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Research Article

Heat Transfer of MHD Non-Newtonian Fluid in Presence of Thermally Stratified Medium

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Abstract:
In this paper, investigated on heat transfer of MHD non-newtonian fluid in presence of thermally stratified medium. The momentum and energy equations associated with the partial differential equations are transformed into highly nonlinear ordinary differential equations by using similarity transformations. Kummer's function is used to represent the analytical solutions to these equations, and obtained the numerical solution of energy equation using analytical solution of flow. Numerical analysis carried out through BVP5C and ND Solver commands, and comparing with analytical method while using wolfram language mathematica and values are accurate. Graphically represented all physical parameters and also represented in Table form and Bar & Contour Graphs. In presence of thermal stratification, it is seen that the rate of surface heat transfer decreases whenever rising in viscoelastic and magnetic parameter, Prandtl number. Temperature distribution exhibits the reverse effect. Applications are geothermal & power plant condensation systems, geological transport, lake thermohydraulics, volcanic flows.

Keywords: *Viscoelastic fluid, Heat transfer, Thermally stratified medium, Stretching sheet, MHD*

1. Introduction

The study of laminar flow and heat transfer across a stretched sheet in a viscous fluid is particularly interesting due to its numerous commercial applications and significant implications for a variety of technological processes. Several industrial production processes produce sheeting material, which includes both metal and polymer sheets. Heat transmission is the transfer of energy from one point to another using a temperature gradient. Nature has three modes of heat transport. i.e., (i) Heat transfer by conduction (ii) Convection and (iii) Radiation. Mass transpose is the transfer of mass/concentration from one point to another point by virtue of concentration difference. Transport of molecules from high chemical potential to low chemical potential is known as Diffusion. The transport of molecules in bulk motion due to bulk flow, called convective mass transfer.

In continuum mechanics and materials science, viscoelasticity refers to materials that exhibit both viscous and elastic properties when deformed. Under pressure, viscous materials such as water resist shear flow and stretch linearly with time. Elastic materials stretch when they are stretched, but quickly revert to their former shape as soon as the tension is released. Since viscoelastic materials have all these properties, their elongation varies with time. In contrast, elasticity is usually caused by the stretching of bonds along crystallographic planes in an ordered solid, while viscosity is caused by the distribution of atoms or molecules in an amorphous material.

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Subhas Abel and Mahesha [1] applying magnetohydrodynamic boundary layer analysis in the presence of thermal radiation and a non-uniform heat source, the flow and heat transfer properties of a non-Newtonian viscoelastic fluid over a flat sheet with a linear velocity were investigated. The results of this study show that the combination of radiation, a non-uniform heat source, and influencing thermal conductivity can considerably impact the rate of heat transfer in the boundary layer domain.

In [2, 3] examined the flow and heat transfer attributes of a non-Newtonian viscoelastic fluid over a stretching sheet while taking into account a number of variables, including a fluid-saturated porous medium, a magnetohydrodynamic boundary layer, and non-uniform heat absorption or desorption. Here, an analysis has been done to look at how a magnetic field affects heat transmission over a stretching sheet with a non-uniform heat source and viscoelastic liquid flow.

The stratification of the medium may be triggered by a change in temperature, which alters the density of the medium. The medium's absorption of thermal energy from heated bodies and other thermal sources frequently causes the phenomenon known as thermal stratification. Thermal stratification is the result of the interaction of two types of steam that have different temperatures. Due to the temperature differential between the two types of water, the warmer, lighter water may float above the colder, heavier water, while the latter may sink to the pipe's bottom.

Tewari and Singh [4], natural convection heat transmission data are reported for a finite-length vertical plate submerged in a fluid-saturated porous medium. The impact of stratification parameters on local and global heat transmission coefficients, velocity, and temperature fields are investigated. [5-7] determined the impact of thermal stratification on porous and nonporous medium in a fluid and presented results regarding effects of Darcy and non-Darcian law phenomena. Takhar et al. [8] imagining the flow of a laminar natural interconnection on border layer over a perpendicular, narrow, isothermal cylinder enclosed in a porous thermal stratified medium. It has been discovered that for accurate values of the ambient stratification parameter, skin friction disappears and the path of heat transport changes. Skin friction and heat transmission declines when ambient stratification, curvature, and inertia factors rise, while increasing with permeability. Rathish Kumar and Shalini [9] investigated free convection in a thermal- stratified porous medium is comparison with the thermally steady porous medium under both darcian and non-darcian considered but in the case of Afify [10] considered only non-Darcian case. In both cases thermal stratification medium behaves different results and they are mentioned in their studies. Ishak et al. [11] overworked the mixed conduction boundary layer's similarity solutions flow along a perpendicular surface that is encased in a porous thermal stratification material. The data indicate that the thermal stratification not only delays boundary layer separation but also significant impact on surface shear stress and surface heat transport. Hassanien and Hamad [12] deliberately established a similarity investigation of micro polar fluid flow on top of a vertical plates with in a thermally stratified unstable free convective boundary layer material. In this study investigates circumstances in which the ambient temperature is uniform, changes with position, and varies exponentially over time.

Bég et al. [13] discussed about the two - dimensional mathematical model of thermal convection developed on a vertically moving plate with non-darcian porous medium saturated with a Newtonian liquid and a thermally stratified medium, but in the case of Neagu [14] studied the perpendicular wavy wall to a non-Darcy porous medium. In both Investigation researchers found good results regarding thermal stratification parameters. Mat Yasin et al. [15] mathematically studied on a perpendicular surface submerged in a thermal-saturated stratified porous medium with nanofluid, the flow of a stable mixed convective boundary layer is observed. It has been found that the addition of nanoparticles increases heat transmission from the surface. Consequently, the type of nanofluid used is crucial for improving heat transfer. Foisal and Alam [16] investigated MHD free convection across an inclined plate in a thermally stratified high porosity material under a magnetic field. According to this study, as thermal stratification rises, local secondary and average shear stress rise, whereas local primary number and average shear stress decline. Analysis of heat and mass transmission in various fluid flows through various geometries like stretching sheet, perpendicular stretching sheet inserted in thermally stratified medium along with convective boundary conditions is examined with including various effects are investigated by [17-21].

Falodun and Omowaye [22] analysis of the heat and mass transmission in double-diffusive MHD non-Darcy convection. It is taken into account over a stretched sheet that is enmeshed in a porous thermally stratified material. The findings revealed that when a magnetic field is added to the flow in a transverse direction, a resistive force known as the Lorentz force is generated. When it comes to heat and mass transmission, this force tends to slow the flow of an electrically conducting fluid. The fluid velocity in the boundary layer drops. Additionally, the suction raises the fluid's concentration, temperature, and velocity. Megahed and Abbas [23] investigated a stretched sheet with heat developed in a porous medium,an other than Newtonian cross liquid flows laminarly and completely developed with heat mass transfer.

All interpenetrating particle species are treated as a single continuous medium in the magnetohydrodynamics (MHD) model of electrically conducting fluids. MHD is often referred to as hydromagnetism or magneto-fluid dynamics. Its main focus is on the large-scale, low-frequency magnetic behavior in liquid metals and plasmas, but it finds applications in many other fields as well, such as engineering, astronomy, and geophysics.

The flow and heat mass transmission of electrically conducting fluid, free convection border-surface flow occurs along a stretching sheet immersed in a thermal-stratified permeable material when a equable normal magnetic field is present is investigated in several studies such as [24, 25]. Murthy et al. [26] developing a convection boundary condition for the interplay of a magnetic field with free communication in non-Darcy porous medium that are thermal-stratified and saturated with nanofluid. It can be seen that the stratification of the medium has a considerable influence on heat transport. This effect is amplified when the parameters for the nanofluid in the medium and the Biot number change due to the convective boundary condition. Mukhopadhyay [27] provides the numerical solutions for heat transfer and steady MHD boundary layer flow across an exponentially stretched surface contained in a thermally stratified medium when suction is present. In a viscous incompressible fluid, the effects of suction and magnetic parameter reduce the velocity field, which increases the skin-friction coefficient. As the magnetic field increases, the rate of transfer decreases. The temperature drops as the stratification parameter values rise. Tamoor [28] provides a numerical solution for the laminar axially-symmetric hydro-magnetic flow of a non-compressible, electrically conducting and viscous fluid transferring around a circular cylinder immersed in a thermally stratified medium. The numerical results obtained for several relevant factors are displayed graphically for the velocity and temperature distribution. Eswaramoorthi et al. [29] thought about the gyro-tactic microbes on a stratified MHD nanofluid movement with heat absorption is emphasized and investigated, and it is introduce that the liquid velocity declined as the rate of the imbibing/inoculation parameter and the Hartmann number enhanced. In this analysis, the fluid velocity decreases when the values of the Hartmann number and the suction/injection parameters increase.The temperature of the fluid increases when the values of the heat generation/absorption parameters increase and decreases when the values of the thermal stratification parameters increase.

