



*Research Article*

## VECTOR FIELD PATH FOLLOWING FOR AN AUTONOMOUS UNDERWATER VEHICLE

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### ABSTRACT:

*The paper presents a method of path following guidance for an autonomous underwater vehicle, which is one of the three fundamental problems in the autonomous system. The line of sight guidance based on the vector field is proposed to generate desired path input to the control laws. The proposed optimal path gives a smooth command transition. The concept of this method is developed by a linear quadratic regulator. The candidate Lyapunov function is then used to show asymptotic decay of path following errors. The simulations have shown that the method gives the underwater vehicle to follow the path and converge to the destination with small error in positions and orientations.*

**Keywords:** Path following, line of sight, vector field

### 1. INTRODUCTION

Autonomous underwater vehicles (AUVs) are playing an important role in underwater exploration allowing humans to explore great depths in various new underwater worlds. AUVs are self-contained and are able to have predefined solutions built into their architecture and to take control actions more accurately and reliably without human intervention. Thus, an AUV is an alternative in complex underwater operations. Examples of such operations are seabed mapping and surveying, studying underwater environment and disasters, underwater inspections and constructions, and under-ice explorations [1]. In marine applications, they are considerable interest in the development of advanced methods for marine control system. Motion control is concentrated into three problems, namely stabilisation, trajectory tracking and path following. Stabilisation refers to stabilising a vehicle at a point in the output space. Trajectory tracking aims to make a vehicle track a desired time-parameterised reference trajectory in the output space. Path following is to make a vehicle converge to and follow a desired spatial path in the output space, without any temporal specification. In this paper, path-following and tracking problem is discussed. Generally speaking for the guidance-control problem, it is usually approached as two separate tasks. The first task denoted the kinematic or path following, is to reach and follow a desired trajectory. In the second task, it is to satisfy dynamic behavior along the path, for example a desired speed. This task is usually specified as an assignment for the speed. It is useful in the development of an approach for steering an AUV along the predefined trajectory with a desired speed for accurate path [2]. In [3], the proposed technique has implicit motion planning in which the trajectory and the control action are not explicitly computed before the motion occurs. One of the examples is a potential field algorithm. This method was developed to specify the robot interaction with the environment and how it responds to the sensory information. It comes to the attention because of its simplicity and less computation.

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However this method has some drawbacks which will be discussed later. To limit the drawbacks, the proposed method of vector field is investigated. The planning using vector field using biologically-inspired strategies has been carried out by [4-6]. The landmark vector model or unit vector represent a position of the robot towards each landmark features. Lyapunov method which is used as a power tool for nonlinear path-following successfully implemented for a land vehicle may not be true for marine vehicle. This is due to the fact that non-dimensional hydrodynamics in the motion. The problem of path following in marine application is discussed in [7, 8]. The method is to understand of surface vessel and the action of helmsman. A way-point representation is designed by reducing 3-DOFs of positions and a heading to 2-DOFs of a yaw and a speed. The similar technique has utilised the Line-of-Sight projection tracking way-point by considering a heading and surge speed.

The paper is organized as follows: The artificial potential field and vector field method are given in section II and III, respectively. In section IV, a path following is proposed. Section V contains the guidance control law. The final section gives the conclusion.

## 2. ARTIFICIAL POTENTIAL FIELD METHOD

The potential field method was first proposed by Khatib [3]. It has become more attractive among researchers in robotics, due to its mathematical simplicity and elegance. The method can be implemented quickly and provide reasonable results without requiring computation resources. It can be applied for real-time control which its method does require only local gradient information. Motion planning using gradient information provides a potential function as an input that drives a vehicle to its desired trajectory whilst avoiding obstacle collision. The artificial potential field technique is an approach that breaks up the free space into a fine grid which is then searched for a free path. Each grid element is assigned a potential, where the goal and neighbouring elements are assigned an attractive potential and obstacles possess a repulsive potential. This ensures that the path created moves towards the goal whilst steering clear of any obstacles.

The potential field approach treats the target position as an attractive well, where the minimum is at the target and treats obstacles as a high potential hill that creates a repulsive force. Assuming the vehicle is a point mass and moves in 2D space. Its position in the workspace is denoted by  $q = [x, y]^T$ . The overall potential is the sum of these two types,

