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Research Article

A LINEAR - SCATTERING PHASE FUNCTION FOR SOLVING RADIATIVE HEAT TRANSFER EQUATION (RTE)

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ABSTRACT:
A scattering-radiation phase function is proposed by the linear equation to be as the optional estimation of the radiative heat transfer equation (RTE) for the case of large diffusely reflective sphere. Simplified mathematics in calculating procedure is a prominent of the present method. Two arbitrary constants of the linear equation including of the slope and the value of the point in which the line cross the y-axis are determined by reproducing hemispherical emittances of isothermal, gray media bounded by a black surface. Thus, the presented equation is in the form of $\Phi(\mu) = -1.616\mu + 1.082$. Here, the Φ is denoted by the quantity of scattering phase function and μ is the cosine of scattering angle. To validate the present linear scattering phase function, the conventional method (exact solution) and the solution of Henyey-greenstein approximation are compared. Agreement between the present equation and two mentioned equations is acceptable in the case of pure radiation problem.

Keywords: *Scattering phase function, radiative heat transfer, linear equation, hemispherical emittances, gray media*

1. Introduction

At high temperature, the heat transfer in the mode of radiation is significant phenomena for modeling on the various types of porous media, such as fluidized and packed beds, catalytic reactors and fibers or foams in spatial thermal shields, etc. In the particular case of combustion within a porous media, it has been emphasized [1] that heat radiation inside the porous media plays an important role in the flame stabilization and that scattering effects cannot be neglected in numerical model. The solution of such model can be obtained by following a single continuum treatment and the solving the radiative heat transfer equation (RTE). Knowledge of radiative properties of the medium, i.e., the absorption coefficient (α), the scattering coefficient (σ) and the scattering phase function (Φ), are also required for solving RTE.

There are two reasons that the modeling of heat radiation at a local scale in a porous structure with an opaque solid phase and a transparent fluid phase, namely the absorption and reflection phenomena are considered by such model, cannot carry out [2]. The first reason is that the medium structure is often only statistically known. For the second reason, the vast calculation times and storage capacities have been required. However, the radiative properties would be determined if a porous medium can be treated as a continuous homogeneous absorbing and scattering medium.

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Many methods of characterization of radiative properties of porous media have been developed by considered as semitransparent media [3-6]. They have used the geometric optics approximation and taken into account a small corrective term associated with diffraction in predicting the radiative properties of porous media. For a randomly packed bed of identical opaque spheres, the arbitrary constants of equivalent absorption, scattering coefficients and a phase function have been proposed by Hendricks and Howell [7]. They used a least square fit approach and a discrete ordinate radiative transfer model from experimental data of hemispherical transmittances and reflectances in identifying the arbitrary constants. Hendricks and Howell [8] have been also studied on ceramics. Based on the investigation of Hendricks and Howell [7], Baillis and Sacadura [9] have been conducted on polyurethane foam. An advantage of Hendricks and Howell [7] method is to be directly applied to an actual porous medium. Unfortunately, there are still drawbacks from such a method that radiative inverse problems are often ill-conditioned and the practical inversion requires to impose a particular type of phase function.

From above-mentioned, radiative properties of the medium consisting of the absorption coefficient (α), the scattering coefficient (σ) and the scattering phase function (Φ), become often problem in solving RTE. Nevertheless, a lot of data of absorption and the scattering coefficient have been presented [10]. Certainly, the theoretical analysis of scattering phase function (Φ) has been found in text books [11-12] such as Reyleigh-Gans method, Heyey-Greenstein phase function, Legrendre polynomial and a double Dirac-delta approximation etc. Most of such function requires advanced mathematics in estimation. Thus, the present paper deals a simplified mathematics to propose the linear scattering phase function for the case of large diffusely reflective sphere. The present phase function can be used conveniently and become the optional estimation of radiative properties in solving RTE.

2. Numerical Analysis

2.1 Physical model and RTE

To propose a linear scattering phase function for solving RTE, the physical model is presented as shown in Fig. 1, and the following assumptions are introduced for the analysis: 1) A plane-parallel, porous plate of optical thickness τ_0 is installed horizontally and is bounded both by an upper vacuum boundary and by a lower opaque diffuse solid one; 2) The porous plate and solid boundary are isothermal and are kept at a constant temperature T_m (K); 3) Irradiation from the outside of the upper boundary can be disregarded; 4) The porous plate is quasi-homogeneous and is capable of emitting, absorbing and anisothopically scattering thermal radiation; 5) Radiative properties are gray and do not depend on temperature.

