r)_a = (\sigma_r)_b = 0$, is utilized in the study. It has application in rotating components where some radial movement at the root is allowed, like gyroscopes, turbine rotors, or flywheels during unconstrained spinning.

$$\varphi_0(r) = \frac{\omega^2 r(3+\mu)}{8} \left[\frac{\rho(\xi)}{E(\xi)} \left\{ (b^2 + a^2)(1-\mu) - \frac{1-\mu^2}{3+\mu} r^2 + \frac{a^2 b^2}{r^2} (1+\mu) \right\} \right] + \frac{\alpha(\xi)}{r} \left[(1+\mu) \int_a^r T(\xi) r dr + \frac{(1-\mu)r^2 + (1+\mu)a^2}{b^2 - a^2} \int_a^b T(\xi) r dr \right] \quad (6)$$

Substituting, the equation thus becomes;

$$\delta \left[\frac{\pi}{1-\mu^2} \int_0^1 E(\xi) \bar{r} \left\{ \frac{(\sum c_i \varphi_i)^2}{\bar{r}^2 + a} + \frac{2\mu}{\bar{r}} \left[\sum c_i \varphi_i \frac{d(\sum c_j \varphi_j)}{d\xi} \right] + \frac{\bar{r}^2 + a}{\bar{r}^2} \left(\frac{d(\sum c_i \varphi_i)}{d\xi} \right)^2 \right\} h d\xi - 2\omega^2 \bar{r} \int_0^1 \rho(\xi) (\bar{r}^2 + a)^2 (\sum c_j \varphi_j) h d\xi - \frac{2}{(1-\mu)} \bar{r} \int_0^1 E(\xi) \alpha(\xi) (\sum c_j \varphi_j) + \frac{(\bar{r}^2 + a)}{\bar{r}} \frac{d(\sum c_j \varphi_j)}{d\xi} T(\xi) h d\xi \right] = 0 \quad (7)$$

The variational operator (δ) is substituted as $\frac{\delta}{\delta c_j}$, $j = 1, 2, 3, 4, \dots, n_j$. By applying Galerkin's error minimization principle. This process yields the following algebraic equation.

$$\frac{\bar{r}}{1-\mu^2} \sum_{i=1}^n \sum_{j=1}^n c_i \int_0^1 E(\xi) \left\{ \frac{\varphi_i \varphi_j}{\bar{r}^2 + a} + \left(\frac{\mu}{\bar{r}} (\varphi_i' \varphi_j + \varphi_i \varphi_j') + \left(\frac{\bar{r}^2 + a}{\bar{r}^2} \right) (\varphi_i' \varphi_j') \right) \right\} h d\xi = \omega^2 \bar{r} \sum_{j=1}^n \int_0^1 \rho(\xi) (\bar{r}^2 + a)^2 \varphi_j h d\xi + \frac{1}{1-\mu} \sum_{j=1}^n \int_0^1 E(\xi) \alpha(\xi) \left\{ \varphi_j \bar{r} + (\bar{r}^2 + a) \varphi_j' \right\} T(\xi) h d\xi \quad (8)$$

Equation (8) can be represented as $\{c\} = [K]^{-1} \{R\}$ and solved numerically for unknown coefficients.

3. Results and discussion

Stress analysis of pure Al, pure Al_2O_3 , and graded Al- Al_2O_3 was carried out for angular velocity (ω) = 300 rad/s. Figure 3(a) shows the uniform thickness disk obtained by substituting $\alpha=0$ in Eq. 9, while the value of $\alpha=0.98$ and $\beta=1$ for a tapered disk profile. The present methodology is validated with Reference [65] and reported in [66]. In [66], The comparison is done for the power law variation of Young's modulus and density, and is omitted here to maintain the brevity of the manuscript.

$$h(r) = h_0 \left[1 - \alpha(\zeta)^\beta \right] \quad (9)$$

The following are the normalized variables used:

$$\bar{r} = \frac{r}{b}, \bar{\rho} = \frac{\rho(r)}{\rho_0}, \bar{E} = \frac{E(r)}{E_0}, \bar{\sigma}_y = \frac{\sigma_y(r)}{\sigma_{y0}}, \bar{\sigma} = \frac{\sigma}{\sigma_{y0}}$$

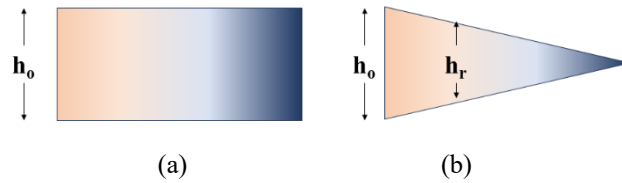


Fig. 3. (a-b) (a) Uniform thickness (b) Tapered disk

3.1 Stresses induced in a uniform thickness disk at constant angular speed

The results of stress analysis of a disk of constant thickness can be seen from Figure 4; the stresses were minimum for metal (Al) and maximum for ceramic (Al_2O_3). The stresses are intermediate in the FG disk between both metal and ceramic disks, which shows the advantages of utilizing a graded disk that optimizes disk performance.

