Transactions of the TSME (2018) Vol. 6, No. 2, 71-81
Journal of Research and Applications in Mechanical Engineering

ISSN 2229-2152 print
DOI: 10.14456/jrame.2018.8

Research Article

IMPROVED RESULTS ON PASSIVITY ANALYSIS OF NEUTRAL-TYPE NEURAL NETWORKS WITH TIME-VARYING DELAYS

A. Klamnoi[1]
N. Yotha[1,*]
W. Weera[2]
T. Botmart[3]
[1] Department of Applied Mathematics and Statistics, Faculty of Science and Liberal Arts, Rajamangala University of Technology Isan, Nakhon Ratchasima 30000, Thailand
[2] Department of Mathematics, University of Pha Yao, Pha Yao 56000, Thailand
[3] Department of Mathematics, Khon Kaen University, Khon Kaen 40000, Thailand

ABSTRACT:
This paper is considered the problem of the robust passivity analysis for neutral-type neural networks with interval time-varying delays. By constructing an augmented Lyapunov-Krasovskii functional and using the double integral inequality with approach to estimate the derivative of the Lyapunov-Krasovskii functionals. Then, the sufficient conditions are established to ensure the robust passivity of the considered neutral-type neural networks with interval time-varying delays. These robust passivity conditions are obtained in terms of linear matrix inequalities, which can be investigated easily by using standard algorithms. Finally, numerical examples are given to demonstrate the effectiveness of the proposed method.

Keywords: *Passivity, neutral-type neural networks, interval time-varying delay, integral inequality*

1. INTRODUCTION

In the past few decades, delayed neural networks (NNs) have been an important issue due to their applications in many areas such as signal processing, pattern recognition, associative memories, fixed-point, computations, parallel computation, control theory and optimization solvers [1-4]. The state estimation problems for NNs with discrete interval, and distributed time-varying delays have been extensively studied in [5-7]. On the other hand, it is well-known that the time delay appears in many dynamic systems such as digital control systems, distributed networks, long transmission time in pneumatic system, remote control systems and manufacturing processes engineering and have extensive applications in various systems [8-10], which caused many poor in performances and even instability. The systems containing the information of past state derivatives are called neutral-type neural networks (NTNNs). The existing work on the state estimator of NTNNs with mixed delays are only [11, 12] at present. Balasubramaniam et al. [13], considered the problem of global passivity analysis of interval neural networks with discrete and distributed delays of neutral type. Consequently, the passivity analysis of NTNNs has also been received considerable attention and lots of works were reported in recent years.

The problem of passivity performance analysis has also been extensively applied in many areas such as signal processing, sliding mode control, and networked control [14-16]. The main idea of the passivity theory is that the passive properties of a system can keep the system internally stable. In [17-21], authors investigated the passivity of neural networks with time-varying delay, and gave some criteria for checking the passivity of neural networks with time-varying delay. Passivity analysis for neural networks of neutral type with Markovian jumping parameters and

* Corresponding author: N. Yotha
E-mail address: narongsak.yo@rmuti.ac.th

time delay in the leakage term have been presented by Balasubramaniam [22]. Robust exponential passive filtering for uncertain neutral-type neural networks with time-varying mixed delays via Wirtinger-based integral inequality has been presented in [23, 24]. Recently, the stability issues with time-delay are considered by the free-matrix-based integral inequality. It has been known that the free-matrix-based integral inequality is less conservative than Jensen inequality [25, 26].

Motivated by above discussing, this paper investigates the robust passivity analysis for NTNNs with interval and neutral time-varying delays. Based on the constructed Lyapunov-Krasovskii functional, free-weighting matrix approach, and double integral inequality for estimating the derivative of the Lyapunov-Krasovskii functional, the delay-dependent passivity conditions are derived in terms of LMIs, which can be easily calculated by MATLAB LMIs control toolbox. Numerical examples are provided to demonstrate the feasibility and effectiveness of the proposed criteria.

2. Preliminaries

Consider the following NTNNs with interval and neutral time-varying delays described by:

$$\begin{cases} \dot{x}(t) = -Ax(t) + Wg(x(t)) + W_1 g\left(x(t-\tau(t))\right) + W_2 \int_{t-k(t)}^{t} g\left(x(s)\right) ds + W_3 \dot{x}(t-h(t)) + u(t), \\ y(t) = g\left(x(t)\right), \\ x(t) = \phi(t), \ \ t \in \left[-\tau_{\max}, 0\right], \quad \tau_{\max} = \max\left\{\tau_2, k_2, h_2\right\}, \end{cases} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)] \in \mathbb{R}^n$ is the state of the neural, $A = \mathrm{diag}(a_1, a_2, \dots, a_n) > 0$ represents the self-feedback term, W, W_1, W_2 and W_3 represents the connection weight matrices, $g(\cdot) = \left(g_1(\cdot), g_2(\cdot), \dots, g_n(\cdot)\right)^T$ represents the activation functions, $\tau(t)$, $k(t)$ and $h(t)$ represents the interval, discrete and neutral time-varying delays, respectively, $u(t)$ and $y(t)$ represents the input and output vectors, respectively. $\phi(t)$ is an initial condition and $\left\|\phi(s)\right\|_{\tau_{\max}} = \max\left\{\sup_{-\tau_{\max} \le s \le 0} \left\|\phi(s)\right\|, \ \sup_{-h_2 \le s \le 0} \left\|\dot{\phi}(s)\right\|\right\}$.

Throughout this paper, we make the following assumption:

$(\mathrm{H1})$**:** The variables $\tau(t)$, $k(t)$ and $h(t)$ satisfy the following conditions:

$$\begin{cases} 0 < \tau_1 \le \tau(t) \le \tau_2, \quad \dot{\tau}(t) \le \tau_3, \\ \quad 0 < k(t) \le k_2, \\ \quad 0 < h(t) \le h_2, \quad \dot{h}(t) \le h_3 < 1, \quad \forall t \ge 0, \end{cases} \tag{2}$$

where are known scalars $\tau_1, \tau_2, \tau_3, k_2, h_2$ and h_3.

$(\mathrm{H2})$**:** The neural activation functions $g_k(\cdot)$, $k = 1, 2, \dots, n,$ satisfy $g_k(0) = 0$ and for $s_1, s_2 \in \mathbb{R}$, $s_1 \ne s_2$:

$$l_k^- \le \frac{g_k(s_1) - g_k(s_2)}{s_1 - s_2} \le l_k^+, \tag{3}$$

where l_k^-, l_k^+ are known real scalars.

