



## Research Article

# Effect of Injection Frequency on Actual Fuel Injection Rate of Piezoelectric Diesel Fuel Injector

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Received 31 August 2023

Revised 9 November 2023

Accepted 13 November 2023

### Abstract:

The objectives of this research was to investigate the effect of injection frequencies (IF) on the actual fuel injection rate (Act.FI) and discrepancy between Act.FI and ideal fuel injection rate (Ideal.FI) of a piezoelectric diesel fuel injector (PDF). IF and duty-times (Dt) of injector control signal (ICS) were varied from 8.33 Hz to 33.33 Hz and 200  $\mu$ s to 2,000  $\mu$ s respectively. Moreover, difference pressure (Dp) and fuel temperature (Ft) were varied from 800 bar to 1,600 bar and 25 °C to 45 °C respectively. It was found that when the IF increased, the Act.FI proportionally increased with a higher gradient compared with an increase of the Ideal fuel injection rate (Ideal.FI) at a certain Dt. The minimum Act.FI was shown at lowest IF equaled to 12.9 g/min appearing at Dp and Ft of 800 bar and 45°C. The maximum Act.FI was shown at highest IF equaled to 190 g/min appearing at Dp and Ft of 1,600 bar and 25°C. The discrepancy was directly proportional to the IF which demonstrated the maximum value of 7.7 g/min or equaled to 4.07% compared to Ideal.FI at IF of 33.32 Hz appearing at Dt, Dp and Ft of 2,000  $\mu$ s, 1,600 bar and 45°C. These were affected by nozzle closure delay which was consequently influenced by the movement of an injector needle. The benefits obtained from this research could be utilized by compensating the Act.FI to be close to the Ideal.FI in the actual operation of a common rail direct injection diesel engines.

**Keywords:** Piezoelectric diesel fuel injector, Injector frequency, Injector characteristic, Fuel injection rate, Common rail direct injection

## 1. Introduction

Diesel common rail direct injection (CRDI) engines were the latest development of diesel engines. These engines provided the highest brake thermal efficiency (BTE) within the internal combustion engine (ICE) category [1]. This was a reason why these engines were utilized supporting the economic growth of each country. Moreover was summarized that the fuel system was a main system affecting to BTE [2]. Piezoelectric diesel fuel injector (PDF) was equipped to previous system for the fuel injection to the combustion chamber for combustion system of diesel CRDI engines [3]. The above fuel system was controlled by an electronic control unit (ECU) which computed the optimal fuel injection rate (Ideal.FI) for the combustion process based on the actual engine load [4, 5]. Then, PDF was controlled by ICS which was a pulse signal creating by ECU [6, 7]. The regulation of Ideal.FI was achieved through IF and Dt of ICS [8]. In contrast, the actual fuel injection of PDF (Act.FI) was influenced by the characteristics of PDF which were caused by the responding capacity of PDF components to ICS [9, 10]. These was caused to the discrepancy between Act.FI and Ideal.FI.

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As per the literature review [11, 12] found that the injector needle (IN) movement for fuel injection process was induced by inertia force according to Newton's first law which influenced the delay time of movement. This could serve as a significant factor influencing the characteristics of PDF. That was caused by several factors which consisted of (i) response time of piezo stack, the movement of IN was controlled by the piezo stack deflection which was followed by IF and Dt of ICS [13, 14]. The movement of IN was affected by inertia force creating from the changing the direction of movement. As above, it was found that the responding time of piezo stack deflection equaled to 6.67  $\mu$ s [15] and that velocity depended on IF of ICS [16]. As mentioned, the responding to ICS of piezo stack was affected by the inertia force that was a one of factor affecting to the delay time of fuel injection process of PDF. So, an accumulated delay time of IN movement for the specific time depended to IF and that was an important parameter affecting the characteristic of PDF. (ii) Hydraulic performance of IN, the IN lifting was impacted by inertia force causing by mass of IN and lifting force from Dp of diesel fuel inside the PDF [17, 18]. These induced the nozzle open delay (NOD) during the start of fuel injection state. According to previous study [19, 20], the NOD of PDF was a constant value of 250  $\mu$ s. That was induced the Act.FI to less than Ideal.FI for 16 percentage. As per the end of injection (EOI) stage, IN was traveled back to valve seat for stop of fuel injection process. In this state, IN was received the resisting force that was provided by Dp of diesel fuel inside the PDF. It caused the nozzle closure delay (NCD) of PDF [21]. As mentioned, that induced the Act.FI to be higher than Ideal.FI. As above, it could be summarized that the cumulative delay time of the IN movement depended on IF. That was one of parameter effecting to characteristic of PDF.