Figure 4(a) illustrates radial stress in a disk. The magnitude of stress is zero at the root and tip of the disk because of the imposed boundary conditions; its magnitude is maximum at $r=0.3$ (approx). Figure 4(b) shows the variation of tangential stress, which is maximum at the root. Similarly, the variation of equivalent stress (von Mises stress) is plotted, which follows the same trend as that of tangential stress, please see figure 4(c). Furthermore, the variation of displacement, radial strain, and tangential strain is plotted in Figure 4(d-f).

3.2 Stresses induced in a tapered disk at constant angular speed

Figure 5(a) shows the variation of radial stress, which follows the same trend as that of a uniform thickness disk. For the case of tangential stress and von Mises stresses, there is a shift in the stresses beyond the value of $r>0.7$, the magnitude of the graded disk becomes maximum, followed by alumina and aluminum, please see Figure 5(b-c). In the strain plot, please see Fig. 5(d-f), a switching of the strain with the graded material and aluminum was seen, but the difference in their magnitude is negligible compared to the disk made up of alumina.

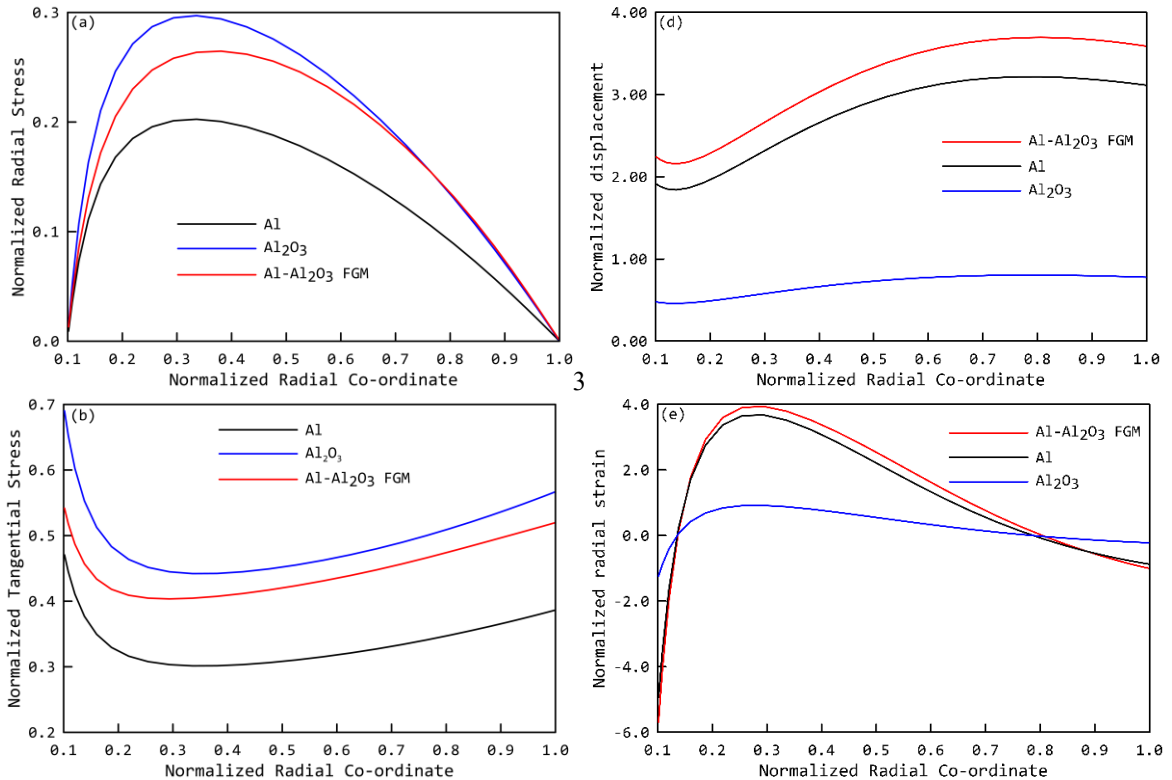


Fig. 4. (a) Radial stress (b) Tangential stress (c) von Mises stress (d) Displacement (e) Radial strain (f) Tangential strain

