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Research Article

LQR AIRCRAFT PITCH CONTROLLER DESIGN FOR HANDLING DISTURBANCE USING DIFFERENTIAL EVOLUTION

Y. Kanokmedhakul
N. Pholdee*
S. Bureerat
N. Panagant
Sustainable Infrastructure Research and Development Center, Department of Mechanical Engineering, Faculty of Engineering, Khon Kaen University, Thailand

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ABSTRACT:
This work presents the use of differential evolution (DE) for tuning a proportional-integral-derivative (PID) controller, linear quadratic regulator (LQR) with an integral action for aircraft pitch control. An optimisation problem for the two controllers are presented to optimise percentage of overshoot, settling time and steady state error while the weighted sum technique is applied. The design variables for the PID controller are control gains while for the LQR controller are the Q and R matrices. Various integral control gain values are employed for the LQR controller leading to a LQR with an integral action controller. The performance of the optimal controllers is investigated based on the single step and multiple steps response while some disturbance is also added. The results showed that PID controller is efficient for response speed while the optimum LQR with integral action controller is efficient for steady state error elimination. Both of the optimum controllers are robust and can handle disturbance rejection.

Keywords: *PID, LQR integral action, DE, Aircraft pitch control*

1. INTRODUCTION

A control system is an important part in a fixed wing unmanned aerial vehicle (UAV) as, in general, the goal of the UAV is to operate the flight without human involving. The control system in a UAV can be separated into two main operations, inner loop and outer loop. The outer loop is mainly assigned to command control surfaces in order to track a flying trajectory while the inner loop is a task to mainly control surface actuators. Normally, design of inner loop control employs linear models while the models can be separated into longitudinal and lateral motions. The pitch control is a longitudinal control designed to control pitch angle in order to stabilise the aircraft when it noses up or down during change of altitude. Rather than the stabilisation requirement, design of a pitch control also needs to satisfy the control handling quality, response speed and accuracy in order to meet the limit of the actuators and precision in attitude tracking.

A pitch controller in a UAV is usually a Proportional, Integral and Derivative (PID) control system [1–5]. Tuning the PID to satisfy the system requirements can be achieved by several techniques such as poles-placement, the Cohen-Coon technique and the Ziegler-Nichols technique [1]. Optimisation techniques can also be applied to the PID tuning problem in order to obtain the optimal control gain [3–5]. However, the PID controller has some limitation in disturbance rejection and uncertainty handling requirement. Therefore, some optimum and robust control techniques such as Linear Quadratic Regulator (LQR) [6], LQR with integrating action, H-infinity [7], or some intelligent control such as fuzzy control, neural network control, etc., are required[8–10].

* Corresponding author: N. Pholdee
E-mail address: nantiwat@kku.ac.th

LQR is an optimal control system which can deal with disturbance as well as eliminate steady state error when being applied with an integral action. For the UAV flight control system, there has been some literature work implemented as reported in [11, 12]. Design of the LQR controller has some difficulty of identifying the matrices Q and R to synthesise the optimum controller. Unfortunately, identifying the Q and R matrices has no conventional technique to implement, thus, expertise and experience are necessary for the LQR design. Studying on the technique to identify the Q and R matrices is still an interesting topic.

In this work, an application of a meta-heuristic (MH) optimisation technique is presented for pitch control design of an aircraft using PID and LQR with integral action controllers. The optimisation problem for PID, and LQR with integrating action controller design is presented while MH is used to find control gains for the PID controller and the matrices Q and R for the LQR controller. The objective functions are posed to minimising percentage of overshoot, settling time and steady state error while the weighted sum technique is used for dealing with the multiple objective functions. A differential evolution (DE) algorithm [13] is used as an optimiser to solve the proposed problem while the performances of all optimum controllers are compared based on single and multiple step response with disturbance being applied.

2. THEORY OF CONTROL AND DE

2.1. Proportional Integral Derivative Control

PID is a controller which contains three elements; Proportional, Integral and Derivative. The overall control function of the PID controller can be expressed as:

$$r(t) = K_p e(t) + K_i \int_0^t e(t)dt + K_d \frac{de(t)}{dt} \tag{1}$$

where K_p, K_i, and K_d are proportional, integral, and derivative control gains respectively while $r(t)$ and $e(t)$ are output response and error signal respectively. Design of the PID controller needs to find control gains of these three elements in order to make the close control system meet the requirement while the traditional techniques to design are, for example, the Cohen-Coon technique, the Ziegler-Nichols technique or applying optimisation.