The effect of characteristic of PDF at above mention was only consideration by discontinuous controlling signal. This contrasted with the actual condition of the fuel injection system which PDF was controlled by continuously ICS with IF referring to the real-time Ne for a specific duration. As mentioned, the objective of this research was to investigate the effect of injection frequencies (IF) on Act.FI and discrepancy between Act.FI and Ideal.FI of PDF. The value of IF was aligned with the actual conditions of the PDF corresponding to the Ne of the diesel CRDI engine. Moreover, the researching scopes of this study were explained at next topic. An experimental procedure initialed with the creation of diesel CRDI test rig which could be adjusted value of IF, Dt, Dp and Ft to coverage the actual condition. Then, Act.FI depending on IF were recorded for each value of Dt, Dp and Ft. After that, the effect of IF on Ideal.FI was calculated by considering based on the theoretical principles. Then, the discrepancy of each IF was computed by the comparison of Act.FI and Ideal.FI for each value of Dt, Dp and Ft. Finally, the discrepancy obtaining from this research could be utilized as adjustment data for Act.FI which led to achieve a closest value to Ideal.FI.

## 2. Research Methodology

This research consisted of varies research methodology topics which could be explained as follows.

### 2.1 Scope of Study

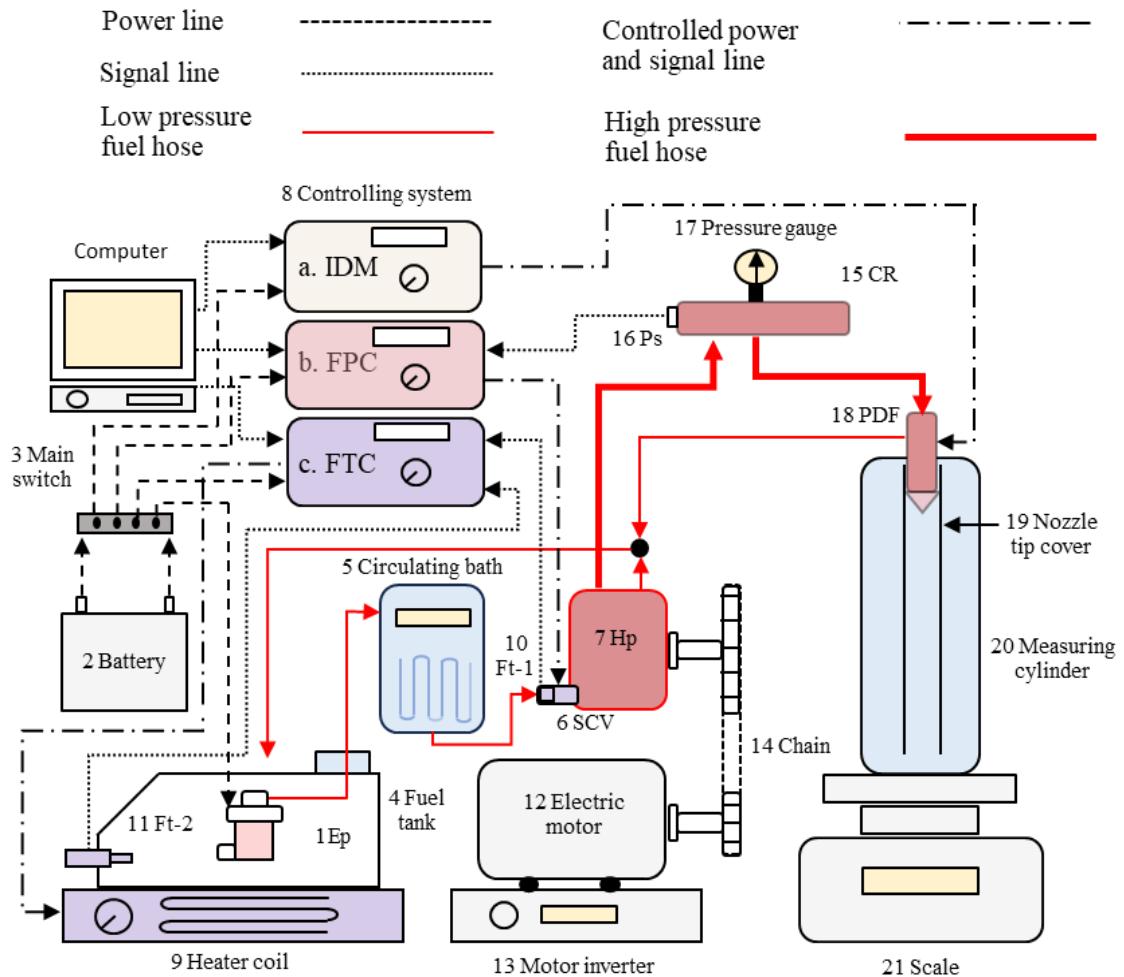
This research commenced by defining the research scope which were detailed in Table 1. The range of mentioned variables was set to cover the actual fuel injection system conditions of the 2012 Ford Ranger Hi-Rider XLT which was a commonly used in this country. The mentioned vehicle was equipped with an inline four-cylinder diesel engine with an engine displacement volume of 2.2 liters, a maximum fuel rail pressure of 1,600 bar, and a compression ratio of 15.7:1. As mentioned, an injector control signal (ICS) which consisted of IF which were varied from 8.33 Hz to 33.33 Hz with an increment of 8.33 Hz and Dt which were varied from 200  $\mu$ s to 2,000  $\mu$ s with an increment of 200  $\mu$ s. Both parameters were defined to cover the actual values of engine speed (Ne) and engine load for the mentioned fuel injection technology. Due to the difference in pressure (Dp) was generated by a high-pressure pump (Hp) utilizing the traction force from the crankshaft rotation. Subsequently, Dp was regulated to maintain a constant value by the electronic control unit (ECU). It was observed that Dp remained inconstant value when the vehicle operated at a low Ne with a high engine load [22]. So, Dp of this research were varied from 800 bar to 1,600 bar by increasing with 400 bar. This range covered to the maximum value of Dp follow by the 2<sup>nd</sup> generation of diesel common rail system [23]. Finally, the properties of the diesel fuel adhered to the fuel properties standards of Euro III-IV. Furthermore, the fuel temperature (Ft) was adjusted within the range of 25°C to 45°C by increasing with 15°C which covered the actual operating Ft in the fuel system of the diesel CRDI engine.