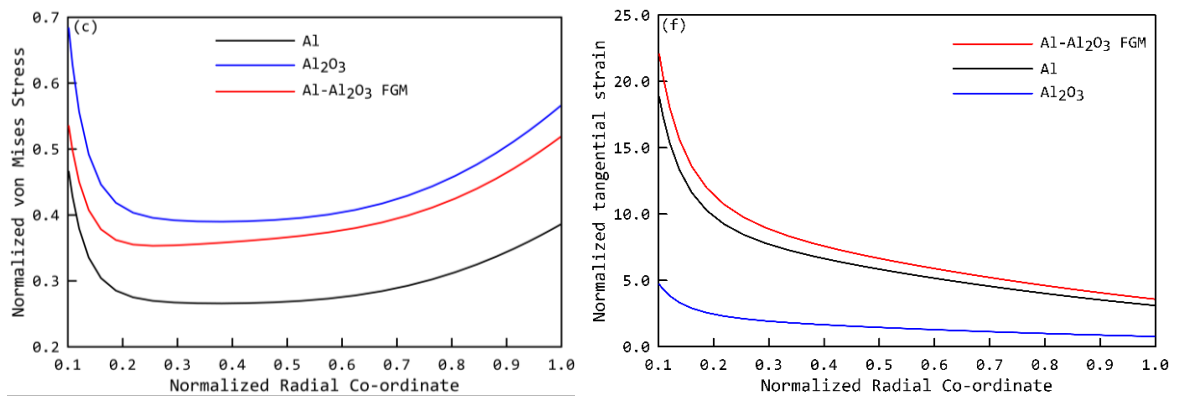


Fig. 4. (a) Radial stress (b) Tangential stress (c) von Mises stress (d) Displacement (e) Radial strain (f) Tangential strain (Continuous)

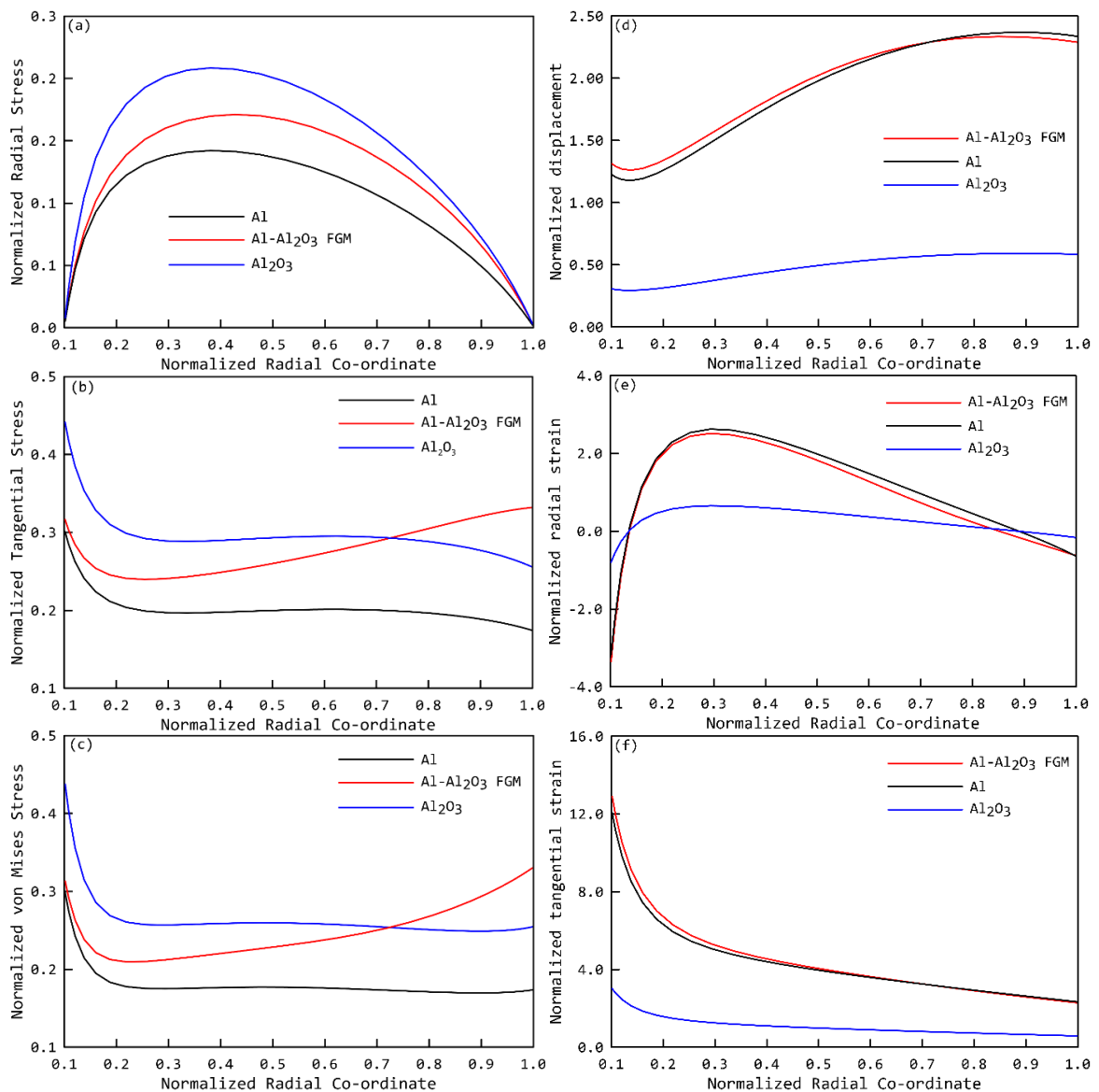


Fig. 5. (a) Radial stress (b) Tangential stress (c) von Mises stress (d) Displacement (e) Radial strain (f) Tangential strain