Let $x\left(t, \phi\right)$ denote the state trajectory of system (1) from the above initial condition and $x\left(t, 0\right)$ the corresponding trajectory with zero initial condition.

Definition 1 [17]: The system (1) is said to be passive if there exists a scalar γ such that for all $t_f \ge 0$:

$$2\int_0^{t_f} y^T(s) u(s)\, ds \ \ge -\gamma \int_0^{t_f} u^T(s) u(s)\, ds,$$

and for all solutions of (1) with $x(t, 0)$.

Lemma 2 [25]: For a positive definite matrix $S>0,$ and any continuously differentiable function $x:[a,b]\longrightarrow\mathbb{R}^n$ the following inequality holds:

$$\int_a^b \dot{x}^T(s)S\dot{x}(s)ds \ \ge\ \frac{1}{b-a}\Pi_1^T S\Pi_1+\frac{3}{b-a}\Pi_2^T S\Pi_2+\frac{5}{b-a}\Pi_3^T S\Pi_3,$$

where

$$\Pi_1=x(b)-x(a),$$
$$\Pi_2=x(b)+x(a)-\frac{2}{b-a}\int_a^b x(s)ds,$$
$$\Pi_3=x(b)-x(a)+\frac{6}{b-a}\int_a^b x(s)ds-\frac{12}{(b-a)^2}\int_a^b\int_\theta^b x(s)ds\,d\theta.$$

Lemma 3 [26]: For a positive definite matrix $S>0,$ and any continuously differentiable function $x:[a,b]\longrightarrow\mathbb{R}^n$ the following inequality holds:

$$\int_a^b\int_\theta^b \dot{x}^T(s)S\dot{x}(s)ds\,d\theta \ \ge\ 2\Pi_4^T S\Pi_4+4\Pi_5^T S\Pi_5+6\Pi_6^T S\Pi_6,$$

where

$$\Pi_4=x(b)-\frac{1}{b-a}\int_a^b x(s)ds,$$
$$\Pi_5=x(b)+\frac{2}{b-a}\int_a^b x(s)ds-\frac{6}{(b-a)^2}\int_a^b\int_\theta^b x(s)ds\,d\theta,$$
$$\Pi_6=x(b)-\frac{3}{b-a}\int_a^b x(s)ds+\frac{24}{(b-a)^2}\int_a^b\int_\theta^b x(s)ds\,d\theta-\frac{60}{(b-a)^3}\int_a^b\int_\theta^b\int_s^b x(\lambda)d\lambda ds\,d\theta.$$

Lemma 4 [18] :Given matrices P, Q and R with $P^T=P$, then

$$P+QF(k)R+\left(QF(k)R\right)^T<0,$$

holds for all $F(k)$ satisfying $F(k)^T F(k)\le I$ if and only if there exists a scalar $\varepsilon>0$ such that

$$P+\varepsilon^{-1}QQ^T+\varepsilon R^T R<0.$$

Lemma 5 [8] *(Schur complement):* Given constant symmetric matrices X, Y, Z with appropriate dimensions satisfying $X=X^T$, $Y=Y^T>0$. Then $X+Z^TY^{-1}Z<0$ if and only if

$$\begin{pmatrix} X & Z^T\\ Z & Y\end{pmatrix}<0 \ \text{ or } \ \begin{pmatrix} -Y & Z\\ Z^T & X\end{pmatrix}<0.$$

3. MAIN RESULTS

For presentation convenience, in the following, we denote

$$L^+ = \mathrm{diag}\left(l_1^+,l_2^+,\ldots,l_n^+\right),\quad L^- = \mathrm{diag}\left(l_1^-,l_2^-,\ldots,l_n^-\right),$$
$$x(t)=\Xi_1 v(t),\quad g(x(t))=\Xi_2 v(t),$$
$$\zeta(t)=\Big[v^T(t),\ v^T\left(t-\tau(t)\right),\ v^T(t-\tau_1),\ v^T(t-\tau_2),\ \int_{t-\tau_1}^t x^T(s)ds,\ \int_{t-\tau_2}^t x^T(s)ds,\ \int_{t-\tau_1}^t\int_\theta^t x^T(s)ds\,d\theta,\ \int_{t-\tau_2}^t\int_\theta^t x^T(s)ds\,d\theta,$$
$$\int_{t-\tau_1}^t\int_\theta^t\int_s^t x^T(\lambda)\,d\lambda ds\,d\theta,\ \int_{t-\tau_2}^t\int_\theta^t\int_s^t x^T(\lambda)d\lambda ds\,d\theta,\ \int_{t-k(t)}^t g^T\left(x(s)\right)ds,\ \dot{x}^T\left(t-h(t)\right),\ \dot{x}(t),\ u^T(t)\Big]^T,$$

where $v(t)=\mathrm{col}\{x(t),\ g(x(t))\}$, $\Xi_1=[I,\ 0]$, $\Xi_2=[0,\ I]$ and $e_i\in\mathbb{R}^{n\times 14n}$ is defined as $e_i=\left[0_{n\times(i-1)n},\ I_n,\ 0_{n\times(14-i)n}\right]$ for $i=1,2,\ldots,14$.

Theorem 1 Under assumptions $(\mathrm{H1})$-$(\mathrm{H2})$, for given scalars $\tau_1,\tau_2,\tau_3,h_2,h_3$ and k_2 the system (1) is passive in Definition 1, if there exist real positive matrices $P\in\mathbb{R}^{7n\times 7n}, Q_i, S_i\in\mathbb{R}^{n\times n}(i=1,2,3)$, real positive diagonal matrices U_1,U_2,T_s,T_{ab} $(s=1,2,3,4;\ a=1,2,3;\ b=2,3,4;\ a<b)$ and real matrices X_1,X_2,X_3 with appropriate dimensions, and a scalar $\gamma>0$ such that the following LMIs holds:

$$\sum=\Omega_1+\Omega_2+\Omega_3+\Omega_4+\Omega_5+\Omega_6+\Omega_7+\Omega_8<0, \tag{4}$$