2.2. Linear-Quadratic Regulator (LQR) and LQR with integral action

LQR is an optimal control technique. Finding the LQR controller, K_r, can be done by minimising the quadratic cost function which can be expressed as.

$$\text{Min: } J(t) = \int_0^t \left[x^T(\tau)Q\,x(\tau) + u^T R u(\tau)\right] d\tau \tag{2}$$

Subjected to

$$\dot{x} = (A - BK_r)x$$
$$u = -K_r x$$

where Q and R are weighting factor matrices need to be defined. The other parameters are defined as follows:

$\dot{x}$ is a state vector
x is a state variable
A is a system matrix
B is an input matrix
u is an input vector
K_r is a system proportional gain.

The control gain K_r which minimising the quadratic cost function can be expressed as;

$$K_r = R^{-1}B^T P \tag{3}$$

where the parameter P can be obtained by solving the Riccati equation as expressed in eq. (4)

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{4}$$

For the LQR with integral action, the integral term is introduced to the system as show in Fig. 1 in order to eliminate the steady state error. The processing to obtain the optimal control gain is similar to the original LQR while the integral control gain K_i can be found in a similar fashion to the PID technique.

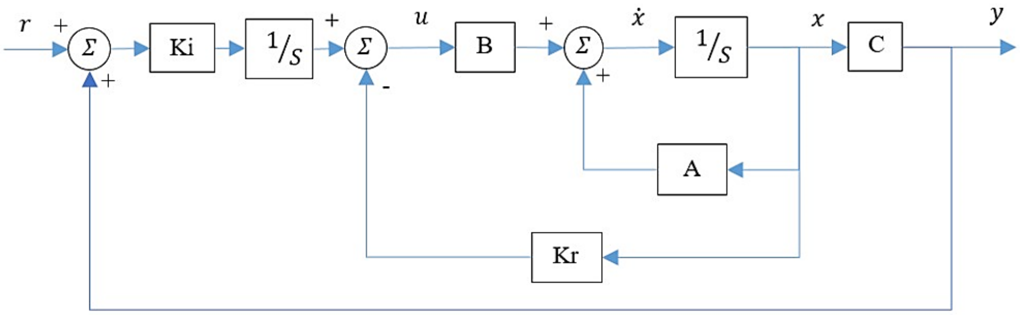

Fig. 1. LQR with integral action in block diagram.

2.3. Differential Evolution (DE)

DE is a meta-heuristic optimiser proposed by [13]. It is a population based algorithm containing two main operators in the reproduction process, which are mutation and crossover. Figure 2 shows the computational search steps of DE, which include initialisation, reproduction and selection as detailed below.

2.3.1 Initialisation

The process of DE starts with randomly generating an initial population (a set of solutions) within the boundary of $[\bar{x}_L, \bar{x}_U]$, upper and lower bounds, while objective function values of the solution member in the population are calculated.

$$P = \{\bar{x}^1, \bar{x}^2, \bar{x}^3 \cdots \bar{x}^N\} \ , \ objF = \{f^1, f^2, f^3 \cdots f^N\} \qquad (5)$$

2.3.2 Mutation

Up to the present time, there have been a number of DE mutation strategies proposed for many applications. In this paper, DE/best/1 is used. Let $\bar{x}^{i_1}$ and $\bar{x}^{i_2}$ be two randomly selected members from the current population and $\bar{x}^{Best}$ be the best solution found so far. A mutant solution can be found as:

$$\bar{u}^i = \bar{x}^{Best} + SF(\bar{x}^{i_1} - \bar{x}^{i_2}) \qquad (6)$$

where *SF* is a scaling factor.

2.3.3 Crossover

The binomial crossover is used with the rate of crossover *CR*. The final solution after the binomial crossover can be found as:

$$v_j^i = \begin{cases} u_j^i \ \ if\ rand \leq CR \\ x_j^i \ \ if\ rand\ > CR \end{cases}.$$

where rand ∈ [0,1] is a uniform random number.

2.3.4 Selection

Having obtained $\bar{v}^i$ with their objective function values, F_v^i, they are then compared to their parents $\bar{x}^i$. $If\ F_v^i < F_x^i$, then $\bar{v}^i$ is selected to the next generation (iteration), otherwise, $\bar{x}^i$ is selected. The reproduction and selection operators are operated repeatedly until meeting a termination condition (usually the maximum number of function evaluations).