**Table 1:** A scope of experimental parameters.

No.	Parameters	Values	Unit
1	Injector control signal (ICS)		
	Injection frequency (IF)	8.33, 16.66, 24.99 and 33.32	Hz
	Duty-time (Dt)	600, 800, 1,000, 1,200, 1,400, 1,600, 1,800 and 2,000	μs
2	Differential pressure (Dp)	800, 1,200 and 1,60	bar
3	Fuel temperature (Ft)	25, 35 and 45	°C
4	Density of diesel fuel	827.50 (at 25°C)	kg/m <sup>3</sup>
		817.39 (at 35°C)	kg/m <sup>3</sup>
		808.57 (at 45°C)	kg/m <sup>3</sup>

## 2.2 Experimental Setup

Due to the absence of technology of a diesel CRDI test rig that could be simulated and adjusted an environment to be closer an actual conditions for the fuel injection process of a PDF. Thus, an experimental setup of this research initiated with design and creating the diesel CRDI fuel injection test rig which referred to the above fuel injection technology as shown in the Fig. 1. The operation of the diesel CRDI test rig initiated with the internal electric pump (Ep, No.1) received the electric power from a battery (12 VDC, 50 Ah, No.2) by using the main switch (No.3) for controlling the current of system. Then, diesel fuel inside the 29L fuel tank (No.4) was transmitted through circulating bath (No.5) to suction control value (SCV, No.6) which located at high pressure pump (Hp, No.7). In case of an unavailability system, diesel fuel flowed back to the fuel tank through the fuel return line locating inside the Hp. After that, it was controlled the Ft for constant value according to the value in the Table 1. As mentioned, the method of Ft controlling were separated into both consisting of (i) Controlling method for Ft exceeding 25 °C, although Ft of this section was higher than the ambient temperature. So, that was controlled the Ft by using the fuel temperature control module (FTC, No. 8C). That system utilized the proportional integral derivative (PID) method for controlling which provided the accuracy of operation of  $\pm 0.15$  percentage for duration operating temperature of -20 °C to 85 °C. The 2,000W AC heater coil (No.9) was employed to generate the heat for the diesel fuel and type K thermocouple was used to measure the Ft at the point of Ft-1 (No.10). Moreover, FTC measured the Ft locating inside the fuel tank by using the same type of thermocouple at the point of Ft-2 (No. 11) for cut-off the system in case of the differential Ft between Ft-1 and Ft-2 more than 10 °C. (ii) Controlling method for Ft below 25°C, due to Ft of this state had lower than ambient temperature. So, diesel fuel were decreased the Ft only by the circulating bath. As per value of Ft at Ft-1 remained constant according to researching scope, SCV was received the controlling signal. Then, diesel fuel was allowed to pass through the Hp cylinder. After that, the traction force of the Hp was generated through the rotation of the electric motor (EM, No.12) operating at a consistent revolution speed of 1,120 rpm (40 Hz). This rotation was controlled by using the Mitsubishi A-700 motor inverter (No.13) and transmitting to the Hp via the driving chain (No. 14). Subsequently, compressed diesel fuel was delivered to the common rail (CR, No.15) to accumulate the differential pressure (Dp) for the fuel injection process. During this phase, Dp of the diesel fuel was regulated and maintained at a constant value in accordance with the research scope by the fuel pressure control module (FPC, No.8b). The PID control system was employed as the control method for this system. An accumulated Dp within the CR was measured by using a pressure sensor (Ps, No.16) which served as feedback data for the operation of the fuel pressure control module (FPC). A previous Dp was compared to measuring data measuring by pressure gauge (No.17) during the PID tuning process. Then, the PDF (No.18) was controlled for fuel injection at specific times by the injector drive module (IDM, No.8a) which employed a bang-bang controller as a controlling method. Then, ICS was generated by the IDM to regulate the fuel injection process of PDF. It was a pulse signal with an electrical voltage level of 140 VDC. The IF of previous signal in unit of Hz were referred to the actual Ne of diesel CRDI engine in unit of rpm which could be found by the Eqs.(1). Furthermore, the consideration period of each fuel injection process was kept constant as one-minute by IDM. Then, diesel spray was de-energized by the nozzle tip cover (No.19) which induced to condense as the liquid phase within the measuring cylinder (No. 20). Finally, the injected diesel fuel within the measuring cylinder was weight by the scale (No.21) and recording in unit of gram per minute.