where

$$\begin{cases}
\Omega_1=\Pi_1^T P\Pi_2+\Pi_2^T P\Pi_1+(\Pi_3+\Pi_4)^T+\Pi_3+\Pi_4,\\
\Omega_2=3(\Xi_1e_1)^T Q_1(\Xi_1e_1)-(\Xi_1e_3)^T Q_1(\Xi_1e_3)-(\Xi_1e_4)^T Q_1(\Xi_1e_4)+2(\Xi_2e_1)^T Q_2(\Xi_2e_1)-(\Xi_2e_3)^T Q_2(\Xi_2e_3)\\
\qquad -(\Xi_2e_4)^T Q_2(\Xi_2e_4)+e_{12}^TQ_3e_{12}-(1-\tau_3)(\Xi_2e_2)^T Q_1(\Xi_2e_2)-(1-h_3)e_{12}^TQ_3e_{12},\\
\Omega_3=(\Xi_2e_1)^T k_2^2S_1(\Xi_2e_1)-e_{11}^TS_1e_{11},\\
\Omega_4=e_{13}^T(\tau_1^2S_2+\tau_2^2S_2)e_{13}-\Pi_5^TS_2\Pi_5-3\Pi_6^TS_2\Pi_6-5\Pi_7^TS_2\Pi_7-\Pi_8^TS_2\Pi_8-3\Pi_9^TS_2\Pi_9-5\Pi_{10}^TS_2\Pi_{10},\\
\Omega_5=e_{13}^T(0.5\tau_1^2S_3+0.5\tau_2^2S_3)e_{13}-2\Pi_{11}^TS_3\Pi_{11}-4\Pi_{12}^TS_3\Pi_{12}-6\Pi_{13}^TS_3\Pi_{13}-2\Pi_{14}^TS_3\Pi_{14}-4\Pi_{15}^TS_3\Pi_{15}-6\Pi_{16}^TS_3\Pi_{16},\\
\Omega_6=\sum_{s=1}^4\left(\Pi_{17}^TT_s\Pi_{18}+\Pi_{18}^TT_s\Pi_{17}\right)+\sum_{a=1}^3\sum_{b=2,b>a}^4\Pi_{19}^TT_{ab}\Pi_{20}+\sum_{a=1}^3\sum_{b=2,b>a}^4\Pi_{20}^TT_{ab}\Pi_{19},\\
\Omega_7=\Pi_{21}\Pi_{22}+\Pi_{22}^T\Pi^T{}_{21},\\
\Omega_8=-\gamma e_{14}^Te_{14}-(\Xi_2e_1)^T e_{14}-e_{14}^T(\Xi_2e_1),
\end{cases} \tag{5}$$

with

$$\Pi_1=\left[(\Xi_1e_1)^T,e_5^T,e_6^T,e_7^T,e_8^T,e_9^T,e_{10}^T\right]^T,$$

$$\Pi_2=\left[e_{13}^T,(\Xi_1e_1)^T-(\Xi_1e_3)^T,(\Xi_1e_1)^T-(\Xi_1e_4)^T,\tau_1(\Xi_1e_1)^T-e_5^T,\tau_2(\Xi_1e_1)^T-e_6^T,0.5\tau_1^2(\Xi_1e_1)^T-e_7^T,0.5\tau_2^2(\Xi_1e_1)^T-e_8^T\right]^T,$$

$$\Pi_3=(\Xi_2e_1)^T(U_1-U_2)e_{13},\qquad \Pi_4=(\Xi_1e_1)^T(L^+U_2-L^-U_1)e_{13},$$

$$\Pi_5=\Xi_1e_1-\Xi_1e_3,\qquad \Pi_6=\Xi_1e_1+\Xi_1e_3-\frac{2}{\tau_1}e_5,\qquad \Pi_7=\Xi_1e_1-\Xi_1e_3+\frac{6}{\tau_1}e_5-\frac{12}{\tau_1^2}e_7,$$

$$\Pi_8=\Xi_1e_1-\Xi_1e_4,\qquad \Pi_9=\Xi_1e_1+\Xi_1e_4-\frac{2}{\tau_2}e_6,\qquad \Pi_{10}=\Xi_1e_1-\Xi_1e_4+\frac{6}{\tau_2}e_6-\frac{12}{\tau_2^2}e_8,$$

$$\Pi_{11}=\Xi_1e_1-\frac{1}{\tau_1}e_5,\qquad \Pi_{12}=\Xi_1e_1+\frac{2}{\tau_1}e_5-\frac{6}{\tau_1^2}e_7,\qquad \Pi_{13}=\Xi_1e_1-\frac{3}{\tau_1}e_5+\frac{24}{\tau_1^2}e_7-\frac{60}{\tau_1^3}e_9,$$

$$\Pi_{14}=\Xi_1e_1-\frac{1}{\tau_2}e_6,\qquad \Pi_{15}=\Xi_1e_1+\frac{2}{\tau_2}e_6-\frac{6}{\tau_2^2}e_8,\qquad \Pi_{16}=\Xi_1e_1-\frac{3}{\tau_2}e_6+\frac{24}{\tau_2^2}e_8-\frac{60}{\tau_2^3}e_{10},$$

$$\Pi_{17}=\Xi_2e_s-L^-\Xi_1e_s,\qquad \Pi_{18}=L^+\Xi_1e_s-\Xi_2e_s,$$

$$\Pi_{19}=(\Xi_2e_a-\Xi_2e_b)-L^+(\Xi_1e_a-\Xi_1e_b),\qquad \Pi_{20}=L^+(\Xi_1e_a-\Xi_1e_b)-(\Xi_2e_a-\Xi_2e_b),$$

$$\Pi_{21}=(\Xi_1e_1)^T X_1+e_{13}^TX_2+e_{14}^TX_3,\qquad \Pi_{22}=A\Xi_1e_1+W\Xi_2e_1+W_1\Xi_2e_2+W_2e_{11}+W_3e_{12}+e_{14}-e_{13}.$$