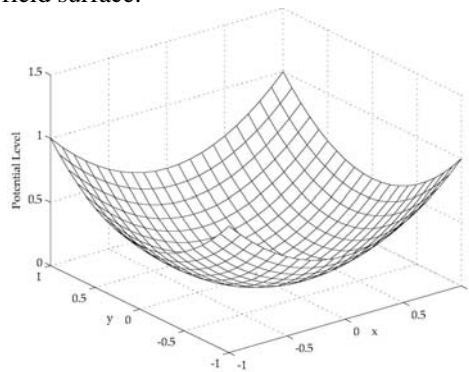
$$U_{total}(q) = U_{att}(q) + U_{rep}(q) \quad (1)$$

where  $U_{att}(q)$  and  $U_{rep}(q)$  denote the attractive and repulsive artificial potential function, thus the field of artificial forces  $\mathcal{F}_{total}(q)$  can be determined by,

$$\mathcal{F}_{total}(q) = -\nabla U_{att}(q) - \nabla U_{rep}(q) \quad (2)$$

### 2.1 Attractive Potential Field

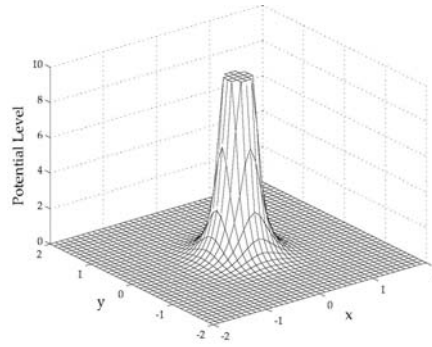
The attractive potential field gives the negative gradient flows pointed toward the target in the workspace. Fig. 1 shows a typical attractive potential field surface.



**Fig. 1.** Attractive potential field plot in 3D.

## 2.2 Repulsive Potential Field

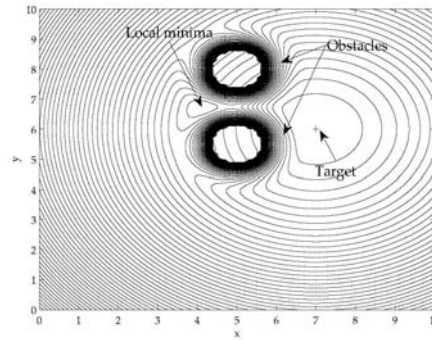
On the other hand, the repulsive potential field gives the negative gradient flows pointed away from the obstacle in the workspace. A repulsive potential field is shown in Fig. 2.



**Fig. 2.** Repulsive potential field plot in 3D.

## 2.3 Limitation

Conventional potential field which is created from some potential functions can have local minima and therefore it creates a trapped situation for the motion of the vehicle. Fundamentally local minima are created at points where the gradient of an attraction function for the target has the same magnitude and is anti-parallel to the gradient of a repulsive function for an obstacle. Local minima can be caused by either one obstacles or combination of obstacle. Fig. 3 shows an example of local minima results from two closely space obstacles.



**Fig. 3.** A local minima in contour plot.

## 3. VECTOR FIELD METHOD

A vector field inspired by a method from the visual based navigation of insects explained by [4] and the average landmark vector model [5] based on a conventional artificial potential field [3] Vector field for predefined trajectory provides more compact representation which is detailed in the following sections.

### Vector Field

From a conventional potential field method, a potential field function is simply considered,

$$U_i(q) = \|q_i - q\| \quad (3)$$

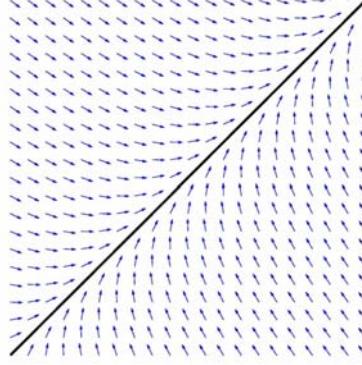
where  $\|\cdot\|$  denotes the Euclidean norm, such that a potential function of  $N$  points derived from Eq. (3) is,

$$U(q) = \sum_{i=1}^N U_i(q) = \sum_{i=1}^N \|q_i - q\| \quad (4)$$

differentiating Eq. (3), gives,

$$Q = \nabla U(q) = - \sum_{i=1}^N \frac{q_i - q}{\|q_i - q\|} \quad (5)$$

From Eq. (5), it can be seen that a vector  $Q$  with unit length points toward each point. Fig. 4 shows an example of the vector field around a straight line. It can be seen that the vector field allows the fields converge to the desired paths.



**Fig. 4.** Vector field for straight line.

#### 4. PATH FOLLOWING

Path following is the problem of making a vehicle converge to and follow a given path, without any temporal demands [9]. In this problem, an entire path is considered rather than a single point. A path is given as a set of coordinates. Now considering  $y_d(\chi)$  defined as a continuous parameterization by the path variable, thus the path can be represented by,

$$Q_p = \{y \in \mathbb{R}^m : \exists \chi \in \mathbb{R} \text{ such that } y = y_d(\chi)\}$$

The path following problem aims to design a function such that an input vector  $u(t)$  in dynamic equation makes  $y(t)$  converges to and track the path with nonzero motion. The Lyapunov method is used as a power tool for nonlinear path-following [10]. Although the proposed method has been successfully implemented for a land vehicle, it may not be true for marine vehicles. This is due to the fact that non-dimensional hydrodynamic terms play a key role in the motion. Therefore, motion control of path-following for marine vehicle requires advance methodologies for accurate path-following that is able to take explicitly into account the nonlinear hydrodynamic terms influenced by ocean currents and wave action.