3.3 Thermo-mechanical stress analysis of a uniform thickness disk at constant angular speed

It is assumed that the Conductivity (k) of the FG disk varies along the radial direction as per the rule of mixture. Further, for a one-dimensional steady-state heat conduction equation, the Temperature variation in a radial direction is computed for the first seven polynomial series terms. Constant temperature boundary conditions are imposed at $r=a$, $T=T_m=25\text{ }^{\circ}\text{C}$ and at $r=b$, $T=T_c=500\text{ }^{\circ}\text{C}$.

Fig. 6(a-b) illustrates the induced radial and tangential stresses. Fig. 6(c) shows the variation of von Mises stress along the disk radius. From the Figure, it can be seen that thermo-mechanical stress induced at the root is maximum for alumina, followed by the FGM disk and aluminum. Beyond a certain value of radius there is reversal of stresses is seen, but it has no role in the performance of the disk because yielding always occurs at the root.

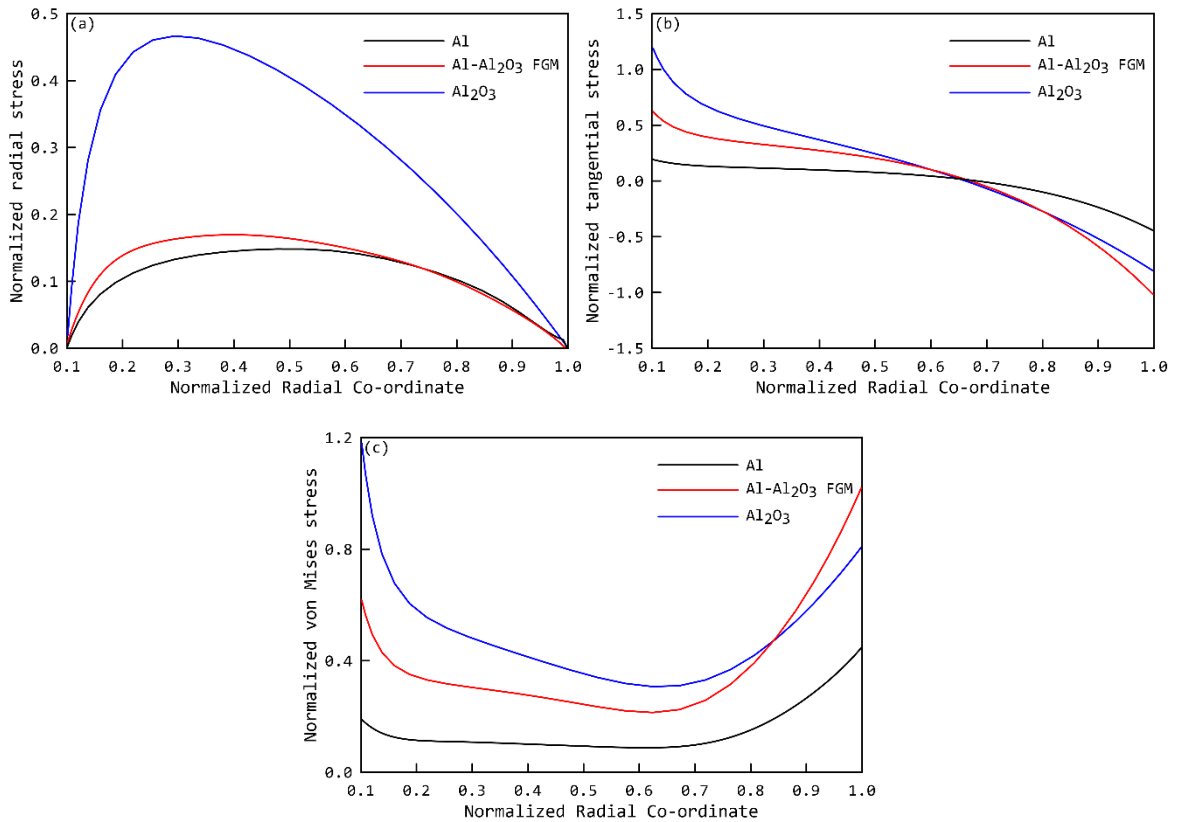


Fig. 6. (a) Radial stress (b) Tangential stress (c) von Mises stress

3.4 Thermo-mechanical stress analysis of a tapered disk at constant angular speed

Compared to centrifugal load only, the stresses induced are higher for thermo-mechanical loadings. Fig. 7 shows the magnitude of stresses induced in a tapered disk, which is lower than that of uniform thickness because the thickness of the root of the disk is larger where the maximum centrifugal load occurs. For the present thermal boundary conditions, the overall sum of stresses is higher, whereas if the boundary conditions reverse, there may be a possibility that it acts as stress relieving, but such boundary conditions are not practically applicable. Also, a constant state of stress was obtained for the case of the FG disk, which indicates a disk of uniform strength can be developed.