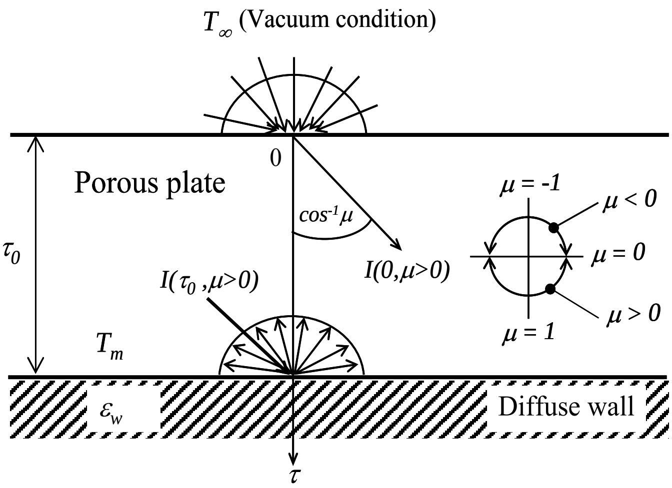

Fig. 1. The physical model of the linear-scattering phase function.

Under these assumptions, the radiative heat transfer equation (RTE) may be written as follows:

$$\mu \frac{dI(\tau,\mu)}{d\tau} + I(\tau,\mu) = \left(1-\omega^*\right)\frac{\sigma T_m^4}{\pi} + \frac{\omega^*}{2}\int_{-1}^{1} P(\mu,\mu')\ I(\tau,\mu')\ d\mu' \tag{1}$$

The boundary conditions to Eq. (1) are given by:

$$\tau = 0:\ I(0, \mu > 0) = 0 \tag{2a}$$

$$\tau = \tau_0:\ I(\tau_0, \mu<0) = \varepsilon_w \frac{\sigma T_m^4}{\pi} + 2\left(1-\varepsilon_w\right)\int_0^1 I(\tau_0,\mu')\ \mu' d\mu' \tag{2b}$$

2.2 Barkstrom's finite difference method

Regarding to Eqs. (1) and (2), the equation of transfer (RTE), together with the associated boundary conditions are solved numerically using Barkstrom's finite difference method [13]. Here, the Barkstorm's method is represented that the radiation intensity I(x, μ) should be separated into two components including of a forward $I^+(x, \mu > 0)$ and backward radiation intensity $I^-(x, \mu < 0)$, and discrete points of the cosine of scattering angle μ is 10 points, the components were rearranged following as:

$$\mu \frac{d\tilde{H}(\tau,\mu)}{d\tau} + \beta^*\tilde{J}(\tau,\mu) = \left(1-\omega^*\right)\frac{\sigma\left[T_s(\tau)\right]^4}{\pi} + \frac{\omega^*}{2}\int_0^1 Q_M(\mu,\mu')\ \tilde{J}(\tau,\mu') d\mu' \tag{3}$$

$$\mu \frac{d\tilde{J}(\tau,\mu)}{d\tau} + \beta^*\tilde{H}(\tau,\mu) = \frac{\beta^*\omega^*}{2}\int_0^1 Q_N(\mu,\mu')\ \tilde{H}(\tau,\mu') d\mu' \tag{4}$$

The quantities of $\tilde{J}(\tau,\mu)$ and $\tilde{H}(\tau,\mu)$ are calculated from:

$$\tilde{J}(\tau,\mu) = \frac{1}{2}\left[\ I^+(\tau,\mu) + I^-(\tau,-\mu)\right] \tag{5}$$

$$\tilde{H}(\tau,\mu) = \frac{1}{2}\left[\ I^+(\tau,\mu) - I^-(\tau,-\mu)\right] \tag{6}$$