#### 5. GUIDANCE CONTROL LAW

Based on various literatures a guidance based control for following the path has been reviewed in previous sections. Considering the maneuvering problem, this constructs an update law for the path and forces the path speed to follow the desired speed. The objective can be then classified into two main problems: kinematics and dynamics. In the maneuvering problem, it can be detailed as follows,

1. Kinematics: Let  $y_d(\chi)$  be a desired output. For any continuous function  $\chi(t)$ , the tracking problem is then to design a control system of the marine vehicle such that makes  $y$  converge to and eventually follow  $y_d(\chi)$  that is,

$$\lim_{t \rightarrow \infty} |y(t) - y_d(\chi(t))| = 0$$

2. Dynamics: Let  $u_d(t)$  be a desired speed. The design of control system is to force the speed of the vehicle to converge to a desired speed  $u_d(\chi, t)$  that is,

$$\lim_{t \rightarrow \infty} |\dot{\chi} - u_d(\chi(t), t)| = 0$$

Various guidance and control strategies are considered in the communities. Two common guidance systems are briefly given:

1. Waypoint guidance is commonly used both for marine and aerial vehicles. A path defined by a series of waypoints in 2D Cartesian coordinates, the waypoints are,

$$\text{WP} = [(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)]$$

2. Line-of-Sight guidance is widely used in marine applications [8]. For a surface vessel, the vector is defined as  $\eta = [x, y, \psi]^T$  where  $[x, y]$  is the position and  $\psi$  is the heading angle. Hence, the desired path for each vessel can be calculated, where the desired heading is computed as,

$$\psi_d(\chi) = \text{atan} \frac{y_d(\chi)}{x_d(\chi)}$$

Now a technique based on the LOS is integrated with the waypoint concept. First, define the position vector,  $p = [x, y, z]^T \in \mathbb{R}^3$  in the inertial reference frame. The position error  $e_d = [e_x, e_y, e_z]^T \in \mathbb{R}^3$  is defined as the transformation between the inertial-frame based position and the inertial-frame based desired position denoted as  $p_d$  and,

$$e_d = R_d^T(p - p_d) \quad (6)$$

where the rotation matrix  $R_d$  is given by,

$$R_d = R_d(\psi)R_d(\theta) \quad (7)$$

where  $R_d(\psi)$ ,  $R_d(\theta)$  are the rotation matrices around  $z$ - and  $x$ - axis respectively, the position error rate  $\dot{e}_p$  is obtained by taking the time derivative of Eq. (6),

$$\dot{e}_d = \dot{R}_d^T(p - p_d) + R_d^T(\dot{p} - \dot{p}_d) \quad (8)$$

substituting Eq. (8) with  $\dot{R}_d = R_d S(\omega)$ , and  $\dot{p}_d = R_d v_d$  and  $\dot{p} = R_p v_p$  and using  $R^T R = I$  and a relative rotation matrix  $R = R_d^T R_p$ , then

$$\begin{aligned} \dot{e}_d &= R_d S(\omega)^T(p - p_d) + R_d^T(\dot{p} - \dot{p}_d) \\ &= R_d S(\omega)^T(p - p_d) + R_d^T R_p v_p - R_d v_d \\ &= S^T(\omega)e_d + R^T v_p - v_d \end{aligned}$$

Given the candidate Lyapunov function,  $V_e = \frac{1}{2}e^T e$ , then its derivative is obtained,

$$\begin{aligned} \dot{V}_e &= e_d^T \dot{e}_d \\ &= e_d^T (S^T(\omega)e_d + R^T v_p - v_d) \\ &= e_d^T (R^T v_p - v_d) \end{aligned}$$

and let's define velocity vector,  $v_p = [u_d, 0, 0]^T$ , thus,

$$\dot{V}_e = e_y u_d \sin \psi_d \cos \theta_d - e_z u_d \sin_d \theta_d \quad (9)$$