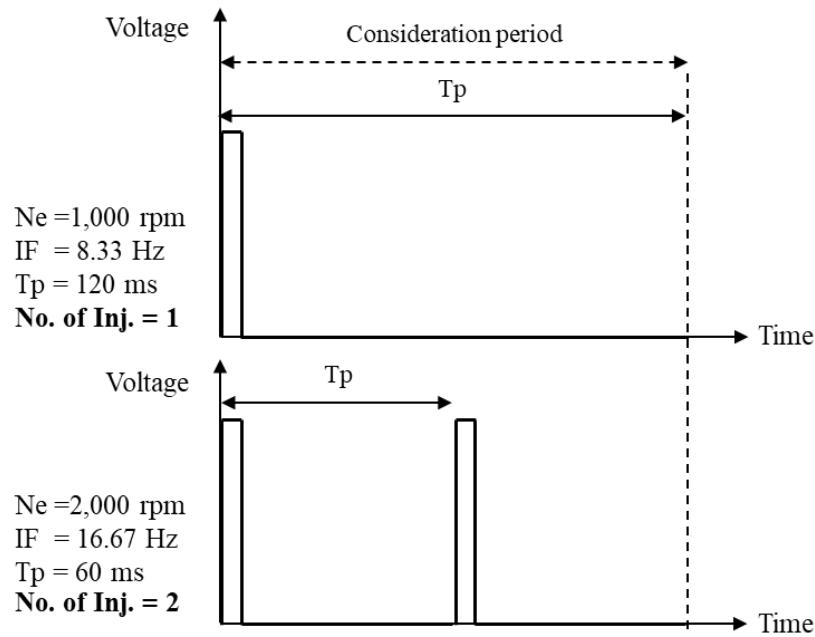


**Fig. 1.** Schematic diagram of diesel CRDI test rig.

$$Ne = (2 \times 60) \times IF \quad (1)$$

### 2.3 Calculation of Ideal Fuel Injection Rate

Ideal fuel injection rate (Ideal.FI) in unit of g/min was compared to Act.FI for each experimental parameter. Due to lack of theoretical for Ideal.FI calculation, these were calculated based on the response characteristics of PDF to the ICS for each value of Dt, Dp and Ft at constant consideration period. As per the above assumption, a time period (Tp) of ICS at IF of 8.33 Hz (1,000 rpm) equaled to 120 ms for each fuel injection. As per the increase of IF for twice or equaled to 16.66 Hz (2,000 rpm), the time period of at this state had to decreased for twice. As per constant value of Dt, Dp, Ft and consideration period, the number of fuel injection at IF of 16.66 Hz had more than the number of fuel injection at IF of 8.33 Hz for twice as per the concept of ICS in the Fig. 2. As mentioned, it could be summarized that the Ideal.FI at IF of 16.66 Hz had to more than the Ideal.FI at IF of 8.33 Hz for twice. Therefore, this research was referred the above assumption to compute the effect of IF on Ideal.FI by using the Act.FI from IF of 8.33 Hz for each Dt, Dp and Ft for Ideal.FI calculation as per Eqs. (2).