Proof: Consider a Lyapunov-Krasovskii functional candidate:

$$V(x(t)) = \sum_{i=1}^{5} V_i(x(t)), \tag{6}$$

where

$$V_1\left(x(t)\right) = \eta^T(t)P\eta(t) + 2\sum_{k=1}^{n}\rho_k \int_0^{x(t)}\left[g_k(s) - l_k^- s ds\right] + 2\sum_{k=1}^{n}\sigma_k \int_0^{x(t)}\left[l_k^+ s - g_k(s)\right]ds,$$

$$V_2\left(x(t)\right) = \sum_{i=1}^{2}\int_{t-\tau_i}^{t}\left[x^T(s)Q_1x(s) + g^T(x(s))Q_2g(x(s))\right]ds + \int_{t-\tau(t)}^{t}x^T(s)Q_1x(s)ds + \int_{t-h(t)}^{t}\dot{x}^T(s)Q_3\dot{x}(s)ds,$$

$$V_3\left(x(t)\right) = k_2\int_{t-k_2}^{t}\int_{\theta}^{t} g^T\left(x(s)\right)S_1\, g\left(x(s)\right)ds\, d\theta,$$

$$V_4\left(x(t)\right) = \sum_{i=1}^{2}\tau_i\int_{t-\tau_i}^{t}\int_{\theta}^{t}\dot{x}^T(s)S_2\,\dot{x}(s)\, ds\, d\theta,$$

$$V_5\left(x(t)\right) = \sum_{i=1}^{2}\int_{t-\tau_i}^{t}\int_{\theta}^{t}\int_{s}^{t}\dot{x}^T(\lambda)S_3\,\dot{x}(\lambda)\, d\lambda\, ds\, d\theta,$$

where

$$\eta(t) = \Big[x^T(t),\ \int_{t-\tau_1}^{t}x^T(s)ds,\ \int_{t-\tau_2}^{t}x^T(s)ds,\ \int_{t-\tau_1}^{t}\int_{\theta}^{t}x^T(s)dsd\theta,\ \int_{t-\tau_2}^{t}\int_{\theta}^{t}x^T(s)dsd\theta,\ \int_{t-\tau_1}^{t}\int_{\theta}^{t}\int_{s}^{t}x^T(\lambda)d\lambda dsd\theta,$$
$$\int_{t-\tau_2}^{t}\int_{\theta}^{t}\int_{s}^{t}x^T(\lambda)\, d\lambda\, ds\, d\theta\Big]^T,$$

$U_1 = \text{diag}\{\rho_1, \rho_2, \ldots, \rho_n\} \geq 0$ and $U_2 = \text{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_n\} \geq 0$ are to be determined.

The time derivative of $V\left(x(t)\right)$ can be computed as follows:

$$\dot{V}_1\left(x(t)\right) = 2\dot{\eta}^T(t)P\eta(t) + 2\sum_{k=1}^{n}\left\{\rho_k\dot{x}(t)\left[g_k(x(t)) - l_k^- x(t)\right] + \sigma_k\dot{x}(t)\left[l_k^+ x(t) - g_k(x(t))\right]\right\}$$
$$\leq \quad \varsigma^T(t)\ \Omega_1\ \varsigma(t),$$

$$\dot{V}_2\left(x(t)\right) \leq 3x^T(t)Q_1x(t) + \dot{x}^T(t)Q_3\dot{x}(t) - x^T(t-\tau_1)Q_1x(t-\tau_1) - x^T(t-\tau_2)Q_1x(t-\tau_2)$$
$$+2g^T\left(x(t)\right)Q_2g\left(x(t)\right) - g^T\left(x(t-\tau_1)\right)Q_2g\left(x(t-\tau_1)\right) - g^T\left(x(t-\tau_2)\right)Q_2g\left(x(t-\tau_2)\right)$$
$$-(1-\tau_3)x^T(t-\tau(t))Q_1x(t-\tau(t)) - (1-h_3)\dot{x}^T(t-h(t))Q_3\dot{x}(t-h(t)),$$
$$= \quad \varsigma^T(t)\ \Omega_2\ \varsigma(t),$$

$$\dot{V}_3\left(x(t)\right) = k_2^2\, g^T\left(x(t)\right)S_1g\left(x(t)\right) - k_2\int_{t-k_2}^{t}g^T\left(x(s)\right)S_1g\left(x(s)\right)ds,$$
$$\leq k_2^2\, g^T\left(x(t)\right)S_1g\left(x(t)\right) - k_2\int_{t-k(t)}^{t}g^T\left(x(s)\right)S_1g\left(x(s)\right)ds,$$
$$\leq k_2^2\, g^T\left(x(t)\right)S_1g\left(x(t)\right) - \int_{t-k(t)}^{t}g^T\left(x(s)\right)ds\, S_1\int_{t-k(t)}^{t}g\left(x(s)\right)ds,$$
$$= \quad \varsigma^T(t)\ \Omega_3\ \varsigma(t),$$

$$\dot{V}_4\left(x(t)\right) = \dot{x}^T(t)\left(\tau_1^2S_2 + \tau_2^2S_2\right)\dot{x}(t) - \tau_1\int_{t-\tau_1}^{t}\dot{x}^T(s)S_2\dot{x}(s)\, ds - \tau_2\int_{t-\tau_2}^{t}\dot{x}^T(s)S_2\dot{x}(s)\, ds,$$
$$\leq \quad \varsigma^T(t)\ \Omega_4\ \varsigma(t),$$

$$\dot{V}_5\left(x(t)\right) = 0.5\dot{x}^T(t)\left(\tau_1^2S_3 + \tau_2^2S_3\right)\dot{x}(t) - \int_{t-\tau_1}^{t}\int_{\theta}^{t}\dot{x}^T(s)S_3\dot{x}(s)\, ds\, d\theta - \int_{t-\tau_2}^{t}\int_{\theta}^{t}\dot{x}^T(s)S_3\dot{x}(s)\, ds\, d\theta,$$
$$\leq \quad \varsigma^T(t)\ \Omega_5\ \varsigma(t),$$

where $\Omega_i, (i=1,2,3,4,5)$ are defined in (5).

From (3), the nonlinear function $g_k(x_k)$ satisfies:

$$l_k^- \le \frac{g_k(x_k)}{x_k} \le l_k^+,\ k=1,2,\ldots,n,\ x_k \ne 0.$$

Thus, for any $t_k > 0,\ (k=1,2,\ldots,\mathrm{n}),$ we have

$$2t_k\left[g_k^T(x(\theta)) - l_k^- x(\theta)\right]\left[l_k^+ x(\theta) - g_k(x(\theta))\right] \ge 0,$$

which $2\left[g^T\left(x(\theta)\right) - x^T(\theta)L^-\right]^T T\left[L^+ x(\theta) - g\left(x(\theta)\right)\right] \ge 0,$ where $T = \mathrm{diag}\{t_1, t_2, \ldots, t_n\}$. Let θ be $t,\ t-\tau(\mathrm{t}),\ t-\tau_1$ and $t-\tau_2$ replace T with $T_s(s=1,2,3,4)$ then, we have $(s=1,2,3,4)$

$$2\varsigma^T(t)\Pi_{17}^T T_s \Pi_{18}\varsigma(t) \ \ge\ 0, \tag{7}$$

Another observation from (3), we have

$$l_k^- \le \frac{g_k(x(\theta_1)) - g_k(x(\theta_2))}{x(\theta_1) - x(\theta_2)} \le l_k^+,\quad k=1,2,\ldots,n.$$

