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Research Article

OPTIMAL SYNTHESIS OF FOUR-BAR LINKAGE PATH GENERATION THROUGH EVOLUTIONARY COMPUTATION

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ABSTRACT:
This paper presents the optimization of path generating of four-bar linkages using evolution algorithms (EAs). The design problem is assigned to minimize the error between desired and obtained coupler curves. Such mechanism synthesis is sometimes called dimensional analysis, which has the design variables as link lengths and other parameters. In this work, three evolutionary algorithms namely differential evolution (DE), hybrid population-based incremental learning and differential evolution (PBIL-DE) and adaptive differential evolution with optional external archive (JADE) are applied for finding the solutions. The results show that DE and JADE are the best methods for synthesizing the path generating of a four-bar linkage.

Keywords: *Mechanism Synthesis, path generation, four-bar linkage, optimization, evolutionary algorithms*

1. INTRODUCTION

Over the last few decades, it has been found that many researchers try to solve the optimization of path generating of four-bar linkages using meta-heuristic methods (MHs) or evolutionary algorithms (EAs) [1]. The path generating or path synthesis is one of the kinematic syntheses of mechanisms [2-7], which basically can be divided into two categories. The first one is called type synthesis [4, 6] where, with a predefined motion transmission, a designer is supposed not to originally know or have sufficient information about a mechanism. This is somewhat equivalent to topology design in structural optimisation. Having finished synthesising, a designer will have a certain type of mechanism. The second synthesis category is called dimensional synthesis, which is set to determine significant dimensions to achieve a given prescribed motion. This synthesis type can be further classified into function generation, path generation and motion generation. The most popular synthesis of this type is the path generation [1-3, 5, 7] where one point on a link must follow a predefined path. Mathematically, all of them can be converted into optimisation problems and MHs are the obvious choice for solving the optimisation problems. Some most frequently used MHs for this task are the differential evolution (DE) algorithm [2-3], the genetic algorithm (GA) [3], the particle swarm optimization (PSO) [3] and the imperialist competitive algorithm (ICA) etc. The work in [3] proposed the comparative performance based on the path error. It was found that the best performance is DE, while the work in [1] shows that ICA is the best algorithm.

This paper is intended as an extension of the above literature. It proposed the optimization of path generating of four-bar linkages using evolution algorithms (EAs).

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The design problem is assigned to minimize the error between desired and obtained coupler curves with three MHs being implemented i.e. differential evolution (DE), hybrid population-based incremental learning and differential evolution (PBIL-DE) and adaptive differential evolution with optional external archive (JADE). The performances of the optimizers are compared and discussed.

The rest of this paper is organized as follows. Section 2 details of the position analysis of a four-bar linkage. All of MHs are presented in Section 3. A design problem and its conditions as well as a numerical experiment are given in Section 4, while the design results are in Section 5. The conclusions and discussion of the study are finally drawn in Section 6.

2. POSITION ANALYSIS OF THE FOUR-BAR LINKAGES AND OPTIMIZATION PROBLEM

The computation of error between desired and obtained coupler curves of the four-bar linkage and optimization problem are detailed in this section.

2.1 Position analysis of the four-bar linkage

The model of the four-bar linkage is shown in Fig. 1. The four-bar linkage is the simplest and most commonly used linkage in many machines. It is a combination of four binary links, which has one link as the frame and connected by four pin joints (denoted by capital letters). The example for this mechanism is a window wiper, door closing mechanism, and rock crushers etc. Based on the Gruebler's equation for a planar mechanism, the mobility or degree of freedom of the mechanism is one. It is a constrained mechanism, fully operated by one driver. The path generating for a four-bar linkage is dimension-based design of the four-bar linkage lengths (r_1, r_2, r_3, r_4) and other parameters, which gives the trace point (x_P, y_P) on coupler link follow the desired path (x_d, y_d).The coupler curve takes place when the pivot link moves with its cycle. From Fig. 1, the coupler point coordinates in the global coordinate can be expressed as

$$x_P = x_{O2} + r_2\cos(\theta_2 + \theta_1) + L_1\cos(\phi_0 + \theta_3 + \theta_1)$$
$$y_P = y_{O2} + r_2\sin(\theta_2 + \theta_1) + L_1\sin(\phi_0 + \theta_3 + \theta_1) \quad (1)$$

where
x_{O2} and y_{O2} are the *x*- and *y*- positions of the O_2 pin joint in the global coordinate.
ϕ_0 can be obtained by considering the couple link *BCP* using the law of cosine, which is expressed as

$$\phi_0 = \cos^{-1}\left[\frac{L_1^2 + r_3^2 - L_2^2}{2L_1 r_3}\right] \quad (2)$$