The investigation of thermally stratified flows is also of great interest in other floating flow systems, e.g. in industrial heat treatment, in geothermal systems, in geological transport, in condensation systems of power plants, in volcanic flows and in the thermohydraulics of lakes. Various application of thermally stratification are discussed in various research field among that here we discussed few of them like that,

Gaggioli et al. [30] discussed an inventive idea was proposed for a thermal energy storage system that stratifies molten salt acts as a heat transpose liquid and a heat accumulator while having an integrated steam generator. The tests carried out at ENEA have shown that the thermal stratification of the molten salt can be kept fairly constant over several hours and that the integrated steam generator actively ensures and maintains the stratification during the operating time by preventing the stratified layers from mixing. Makinde and Mishra [31] studied the combined effects of viscosity changes, Joule heating, magnetic field, buoyancy forces, heat source, nth order chemical reaction and viscous dissipation on mixed convection. The investigation focuses on the Blasius flow of a conducting fluid over a permeable plate that is convectively heated and submerged in a porous medium. In the existence of a magnetic field, the skin friction is found to decline, while the Nusselt and Sherwood numbers rises with decreasing fluid viscosity. Mishra et al. [32] investigates the influence of radiation and a heat source on an electrically conducting micropolar MHD fluid flow along a semi-infinite horizontal plate. The main flow direction was influenced by the uniform magnetic field. The inclusion of the coupling parameter and the inertial effect leads to a decline in the thickness of the boundary layer, while the magnetic parameter causes an increase in the velocity profile. Baag et al. [33] investigation of flow and heat transfer in the MHD boundary layer flow over an expanding plate through a porous medium under the influence of a changing magnetic field in the existence of a constant heat source. The Lorentz force slows down the velocity profile at all points of the velocity boundary layer, and an increase in the heat source parameter in the thermal layer leads to an increase in the temperature profile — these are the two main findings. Irfan et al. [34] examines the reactions of compliant walls on the heat transmission analysis within the electro osmotic flow of the Ellis fluid model and provides a new perspective. Silver nanoparticles are also included, which could be used therapeutically due to their antibacterial properties. Nazeer et al. [35] investigation focuses on the peristaltic transmission of a pair of stress fluids with heat transmission via flexible channel walls equipped with hair-like features. Metachronal (MCW) and peristaltic (P) waves propagate simultaneously and cause locomotion of the biological fluid. Nazeer et al. [36], in this study, a mathematical model is established to investigate the impacts of mass and heat transmission in a tangential hyperbolic fluid bound in a horizontal plane. The results show that the channel of the concentration field decreases and the velocity and temperature fields increase as a function of the pressure gradient and power-law index parameters.

A close inspection of above recent published works reveals that the no one research has been done on the Heat transmission of MHD non-Newtonian fluid in existence of thermally stratified medium including different analytical methods, numerical methods. Therefore in our research included comparison of numerical and analytical methods. In this investigation, the arising new research questions for our study serve as motivation to suggest novel applications of research questions and their implementation in our studies, are presented magnificently, and represent the application of our studies. A Fourth-order ordinary differential equation that corresponds to the momentum equation are obtained using the appropriate transformations. Kummer's function is used to represent the analytical solutions to these equations also obtained the numerical solution of energy equation using analytical solution of flow.Numerical analysis carried out through BVP5C and ND Solver commands, and comparing with analytical method while using wolfram language mathematica. For the problem our solution is to find out the analytical and numerical solution and validate the result by comparing the analytical, numerical methods and presented results in the form 2D graphs and Contour graphs. For the purpose of graphically displaying the findings, numerical calculations using the shooting technique were performed for distinct values of the problem's non-dimensional parameters up to the appropriate degree of precision. The study of the findings demonstrates that, in the presence the stratification parameter significantly affects the flow field and also thermal stratification effect on Magnetic parameter, Prandtl number, Viscoelastic parameter. Heat transmission decreases due to variation of fluid flow over a stretching sheet and also showing different environmental changes due to their property of thermal behavior of stratification. This research also includes an estimation of the heat transfer coefficient, which is crucial from the perspective of industrial applications. It is believed that the findings would complement the findings of earlier research in addition to offering important information for applications and also researchers are used our problem by including new parameters ,new geometries and new innovative ideas. Our investigation is helpful for future research and emphasizes new ideas to implement in terms of various fluids.

2. Physical Interpretation of the Model

2.1 Walter's Liquid B Model

Viscoelastic fluids are one of the significant subclasses of non-Newtonian fluids that have attracted a lot of attention from academics lately. The reason for this is that a wide range of scientific and engineering fields, especially chemical engineering, use these kinds of investigations. Walters has developed a sophisticated model known as the Walters'-B viscoelastic fluid model. This rheological model can faithfully replicate the intricate flow behavior of a variety of polymer solutions, hydrocarbons, paints, and other industrial liquids. The highly nonlinear partial differential equations produced by the boundary layer flow of the Walters-B fluid model are one order higher than the equations governing Newtonian fluids. Additionally, it takes into account the fluid's elastic characteristics, which are important in the extensional behavior of polymers.

An application of Walter's liquid B model is not compressible, electronically conducting, viscous with an elastic liquid flow in a steady state in two dimensions caused by a stretched sheet, is taken into consideration. (See Fig. 1). The sheet is stretched along the x-axis while maintaining the origin stationary by applying force to the y coordinate. The x coordinate is taken along the stretch plate. Assume that the normal stress contribution is of the equal magnitude as the shear stress contribution and use the border layer approximation. A variable magnetic field $B_0$is applied normally to the plate, B_0 is consistent. The plate has a temperature of $T_W(x)$ and is immersed in a thermally stratified medium with a variable ambient temperature of $T_\infty(x)$.Where, $T_W(x) > T_\infty(x)$ and presumed that $T_W(x) = T_0 + cx^2$, $T_\infty(x) = T_0 + dx^2$ where T_0 is the reference temperature, $c > 0, d \geq 0$ are constant.

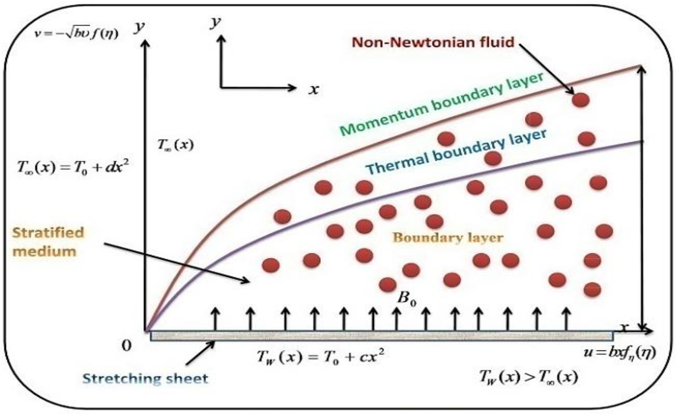

Fig. 1. Sketch of the physical problem.

2.2 Thermal Stratification (Correlation between Temperature and Stratified Environment in Nature)

When two vapour forms with different temperatures come into contact, thermal stratification occurs. Due to their different temperatures, the warmer, lighter water can float above the colder, heavier water, which settles at the bottom of the pipe. Example:-Anyone who has been swimming in the summer and walked through the warm water at the surface to feel the icy water a few meters below has felt the most obvious effect of thermal stratification. This stratification is a natural phenomenon in any static body of water. From Figs. 2-5 show that the effect of thermal stratification during winter, spring, summer, Fall seasons.

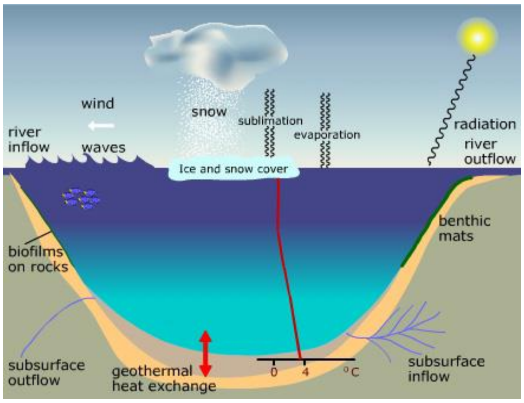

Fig. 2. Stratification and density inversion during winter.

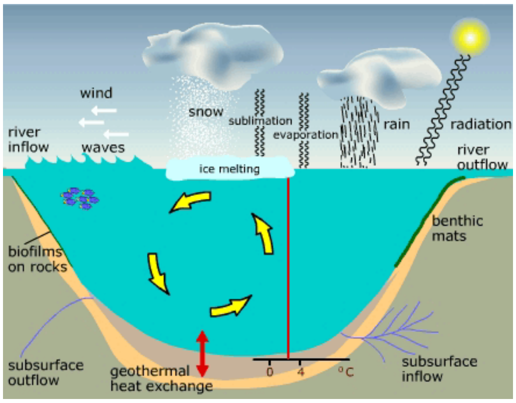

Fig. 3. Spring shows an absence of thermal stratification.

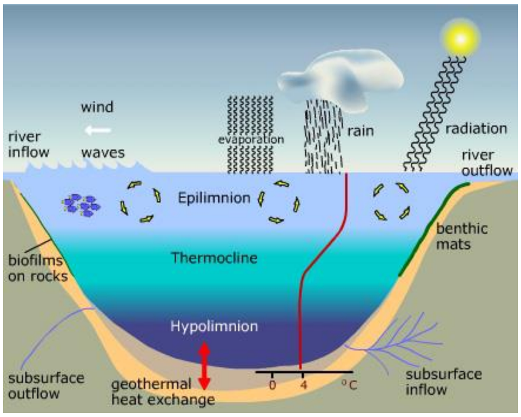

Fig. 4. Summer stratification of lake temperatures.

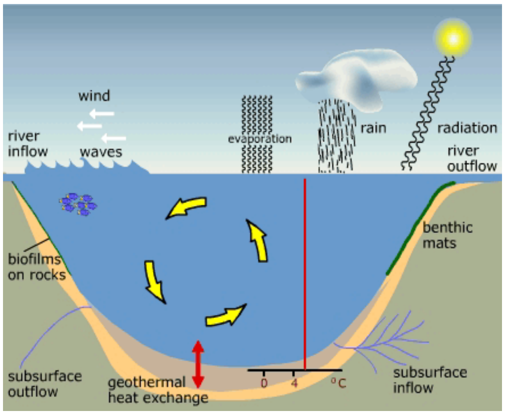

Fig. 5. Fall again shows an absence of thermal stratification.

2.2.1 Stratification and Density Inversion during Winter

In the winter, the water temperature beneath the ice is 0°C. This causes a density inversion (water at 0°C is lighter than water at 4°C), stratifying the water body in winter. The hypolimnion might turn anoxic, with nutrients stimulated in the sediment and accumulating in the deeper water layers. After the ice has melted in the springtime, strong winds, lower surface temperatures (which means higher density and viscosity), and the lack of thermal stratification enable the entire water body to mix completely. The mixing extracts nutrients from the lake bottom. Anaerobic surroundings at the lower, deepest parts are converted to aerobic environments.