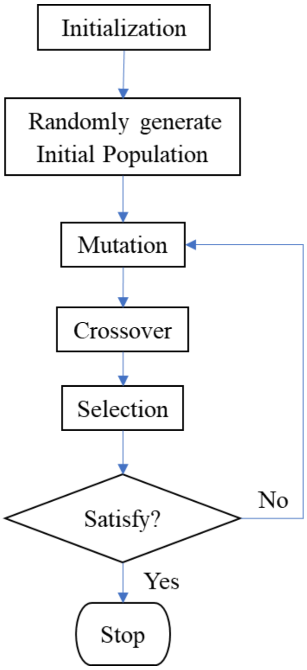

Fig. 2. DE procedure flow chart

2.4 Modelling of pitch controller for altitude hold

Figure 3 shows the aircraft standard (body) axes, related components, and their control parameter. The aircraft control system is usually separated into longitudinal and lateral/directional motions which, in this paper, will consider only the longitudinal motion. The longitudinal motion consists of four components including

1. Forces represented by X, Y, Z for the forces in the directions of x, y, z axes respectively.
2. Angular velocities represented by p, q, r for the rotational velocities in the directions of x, y, z axes respectively.
3. Velocities represented by u, v, w for the x, y, z –axis velocities respectively.
4. Moments represented by L, M, N for the moments in the directions of x, y, z axes respectively. They are also respectively called rolling, pitching, and yawing moments.

For simplicity, the assumption is made. The forward speed ($\boldsymbol{u_0}$) of an aircraft is considered to be constant, neglecting all effect of control surfaces except for the elevator. The derivative term such as $dX/(m \cdot du)$ is reduced to X_u and applied to all variables while being divided by mass for convenience [14].

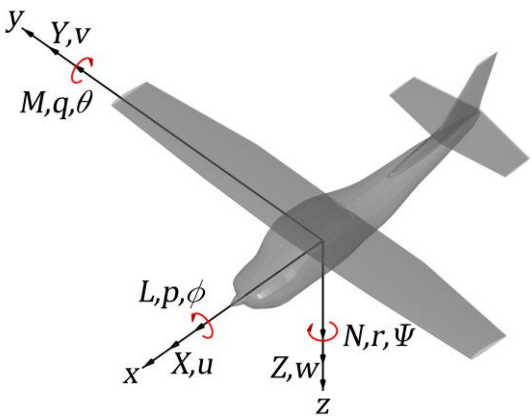

Fig. 3. Aircraft standard axes for angles, forces, velocities, rotational velocities and moments around x, y, z axes

The longitudinal force and moment equations of an aircraft can be expressed as given in eq. (7-9).

$$X - mg\sin\theta = m(\dot{u} + qv - rv) \tag{7}$$

$$Z + mg\cos\theta\cos\emptyset = m(\dot{w} + pv - qu) \tag{8}$$

$$M = I_y\dot{q} + rq(I_x - I_z) + I_{xz}(p^2 - r^2) \tag{9}$$

All the variables in the equation of motion are replaced by reference value plus by perturbation or disturbance

$$\begin{array}{lll} u = u_0 + \Delta u, & v = v_0 + \Delta v, & w = w_0 + \Delta w \\ p = p_0 + \Delta p, & q = q_0 + \Delta q, & r = r_0 + \Delta r \\ X = X_0 + \Delta X, & Z = Z_0 + \Delta Z, & M = M_0 + \Delta M \end{array} \tag{10}$$

Assume that the flight condition is symmetric and the propulsive forces are constant, so the parameters $v_0,\ p_0,\ q_0,\ r_0,\ \emptyset_0$ are set to 0 which lead the equations of motion to eq. (11) to (13).