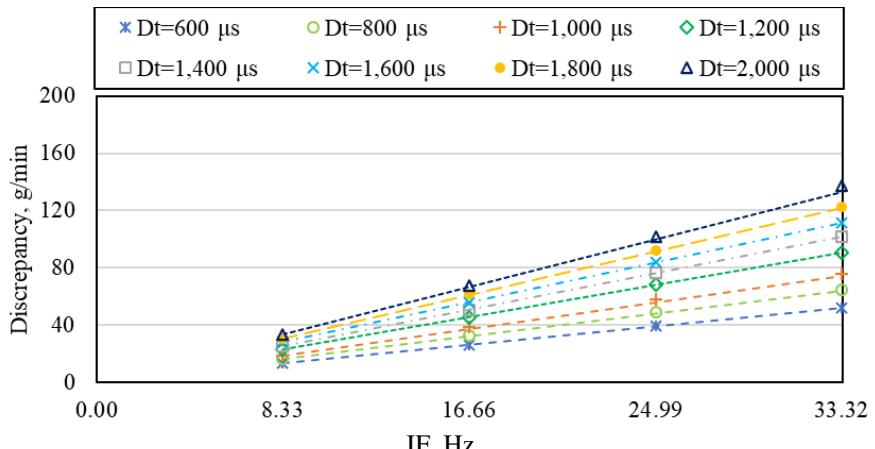


**Fig. 2.** Concept of fuel injection control signal at constant values of  $Dt$ ,  $Dp$  and  $Ft$ .

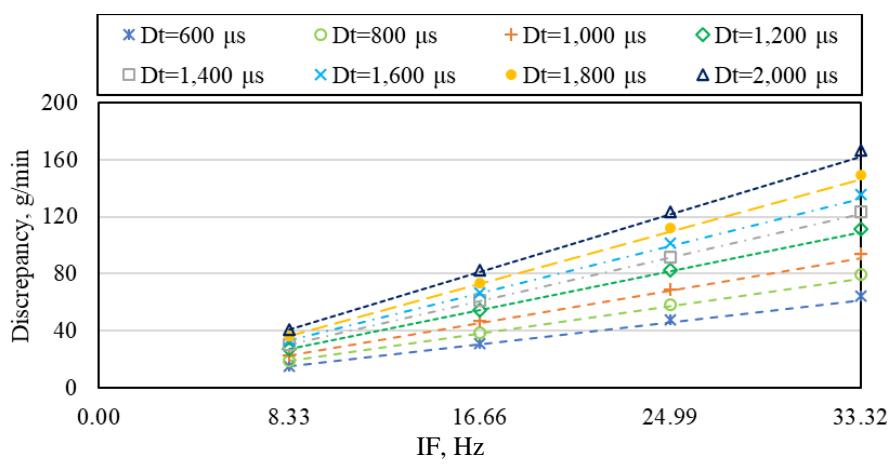
$$Ideal.FI = Act.FI_{8.33} \times \frac{Ne}{1,000} \quad (2)$$

### 3. Results and Discussions

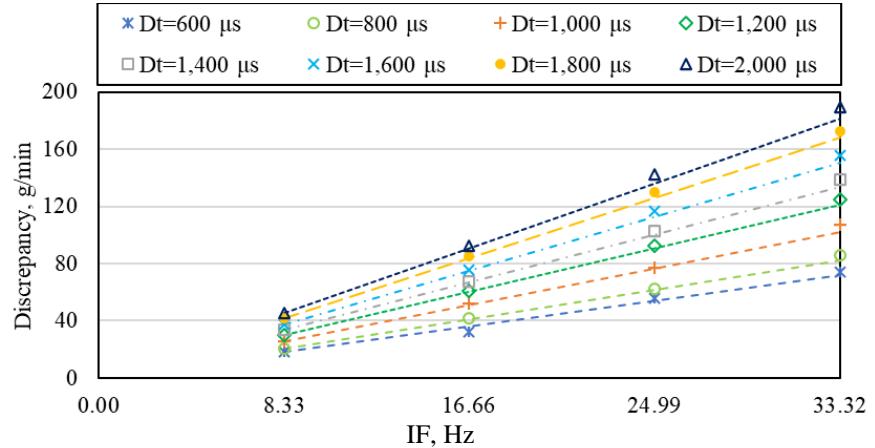
This research examined the effect of IF on Act.FI of PDF which was operated by simulating the fuel injection system of diesel CRDI. IF were referred to an actual  $Ne$  of diesel CRDI engine covering to an actual operation conditions which were varied from 8.33 Hz to 33.33 Hz increasing by 8.33 Hz. As per the results of experimental were found that the effect of IF on Act.FI were presented the same trend for each  $Dt$ ,  $Dp$  and  $Ft$ . However, it was observed that the effect of IF on Act.FI was particularly significant at  $Ft$  of 45 °C for each  $Dt$  and  $Dp$ . Therefore, this research selectively presented the experimental results only for  $Ft$  equaled to 45°C as illustrated in Fig. 3. Additionally, Ideal.FI which were calculated from Eqs. (2) were represented by dashed lines. However, the summarized outcomes were still based on all experimental results. The findings demonstrated that Act.FI varied with IF in each size of  $Dt$ ,  $Dp$  and  $Ft$ . In case of lowest IF or equaled to 8.33 Hz, it was shown the minimum Act.FI or equaled to 12.9 g/min appearing at  $Dp$  and  $Ft$  of 800 bar and 45°C respectively. In case of highest IF or equaled to 33.32 Hz, it was shown the maximum Act.FI or equaled to 190.0 g/min appearing at  $Dp$  and  $Ft$  of 1,600 bar and 25°C respectively. As per the calculation of Ideal.FI by comparing with  $Act.FI_{8.33}$ , it was observed that Ideal.FI varied with IF in each size of  $Dt$ ,  $Dp$ , and  $Ft$ . Furthermore, it was noted that the maximum discrepancy between Act.FI and Ideal.FI was evidenced at the highest IF of 33.32 Hz for each  $Dp$  and  $Ft$  size. At  $Dp$  equaled to 800 bar, the maximum discrepancy of 2.3 g/min was shown at  $Dt$  and  $Ft$  equaled to 2,000  $\mu$ s and 45°C respectively as illustrated in Fig. 3(A). At  $Dp$  equaled to 1,200 bar, the maximum discrepancy of 3.9 g/min was shown at  $Dt$  and  $Ft$  equaled to 2,000  $\mu$ s and 45°C respectively as illustrated in Fig. 3(B). Finally at  $Dp$  equaled to 1,600 bar, the maximum discrepancy of 7.7 g/min was shown at  $Dt$  and  $Ft$  equaled to 2,000  $\mu$ s and 45°C respectively as illustrated in Fig. 3(C).