2.2.2 Spring Shows an Absence of Thermal Stratification

Strong winds, lower surface temperatures (greater density and viscosity), and a lack of thermal stratification allow for thorough mixing of the whole body of water. The mixing extracts nutrients from the lake bottom. Anaerobic atmospheres at lower depths are converted to aerobic atmospheres.

2.2.3 Summer Stratification of Lake Temperatures

The rising level of sun radiation warmed the water's highest layers, reducing its density and viscosity. The lake stratifies into an epilimnion (the upper section of the water column) and a hypolimnion (the deeper part of the water column). At the thermocline (the region of maximum temperature variations), the metalimnion separates the epilimnion from the hypolimnion. The substantial temperature gradient between the epilimnion and the hypolimnion stops the two levels from mixing. Algae can bloom in areas with high levels of light. During summer stagnation, bacteria decompose dead algae, resulting in anoxic conditions at the lake's bottom and in surface sediment layers (boosted oxygen demand).In several alpine zones, cold and snowy seasons occur on a frequent basis during the summer, causing the water column to mix at least to a particular level.

2.2.4 Fall Again Shows an Absence of Thermal Stratification

Winds, colder surface water, and a lack of thermal stratification allow the lake to mix. In high-mountain lakes that are not typically deep, mixing may influence every layer of the water column. During freezing, mix the water column to reestablish the aerobic state and disperse sediment-bound nutrients uniformly. The varied water temperatures at various depths represent a lake's thermal balance. High alpine lakes, including other lakes, are impacted by seasonal variations in the surrounding environment. The organisms must be able to adapt to these fluctuations.

The primary factors for thermal behavior are:

- Water density is maximum at 4°C, surface water is lighter at $T > 4°C$ in summer and $T < 4°C$ below the ice cover.
- Summer sees an increase in heat inflow from the atmosphere.
- Snowfall during the summer can quickly disrupt thermal stratification.
- Windstorms can mix homothermal water layers throughout the summer.

By considering all seasonal changes occurred in environment with presence of thermal stratification, same environment is occurred in our model here we are considering walter’s liquid B model with presence of non-newtonian liquid.

2.3 Correlations of Thermal Stratification with Temperature Studied in Our Investigation

Whenever Non–newtonian fluid is moving through stratified medium then some certain changes are occur due to two kinds of steam with two different temperature contact while electrically pass some magnetic field then fluids acts as a conducting fluid then its converts into Magnetohydrodynamic field .In this condition viscoelastic property of the fluid will changes and also some stratified environment of the fluid will also shows some certain variation on the heat and mass transfer of the fluid and its impacts on the momentum and thermal diffusivity of the fluid then prandtl number of the fluid will varies and also viscoelastic property of the fluid. From this model fluid can be used in a many industrial applications like that geothermal systems, geological transport, power plant condensation system, lake thermohydraulics, and volcanic flows etc.

3. Mathematical Model Construction of the Fluid

Considered are the governing equations for heat transmission and flow problem are given by,

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Subhas Abel and Nandeppanavar [2] introduce below equation,

Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\}-\frac{\sigma B_0{}^2 u}{\rho} \tag{2}$$

Mukhopadhyay [27] introduce below equation,

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Where x and y directional velocity components are denoted by u, v respectively.
$v=\frac{\mu}{\rho}$ is kinematic viscosity, k_0 is the viscoelastic parameter, B_0 is uniform magnetic field and σ is fluids electrical conductivity, ρ is density of fluid, μ is the fluid dynamic viscosity , c_p is Specific heat capacitance and k is the fluids thermal conductivity.

3.1 The Boundary Conditions for Velocity and the Solution of the Momentum Equation

The proper boundary conditions for the problem are given by,

$$\left.\begin{array}{llll} u=bx & v=0 & at & y=0 \\ u\to 0 & u_y & as & y\to\infty \end{array}\right\} \tag{4}$$

Here, differentiation with regard to y is represented by the subscript y and $b>0$ known as the linear stretched rate constants. The similarity transformation for the movement of a viscous with elastic fluid of Walter's liquid B type across an impermeable stretched sheet is used to solve the governing boundary layer momentum Eq. (2), which are

$$v=-\sqrt{bv}f(\eta),\ u\ =\ bxf_{\eta}(\eta)\ \ ,\eta=\sqrt{\frac{b}{v}}y,\ \theta(\eta)\ =\ \frac{T-T_\infty}{T_W-T_0} \tag{5}$$

Where η is the similarity non-constant value and f is the without dimension stream function .Substituting Eq. (5), in Eq. (2), a nonlinear differential equation of fourth order is discovered as,

$$f_\eta^2-ff_{\eta\eta}=f_{\eta\eta\eta}-k_1\{\ 2f_\eta f_{\eta\eta\eta}-ff_{\eta\eta\eta\eta}-f_{\eta\eta}^2\ \}-\ Mn\ f_\eta \tag{6}$$

Where $k_1=\frac{k_0 b}{v}$ the viscous with an elastic parameter is, $Mn=\frac{\sigma B_0^2}{b\rho}$ is magnetic parameter.

By using boundary condition Eq. (4), takes the form

$$\left.\begin{array}{llll} f_\eta(\eta)=1, & f(\eta)\ =0 & at & \eta=0 \\ f_\eta(\eta)\to 0, & f_{\eta\eta}(\eta)\to 0 & as & \eta\to\infty \end{array}\right\} \tag{7}$$

We arrived at a feasible solution of Eq. (6), using the boundary condition Eq. (7) as

$$f(\eta) = \frac{1 - Exp[e^{-\alpha\eta}]}{\alpha} \tag{8}$$

Where $\alpha = \sqrt{\frac{1+Mn}{1-k_1}}$, $(0 \leq \alpha < \infty$ and $0 < k_1 < 1)$

Then the velocity constituents are,

$$\left. u = bxe^{-\alpha\eta}, \quad v = -\sqrt{bv}\left(\frac{1 - e^{-\alpha\eta}}{\alpha}\right) \right\} \tag{9}$$

3.2 Boundary Condition on Temperature and its Solution (Analytical Method)

The following are the suitable boundary constraints for the temperature are given below

$$\left. \begin{array}{lll} T = T_W(x) & at & y = 0 \\ T = T_\infty(x) & as & y \to \infty \end{array} \right\} \tag{10}$$

$T_W(x)$ is the sheet temperature and is submerged in a thermally stratified medium with fluctuating ambient temperature $T_\infty(x)$

Here $T_W(x) > T_\infty(x)$

It is assumed that $\left. \begin{array}{l} T_W(x) = T_0 + cx^2 \\ T_\infty(x) = T_0 + dx^2 \end{array} \right\}$ (11)

Where, T_0 is reference temperature$c > 0$, $d \geq 0$ are sustained values.

Using Eq. (5), and Eq. (10), Eq. (11), in Eq. (3) and its reduces to

$$\theta_{\eta\eta} + \mathrm{Pr}(f\theta_\eta - 2f_\eta\theta) - 2\mathrm{Pr}St\, f_\eta = 0 \tag{12}$$

Where $\mathrm{Pr} = \frac{\mu c_p}{k}$ (Prandtl Number), $St = \frac{d}{c}$ (Stratification parameter)

Corresponding boundary conditions are,

$$\left. \begin{array}{lll} \theta(\eta) = 1 - St & at & \eta = 0 \\ \theta \to 0 & as & \eta \to \infty \end{array} \right\} \tag{13}$$

Here, $St > 0$ shows that the stable stratified environment but $St = 0$ Shows an unstratified environment.
Now we proceed to obtain solution of Eq. (12).
Introduce a new variable

$$\left. \begin{array}{l} \xi = -\frac{\mathrm{Pr}}{\alpha^2} e^{-\alpha\eta} \\ \theta(\eta) = \theta(\xi) \end{array} \right\} \tag{14}$$

Using Eq. (14), in Eq. (12). We obtain,

$$\xi\theta_{\eta\eta}(\xi) + (1 - \frac{\mathrm{Pr}}{\alpha^2} - \xi)\theta_\eta(\xi) + 2\theta(\xi) = -2St \tag{15}$$

The suitable boundary conditions are used in Eq. (15), are

$$\left.\begin{array}{lll}\theta(\xi) = 1 - St & at & \xi = 0 \\ \theta(\xi) \to 0 & as & \xi \to \infty\end{array}\right\} \tag{16}$$

The solution of Eq. (15) is assumed in the form,

$$\theta(\xi) = \theta_{c}(\xi) + \theta_p(\xi) \tag{17}$$

Where $\theta_{c}(\xi)$ denotes the complementary solution and $\theta_p(\xi)$ the specific integral, we derive a complementary solution to Eq. (15), in the following form of a confluent hyper-geometric function by using the border conditions Eq. (16).

$$\theta_c(\xi) = \xi^{\mathrm{Pr}/\alpha^2} M\left(\frac{\mathrm{Pr}}{\alpha^2} - 2, 1 - 3\left(\frac{\mathrm{Pr}}{\alpha^2}\right), \xi\right) \tag{18}$$

And particular integral solutions is,

$$\theta_p(\xi) = \frac{-2St}{\xi\left(4 - 2\left(\frac{\mathrm{Pr}}{\alpha^2}\right)\right)}\xi^2 \tag{19}$$

Utilizing the border constraints of Eq. (16), and redrafting solution as a variable η
We get $\theta(\eta) = \theta_c(\eta) + \theta_p(\eta)$

$$\theta(\eta) = C_1\left(e^{-\alpha\eta}\right)^{\mathrm{Pr}/\alpha^2} M\left(\frac{\mathrm{Pr}}{\alpha^2} - 2, 1 - 3\frac{\mathrm{Pr}}{\alpha^2}, -\frac{\mathrm{Pr}}{\alpha^2}\left(e^{-\alpha\eta}\right)\right) + C_2\left(e^{-\alpha\eta}\right) \tag{20}$$

Where,

$$\left.C_1 = \frac{1 - St - C_2}{M\left(\frac{\mathrm{Pr}}{\alpha^2} - 2, 1 - 3\frac{\mathrm{Pr}}{\alpha^2}, -\frac{\mathrm{Pr}}{\alpha^2}\right)} \quad and \quad C_2 = \frac{St\left(\frac{\mathrm{Pr}}{\alpha^2}\right)}{\left(2 - \frac{\mathrm{Pr}}{\alpha^2}\right)}\right\} \tag{21}$$