$$\left(\frac{d}{dt} - X_u\right)\Delta u - X_w\Delta w + (g\cos\theta_0)\Delta\theta = X_{\delta_e}\Delta\delta_e + X_{\delta_T}\Delta\delta_T \tag{11}$$

$$-Z_u\Delta u - \left[(1 - Z_{\dot{w}})\frac{d}{dt} - Z_w\right]\Delta w - \left[(u_0 + Z_q)\frac{d}{dt} - g\sin\theta_0\right]\Delta\theta = Z_{\delta_e}\Delta\delta_e + Z_{\delta_T}\Delta\delta_T \tag{12}$$

$$-M_u\Delta u - \left(M_{\dot{w}}\frac{d}{dt} + M_w\right)\Delta w + \left(\frac{d^2}{dt^2} - M_q\frac{d}{dt}\right)\Delta\theta = M_{\delta_e}\Delta\delta_e + M_{\delta_T}\Delta\delta_T \tag{13}$$

Since Z_q and $Z_{\dot{w}}$ are very small, they can be neglected. By rearranging eq. (13) to the state space representation, it yields.

$$\begin{bmatrix}\Delta\dot{\alpha}\\ \Delta\dot{q}\\ \Delta\dot{\theta}\end{bmatrix} = \begin{bmatrix} Z_\alpha/u_0 & 1 & 0\\ M_\alpha + M_{\dot{\alpha}}Z_\alpha & M_q + M_{\dot{\alpha}} & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\Delta\alpha\\ \Delta q\\ \Delta\theta\end{bmatrix} + \begin{bmatrix} Z_{\delta_e}/u_0\\ M_{\delta_e}\\ 0\end{bmatrix}[\Delta\delta_e] \tag{14}$$

3. NUMERICAL STUDIES

In this work, the pitch control model is formulated based on eq. (14) and the stability derivative parameters shown in Table 1. The state space model can be expressed in eq. (15) while the transfer function between the pitch angle and the aileron defection is expressed in eq. (16).

$$x = \begin{bmatrix}\Delta\alpha\\ \Delta q\\ \Delta\theta\end{bmatrix},\quad A = \begin{bmatrix} -2.02 & 1 & 0\\ -6.9868 & -2.9476 & 0\\ 0 & 1 & 0\end{bmatrix},\quad B = \begin{bmatrix}0.16\\ 11.7304\\ 0\end{bmatrix},\quad C = [0 \quad 0 \quad 1] \tag{15}$$

$$\frac{d\theta}{d\delta} = \frac{11.7304s + 22.5775}{s^3 + 4.9676s^2 + 12.9410s} \tag{16}$$

Two optimisation problems for optimum PID and LQR tuning are presented. The objective function is assigned to simultaneously minimise the percentage of overshoot (*OS*), settling time (*ST*) and steady state error (*SSE*) of step response where the command step input is 0.2 radian or about 11.5 degree is applied. The weighted sum is used to combine the three objective functions while the weighting values are set to be $w_1 = 1, w_2 = 10,\ w_3 = 10$. The equivalent single objective function can be expressed as follows:

$$objF = w_1 * OS + w_2 * ST + w_3 * SSE \tag{17}$$

Table 1: Necessary longitudinal derivatives parameters

Longitudinal Derivatives	Dynamics Pressure and Dimensional Derivative $Q = 36.8\ lb/ft^2$, $QS = 6771\ lb$, $QS\bar{c} = 38596\ ft \cdot lb$, $\bar{c}/2u_0 = 0.016\ s$		
	Components		
	X-Force	Z-Force	Pitching Moment
Rolling Velocities	$X_u = -0.045$	$Z_u = -0.369$	$M_u = 0$
Yawing Velocities	$X_w = 0.03$	$Z_w = \frac{Z_\alpha}{u_0} = -0.202$	$M_w = 0.05$ $M_{\dot{w}} = 0.051$
Angle of attack	$X_\alpha = 0$ $X_{\dot{\alpha}} = 0$	$Z_\alpha = -355.42$ $Z_{\dot{\alpha}} = 0$	$M_\alpha = -8.8$ $M_{\dot{\alpha}} = -0.8976$
Pitching rate	$X_q = 0$	$Z_q = 0$	$M_q = -2.05$
Elevator Deflection	$X_{\delta_e} = 0$	$Z_{\delta_e} = -28.15$	$M_{\delta_e} = -11.7304$

DE is used to solve the proposed optimisation problem while the DE scaling factor (SF) and cross over rate (CR) are set to be 0.5 and 0.7 respectively. The population size is set to be 25 while number of iterations is set to be 100. For the PID case, the design variables are K_p, K_i and K_d. The pure derivative term on K_d is not used but the low pass filter with fixed $N = 100$ is employed instead. The lower and upper bounds of three gain values $[\bar{x}_L, \bar{x}_U]$ are set to [-100,100] respectively. For LQR with integral action case, the design variables are set to be the Q and R matrices, which can be expressed as:

$$Q = \begin{bmatrix} Q_1 & 0 & 0 \\ 0 & Q_2 & 0 \\ 0 & 0 & Q_3 \end{bmatrix}, R = r. \qquad (18)$$