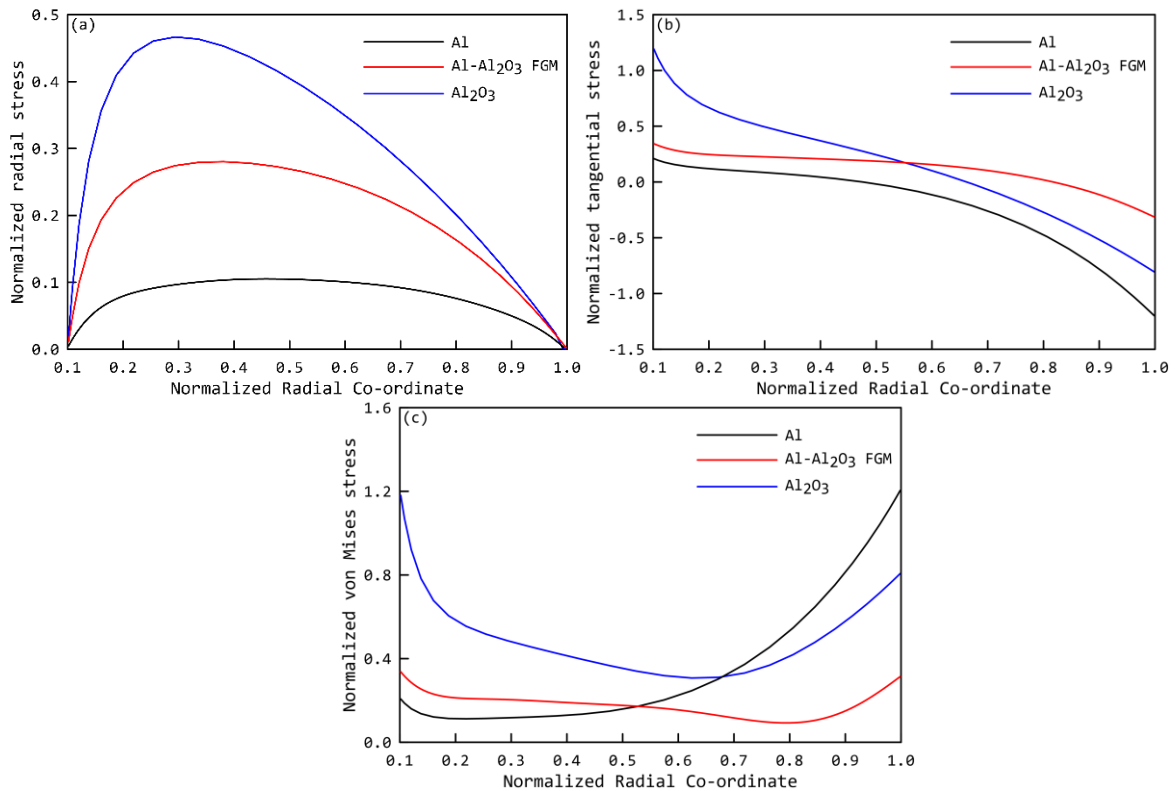


Fig. 7. (a) Radial stress (b) Tangential stress (c) von Mises stress

4. Conclusion

Following a thorough analysis of the available literature, several material models were explored to determine the best one for correctly capturing experimental data so that the obtained solution, i.e., the stresses and strains, can be considered as accurate ones. As a result, the Halpin-Tsai model was used to accurately estimate mechanical parameters. The Halpin-Tsai model for FGMs is simple to apply; it has just one variable that is determined by the form of the particle. For analysis, the particle's form is considered to be spherical. Depending on the application in rotating axisymmetric constructions, two alternative materials can be used to provide optimal performance. In the case of rotating disks, metals are employed at the root and ceramics at the tip, with graded materials in between. Because metal will provide the necessary hardness, and ceramics. The current analysis is carried out taking the grading index (n) as unity. Varying the grading index by higher than or lower than unity either increases or decreases the performance of the component further, depending on whether the graded mixture is rich in metal or ceramic. Therefore, the present study is important for design engineers in estimating the performance of functionally graded rotating structures under such conditions. This analysis provides valuable insights into the distribution of stresses across different disk profiles under constant speed conditions and thermal loadings, highlighting the variations in stress patterns and their implications for structural integrity and performance.