(A) Dp and Ft equaled to 800 bar and 45 °C.



(B) Dp and Ft equaled to 1,200 bar and 45 °C.



(C) Dp and Ft equaled to 1,600 bar and 45 °C.

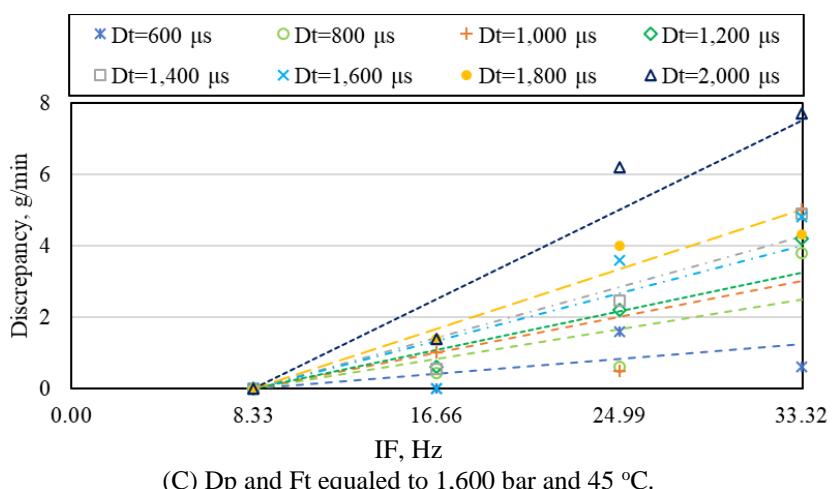
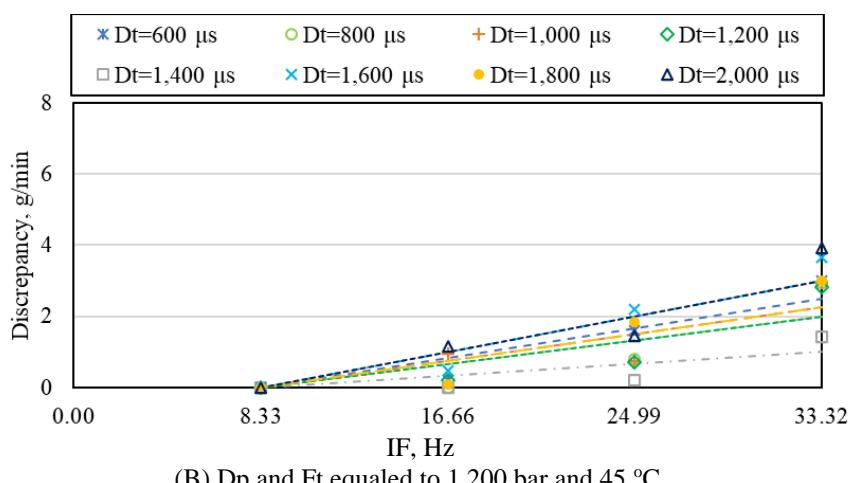
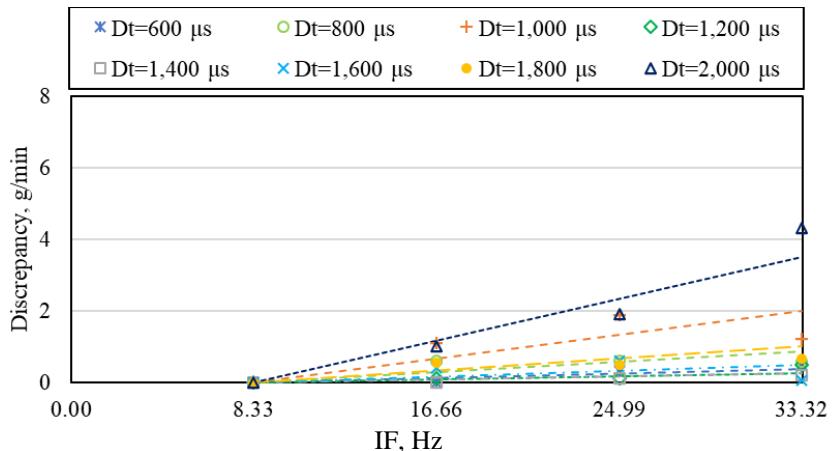
**Fig. 3.** Effect of IF on Act.FI of PDF.