Here, Kummer's function is denoted by M and is described by,

$$1 + \sum_{n=1}^{\infty} \frac{(a_0)_n z^n}{(b_0)_n n!} = M(a_0, b_0, z)$$

$$(a_0)_n = a_0(a_0 + 1)(a_0 + 2)\ldots\ldots\ldots\ldots(a_0 + n - 1)$$

$$(b_0)_n = b_0(b_0 + 1)(b_0 + 2)\ldots\ldots\ldots\ldots(b_0 + n - 1)$$

The dimensionless wall temperature gradient interpreted as from Eq. (20) is given below,

$$\left.\begin{array}{l}\theta_\eta(\eta) = C_1\left(\frac{\mathrm{Pr}}{\alpha^2}\right)(-\alpha)\left(e^{-\alpha\eta}\right)^{\mathrm{Pr}/\alpha^2} M\left(\frac{\mathrm{Pr}}{\alpha^2} - 2, 1 - 3\frac{\mathrm{Pr}}{\alpha^2}, -\frac{\mathrm{Pr}}{\alpha^2}\left(e^{-\alpha\eta}\right)\right) + \\ \left(\frac{\frac{\mathrm{Pr}}{\alpha^2} - 2}{1 - 3\frac{\mathrm{Pr}}{\alpha^2}}\right)\left(\frac{\mathrm{Pr}}{\alpha}\right) M\left(\frac{\mathrm{Pr}}{\alpha^2} - 1, 2 - 3\frac{\mathrm{Pr}}{\alpha^2}, -\frac{\mathrm{Pr}}{\alpha^2}\left(e^{-\alpha\eta}\right)\right) \\ + C_2\left(e^{-\alpha\eta}\right)(-\alpha)\end{array}\right\} \tag{22}$$

Nusslet Number:-
The rate of heat transmission in terms of the Nusslet number at the plate is given by,

$$Nu = \frac{q}{k(T_w - T_0)}$$

$$q = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

Where,

$$T_w - T_0 = bx^2 \quad , \quad \frac{\partial T}{\partial y} = bx^2\theta_\eta(\eta)\sqrt{\frac{b}{v}}$$

$$q = -k(bx^2\theta_\eta(\eta)\sqrt{\frac{b}{v}})$$

$$Nu = \frac{-k(bx^2\theta_\eta(\eta)\sqrt{\frac{b}{v}})}{kbx^2}$$

$$Nu = -\theta_\eta(\eta)\sqrt{\frac{b}{v}}) \tag{23}$$

$$Nu = -\theta_\eta(\eta)\ \left(\frac{b}{v}\right)^{1/2}$$

$\mathrm{Re} = \frac{b}{v}$ is the local Reynolds number

$$Nu\mathrm{Re}^{-1/2} = -\,\theta_\eta(\eta) \tag{24}$$

4. Method of Solution

4.1 Analytical Methods

In applied mathematics, the phrase "analytical methods" describes the manipulation of formulae and equations (using algebraic, trigonometric, and calculus principles, etc.) to solve for a particular variable and provide oneself with a straightforward expression to ascertain its value. Analytical methods, such as the anti-derivative, involve integrating to determine the numerical value of definite integrals.

However, numerical approaches typically give some level of mathematical approximation. Analytical procedures, such as series approximations, may also entail approximations in mathematics. In both cases, it is routinely used to approximate the physics (for example, by leaving out forces that are unimportant) in order to solve a problem. Back-of-the-envelope math is a blatant illustration of this. When an analytical approximation is produced, numerical approaches are usually used to gain greater accuracy.

4.2 Numerical Methods

Numerical methods are ways to approximate mathematical operations. We need approximations because, for instance, solving a set of 1,000 simultaneous linear equations for 1,000 unknowns is difficult or we can't complete the process analytically. Numerical methods are mathematical approaches developed to deal with numerical problems. In computer languages, numerical algorithms are used to develop numerical techniques with the appropriate convergence tests.

BVP5C:- The resulting set of coupled linear ordinary differential equations (ODEs) characterizing the problem is numerically solved using MATLAB's built-in BVP5C method.BVP5C is a finite difference algorithm that establishes the Lobatto IIIa formula across four stages. This collocation formula yields a fifth-order precise uniform C1-continuous solution in [a, b] using the collocation polynomial.

In this section, we show the numerical explanation of the coupled nonlinear Eq. (6) and Eq. (12), with boundary conditions Eq. (7) and Eq. (13) by using RKF45 shooting algorithm.
The equations are work out by using shooting technique implemented in MATLAB software. Initially all higher order equations are transforming into system of first order equations. Let us considered as,

$$f = F_1, \quad f_\eta = F_2, \quad f_{\eta\eta} = F_3, \quad F_{\eta\eta\eta} = F_4, \quad \theta = F_5, \quad \theta_\eta = F_6$$

Then system of differential equation can be written as

$$\left.\begin{aligned} F_1' &= F_2 \\ F_2' &= F_3 \\ F_3' &= F_4 \\ F_4' &= \left((F_2{}^2 - F_1F_3 - F_4 + k_1(2 * F_2 * F_4 - F_3^2) + Mn\ F_2) \big/ k_1\ F_1 \right) \\ F_5' &= F_6 \\ F_6' &= 2\ \Pr\ St\, F_2 - \Pr(F_1 F_6 - 2\ F_2 F_5) \end{aligned}\right\} \quad (25)$$

Corresponding boundary conditions are applied in MATLAB software as,

$$\left.\begin{aligned} &F_2 = 1, \quad F_1 = 0, && at \quad \eta = 0 \\ &F_2 = 0, \quad F_3 = 0 && as \quad \eta \to \infty \\ &F_5 = (1 - St) && at \quad \eta = 0 \\ &F_5 = 0 && as \quad \eta \to \infty \end{aligned}\right\} \quad (26)$$

By using equation (25) and (26) we obtained graphs for all parameters in MATLAB software and are discussed in detail.

4.3 Shooting Technique

The shooting method is a numerical analysis methodology that reduces boundary value problems to starting value problems. It involves working through the initial value problem under different starting points until a solution is found that simultaneously fixes the border conditions of the boundary value problem. Eq. (12), with boundary condition Eq. (13), are solved Analytically and obtained the solution as in Eq. (15).Further this solution is utilized to determine the numerical solution of heat transmission Eq. (15), with boundary condition Eq. (16), as we see the boundary condition Eq. (16), there is a missing boundary condition in heating conditions, that is $\theta'(0)$ (after converting BVP to IVP). This condition is necessary for obtaining the solution of energy equation Eq. (12), which is done by using shooting technique (using Newton-Raphson Method) after obtaining the missing boundary conditions the solution of energy equation by Runge-Kutta fourth order method. The tolerance 10^{-5} is taken for the computation.

4.4 ND Solver

The ND Solver can be used to solve a wide range of ordinary differential equations (ODEs), some partial differential equations (PDEs), and some differential algebraic equations (DAEs) numerically.
General Syntax:-
ND Solve [*eqns*, u,{x, x_{min},x_{max}}]
Determines the numerical solution to the function u with the independent variable x in the range of x_{min} to x_{max} for the ordinary differential equations.

5. Result and Discussion

Heat transmission of MHD non-Newtonian liquid in presence of thermally stratified medium is studied .In this test the system of ordinary differential equations have been taken through the BVP5C (Boundary value problems of fifth order Command).We first describe the illustration of different physical parameters with graphical representation. The distribution for dimensionless values with transformed equations of Temperature profiles for various parameters is shown in Figs 6(a)-9(d), Fig. 10, Fig. 11 and shows in Table 1, Table 2 respectively and also represented in Bar gaphs. Table 3 shows the nusslet number of all physical parameters and is shown in bar graph Fig. 12. The all physical parameters effects are shown with the help of 2D graphs and here we compare the graphs of same values in analytical and numerical methods. Here we used kummer's function to solve energy equation and drawn graphs with the help of wolfram language mathematica and also here we used another two numerical methods one is BVP5C (Boundary value problems of fifth order Command) & another one is ND Solver (Numerical Differential Equations Solver) are used .From all three methods we are getting same graphs, converging of graphs are varies but nature of graphs are same. From this we concluded that physical interpretation of the temperature profile for stratified parameter , viscoelastic parameter, Prandtl number, Magnetic parameter are same.

Contour map:-A contour line, also known as an isoline, isopleth, isoquant, or isarithm, is a curve that links points having the same value as a function of two variables. This is the point where the (x, y) plane and the three-dimensional graph of the function $f(x, y)$ cross. More broadly speaking, a curve connecting places where the function has the same value is called a contour line for a function of two variables. Within the field of cartography, a contour line—which is sometimes referred to as a "contour"—links points that are equally high above a specific threshold, such as the mean sea level. A contour map is a map that uses contour lines to depict its features, such as a topographic map, which can display hills, valleys, and the steepness or smoothness of slopes. The height difference between consecutive contour lines on a contour map is called the contour interval.