References

- [1] Miyamoto Y, Kaysser WA, Rabin BH, Kawasaki A, Ford RG. Functionally Graded Materials. Materials Technology Series. Boston: Springer US; 1999;5
- [2] Madan R, Bhowmick S. A review on application of FGM fabricated using solid-state processes. *Advances in Materials and Processing Technologies*. 2020:1–12.
- [3] Al-Sanabani FA, Madfa AA, Al-Qudaimi NH. Alumina ceramic for dental applications: A review article.
- [4] Almasi D, Sadeghi M, Lau WJ, Roozbahani F, Iqbal N. Functionally graded polymeric materials: A brief review of current fabrication methods and introduction of a novel fabrication method. *Materials Science and Engineering C*. 2016;64:102–107.

- [5] Durodola JF, Attia O. Deformation and stresses in functionally graded rotating disks. *Composites Science and Technology*. 2000;60(7):987–995.
- [6] Bayat M, Saleem M, Sahari BB, Hamouda AMS, Mahdi E. Analysis of functionally graded rotating disks with variable thickness. *Mechanics Research Communications*. 2008;35(5):283–309.
- [7] Peng XL, Li XF. Thermal stress in rotating functionally graded hollow circular disks. *Composite Structures*. 2010;92(8):1896–1904.
- [8] Allam MNM, Tantawy R, Zenkour AM. Thermoelastic stresses in functionally graded rotating annular disks with variable thickness. *Journal of Theoretical and Applied Mechanics*. 2018:1029.
- [9] Tutuncu N, Temel B. An efficient unified method for thermoelastic analysis of functionally graded rotating disks of variable thickness. *Mechanics of Advanced Materials and Structures*. 2013;20(1):38–46.
- [10] Shahriari B, Safari M. Stress analysis of FGM rotating disk subjected to mechanical and thermal loads in aircraft gas turbine engine. *Mechanics of Advanced Composite Structures*. 2019.
- [11] Nie GJ, Batra RC. Stress analysis and material tailoring in isotropic linear thermoelastic incompressible functionally graded rotating disks of variable thickness. *Composite Structures*. 2010;92(3):720–729.
- [12] Abdalla HMA, Casagrande D, Moro L. Thermo-mechanical analysis and optimization of functionally graded rotating disks. *The Journal of Strain Analysis for Engineering Design*. 2020;55(5–6):159–171.
- [13] Madan R, Saha K, Bhowmick S. Limit speeds and stresses in power law functionally graded rotating disks. *Advances in Materials Research*. 2020;9(2):115–131.
- [14] Rabia B, Tahar HD, Abderezak R. Interfacial stresses in porous PFGM-RC hybrid beam. *Advances in Materials Research*. 2024;13(1):37–53.
- [15] Madan R, Bhowmick S. Material modelling and limit angular speed analysis of porous trigonometric functionally graded rotating disk. *Advances in Materials and Processing Technologies*. 2022:1–13.
- [16] Garg A, Belarbi MO, Li L, Chalak HD, Tounsi A, Zenkour AM. Comparative study on the buckling analysis of exponential, power and sigmoidal sandwich FGM plates under hygro-thermal conditions. *Advances in Materials Research*. 2024;13(6):431–462.
- [17] Suresh S, Mortensen A. Functionally graded metals and metal-ceramic composites: Part 2 Thermomechanical behaviour. *International Materials Reviews*. 1997;42(3):85–116.
- [18] Bruck HA, Rabin BH. Evaluation of rule-of-mixtures predictions of thermal expansion in powder-processed Ni-Al₂O₃ composites. *Journal of the American Ceramic Society*. 2004;82(10):2927–2930.
- [19] Mori T, Tanaka K. Average stress in matrix and average elastic energy of materials with misfitting inclusions. *Acta Metallurgica*. 1973;21(5):571–574.
- [20] Nakamura T, Wang T, Sampath S. Determination of properties of graded materials by inverse analysis and instrumented indentation. *Acta Materialia*. 2000;48(17):4293–4306.
- [21] Bhattacharyya M, Kapuria S, Kumar AN. On the stress to strain transfer ratio and elastic deflection behavior for Al/SiC functionally graded material. *Mechanics of Advanced Materials and Structures*. 2007;14(4):295–302.
- [22] Madan R, Bhowmick S. Modeling of functionally graded materials to estimate effective thermo-mechanical properties. *World Journal of Engineering*. 2021.
- [23] Das P, Benslimane A, Islam MA, Mondal D, Nazim MS. A thermo-mechanically loaded rotating FGM cylindrical pressure vessels under parabolic changing properties: An analytical and numerical analysis. *Heliyon*. 2024.
- [24] Kadum Njim E, et al. Numerical and analytical investigation of free vibration behavior of porous functionally graded sandwich plates. *Mechanics of Advanced Composite Structures*. 2024.
- [25] Alhous ZFA, Jweeg MJ, Njim EK, Mouthanna A, Flayyih MA, Madan R, Khobragade P, Rai PK. Nonlinear frequency and dynamic response of PLA polymeric imperfect FG sandwich plates under hygrothermal conditions. *Coupled Systems Mechanics*. 2025;14(1):1–19.
- [26] Abbas EN, Abud Ali ZAA, Njim EK, Flayyih MA, Madan R. Analytical and numerical investigation of free vibration of nanoparticle-reinforced composite cylindrical shells. *Diagnostyka*. 2025:1–12.
- [27] Njim EK, Hasan HR, Jweeg MJ, Al-Waily M, Hameed AA, Youssef AM, Elsayed FM. Experimental and numerical analysis of uni-axial buckling of single-phase functionally graded porous polymeric sandwich plates. *BIO Web of Conferences*. 2024;97:00002.
- [28] Njim EK, Bakhy SH, Al-Waily M. Analytical and numerical investigation of free vibration behavior for sandwich plate with functionally graded porous metal core. *Journal of Science and Technology*. 2021;29(3).
- [29] Njim EK, et al. Mechanical properties of sandwiched construction with composite and hybrid core structure. *Advances in Polymer Technology*. 2024:3803199.