The previous results indicated that an increase in IF led to more discrepancy. Therefore, the effect of IF on discrepancy of PDF in each Dt, Dp, and Ft were described in this section. These were observed that the experimental results provided a consistent trend for each Ft. However, it was observed that the effect of IF on discrepancy was particularly significant at Ft of 45 °C for each Dp. Therefore, this research selectively presented the experimental results only for

$F_t$  equaled to 45°C as illustrated in Fig. 4 which consisted of the result of experimental at  $D_p$  equaled to 800 bar as per Fig. 4(A), 1,200 bar as per Fig. 4(B) and 1,600 bar as per Fig. 4(C) respectively. Additionally, the trends of these effects were represented by dashed lines. In case of considering at the lowest  $D_p$  of 800 bar and constant value of  $D_t$  of 2,000  $\mu s$  it was shown that the increase of IF for twice or varying from 16.66 Hz to 33.32 Hz led to the maximum change in discrepancy of 2.5 times shifting from 0.6 g/min to 1.5 g/min, 2.7 times shifting from 0.7 g/min to 1.9 g/min and 3.2 times shifting from 1.0 g/min to 3.2 g/min appearing at  $F_t$  equaled to 25 °C, 35 °C and 45 °C respectively. As per consideration at increasing of  $D_p$  for twice or equaled to 1,600 bar and constant value of  $D_t$  equaled to 2,000  $\mu s$  it was shown that an increase of IF for twice or varying from 16.66 Hz to 33.32 Hz led to the maximum change in discrepancy of 3.3 times shifting from 1.8 g/min to 6.0 g/min, 4.9 times shifting from 1.4 g/min to 6.8 g/min and 5.5 times shifting from 1.4 g/min to 7.7 g/min appearing at  $F_t$  equaled to 25 °C, 35 °C and 45 °C respectively. As per consideration at low  $D_t$  of 1,000  $\mu s$  and constant value of  $D_p$  of 1,600 bar, it was shown that an increase of IF for twice or varying from 16.66 Hz to 33.32 Hz led to the maximum change in discrepancy of 3.0 times shifting from 1.4 g/min to 4.2 g/min, 4.2 times shifting from 0.9 g/min to 3.8 g/min and 5.0 times shifting from 1.0 g/min to 5.0 g/min appearing at  $F_t$  equaled to 25 °C, 35 °C and 45 °C respectively. In the same way to considering at increasing of  $D_t$  for twice of 2,000  $\mu s$  and constant value of  $D_p$  of 1,600 bar it was shown that an increase of IF for twice or varying from 16.66 Hz to 33.32 Hz led to the maximum change in discrepancy of 3.3 times shifting from 1.8 g/min to 6.0 g/min, 4.9 times shifting from 1.4 g/min to 6.8 g/min and 5.5 times shifting from 1.4 g/min to 7.7 g/min appearing at  $F_t$  equaled to 25 °C, 35 °C and 45 °C respectively as per Fig. 4(C). As mentioned, it was demonstrated that the discrepancy varied with IF. That was shown the most prominently evident at the maximum value of  $D_t$ ,  $D_p$ , and  $F_t$  equaled to 2,000  $\mu s$ , 1,600 bar and 45 °C respectively.