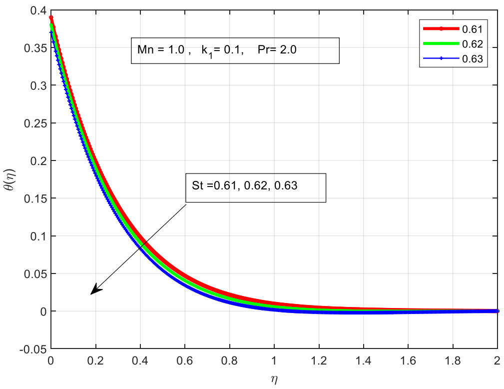

Fig. 6 (a) Impact of St on $\theta(\eta)$ (BVP5C Commands)

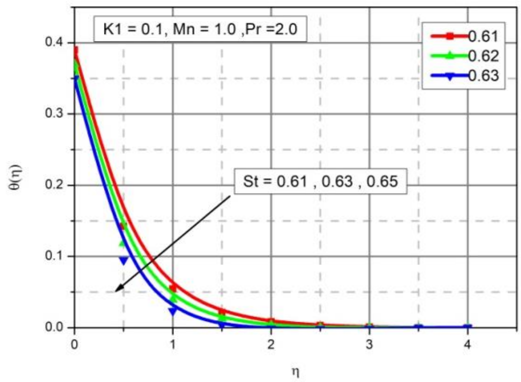

Fig. 6 (b) Impact of St on $\theta(\eta)$ (Analytical Method - Wolfram Language Mathematica)

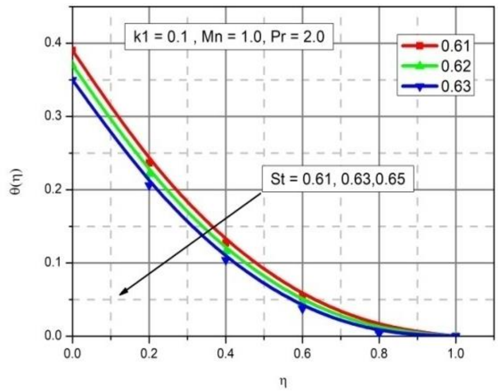

Fig. 6 (c) Impact of St on $\theta(\eta)$ (ND Solver Commands)

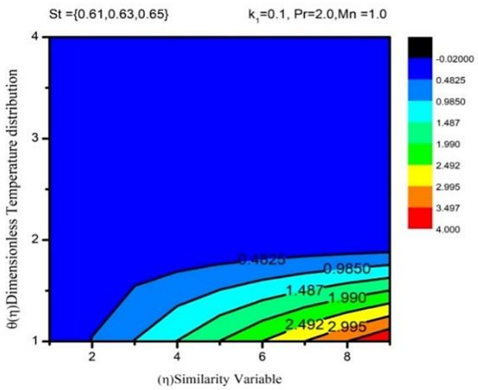

Fig. 6 (d) The impact of St on the temperature profile is shown in the contour plot.

Analysis of Figs. 6(a-d), this figure represents the impacts of stratification parameter (St) on Temperature for various values of St in presence of viscoelastic parameter (k_1), prandtl number (Pr) and (Mn) Magnetic parameter. Examine the variation of the temperature field for different thermal stratification parameters St. It is observed that the fluid temperature is a declining function of thermal stratification as the thickness parameter increases. The effective convection potential that exists between the linear stretch film and the ambient temperature decreases with an increase in St. In view of this, the fluid temperature and the thickness of the thermal boundary layer decrease with higher thermal stratification. The heat transfer rate at the surface increases with increasing thermal stratification. Figure shows that St increases the free-stream temperature by a similar amount. With an St increase in values, the thickness of the thermal boundary layer decreases. All profiles converge at the boundary layer's outer edge, decaying from their highest value in the free stream drops to zero at the wall. This figure shows that St values is 0.61 increases for the temperature zeros wall and decreases the St values is 0.63, 0.65 at thickness of the boundary layer. Here we observed the range of St is $0 < St < 0.8$, if $St = 0$ (Unstratified environment) and (stratified environment) $St > 0$. In Fig. 6(d) effect of St on thermal stratification shows in contour plot.

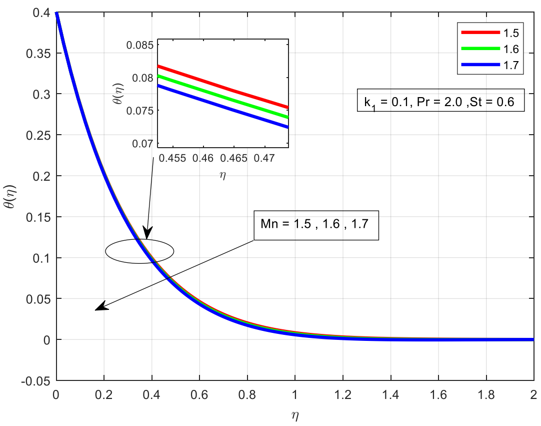

Fig. 7 (a) Impact of Mn on $\theta(\eta)$ (BVP5C Commands)

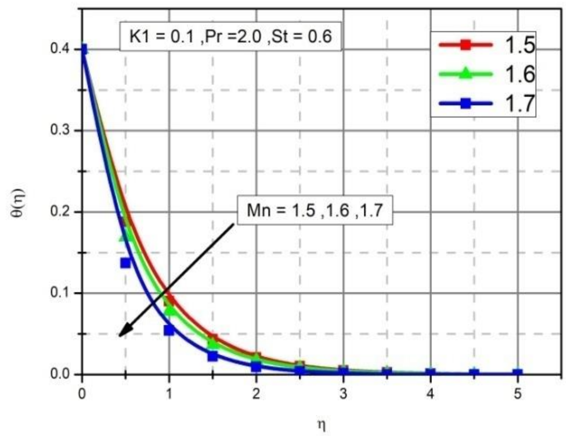

Fig. 7 (b) Impact of Mn on $\theta(\eta)$ (Analytical Method-Wolfram Language Mathematica)

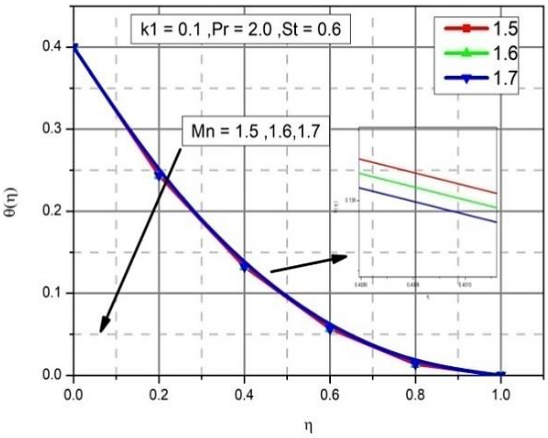

Fig. 7 (c) Impact of Mn on $\theta(\eta)$ (ND Solver Commands)

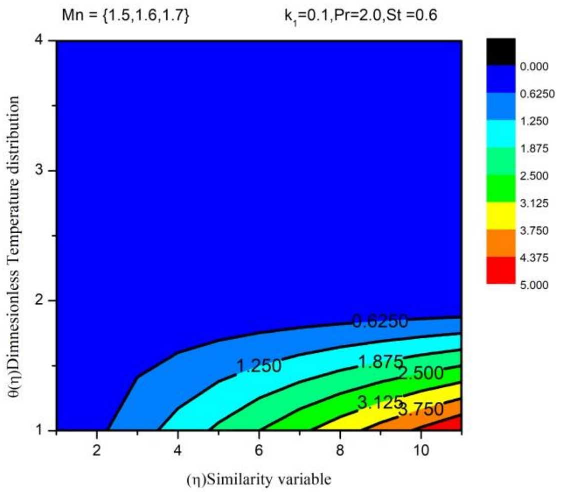

Fig. 7 (d) The impact of Mn on the temperature profile is shown in the contour plot.

Analysis of Figs. 7(a-d), this figure represents the consequence of Magnetic parameter (Mn) on the temperature for distinct values of Mn in existence of viscoelastic parameter (k_1), prandtl numerical value (Pr), and stratification parameter (St).The findings demonstrate that as Mn values rise, the thermal boundary layer thickness also rises. The thickening of the thermal boundary layer results from a rises in frictional drag caused by the Lorentz force. Magnetic parameter is directly proportional to the electrical conductivity of the fluid, magnetic field strength and indirectly proportional to the linear stretching sheet, density of the fluid. As a viscosity of the fluid increases the density of the fluid increases hence Magnetic parameter decreases. It is evident that whenever the temperature rises, the dimensionless temperature falls. This is because the temperature drops because the applied transverse magnetic field creates a drag known as the Lorentz force, which opposes motion. This figure shows that Mn values is 1.5 increases for the temperature zeros wall and decreases the Mn values is 1.6, 1.7 at thickness of the boundary layer. Here we observed the range of Mn is $0.9 \leq Mn \leq 3.9$. In Fig. 7(d) effect of Mn on thermal stratification shows in contour plot.

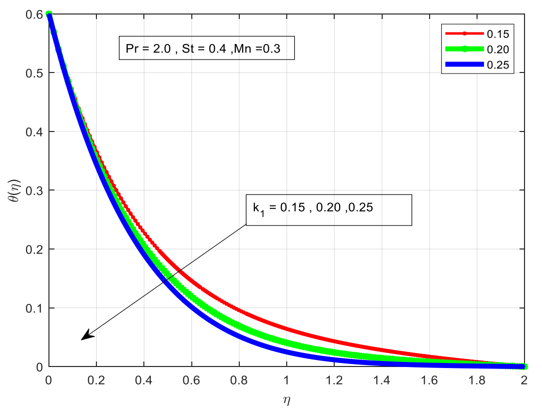

Fig. 8 (a) Impact of k_1 on $\theta(\eta)$ (BVP5C Commands)

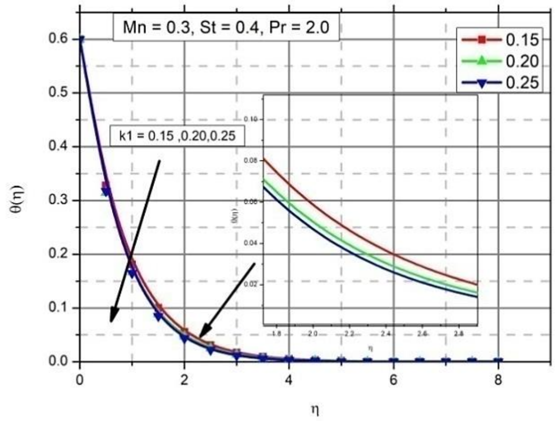

Fig. 8 (b) Impact of k_1 on $\theta(\eta)$ (Analytical Method-Wolfram Language Mathematica)