Thus, for any $t_k > 0,\ (k=1,2,\ldots,n)$ and $\Lambda = g_k\left(x(\theta_1)\right) - g_k\left(x(\theta_2)\right)$, we have

$$2t_k\left[\Lambda - l_k^-(x(\theta_1) - x(\theta_2))\right]\left[l_k^+(x(\theta_1) - x(\theta_2)) - \Lambda\right] \ge 0,$$

which $2\left[\Lambda - L^-(x(\theta_1) - x(\theta_2))\right]^T T\left[L^+(x(\theta_1) - x(\theta_2)) - \Lambda\right] \ge 0,$ where $\Lambda = \mathrm{col}\{\Lambda_1, \Lambda_2, \ldots, \Lambda_n\}$. Let θ_1 and θ_2 take values in $t,\ t-\tau(t),\ t-\tau_1$ and $t-\tau_2$ and replace T with $T_{ab}\left(a=1,2,3; b=2,3,4;\ b>a\right)$ then, we have

$$2\varsigma^T(t)\Pi_{19}^T\ T_{ab}\ \Pi_{20}\varsigma(t) \ge 0,\ a=1,2,3,\ b=2,3,4,\ b>a. \tag{8}$$

From (7) and (8), it can be shown that

$$\varsigma^T(t)\ \Omega_6\ \varsigma(t) \ge 0, \tag{9}$$

where Ω_6 is defined in (5).

On the other hand, for any matrices X_1, X_2 and X_3 with appropriate dimensions, it is true that

$$\begin{aligned} 0 &= 2\left[x^T(t)X_1 + \dot{x}^T(t)X_2 + u^T(t)X_3\right]\left[-Ax(t) + Wg\left(x(t)\right) + W_1 g\left(x(t-\tau(t))\right) + W_2\int_{t-k(t)}^{t} g\left(x(s)\right)ds + W_3\dot{x}\left(t-h(t)\right)\right. \\ &\quad \left. + u(t) - \dot{x}(t)\right], \\ &= \varsigma^T(t)\ \Omega_7\ \varsigma(t), \end{aligned} \tag{10}$$

where Ω_7 is defined in (5).

Therefore, we conclude that

$$\begin{aligned} \dot{V}(x(t)) - \gamma u^T(t)u(t) - 2y^T(t)u(t) &\le \sum_{i=1}^{5} V_i\left(x(t)\right) + \Omega_6 + \Omega_7 - \gamma u^T(t)u(t) - 2y^T(t)u(t), \\ &= \varsigma^T(t)\ \Sigma\ \varsigma(t), \end{aligned} \tag{11}$$

where Σ is defined in (4). If we have $\Sigma < 0$, then

$$\dot{V}(x(t)) - \gamma u^T(t)u(t) - 2y^T(t)u(t) \;\le\; 0, \tag{12}$$

for any $\varsigma(t) \neq 0.$ Since $V(x(0)) = 0$ under zero initial condition, let $x(t) = 0$ for $t \in [\tau_{\max},\ 0]$ after integrating (12) with respect to t over the time period from 0 to t_f, we get

$$2\int_0^{t_f} y^T(s)u(s)\,ds \;\ge\; V(x(t_f)) - V(x(0)) - \gamma\int_0^{t_f} u^T(s)u(s)\,ds,$$
$$\ge\; -\gamma\int_0^{t_f} u^T(s)u(s)\,ds.$$

Thus, the NTNNs (1) is passive in the sense of Definition 1. This completes the proof.

In the following, it is interesting to consider the passivity condition of passivity analysis for uncertain NTNNs with interval time-varying delays:

$$\begin{cases} \dot{x}(t) = -(A + \Delta A(t))x(t) + (W + \Delta W(t))g(x(t)) + (W_1 + \Delta W_1(t))g(x(t-\tau(t))) \\ \qquad + (W_2 + \Delta W_2(t))\int_{t-k(t)}^{t} g(x(s))\,ds + (W_3 + \Delta W_3(t))\dot{x}(t-h(t)) + u(t), \\ y(t) = g(x(t)), \\ x(t) = \phi(t), \quad t \in [-\tau_{\max}, 0], \quad \tau_{\max} = \max\{\tau_2, k_2, h_2\}, \end{cases} \tag{13}$$

where $\Delta A(t), \Delta W(t), \Delta W_1(t), \Delta W_2(t)$ and $\Delta W_3(t)$ represent the time- varying parameter uncertainties that are assumed to satisfy following conditions:

$$[\Delta A(\mathrm{t})\ \ \Delta W(t)\ \ \Delta W_1(t)\ \ \Delta W_2(t)\ \ \Delta W_3(t)] = HF(t)[E_A\ \ E_W\ \ E_{W1}\ \ E_{W2}\ \ E_{W3}], \tag{14}$$

where $H, E_A, E_W, E_{W1}, E_{W2}$ and E_{W3} are known real constant matrices, and $F(\cdot)$ is and unknown time- varying matrix function satisfying $F^T(t)F(t) \le I.$

Then we have the following result.

Theorem 2 Under assumptions $(\mathrm{H1})$-$(\mathrm{H2})$, for given scalars $\tau_1, \tau_2, \tau_3, h_2, h_3$ and k_2 the system (1) is passive in Definition 1, if there exist real positive matrices $P \in \mathbb{R}^{7n\times 7n}$, Q_i, $S_i \in \mathbb{R}^{n\times n}$ $(i = 1,2,3)$, real positive diagonal matrices U_1, U_2, T_s, T_{ab} $(s = 1,2,3,4;\ a = 1,2,3;\ b = 2,3,4;\ a < b)$ and real matrices X_1, X_2, X_3 with appropriate dimensions, and scalars $\gamma > 0$ and $\varepsilon > 0$ such that the following LMIs holds:

$$\begin{bmatrix} \Sigma + \varepsilon\Theta_2^T\Theta_2 & \Theta_1^T \\ \Theta_1 & -\varepsilon I \end{bmatrix} < 0, \tag{15}$$

where

$$\Theta_1 = (\Xi_1 e_1)^T X_1 H + e_{13}^T X_2 H + e_{14}^T X_3 H,$$
$$\Theta_2 = -E_A(\Xi_1 e_1) + E_W(\Xi_2 e_1) + E_{W1}(\Xi_2 e_2) + E_{W2}e_{11} + E_{W3}e_{12},$$

and Σ is defined in Theorem 1.