- [30] Hadji L, Avcar M, Civalek Ö. An analytical solution for the free vibration of FG nanoplates. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*. 2021;43(9):418.
- [31] Viet NV, Zaki W, Wang Q. Free vibration characteristics of sectioned unidirectional/bidirectional functionally graded material cantilever beams based on finite element analysis. *Applied Mathematics and Mechanics (English Edition)*. 2020;41(12):1787–1804.
- [32] Lu Y, Chen X. Nonlinear parametric dynamics of bidirectional functionally graded beams. *Shock and Vibration*. 2020;2020:1–13.
- [33] Hadji L, Madan R, Bernard F. Thermal buckling in multi-directional porous plates: The effects of material grading and aspect ratio. *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*. 2024;238(4):412–426.
- [34] Hadji L, Plevris V, Madan R, Ait Atmane H. Multi-directional functionally graded sandwich plates: Buckling and free vibration analysis with refined plate models under various boundary conditions. *Computation*. 2024;12(4):65.
- [35] Hadji L, Plevris V, Madan R. A static and free vibration analysis of porous functionally graded beams. *The 2nd International Conference on Civil Infrastructure and Construction*. 2023:433–441.
- [36] Madan R, Bhowmick S, Hadji L, Alnujaie A. Limit angular speed analysis of porous functionally graded rotating disk under thermo-mechanical loading. *Materials and Manufacturing Processes*. 2023.
- [37] Raad H, Najim E, Jweeg M, AlWaily M, Hadji L, Madan R. Vibration analysis of sandwich plates with hybrid composite cores combining porous polymer and foam structures. *Journal of Computational Applied Mechanics*. 2024;55(3).
- [38] Hangai Y, Ishihara A, Utsunomiya T, Kuwazuru O, Yoshikawa N. Fabrication of two layers porous aluminum varying porosity by friction powder sintering process and its compression properties. *Journal of the Japan Institute of Metals*. 2016;80(6):390–393.
- [39] Madan R, Bhowmick S. Fabrication, microstructural characterization and finite element analysis of functionally graded Al-Al₂O₃ disk using powder metallurgy technique. *Materials Today Communications*. 2022;32:103878.
- [40] Hadji L, Bernard F, Safa A, Tounsi A. Bending and free vibration analysis for FGM plates containing various distribution shape of porosity. *Advances in Materials Research*. 2021;10(2):115–135.
- [41] Njim EK, Al-Maamori MH, Madan R, Bakhy SH, Al-Waily M, Khobragade P, Hadji L. Numerical and analytical investigation of free vibration behavior of porous functionally graded sandwich plates. *Mechanics of Advanced Composite Structures*. 2025;12(3):555–568.
- [42] Sondhi L, Sahu RK, Kumar R, Yadav S, Bhowmick S, Madan R. Functionally graded polar orthotropic rotating disks: investigating thermo-elastic behavior under different boundary conditions. *International Journal on Interactive Design and Manufacturing*. 2023.
- [43] Parihar RS, Setti SG, Sahu RK. Recent advances in the manufacturing processes of functionally graded materials: a review. *Science and Engineering of Composite Materials*. 2018;25(2):309–336.
- [44] Verma RK, Parganiha D, Chopkar M. A review on fabrication and characteristics of functionally graded aluminum matrix composites fabricated by centrifugal casting method. *SN Applied Sciences*. 2021;3(2):227.
- [45] Madan R, Bhowmick S. A review on application of FGM fabricated using solid-state processes. *Advances in Materials and Processing Technologies*. 2020;6(3):608–619.
- [46] Fujii T, Murakami R, Kobayashi N, Tohgo K, Shimamura Y. Uniform porous and functionally graded porous titanium fabricated via space holder technique with spark plasma sintering for biomedical applications. *Advanced Powder Technology*. 2022;33(6).
- [47] Li Y, Feng Z, Hao L, Huang L, Xin C, Wang Y, Bilotti E, Essa K, Zhang H, Li Z, Yan F, Peijs T. A review on functionally graded materials and structures via additive manufacturing: From multi-scale design to versatile functional properties. *Advanced Materials Technologies*. 2020;5(6):1900981.
- [48] Bobbio LD, Bocklund B, Simsek E, Ott RT, Kramer MJ, Liu ZK, Beese AM. Design of an additively manufactured functionally graded material of 316 stainless steel and Ti-6Al-4V with Ni-20Cr, Cr, and V intermediate compositions. *Additive Manufacturing*. 2022;51.
- [49] Deshmukh PS, Sathiaraj D. A review on twin wire arc additive manufacturing of metals and alloys: Microstructure and mechanical properties. *Lecture Notes in Mechanical Engineering*. 2022:67.
- [50] Dielemans G, Briels D, Jaugstetter F, Henke K, Dörfler K. Additive manufacturing of thermally enhanced lightweight concrete wall elements with closed cellular structures. *Journal of Facade Design and Engineering*. 2021;9(1):59–72.
- [51] Dong G, Tang Y, Li D, Zhao YF. Design and optimization of solid lattice hybrid structures fabricated by additive manufacturing. *Additive Manufacturing*. 2020;33.