$Q_M(\mu,\mu')$ and $Q_N(\mu,\mu')$ denote the metrics of the scattering phase function as expressed by

$$Q_M(\mu,\mu') = P(\mu,\mu') + P(\mu,-\mu') \tag{7}$$

$$Q_N(\mu,\mu') = P(\mu,\mu') - P(\mu,-\mu') \tag{8}$$

Furthermore, boundary conditions for solving Barkstrom's method are given by

$$\tau = 0:\ \tilde{J}(0,\mu') + \tilde{H}(0,\mu') = 0 \tag{9a}$$

$$\tau = \tau_0:\ \tilde{J}(\tau_0,\mu') - \tilde{H}(\tau_0,\mu') = \varepsilon_w\left(\sigma T_m^4/\pi\right) + 2\left(1-\varepsilon_w\right)\int_0^1\left[\tilde{J}(\tau,\mu') + \tilde{H}(\tau,\mu')\right] d\mu' \tag{9b}$$

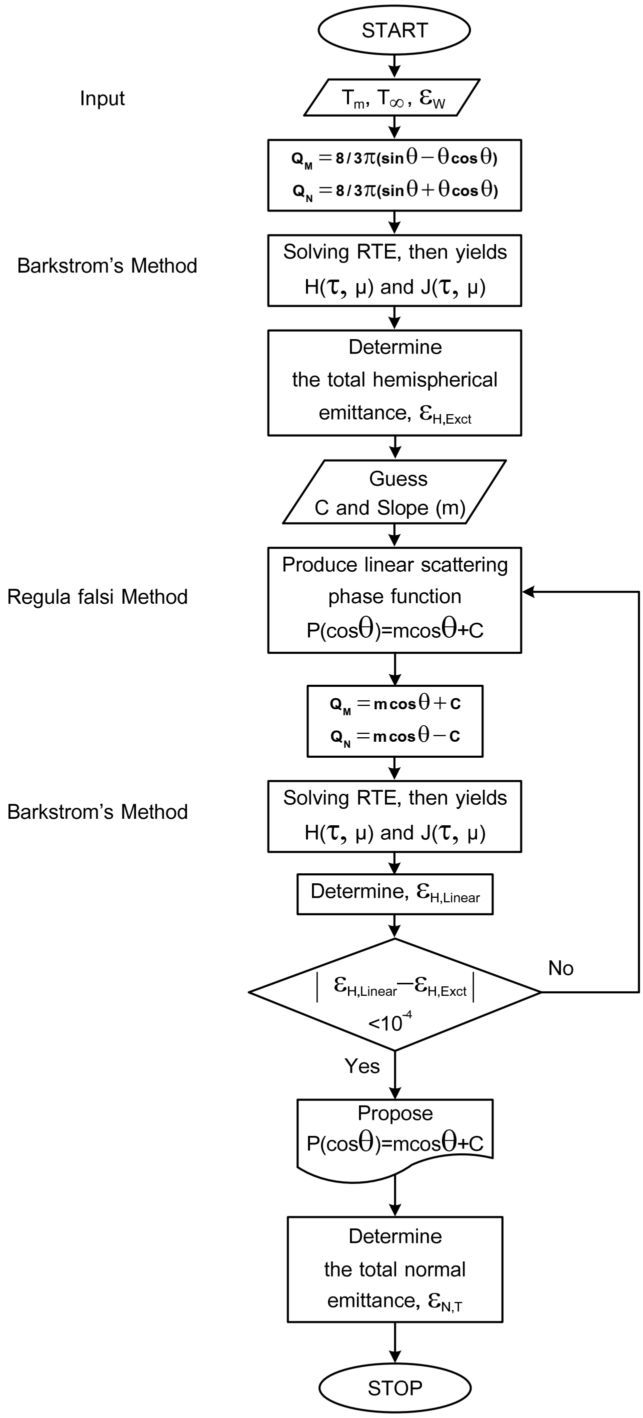

Fig. 2. Flowchart of estimation the linear - scattering phase function.

2.3 Numerical procedure

In the presented linear equation, (the straight-line equation of scattering phase function, the y-axis and x-axis are denoted by the quantity of scattering phase function (Φ) and the cosine of scattering angle (θ) respectively. Two arbitrary constants of the linear equation including of the slope (m) and the value of the point in which the line cross the y-axis (c) are required. Thus, the governing equations (RTE) and boundary conditions have been solved by Barkstrom's finite difference method [13].