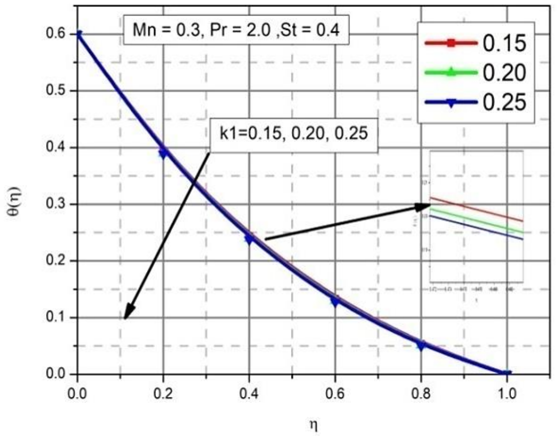

Fig. 8 (c) Impact of k_1 on $\theta(\eta)$ (ND Solver Commands)

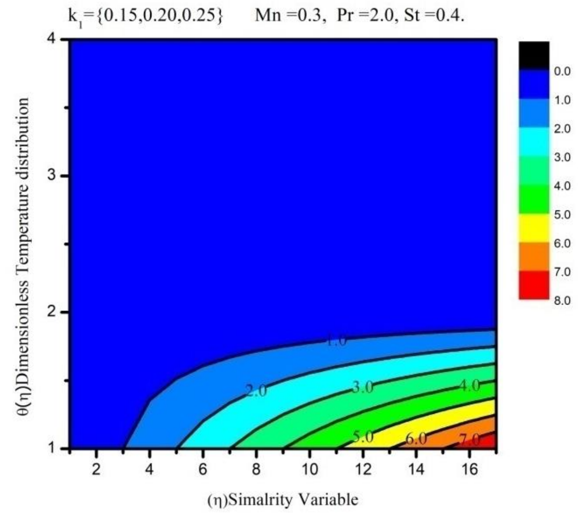

Fig. 8 (d) The impact of k_1 on the temperature profile is shown in the contour plot.

Analysis of Figs. 8(a-d), this graph shows the relationship between the viscoelastic parameter (k_1) and temperature for distinct values when the Prandtl number (Pr), magnetic parameter (Mn), and stratification parameter (St) are present. This figure shows that increment of viscous with elastic parameter (k_1) leads to a rising of temperature profiles $\theta(\eta)$in the border layer. This is due to the fact that the stiffness of the thermal border layer occurs due to other than Newtonian viscous with elastic normal stresses. This figure shows that k_1 values is 0.15 increases for the temperature zeros wall and decreases the $k_1$values is 0.20, 0.25 at thickness of the boundary layer. Here $k_1 = {k_0 b}/{\upsilon}$visco elastic parameter is directly proportional to the elastic parameter and linear stretching sheet ,as the elastic parameter and linear stretching parameter increases automatically viscoelastic parameter also increase but in the case of viscosity of the fluid decreases because its indirectly proportional to visco-elastic parameter. In this instance, the relationship between stress and strain rate is linear. Non-Newtonian fluid is defined as a substance that responds nonlinearly to the strain rate. Another intriguing scenario is one in which the viscosity falls but the shear or strain rate stays constant. Thixotropic describes a substance that behaves in this way. Additionally, the material displays plastic deformation when the stress is unrelated to this strain rate. The thermodynamic theory of polymer elasticity explains the rubber-like behaviour seen in many viscoelastic materials. Amorphous polymers, semi crystalline polymers, biopolymers, metals at extremely high temperatures, and Bitumen materials are a few examples of viscoelastic materials. Here we observed the range of k_1 is $0 < k_1 < 1$viscoelatic parameter range depend on the shear and shear strain of the fluid. In Fig. 8(d) effect of $k_1$on thermal stratification shows in contour plot.

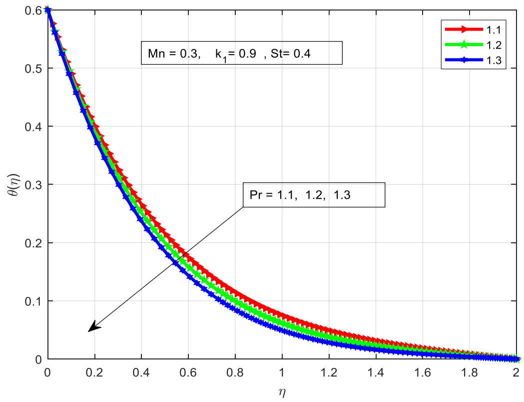

Fig. 9 (a) Impact of Pr on $\theta(\eta)$ (BVP5C Commands)

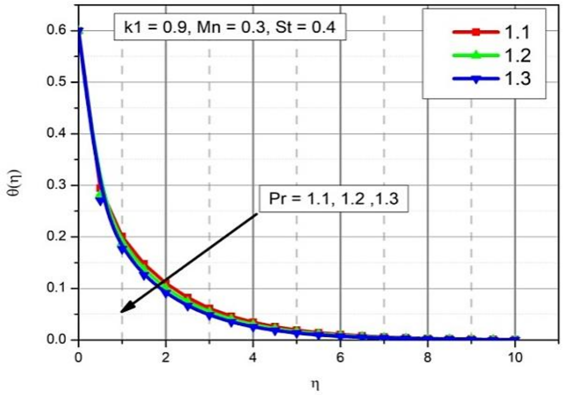

Fig. 9 (b) Effect of Pr on $\theta(\eta)$ (Analytical Method-Wolfram Language Mathematica)

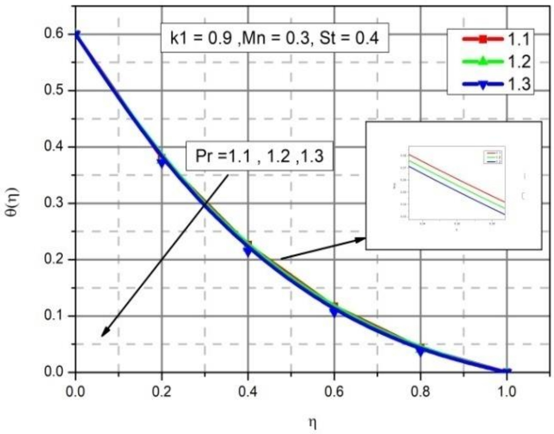

Fig. 9 (c) Impact of Pr on $\theta(\eta)$ (ND Solver Commands)

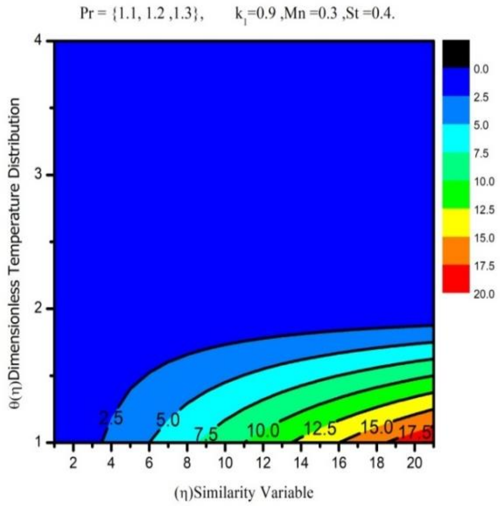

Fig. 9 (d) The impact of Pr on the temperature profile is shown in the contour plot.

Analysis of Figs. 9(a-d), this figure represents the effects of prandtl numerical value (Pr) on Temperature for different values of Pr in existence of viscoelastic parameter (k_1), Magnetic parameter (Mn), and stratification parameter (St). Figure shows that increment in the prandtl numerical value (Pr) will cause the temperature to fall because of its effect on heat transport. In other words, a drop in the prandtl number will result in a thinner thermal border layer. This figure shows that Pr values is 1.1 increases for the temperature zeros wall and decreases the Pr values is 1.2, 1.3 at thickness of the boundary layer. The graph shows a reduction in temperature and thermal boundary thickness when Prandtl number Pr increases at a given value of η. This is because higher Pr values are inversely proportional to the fluid's thermal diffusivity and directly proportional to the fluid's momentum diffusivity. Additionally, higher Prandtl number fluids have relatively low thermal conductivity, which reduces conduction and, as a result, the thickness of the thermal boundary layer, leading to a decrease in temperature. Increasing Pr causes the temperature gradient at the surface to rise, which increases the heat transfer rate. Here we observed the range of Pr is $0 \leq \mathrm{Pr} < 3$, Pr range is depends upon the viscosity and thermal diffusivity of the fluid. In Fig. 9(d) effect of Pr on thermal stratification shows in contour plot.

Table 1: Various values of St, $k1$, Mn, Pr for $\theta(0)$ in Analytical Method, BVP5C and ND Solver Commands.

St	k_1	Mn	Pr	Analytical Method – Wolfram Language Mathematica	Numerical Method- BVP5C Command- RKF45 Algorithm used	Numerical Method – ND Solver Command- Wolfram Language Mathematica
0.61				0.39	0.39	0.39
0.63				0.37	0.37	0.37
0.65				0.35	0.35	0.35
	0.15			0.6	0.6	0.6
	0.20			0.6	0.6	0.6
	0.25			0.6	0.6	0.6
		1.5		0.4	0.4	0.4
		1.6		0.4	0.4	0.4
		1.7		0.4	0.4	0.4
			1.1	0.6	0.6	0.6
			1.2	0.6	0.6	0.6
			1.3	0.6	0.6	0.6

Table 2: Wall Temperature gradient values for different values of governing parameters.

k_1	Mn	Pr	St	$\theta_\eta(0)$ (Analytical solution) –Wolfram Language Mathematica	$\theta_\eta(0)$ (Numerical Solution) shooting technique (using Newton-Raphson Method)
0.1	0.5	1.0	0.2	-3.83825	-3.83832
0.2				-3.60227	-3.60225
0.3				-2.94361	-2.94360
0.2	0.0	1.0	0.2	0.198825	0.198826
	0.5			-3.60227	-3.60226
	1.0			-2.38569	-2.38566
0.1	0.2	1.0	0.2	-0.138564	-0.138562
		1.5		-0.468783	-0.468795
		2.0		-0.943669	-0.943670
0.2	0.5	1.0	0.0	-4.8162	-4.816200
			0.2	-3.60227	-3.602250
			0.4	-2.38835	-2.388352

Table 3: Various values of St, k_1 , Mn , Pr for $-\theta_\eta(0)$ (Nusslet number).