- [52] Mallick A, Gangi Setti S, Sahu RK. Centrifugally cast functionally graded materials: Fabrication and challenges for probable automotive cylinder liner application. *Ceramics International*. 2023;49(6):8649–8682.
- [53] Madan R, Bhowmick S. Fabrication, microstructural characterization and finite element analysis of functionally graded Al-Al₂O₃ disk using powder metallurgy technique. *Materials Today Communications*. 2022;32.
- [54] Madan R, Bhowmick S. Fabrication and microstructural characterization of Al-SiC based functionally graded disk. *Aircraft Engineering and Aerospace Technology*. 2022.
- [55] Ansari M, Jabari E, Toyserkani E. Opportunities and challenges in additive manufacturing of functionally graded metallic materials via powder-fed laser directed energy deposition: A review. *Journal of Materials Processing Technology*. 2021;294:117117.
- [56] Ayan Y, Kahraman N. Fabrication and characterization of functionally graded material structure containing two dissimilar steels (ER70S-6 and 308LSi) by wire arc additive manufacturing. *Materials Today Communications*. 2022;33:104457.
- [57] Singh RK, Singh RC. Evaluation of the mechanical and microstructural properties of an Al7075/B4C/Gr FGM plate fabricated using a stir casting process. *Journal of Mechanical Science and Technology*. 2024;38(10):5389–5398.
- [58] Kumar V, Kumari P. Fabrication and characterization of Cu/SiC-based axially functionally graded beam. *Advanced Engineering Materials*. 2022;24(5).
- [59] Wei C, Cheng J, Zhang M, Zhou R, Wei B, Yu X, Luo L, Chen P. Fabrication of diamond/W–Cu functionally graded material by microwave sintering. *Nuclear Engineering and Technology*. 2022;54(3):975–983.
- [60] Cho JR, Ha DY. Volume fraction optimization for minimizing thermal stress in Ni–Al₂O₃ functionally graded materials. *Materials Science and Engineering A*. 2002;334(1–2):147–155.
- [61] Maleki Jebeli S, Shariat Panahi M. An evolutionary approach for simultaneous optimization of material property distribution and topology of FG structures. *Engineering Computations*. 2015;32(2):234–257.
- [62] Jiang F, Zhao W, Chen L, Zheng C, Chen H. Combined shape and topology optimization for sound barrier by using the isogeometric boundary element method. *Engineering Analysis with Boundary Elements*. 2021;124:124–136.
- [63] Mostafa Shaaban A, Anitescu C, Atroshchenko E, Rabczuk T. Shape optimization by conventional and extended isogeometric boundary element method with PSO for two-dimensional Helmholtz acoustic problems. *Engineering Analysis with Boundary Elements*. 2020;113:156–169.
- [64] Madan R, Saha K, Bhowmick S. Limit elastic analysis of rotating annular disks having sigmoid-FGM composition based on MROM. *World Journal of Engineering*. 2019;16(6):806–813.
- [65] Nejad MZ, Rastgoo A, Hadi A. Exact elasto-plastic analysis of rotating disks made of functionally graded materials. *International Journal of Engineering Science*. 2014;85:47–57.
- [66] Lakshman S, Subhashis S, Kashinath S, Shubhankar B. Limit elastic speeds of functionally graded annular disks. *FME Transactions*. 2018;46(4):603–611.