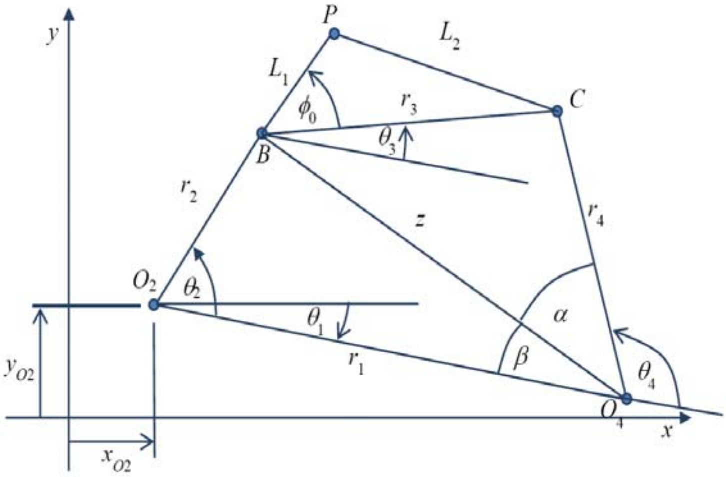


Fig. 1. Four-bar linkage in global coordinate system.

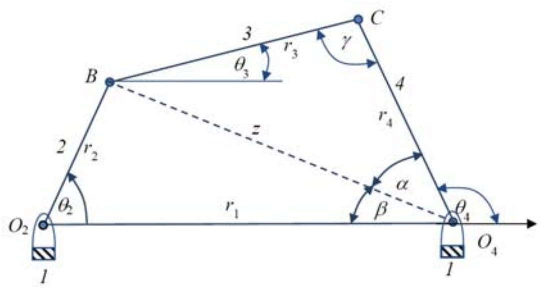


Fig. 2. Four-bar linkage in local coordinate.

The interior angles (θ_3, θ_4, and γ) for the known link lengths (r_1, r_2, r_3, r_4) at any crank angle (θ_2) can be obtained by considering Fig. 2. These angles are determined as follows:

$$z^2 = r_1^2 + r_2^2 - 2r_1\, r_2\, \cos\theta_2$$

$$z^2 = r_3^2 + r_4^2 - 2r_3\, r_4\, \cos\gamma$$

$$\gamma = \cos^{-1}\left[\frac{r_3^2 + r_4^2 - r_1^2 - r_2^2 + 2r_1 r_2 \cos\theta_2}{2r_3 r_4}\right]$$

$$\gamma = \cos^{-1}\left[\frac{r_3^2 + r_4^2 - z^2}{2r_3 r_4}\right] \tag{3}$$

$$\alpha = \cos^{-1}\left[\frac{z^2 - r_3^2 + r_4^2}{2zr_4}\right] \tag{4}$$

$$\beta = \cos^{-1}\left[\frac{z^2 + r_1^2 - r_2^2}{2zr_1}\right] \tag{5}$$

$$\theta_3 = \pi - (\alpha + \beta + \gamma) \tag{6}$$

$$\theta_4 = \pi - (\alpha + \beta) \tag{7}$$

These equations will be used for objective function evaluations of our optimization problem.

2.2 Optimization problem

The objective optimization of this problem is assigned to minimize the error between desired and obtained coupler curves, thus, the objective function is the position error expressed as the sum of squares of the distances between each P_d and the corresponding P_o where P_d is a set of desired points, and P_a is a set of obtained points. The design variables x for this problem include r_1, r_2, r_3, r_4, L_1, and L_2, the global position of the O_2 (x_{O2}, y_{O2}), and the angle of frame 1 (θ_1). The four-bar linkage that is obtained as the final result should meet two conditions or constraints. The first constraint is the input crank can rotate with a complete revolution in either direction (clockwise or counter clockwise). The second constraint is at least one link in the linkage can rotate completely. This implies that the linkage must satisfy the Grashof's criterion i.e. the sum of the shortest and longest links of the linkage must be lower than the sum of two remaining links in order to have a crank-rocker four-bar linkage. The optimization problem can then be written as:

$$\min f(\mathbf{x}) = \sum_{i=1}^{N}\left[(x - x_P)^2 + (y - y_P)^2\right] \tag{8}$$

subject to:

min(r_1, r_2, r_3, r_4) = crank
2 min(r_1, r_2, r_3, r_4) + 2 max(r_1, r_2, r_3, r_4) < (r_1+ r_2+ r_3+ r_4)
$\mathbf{x}_l \leq \mathbf{x} \leq \mathbf{x}_u$.

where $\mathbf{x} = \{x_{O2}, y_{O2}, r_1, r_2, r_3, r_4, L_1, L_2, \theta_1\}^T$, and N is the number of points on the prescribed or desired curve.

3. EVOLUTIONARY ALGORITHMS

3.1 Differential evolution (DE)

DE is one of the most popular and powerful EAs which is a population-based stochastic optimization method. It starts with an initial population, which is randomly generated when no preliminary knowledge about the solution space is available. The DE operators include mutation, crossover, and selection. These operators are used to maintain population diversity, as well as to avoid premature convergence. The DE scheme is used in this study can be classified as the standard DE or DE/best/2/bin algorithm [8].