St	$k1$	Mn	Pr	Nusslet Number
0.61				1.289109
0.63				1.277328
0.65				1.265546
	0.15			1.506989
	0.20			1.579839
	0.25			1.626652
		1.5		1.316794
		1.6		1.321538
		1.7		1.326425
			1.1	1.191572
			1.2	1.259357
			1.3	1.325232

Table 1, Table 2 and Table 3: Various values of $St, k1$, Mn , Pr for $\theta(0)$in Analytical Method, BVP5C and ND Solver Commands are discussed in Table 1, we observed that in analytical methods and numerical methods we are getting same values and accurate it shows that nature of graph is same but convergence of a graph is varies. Wall temperature gradient values for different values of governing parameters are shown in Table 2. In this table we observed that various values of St, $k1$, Mn, Pr are getting nearest values in analytical solution (Kummer's function and numerical solution(shooting technique),with this concluded that thermal stratification behavior are accurate in both methods. Table 3 shows the Various values of St, $k1$, Mn , Pr shows the ratio of convective to conductive heat transfer at a boundary in a fluid. Table 1, Table 2 and Table 3 are represented in Fig. 10, Fig. 11 and Fig. 12 respectively.

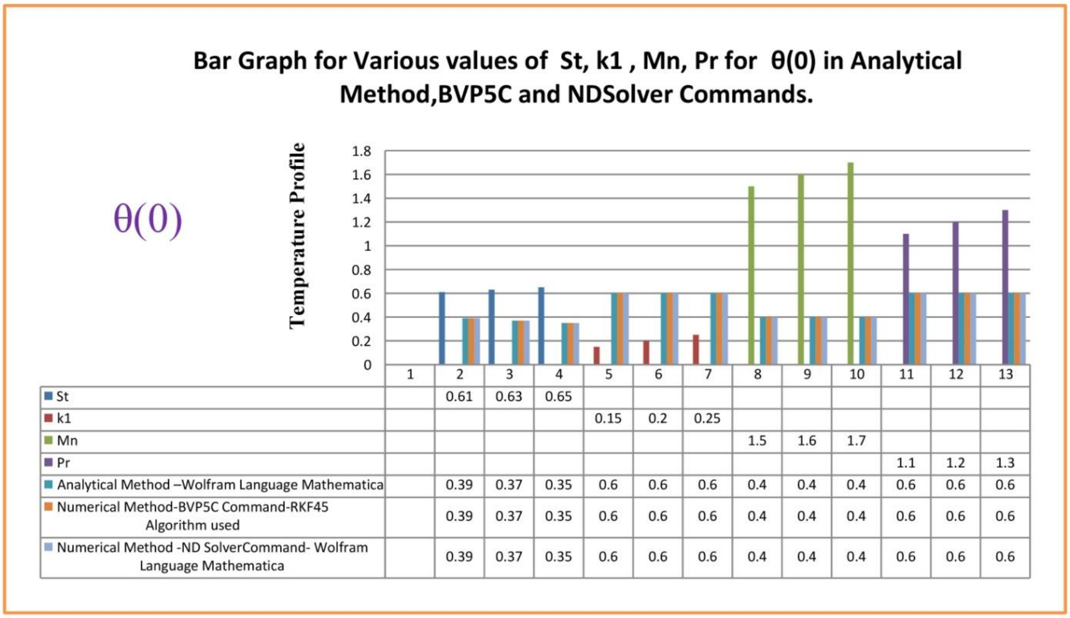

Fig. 10. Various values of $St, k1$, Mn , Pr for $\theta(0)$in Analytical Method, BVP5C and ND Solver Commands.

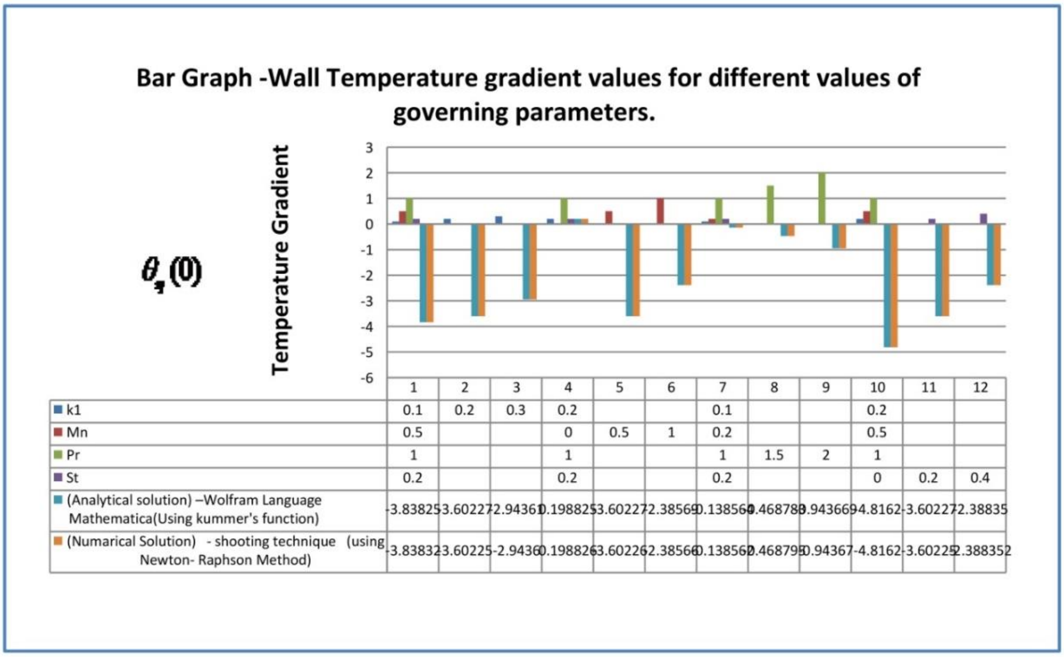

Fig. 11. Wall temperature gradient values for different values of governing parameters.

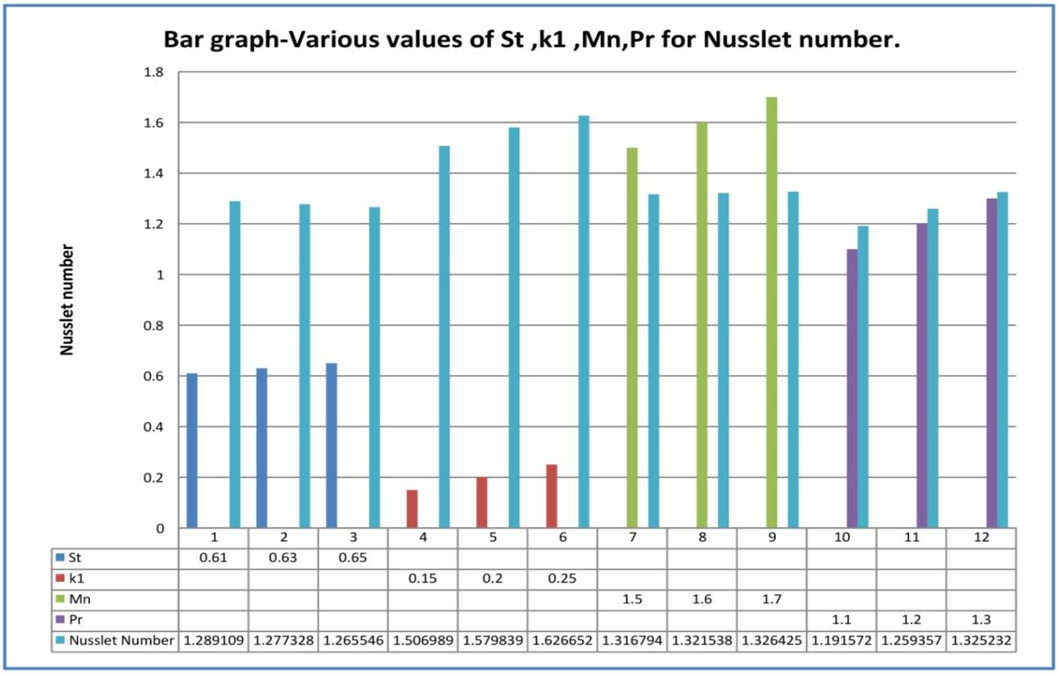

Fig. 12. Various values of St, $k1$, Mn , Pr for $\theta(0)$in analytical method, BVP5C and ND solver commands.

6. Applications of Thermal Stratification

Some applications of thermal stratification are covered here, including geothermal systems, geological transport, surface condensers in thermal power plants, and lake thermohydraulics.

6.1 Geothermal Systems

A geothermal system is a natural heat transfer that takes place within a limited volume of the earth's crust and transports heat from a heat source to a heat sink, which is often the free surface (Fig. 13). This process can manifest itself on the surface in the form of hot springs, geysers and mud volcanoes.

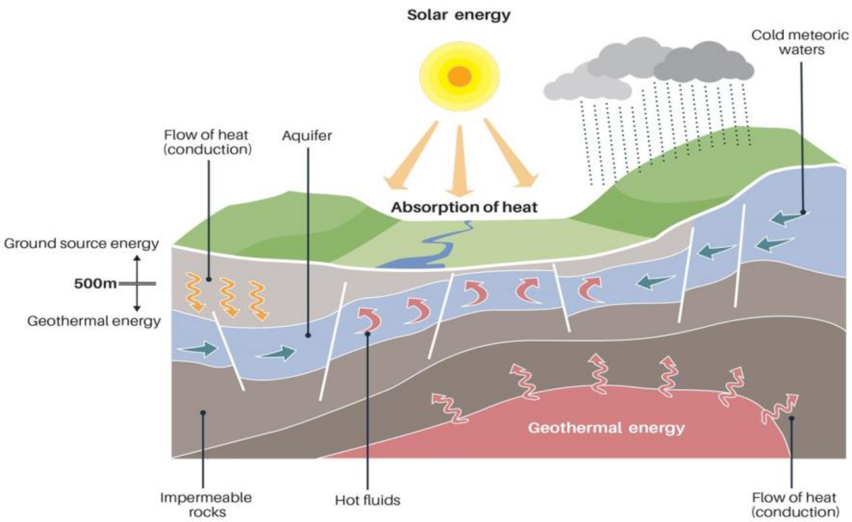

Fig. 13. Effect of thermal stratification in a geothermal system.