Once the equation of transfer (RTE) is solved and the radiation field is specified, then the total hemispherical emittance $\varepsilon_{H,T}$ is evaluated as follows:

$$\varepsilon_{H,T} = \frac{q_R^{\uparrow}}{\sigma T_m^4} = \frac{4\int_0^1 \tilde{H}(x,\mu')\,\mu' d\mu'}{\sigma T_m^4} \tag{10}$$

Then, two arbitraries of m and c can be determined empirically from such total hemispherical emittances (Eq. (10)) if the predicted value of $\varepsilon_{H,T}$ agree with the results obtained by exact phase function (Diffuse sphere) [12]. Moreover, a regula falsi method [14] is adapted to adjust the optimal value of m and c. After obtaining the solution of RTE, intensity of radiation (I (τ,μ)), the total normal emittances have been readily evaluated to validate the present linear scattering phase function as follows:

$$\varepsilon_{N,T} = \frac{I(0,-1)}{\sigma T_m^4/\pi} \tag{11}$$

To deeply understand on the procedure of determination the linear equation of scattering phase function, the flowchart of mathematic model is proposed in Fig. 2

3. Results and Discussion

3.1 The linear scattering phase function

Fig. 3 indicates relation between the total hemispherical emittance ε_H and the albedo ω that the porous media is bounded by a black surface (ε_w = 1.0) with optical thickness (τ) of 2.5 and asymmetry factor ($\tilde{g}$) of 0.8. Excellent agreement between the predictions from the exact diffuse sphere phase function and the present equation is obtained. Therefore, the value of m and c proposed as the simplified linear scattering phase function in the present study may be reasonable.

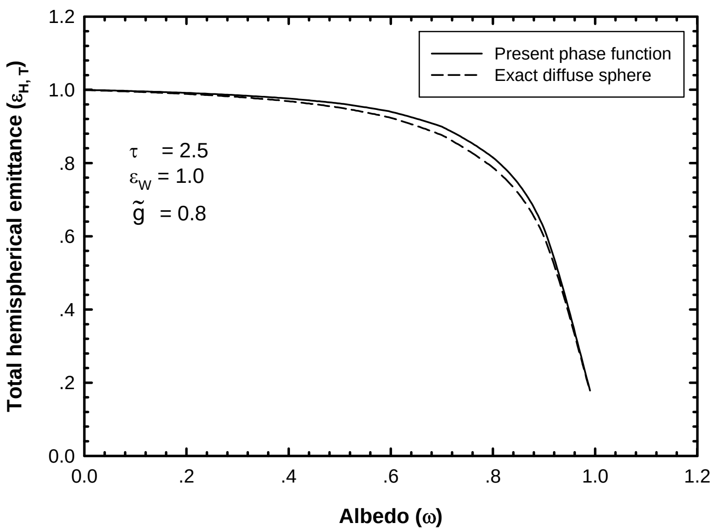

Fig. 3. Relation between the total hemispherical emittance ($\varepsilon_{H,T}$) and albedo (ω).

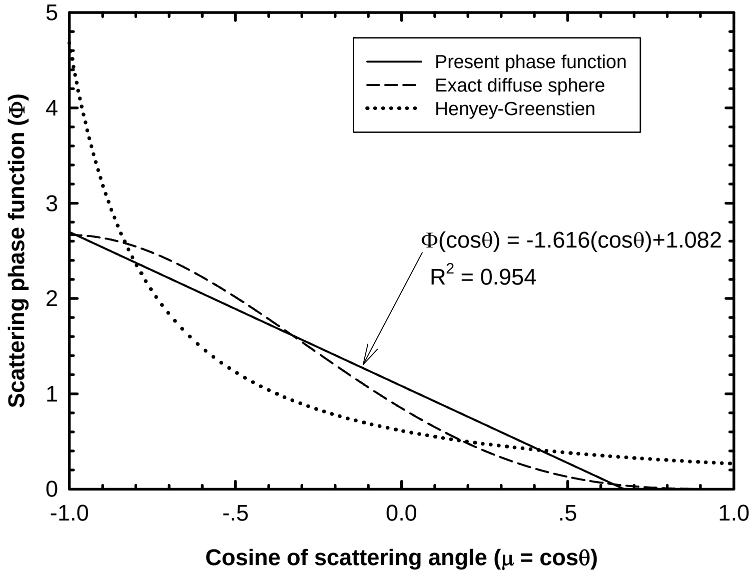

Fig. 4. Comparison of the present phase functions with the exact diffuse sphere and Henyey-Greenstien approximation.