There are totally 4 design variables, $[r, Q_1, Q_2, Q_3]$ for this case. The upper and lower bounds of Q and R are set equally between 0.0001 and 1000. Various integral gain (K_i) of 10^{-1}, 10^{0}, 10^{1},…, 10^{4}, are used for integral action.

4. RESULT AND DISCUSSION

4.1. PID optimum tuning by DE

After performing an optimisation run, the optimum PID gains are shown in Table 2 while a step response is reported in Table 3. The system time response shows the performance with rise time and settling time respectively equal to 0.0432 and 0.0655 with 0.8208 overshoot and 0.0152 steady state error. Figure 4 shows how the system response to the reference using the optimal gain.

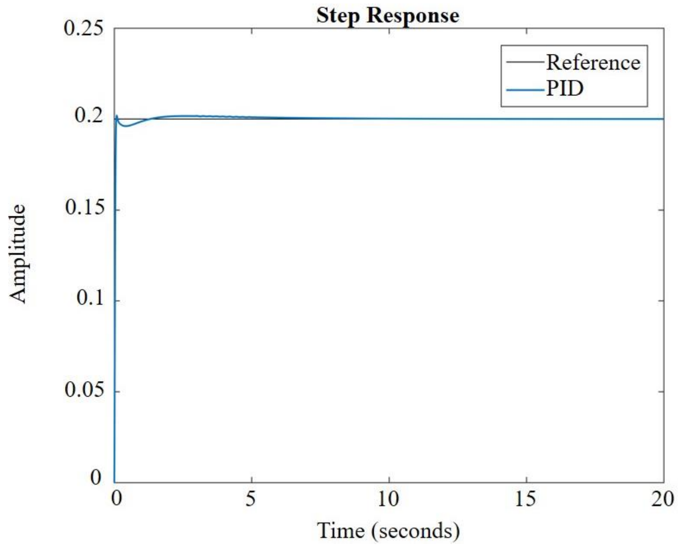

Fig. 4. Step response of the system with PID optimising gain.

Table 2: The best of 20 run times for PID tuning

Design variables	*K*p	10.3011
	*K*i	2.7423
	Kd	3.0046
Result	Rise Time	0.0432
	Settling Time	0.0655
	Overshoot	0.8208
	Steady state error	0.0152

4.2. LQR tuning by DE

After perform optimisation based on various integration gain, the results are shown in Table 3 and Fig. 5. It was found that when K_i increases, the rise time, settling time, overshoot and steady state error tend to be decreased leading to a better LQR controller. The best obtained LQR controller found when the K_i is 10000. When comparing with the optimum obtained PID controller, only the LQR with K_i = 1000 perform better in terms of steady state error elimination while the PID is better in term of response speed.

Figure 6 shows the multi-step response of the optimal PID controller and the LQR with K_i = 1000. Multiple steps input was used as a reference signal which has the magnitude of 0.2 radian at the start then decrease to 0.1 radian at 5 seconds and then increase again to 0.15 radian at 10 seconds. Disturbance of 0.05 radian was ejected at 7 seconds to test system stability. The rise time, settling time and steady state error values obtained from the PID control system are 0.0432, 0.0655 and 0.8208, respectively while those obtained from the LQR with integral action control system are 0.1257, 0.2118 and 0, respectively. It can be said that the optimum PID obtained is efficient in term of response speed whereas the optimum LQR with integral action obtained in this study is efficient in term of steady state error elimination. Both controllers can deal with disturbance of the system.