Proof : Replacing A, W, W_1, W_2 and W_3 in LMIs (4) with $A+HF(t)E_A$, $W+HF(t)E_W$, $W_1+HF(t)E_{W1}$, $W_2+HF(t)E_{W2}$ and $W_3+HF(t)E_{W3}$ respectively, so we have

$$\Sigma + \Theta_1^T F(t)\Theta_2 + \Theta_2^T F(t)\Theta_1 < 0. \tag{16}$$

By Lemma 4, it can be deduced that $\varepsilon > 0$ and

$$\Sigma + \varepsilon^{-1}\Theta_1^T\Theta_1 + \varepsilon\Theta_2^T\Theta_2 < 0, \tag{17}$$

is equivalent to (15) in the sense of the Schur complements Lemma 5. The proof is complete.

Remark 1 Theorem 1 presents estimating of the integral terms by Lemma 2 and Lemma 3, which provided a tighter lower bound than Wirtinger-based integral inequality [24].

4. NUMERICAL EXAMPLE

In this section, we present two examples to illustrate the effectiveness and the reduced conservatism of our result.

Example 1 Consider the NTNNs (1) with the following parameters:

$$A=\begin{bmatrix}1.2 & 0\\ 0 & 1.8\end{bmatrix},\ W=\begin{bmatrix}-0.2 & 0.2\\ 0.26 & 0.1\end{bmatrix},\ W_1=\begin{bmatrix}-0.1 & -0.2\\ 0.2 & 0.1\end{bmatrix},\ W_2=\begin{bmatrix}-0.52 & 0\\ 0.2 & -0.09\end{bmatrix},\ W_3=\begin{bmatrix}-0.2 & 0\\ 0.2 & -0.1\end{bmatrix}.$$

The activation functions are assumed to be $g_1 = g_2 = \tanh(s)$ with $l_1^- = 0,\ l_1^+ = 1,\ l_2^- = 0$ and $l_2^+ = 1$. For $0.5 \le \tau(t) \le 3.5,\ \tau_3 = 0.7,\ 0 < h(t) \le 3.5,\ 0 < k(t) \le 1,$ and $h_3 = 0.5$ by using MATLAB LMIs control toolbox and by solving the LMIs in Theorem 1, in our paper we obtain the feasible solutions:

$$P=\begin{bmatrix}
9.7468 & 3.1406 & 0.1768 & 0.0075 & -0.0066 & 0.0546 & -0.7557 & 0.2706 & -0.0296 & 0.0151 & 0.3430 & 0.0691 & -0.0013 & 0.0015\\
3.1406 & 10.6928 & 0.0142 & 0.0900 & 0.0097 & 0.0489 & 0.0817 & -0.2262 & -0.0045 & -0.0025 & 0.2054 & 0.3565 & -0.0001 & 0.0009\\
0.1768 & 0.0142 & 5.8693 & -0.2743 & -0.4111 & -0.0256 & 1.2352 & 0.0126 & -0.1472 & 0.0226 & -0.4700 & -0.0432 & 0.0000 & -0.0002\\
0.0075 & 0.0900 & -0.2743 & 5.2888 & -0.0029 & -0.4230 & 0.0695 & 1.3149 & 0.0264 & -0.0899 & -0.0580 & -0.5500 & 0.0004 & 0.0006\\
-0.0066 & 0.0097 & -0.4111 & -0.0029 & 4.9310 & 0.2786 & -0.0426 & -0.0519 & 0.0746 & 0.0393 & 0.0974 & -0.0160 & 0.0008 & -0.0008\\
0.0546 & 0.0489 & -0.0256 & -0.4230 & 0.2786 & 5.5436 & -0.0343 & -0.1567 & 0.0434 & 0.1640 & -0.0144 & 0.0849 & 0.0001 & 0.0001\\
-0.7557 & 0.0817 & 1.2352 & 0.0695 & -0.0426 & -0.0343 & 4.7710 & 0.0573 & -0.0578 & 0.0041 & -1.8054 & -0.0133 & -0.0044 & 0.0002\\
0.2706 & -0.2262 & 0.0126 & 1.3149 & -0.0519 & -0.1567 & 0.0573 & 4.8307 & -0.0025 & -0.0587 & 0.0075 & -1.7558 & -0.0003 & -0.0046\\
-0.0296 & -0.0045 & -0.1472 & 0.0264 & 0.0746 & 0.0434 & -0.0578 & -0.0025 & 0.0240 & 0.0056 & 0.0249 & -0.0045 & 0.0010 & 0.0004\\
0.0151 & -0.0025 & 0.0226 & -0.0899 & 0.0393 & 0.1640 & 0.0041 & -0.0587 & 0.0056 & 0.0355 & -0.0049 & 0.0182 & 0.0005 & 0.0019\\
0.3430 & 0.2054 & -0.4700 & -0.0580 & 0.0974 & -0.0144 & -1.8054 & 0.0075 & 0.0249 & -0.0049 & 4.6585 & -0.0043 & -0.0004 & -0.0002\\
0.0691 & 0.3565 & -0.0432 & -0.5500 & -0.0160 & 0.0849 & -0.0133 & -1.7558 & -0.0045 & 0.0182 & -0.0043 & 4.5942 & -0.0001 & -0.0005\\
-0.0013 & -0.0001 & 0.0000 & 0.0004 & 0.0008 & 0.0001 & -0.0044 & -0.0003 & 0.0010 & 0.0005 & -0.0004 & -0.0001 & 0.0008 & 0.0000\\
0.0015 & 0.0009 & -0.0002 & 0.0006 & -0.0008 & 0.0001 & 0.0002 & -0.0046 & 0.0004 & 0.0019 & -0.0002 & -0.0005 & 0.0000 & 0.0008
\end{bmatrix},$$

$$Q_1=\begin{bmatrix}3.1810 & 1.1655\\ 1.1655 & 5.8872\end{bmatrix},\ Q_2=\begin{bmatrix}0.36015 & -0.1414\\ -0.1414 & 0.6644\end{bmatrix},\ Q_3=\begin{bmatrix}7.0514 & 1.0326\\ 1.0326 & 4.5277\end{bmatrix},\ S_1=\begin{bmatrix}7.1153 & -0.4366\\ -0.4366 & 2.1361\end{bmatrix},\ S_2=10^{-5}\times\begin{bmatrix}0.0016 & -3.3988\\ -3.3988 & 0.0015\end{bmatrix},$$