6.2 Geological Transport

The term transportation refers to the way in which sediments are transferred, e.g. stones rolling across a river bed or sea shore, grains of sand stirred up by the wind and salts in solution (Fig. 14).

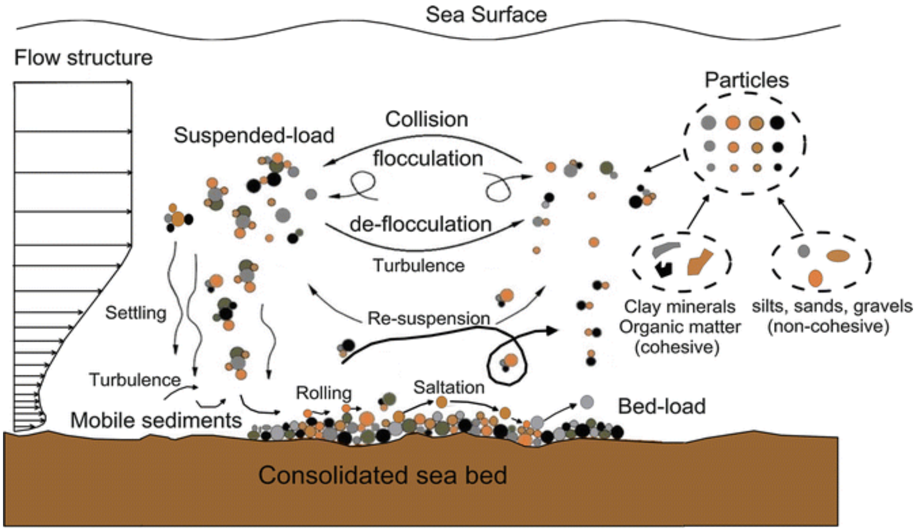

Fig. 14. Effect of thermal stratification in a geological Transport.

6.3 Surface Condenser in a Thermal Power Plant

A surface condenser is a water-cooled shell-and-tube heat transfer system that condenses exhaust steam from a steam turbine in thermal power plants (Figs. 15(a-b)). These condensers are heat transfer systems that transfer steam from a gaseous to a liquid state at pressures lower than atmospheric pressure. Air-cooled condensers are commonly utilized in areas with limited cooling water supplies. However, an air-cooled condenser is far more expensive and cannot reach the same low steam turbine exhaust pressure (or temperature) as a water-cooled surface condenser. Surface condensers are utilized in a wide range of applications and industries, including the condensation of steam turbine exhaust gases in power plants.

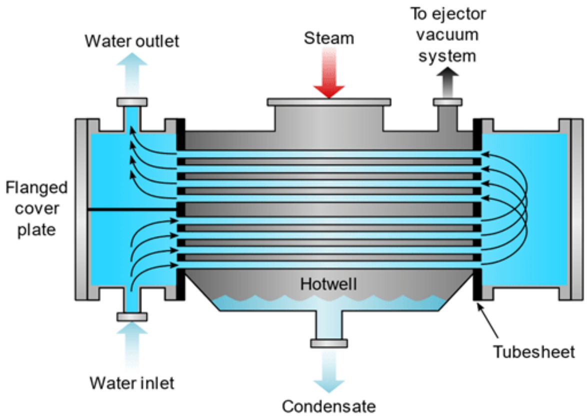

Fig. 15 (a) Physical Model of Surface Condenser.

Fig. 15 (b) Surface Condenser in a thermal power Plant.

6.4 Lake Thermohydraulics

Thermal hydraulics (or thermohydraulics) is the study of hydraulic flow in thermal fluids. The field can be separated into three categories: thermodynamics, fluid mechanics, and heat transport; however, they are frequently interconnected. A common example is steam generation in power plants, which involves the transfer of energy to mechanical motion as well as the change of state of the water during the process. Thermal-hydraulic analysis can help establish essential characteristics for reactor design, such as plant efficiency and system cooling. The typical adjectives include "thermohydraulic," "thermal-hydraulic," and "thermal-hydraulic.". Temperature conditions have a considerable influence on the hydraulic conditions in lakes, partly due to the density distribution. The vertical stability of stratification in lakes is mainly determined by the temperature distribution (Fig. 16).

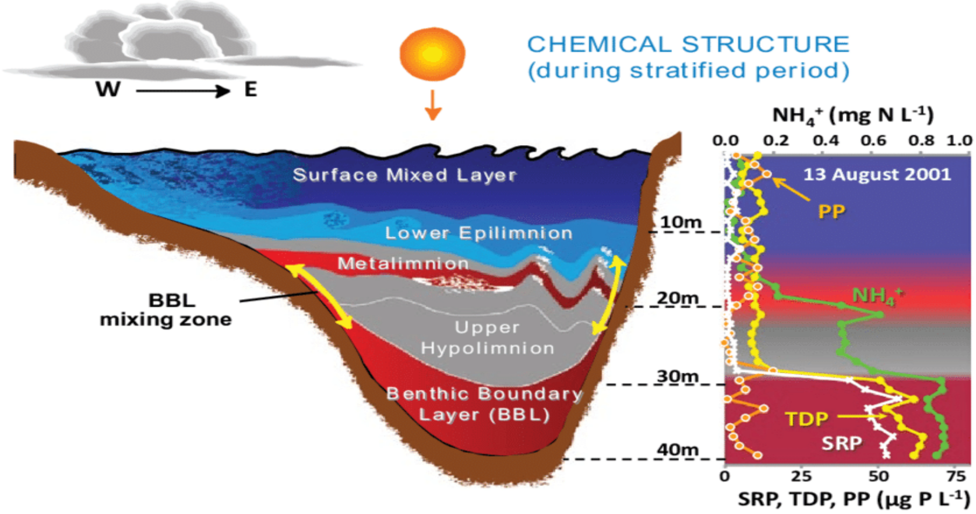

Fig. 16. Thermal stratification occurred in a lake.

6.5 Lava Flows

A lava flow is the outpouring of lava during an effusive eruption. (Unlike lava flows, an explosive eruption creates a mixture of volcanic ash and other pieces known as tephra.) Most lava has a viscosity similar to that of ketchup, which is around 10,000 to 100,000 times that of water. Despite this, lava can travel long distances before solidifying due to the rapid formation of a solid crust that insulates the remaining liquid lava, allowing it to remain hot and inviscid enough to flow (Fig. 17).

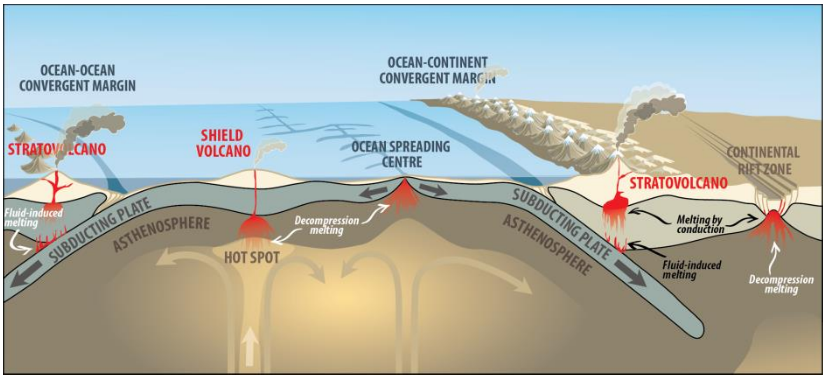

Fig. 17. Thermal stratifications effect in a lava flows.

7. Conclusion

The current study provides numerical solutions for heat transfer analysis and MHD viscoelastic fluid flow over thermally stratified media. The governing equations are non linearized versions of momentum equations, energy equations, and continuity equations. Using similarity transformations to solve linear differential equations. Kummer's function is used for to find the solution for temperature $\theta(\eta)$and Temperature gradient $\theta_\eta(\eta)$and also used two different numerical methods BVP5C and ND Solver Commands to compare the physical parameters with analytical method (Kummer's function).

From our analysis, we discovered the following results:

- The effect of viscoelastic parameter (k_1), prandtl number (Pr), and stratification parameter (St) and Magnetic parameter (Mn) on Temperature $\theta(\eta)$ shows same nature of graphs in three methods.
- In Temperature gradient $\theta_\eta(\eta)$shooting technique used and compare the values with analytical method and their values are accurate.
- Numerical analysis carried out through BVP5C and ND Solver commands, and comparing with analytical method while using wolfram language mathematica. It was discovered that ND Solver and BVP5C were both accurate.
- In presence of thermal stratification, it is seen that the rate of surface heat transfer decreases whenever rising in viscoelastic and magnetic parameter values, Prandtl number.
- Temperature distribution exhibits the reverse effect.
- Applications of thermal stratification are discussed and presented the outcomes of the presented work.

Nomenclature

Mn	Magnetic parameter
Pr	Prandtl number
St	Stratification parameter
T	Temperature of the fluid
b	Linear stretching rate constant
c, d	Constant values
$T_W(x)$	Prescribed surface temperature
$T_\infty(x)$	Variable free-stream temperature
u, v	Components of velocity in x and y directions
c_p	Heat capacity at constant pressure
B_0	Uniform magnetic field
k	Thermal conductivity of the fluid.
T_W	Wall temperature
T_∞	Temperature far away from the plate
k_0	Elastic parameter
k_1	Viscoelastic parameter
α	Thermal diffusivity of the fluid

Greek Symbols

θ	Dimensionless temperature
η	Similarity variable
υ	Kinematic viscosity
ρ	Density of the fluid
μ	Dynamic viscosity
ξ	New variable

Subscripts

W	Properties at the plate
∞	Free stream condition

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