Table 3: The optimum results obtained for vary *Ki*

***K*i**		**0.1**	**1**	**10**	**100**	**1000**	**10000**
Design variables	*Q*1	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
	*Q*2	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
	*Q*3	1.4101	4.4385	13.8071	60.8477	86.6876	348.9385
	r	10.4544	3.2774	0.5958	0.1194	0.0079	0.0015
Result	Rise Time	5.7107	1.6547	0.5813	0.2699	0.1257	0.0584
	Settling Time	8.6238	2.6793	0.9795	0.4551	0.2118	0.0984
	Overshoot	1.9176	1.9125	0.1708	0.0000	0.0000	0.0001
	Steady state error	0.1030	0.0340	0.0143	0.0067	0.0031	0.0014
		-0.1582	-0.3289	-0.4992	-0.5706	-0.5897	-0.5942
	Kr	0.0630	0.2021	0.6429	1.7023	3.9772	8.8684
		0.3673	1.1637	4.8138	22.5701	105.0021	487.6114

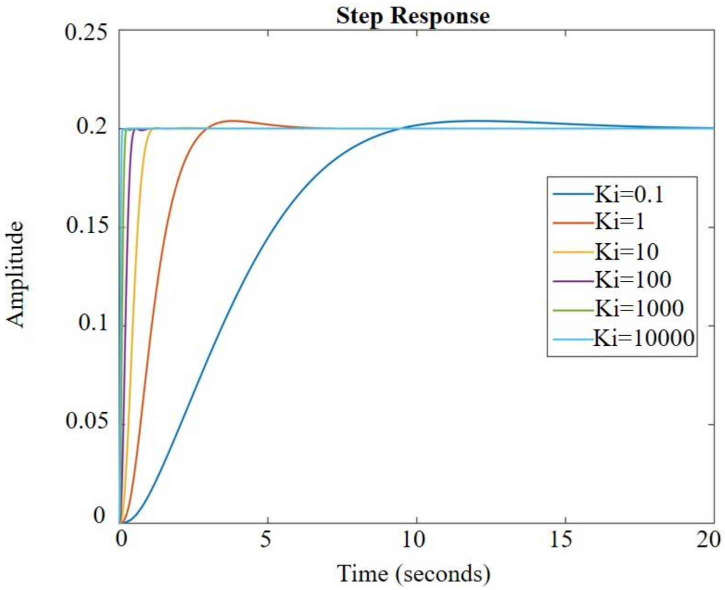

Fig. 5. Step response 0.2 radian with vary Ki

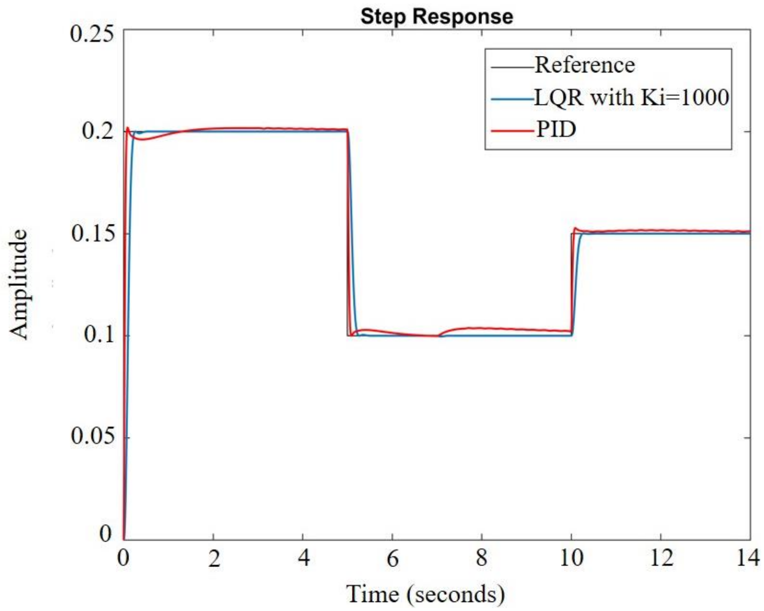

Fig. 6. Multi-step input and disturbance is generated to test system stability

5. CONCLUSION

In this work, DE is successfully applied for optimisation of pith control design of a UAV based on using PID and LQR with integral action controller. The optimisation problem is posed to minimise percentage of overshoot, settling time and steady state error while weighted sum technique is applied. The design variable are the control gains are for the PID controller while Q and R matrices are the design variables for the LQR with integral action controller. Various integral control gain values are employed for the LQR controller leading to the LQR with an integral action controller. Performance of the controllers are investigated based on the single step and multiple steps response. The results obtained reveal that the optimum PID controller is efficient for response speed while the optimum LQR with integral action controller is efficient for steady state error elimination. Both of the optimum controllers are robust and can handle disturbance rejection.

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