$$S_3=10^{-9}\times\begin{bmatrix}7236.1573 & -9.2006\\ -9.2006 & 7062.0086\end{bmatrix},\ X_1=\begin{bmatrix}9.12346 & 1.9938\\ 1.9938 & 9.6625\end{bmatrix},\ X_2=\begin{bmatrix}5.6121 & 1.0238\\ 1.0238 & 4.5991\end{bmatrix},\ X_3=\begin{bmatrix}8.0686 & 1.1923\\ 1.1923 & 8.1750\end{bmatrix},$$

$$U_1=\begin{bmatrix}7.8148 & 0\\ 0 & 5.7281\end{bmatrix},\ U_2=\begin{bmatrix}3.9675 & 0\\ 0 & 5.0184\end{bmatrix},\ T_1=\begin{bmatrix}5.2041 & 0\\ 0 & 2.9493\end{bmatrix},\ T_2=\begin{bmatrix}0.4138 & 0\\ 0 & 0.7378\end{bmatrix},\ T_3=\begin{bmatrix}1.6358 & 0\\ 0 & 1.9877\end{bmatrix},$$

$$T_4=\begin{bmatrix}1.6286 & 0\\ 0 & 1.9856\end{bmatrix},\ T_5=\begin{bmatrix}0.1862 & 0\\ 0 & 0.4702\end{bmatrix},\ T_6=\begin{bmatrix}0.5543 & 0\\ 0 & 0.8760\end{bmatrix},\ T_7=\begin{bmatrix}0.5436 & 0\\ 0 & 0.8681\end{bmatrix},\ T_8=\begin{bmatrix}0.2092 & 0\\ 0 & 0.3650\end{bmatrix},$$

$$T_9=\begin{bmatrix}0.2099 & 0\\ 0 & 0.3639\end{bmatrix},\ T_{10}=\begin{bmatrix}0.6603 & 0\\ 0 & 0.9218\end{bmatrix},\quad \gamma = 28.8384.$$

In this example, Figure 1 gives the state trajectory of the NTNNs (1) under zero input, and the initial condition $\begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T = \begin{bmatrix} 0.2 & -0.3 \end{bmatrix}^T$.

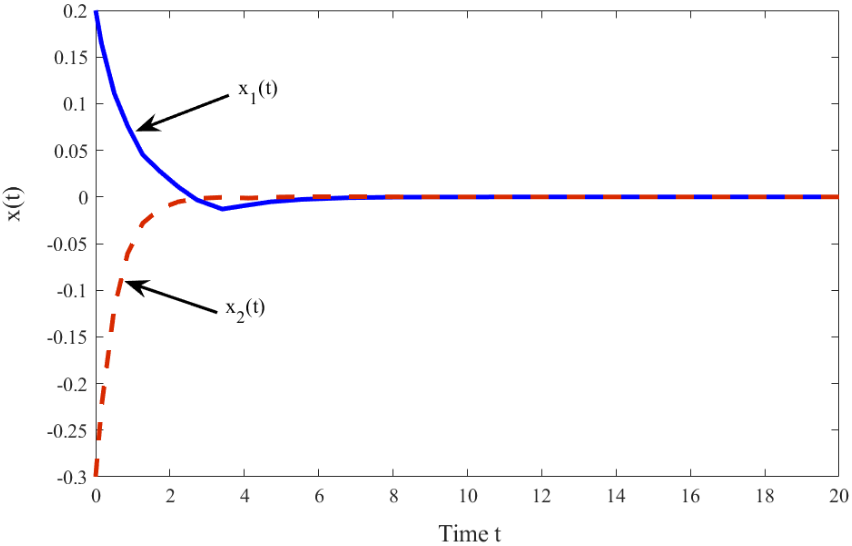

Fig. 1. State trajectories of x_1 and x_2 for Example 1.

Example 2 Consider the NTNNs (13) with the following parameters:

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}, \quad W = \begin{bmatrix} -0.4 & 0 \\ -0.1 & 0.1 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 0.1 & 0.2 \\ -0.15 & -0.18 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0.41 & -0.5 \\ 0.69 & 0.31 \end{bmatrix}, \quad W_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$H = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.7 \end{bmatrix}, E_A = \begin{bmatrix} 0 & 0 \\ 0.1 & 0.1 \end{bmatrix}, \quad E_{W1} = \begin{bmatrix} 0 & 0 \\ 0.02 & 0.03 \end{bmatrix}, \quad E_{W2} = \begin{bmatrix} 0 & 0 \\ 0.02 & 0.02 \end{bmatrix}, \quad E_{W3} = \begin{bmatrix} 0 & 0 \\ 0.001 & 0.001 \end{bmatrix}.$$

The activation functions are assumed to be $g_1 = g_2 = \tanh(s)$ with $l_1^- = 0,\ l_1^+ = 1,\ l_2^- = 0$ and $l_2^+ = 1$. For $0.5 \le \tau(t) \le 4,\ \tau_3 = 0.5,\ 0 < h(t) \le 3.5,\ 0 < k(t) \le 0.9,$ and $h_3 = 0.7$ by using MATLAB LMIs control toolbox and by solving the LMIs in Theorem 2, in our paper we obtain the feasible solutions:

$$P = \begin{bmatrix}
84.4165 & 3.5263 & -0.0836 & -0.1201 & -0.1852 & -0.3277 & 0.1583 & -0.8058 & -0.0345 & -0.0590 & 0.7655 & 0.1168 & -0.0030 & -0.0045 \\
3.5263 & 95.4586 & 0.1045 & -0.1223 & 0.3276 & -0.1652 & 0.8402 & 0.2395 & 0.0601 & -0.0297 & -0.0179 & 0.7775 & 0.0046 & -0.0026 \\
-0.0836 & 0.1045 & 251.7313 & 0.0564 & -17.9726 & 0.2688 & 69.3635 & 0.2195 & -2.1684 & 0.0546 & -38.5787 & -0.3483 & 0.0154 & 0.0054 \\
-0.1201 & -0.1223 & 0.0564 & 251.5809 & 0.1162 & -17.1330 & -0.1580 & 69.4392 & 0.0246 & -1.9769 & -0.2371 & -39.7804 & 0.0025 & 0.0278 \\
-0.1852 & 0.3276 & -17.9726 & 0.1162 & 323.1948 & -0.3806 & -7.6092 & 0.0960 & 11.5753 & -0.0569 & 5.2763 & 0.0423 & -0.0243 & -0.0028 \\
-0.3277 & -0.1652 & 0.2688 & -17.1330 & -0.3806 & 325.5460 & -0.0545 & -7.4640 & -0.0894 & 11.7412 & -0.0599 & 5.2954 & -0.0078 & -0.0357 \\
0.1583 & 0.8402 & 69.3635 & -0.1580 & -7.6092 & -0.0545 & 220.2724 & -0.3687 & -1.7341 & -0.0016 & -80.4882 & -0.0355 & -0.1296 & 0.0011 \\
-0.8058 & 0.2395 & 0.2195 & 69.4392 & 0.0960 & -7.4640 & -0.3687 & 220.5678 & 0.0267 & -1.6718 & -0.1862 & -81.9804 & 0.0030 & -0.1219 \\
-0.0345 & 0.0601 & -2.1684 & 0.0246 & 11.5753 & -0.0894 & -1.7341 & 0.0267 & 1.8162 & -0.0188 & 0.8105 & 0.0017 & 0.1038 & -0.0014 \\
-0.0590 & -0.0297 & 0.0546 & -1.9769 & -0.0569 & 11.7412 & -0.0016 & -1.6718 & -0.0188 & 1.8264 & -0.0158 & 0.8186 & -0.0018 & 0.1038 \\
0.7655 & -0.0179 & -38.5787 & -0.2371 & 5.2763 & -0.0599 & -80.4882 & -0.1862 & 0.8105 & -0.0158 & 229.9570 & 1.0551 & 0.0513 & -0.0006 \\
0.1168 & 0.7775 & -0.3483 & -39.7804 & 0.0423 & 5.2954 & -0.0355 & -81.9804 & 0.0017 & 0.8186 & 1.0551 & 233.9030 & 0.0003 & 0.0554 \\
-0.0030 & 0.0046 & 0.0154 & 0.0025 & -0.0243 & -0.0078 & -0.1296 & 0.0030 & 0.1038 & -0.0018 & 0.0513 & 0.0003 & 0.0259 & -0.0002 \\
-0.0045 & -0.0026 & 0.0054 & 0.0278 & -0.0028 & -0.0357 & 0.0011 & -0.1219 & -0.0014 & 0.1038 & -0.0006 & 0.0554 & -0.0002 & 0.0262
\end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 324.7688 & 19.4190 \\ 19.4190 & 358.8293 \end{bmatrix}, \ Q_2 = \begin{bmatrix} 52.6405 & 3.0286 \\ 3.0286 & 47.5685 \end{bmatrix}, \ Q_3 = \begin{bmatrix} 11.8541 & 0.2194 \\ 0.2194 & 10.3829 \end{bmatrix}, \ S_1 = \begin{bmatrix} 405.3284 & -3.7323 \\ -3.7323 & 327.1213 \end{bmatrix}, \ S_2 = \begin{bmatrix} 0.0563 & -0.0005 \\ -0.0005 & 0.0552 \end{bmatrix},$$

$$S_3 = 10^{-4} \times \begin{bmatrix} 5.0981 & 0.1213 \\ 0.1213 & 5.5087 \end{bmatrix}, X_1 = \begin{bmatrix} 292.9350 & 6.1759 \\ 6.1759 & 259.4640 \end{bmatrix}, \ X_2 = \begin{bmatrix} 35.4156 & -0.0499 \\ -0.0499 & 33.7562 \end{bmatrix}, \ X_3 = \begin{bmatrix} 190.3478 & 3.1112 \\ 3.1112 & 160.5565 \end{bmatrix},$$

$$U_1 = \begin{bmatrix} 180.2987 & 0 \\ 0 & 167.7276 \end{bmatrix},\ U_2 = \begin{bmatrix} 142.9120 & 0 \\ 0 & 169.5647 \end{bmatrix}, T_1 = \begin{bmatrix} 245.6991 & 0 \\ 0 & 229.8644 \end{bmatrix}, T_2 = \begin{bmatrix} 75.0977 & 0 \\ 0 & 74.4702 \end{bmatrix},\ T_3 = \begin{bmatrix} 98.5068 & 0 \\ 0 & 99.9682 \end{bmatrix},$$

$$T_4 = \begin{bmatrix} 98.4898 & 0 \\ 0 & 99.9658 \end{bmatrix},\quad T_5 = \begin{bmatrix} 40.6991 & 0 \\ 0 & 41.5240 \end{bmatrix},\quad T_6 = \begin{bmatrix} 52.8268 & 0 \\ 0 & 55.0384 \end{bmatrix},\quad T_7 = \begin{bmatrix} 52.6506 & 0 \\ 0 & 54.8779 \end{bmatrix},\ T_8 = \begin{bmatrix} 37.4250 & 0 \\ 0 & 38.9042 \end{bmatrix},$$

$$T_9 = \begin{bmatrix} 37.4342 & 0 \\ 0 & 38.9422 \end{bmatrix},\quad T_{10} = \begin{bmatrix} 48.1412 & 0 \\ 0 & 49.4504 \end{bmatrix},\quad \gamma = 1242.1687,\ \varepsilon = 227.2216.$$

5. CONCLUSION

In this paper, the robust passivity analysis for NTNNs with interval and neutral time-varying delays have been studied. By employing the Lyapunov-Krasovskii functional method, and double integral inequality was developed to guarantee the passivity performance of NTNNs. A new passivity analysis criterion has been given in terms of LMIs, which depended on the time-varying delays. Finally, numerical examples have been presented which illustrate the effectiveness and usefulness of the proposed method.

NOMENCLATURE

$\mathbb{R}^n$	The $n-$dimensional Euclidean space
$\mathbb{R}^{n\times n}$	The set of $m\times n$ real matrices
I_n	The $n-$dimensional identity matrix
$\lambda(A)$	The set of all eigenvalues of A
$\lambda_{\max}(A)$	$\max\{\operatorname{Re}\lambda; \lambda \in \lambda(A)\}$
$\lVert\cdot\rVert$	The Euclidean vector norm
$C([0,t],\mathbb{R}^n)$	The set of all $\mathbb{R}^n-$ valued continuous functions on $[0,t]$
$X \geq 0$	X positive semi-definite
$X > 0$	X positive definite
$\operatorname{diag}(\dots)$	A block diagonal matrix

ACKNOWLEDMENT

This project was supported by the Faculty of Science and Liberal Arts, Rajamangala University of Technology Isan.

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