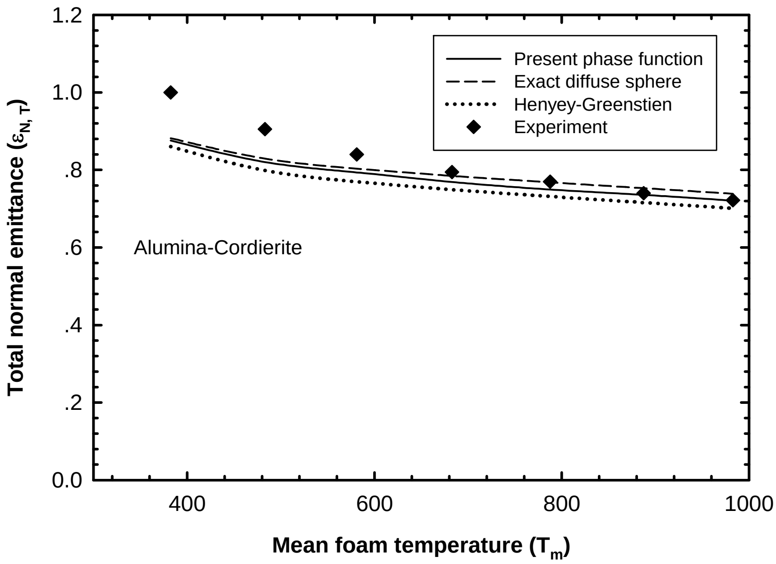

Fig. 5. Variations of the total normal emittances ($\varepsilon_{N,T}$) of the Alumina-Cordierite foam against mean foam temperature (T_m).

Fig. 4 shows a comparison of the present phase functions with the exact diffuse sphere. The arbitrary constant of m and c of the linear equation are -1.616 and 1.082 respectively: $\Phi(\cos\theta) = -1.616\cos\theta + 1.082$. Agreement between the exact function and the present equation is acceptable but there is a little different. The present equation is higher than the exact function for $\mu < -0.65$ and then is lower than the exact function. This is fact that is nature of the straight-line equation. In addition, the approximation phase function of Henyey-Greenstien is also compared. It is found that the present scattering phase function agree with the exact diffuse sphere better than the Henyey-Greenstien approximation. From this result, it can be said that the present phase function is a simple mathematics but has a high performance in predicting the radiative property particular in gray media.

3.2 The total normal emittances

Fig. 5 depicts variations of the total normal emittances ($\varepsilon_{N,T}$) of the Alumina-Cordierite foam against mean foam temperature (T_m). In accord with temperature variations of the total hemispherical emissivity, i.e., 1-ρ_H, the total normal emittance decreases with an increase in foam temperature. There exist some discrepancies between theories (three scattering phase function) and experiment (shown by the black symbols). Agreement between the predictions based on exact diffuse sphere and the present phase function is acceptable. Small discrepancies observed in a temperature region less than about 600 K are appeared due to low sensivity of a radiation detector (thermopiles) used in the experiment, on the other hand, in a temperature region greater than 600 K, good agreement is achieved. Obviously, below prediction of the Henyey-Greenstien approximation is obtained. The present linear equation of scattering phase function can be used conveniently in prediction some pure radiation problems.

5. CONCLUSIONS

The major conclusions that can be derived from the present study are summarized as follows:

1) The simplified linear equation of the scattering phase function is proposed by $\Phi(\cos\theta) = -1.616\cos\theta + 1.082$. Agreement between the exact function and the present equation is acceptable and is better than the Henyey-Greenstien approximation.
2) The present phase function proposed by the straight-line equation yields good results in predicting total normal emittances ($\varepsilon_{N,T}$) of isothermal, Alumina-Cordierite open-cell foams.

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NOMENCLATURE

$\tilde{g}$	asymmetry factor
$I(\tau,\mu)$	intensity of radiation
$P(\mu,\mu')$	scattering phase function
$P_n(\mu)$	Legendre function
q_R	net radiative heat flux
T_m	isothermal wall temperature
β	extinction coefficient
$\varepsilon_{H,T}$	total hemispherical emittance
$\varepsilon_{N,T}$	total normal emittance
ε_W	emissivity at solid boundary
$\Phi(\mu)$	scattering phase function
μ	cosine of scattering angle
σ	Stefan-Boltzmann's constant
τ	optical thickness
ω	albedo