The aim of path following method is to allow an AUV to follow waypoints which is represented by a series of vehicle's coordinates joined by line segments. As described earlier a number of techniques have been developed to solve this problem. The Line-of-Sight guidance technique [10] is intuitive and widely used in the application for path following of an underwater vehicle. Line-of-sight guidance is chosen and can be given by,

$$\psi_d = \text{atan2}\left(\frac{-e_y}{\Delta}\right) \quad (10a)$$

$$\theta_d = \text{atan2}\left(\frac{e_z}{\sqrt{e_y^2 + \Delta^2}}\right) \quad (10b)$$

where  $\Delta$  is a look ahead distance, substituting Eqs. (10a) and (10b) into Eq. (9), found that

$$\begin{aligned} \dot{V}_e &= e_y u_d \sin \psi_d \cos \theta_d - e_z u_d \sin \theta_d \\ &\leq e_y |u_d| |\sin(\text{atan2}\frac{-e_y}{\Delta})| |\cos(\text{atan2}\frac{e_z}{\sqrt{e_y^2 + \Delta^2}})| \\ &\quad - e_z |u_d| |\sin(\text{atan2}\frac{e_z}{\sqrt{e_y^2 + \Delta^2}})| \\ &\leq -e_y |u_d| |\sin(\text{atan2}\frac{e_y}{\Delta})| \\ &\quad - e_z |u_d| |\sin(\text{atan2}\frac{e_z}{\sqrt{e_y^2 + \Delta^2}})| \\ &\leq -e_y^2 |\gamma_\psi| - e_z^2 |\gamma_\theta|. \end{aligned}$$

since  $\dot{V}_e$  is negative definite, the error vector  $e$  is rendered uniformly globally asymptotically stable if  $u_d > 0$ ,  $\psi_d$  and  $\theta_d$  are defined as in Eqs. (10a)-(10b) for all initial conditions. Algorithm 1 shows how the Line-of-Sight guidance can be computed. It is to determine the commanded yaw and pitch angle for an AUV.

#### Algorithm 1: LOS path following guidance

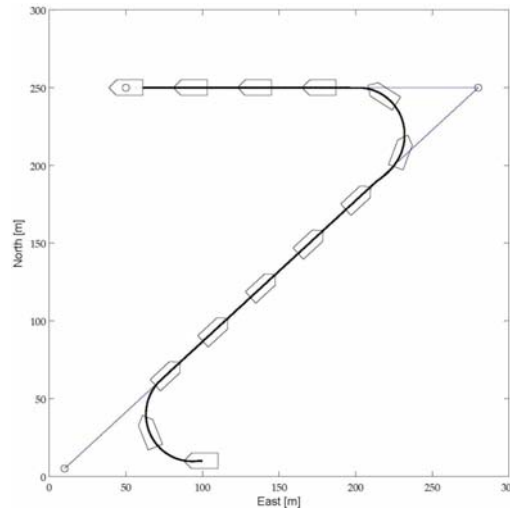
Given a set of waypoints  $p_d = [w_{xi}, w_{yi}] \in \mathbb{R}^2$  determine commanded yaw and pitch angle as follows:

1.  $\psi \leftarrow \text{atan2}\left(-\frac{w_{y2} - w_{y1}}{w_{x2} - w_{x1}}\right)$
2.  $\theta \leftarrow \text{atan2}\left(\frac{w_{z2} - w_{z1}}{\sqrt{(w_{x2} - w_{x1})^2 + (w_{y2} - w_{y1})^2}}\right)$
3.  $e \leftarrow R^T(p - p_d)$
4.  $\psi_d \leftarrow \text{atan2}\left(-\frac{e_y}{\Delta}\right)$
5.  $\theta_d \leftarrow \text{atan2}\left(\frac{e_z}{\sqrt{e_y^2 + \Delta^2}}\right)$
6.  $p^* \leftarrow \frac{(p - w_1)^T(w_2 - w_1)}{\|w_2 - w_1\|}$
7. if  $p^* > 1$  then
8. switch to next waypoint
9. else
10.  $\psi^c \leftarrow \psi - \psi_d$
11.  $\theta^c \leftarrow \theta - \theta_d$
12. endif

By combining the control law (in this case the sliding mode control is developed) and the guidance law, an AUV is able to follow the predefined path with a desired surge speed. It is to determine the commanded yaw and pitch angle of an AUV. Given the waypoints by the path generation with the vector field concept, the LOS connects the current waypoint and the next waypoint in the system. Fig. 5 shows snapshots of an AUV following the three consecutive way-points.

## 6. CONCLUSIONS

The development of a path following in AUV's guidance system is presented in this work. A proposed vector field method improves the conventional motion planning in which the technique of Lyapunov stability shows asymptotic decay of errors. As shown in the program simulation results, the proposed technique provides significantly better following, an AUV is able to follow and converge to the path closely aligning in both position and orientation to the destination.



**Fig. 5.** An AUV follows the path in three consecutive points (denoted as  $\odot$ ) with the use of waypoint guidance law and LOS technique. An AUV is moving with a constant speed at 1.3 m/s.

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