As per results, it was found that  $Act.FI$  varied with IF due to an increase of IF led to higher frequency fuel injection by PDF for each value of  $D_t$ ,  $D_p$  and  $F_t$ . These were consistent with the findings of this research which demonstrated the highest accumulated  $Act.FI$  at IF of 33.32 Hz. Furthermore, it was observed that the discrepancy obtaining from the fuel injection system of PDF varied with IF for each  $D_t$ ,  $D_p$  and  $F_t$ . In case of considering the response characteristics of the PDF to IF, it was observed that PDF was controlled by ICS which consisted of high state for start of injection (SOI) and low state for end of injection (EOI) [24]. As per the SOI state, the internal components of the PDF were moved by electromagnetic force which caused to non-pressure balancing occurring within the PDF. Then, the IN was lifted by  $D_p$  of diesel fuel within the PDF for initiate the fuel injection process. As mentioned, it was found that the response of the IN to IF was influenced by the inertia force. That effect contributed to the occurrence of the nozzle open delay (NOD) during the fuel injection event. The mentioned phenomenon caused to the  $Act.FI$  of this state being lower than the  $Ideal.FI$  at each IF. During the EOI state, it was found that the internal components of the PDF traveled back to the initial positions [25]. Subsequently, IN was pushed back to the valve seat for closing the nozzle holes to stop the fuel injection process. The injector needle movement was affected by the inertia force from the changing the direction of movement with the similar as in the previous state. Moreover, IN was affected by the resistance force from  $D_p$  of diesel fuel within the PDF which depended on  $D_p$  and IN lifting distance which depended on  $D_t$  [26]. As indicated above, it was evident that the NOD was larger than the NOD. As per a continuous fuel injection process was found that the discrepancy depended on IF resulting from the impact of the inertia force to the IN movement. Furthermore, maximum discrepancy was observed at the maximum values of  $D_p$  and  $D_t$  due to IN was most significantly impacted by the inertia force. This corresponded to the results obtained in this research.

Although the operational system of the diesel CRDI engine was influenced by hydraulic losses in both the fuel supply and fuel injection systems causing to pressure drop to the fuel injection system. As per the calculation demonstrated that the above effect had a minimal impact on previous system equaling an average about 0.03% of  $D_p$  in the previous configuration. That was performed using Bernoulli's principle by the assumption of the same high of inlet and outlet fuel of the PDF including the fuel injection process to ambient pressure. Moreover,  $D_p$  which were above calculation consisted of 800 bar, 1,200 bar and 1,600 bar. As mentioned, this effect had a negligible impact on  $Act.FI$ . Consequently, the impact of hydraulic losses was disregarded in this study.



**Fig. 4.** Effect of IF on discrepancy of PDF.

## 4. Conclusion

An increase of IF led to a rising of Act.FI. Due to the increasing of IF, there was caused to rise of an accumulation of inertia force which leads to an increasing of NCD accumulation. This resulted in Act.FI being higher than Ideal.FI for each IF. In contrast, although the accumulation of NOD increased but an impact of this parameter was less than the effect of accumulation of NCD because the accumulation of NOD was less than accumulation of NCD. As mentioned, it could be concluded that increasing of IF influenced on rising of discrepancy between Ideal.FI and Act.FI. The experimental results were found that the discrepancy was directly proportional to the IF which demonstrated the maximum value of 7.7 g/min or equaled to 4.07% compared to Ideal.FI at IF of 33.32 Hz appearing at Dt, Dp and Ft of 2,000  $\mu$ s, 1,600 bar and 45°C. Furthermore, it could be summarized that the prominence of Act.FI and discrepancy increased with the increasing of Dt, Dp, and Ft. Due to a higher Dt, this led to a greater IN lift comparing with a lower value which caused to increase friction during the IN movement at the EOI state. A high Dp caused to high friction to the IN movement at the EOI state. These parameters induced to high inertia of IN which induced to high NCD. Additionally, hydraulic losses in both the fuel supply and fuel injection systems caused to pressure drop to the fuel injection system. Nevertheless, this effect had a minimal impact on Act.FI. Finally, high Ft led to the reduction of diesel viscosity. As mentioned, these were found that the highest of Dt, Dp and Ft caused to highest discrepancy.

## 5. Acknowledgement

This research was supported by the Research and Researchers for Industries Program (RRi) of the Thailand Research Fund (TRF) (contract no. PHD61I0043), Engineo Co., Ltd., the Vehicular System Laboratory (VSLab), the Department of Mechanical Engineering, the Faculty of Engineering, and The Chiang Mai University. The authors would like to express their sincere appreciation for all the support provided.

## Nomenclature

Act.FI	Actual fuel injection rate, g/min
BTE	Brake thermal efficiency
CR	Common rail
CRDI	Common rail direct injection
Dt	Duty-time, $\mu$ s
Dp	Differential pressure, bar
Ep	Electric power
ECU	Electronic control unit
FPC	Fuel pressure control module
Ft	Fuel temperature, °C
Ft-1	First measuring point of fuel temperature.
Ft-2	Second measuring point of fuel temperature.
FTC	Fuel temperature control module
Hp	High pressure pump
Ideal.FI	Ideal fuel injection rate, g/min
IF	Injection frequency, Hz
ICS	Injector control signal
IN	Injector needle
IDM	Injector drive module
NOD	Nozzle open delay
NCD	Nozzle closure delay
PDF	Piezoelectric diesel fuel injector
PID	Proportional integral derivative
Ps	Pressure sensor
SCV	Suction control valve
VDC	Direct current

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