3.2 JADE

It has also been found that optimisation parameter settings of DE are some of the most influential factors affecting its performance. For example, the scaling factor (F) used in DE mutation and the crossover probability (CR) are crucial parameters for its performance and is also problem-dependent. As a consequence, the development of self-adaptive DE began, which to a great extent can improve DE search performance. Adaptive schemes of JADE can update the control parameters based on their historical record of success [9].

3.3 population-based incremental learning (PBIL)

The original PBIL algorithm was governed by binary searching similar to genetic algorithms (GA) without crossover operation [10]. Later work was focused on the development of PBIL for continuous and discrete design spaces. The principle of PBIL search can be thought of as iteratively limiting the design space depending on current best design variables and random process. The design space is iteratively narrowed until approaching the optimum. Rather than keeping all binary genes or a population as with GA, the population in PBIL is represented by the probability vector of having '1' at each bit position in the binary strings. In this paper the hybrid version of PBIL-DE [11] is used to solve the optimization problem.

4. NUMERICAL EXPERIMENT

The case studies discussed herein are path synthesis problems in which a set of target points are predefined by a designer to generate an optimization problem. The target or prescribed points of this study are a straight line, a circle sector, and a cosine curve. For all cases, the objective function is computed by Equation (8) while the design problem is minimized by DE, PBIL and JADE. To study the performance of the EAs, the parameter settings of all algorithms are given in Table 1.

4.1 Case-1: linear

The first problem has the target points as a straight line as

$$y_{d,i} = 2.5x_{d,i},\ i = 1,\ldots,20 \tag{9}$$

where $50 \leq x_i \leq 80$

Bounds of the design variables:

$-10 \leq x_{O2}, y_{O2} \leq 20$
$100 \leq r_1 \leq 270$
$50 \leq r_2 \leq 160$
$100 \leq r_3, r_4 \leq 250$
$80 \leq L_1 \leq 240$
$70 \leq L_2 \leq 220$
$-30 \leq \theta_1 \leq 30$

4.2 Case-2: circle sector

The second problem has the target points as a circle sector

$$x_{d,i} = 50 + R_0\cos(\theta_i), y_{d,i} = 150 + R_0\sin(\theta_i),\ i = 1,\dots,20 \quad (10)$$

where $-30 \le \theta_i \le 60$, $R_0 = 30$

Limits of the variables:
$-10 \le x_{O2}, y_{O2} \le 20$
$100 \le r_1 \le 270$
$50 \le r_2 \le 160$
$100 \le r_3, r_4 \le 250$
$80 \le L_1 \le 240$
$70 \le L_2 \le 220$
$-30 \le \theta_1 \le 30$

4.3 Case-3: cosine curve

The second problem has the target points as a cosine curve

$$y_{d,i} = 120 + 20\cos(m_0 x_{d,i} + c_0),\ i = 1,\dots,20 \quad (11)$$

where $50 \le x_i \le 80$, $m_0 = 60$, $c_0 = 50m_0$

Limits of the variables:
$-10 \le x_{O2}, y_{O2} \le 20$
$100 \le r_1 \le 270$
$50 \le r_2 \le 160$
$100 \le r_3, r_4 \le 250$
$80 \le L_1 \le 240$
$70 \le L_2 \le 220$
$-30 \le \theta_1 \le 30$

Table 1: Parameters of DE, PBIL and JADE

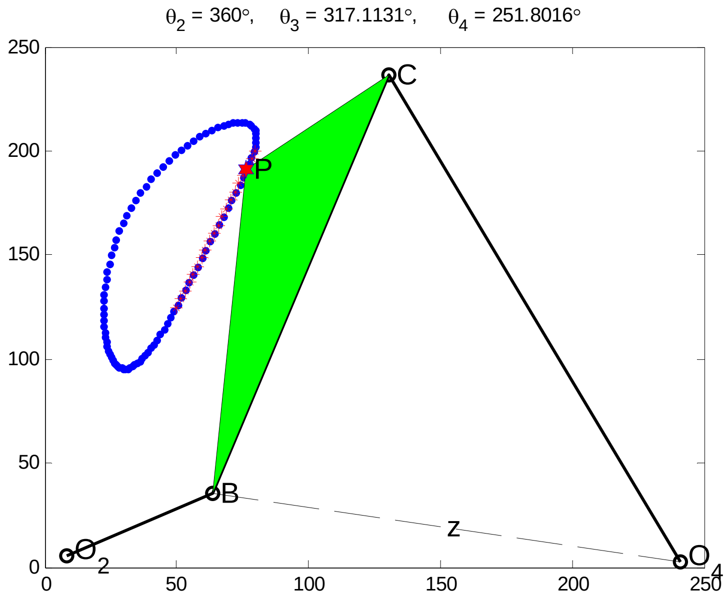
Parameters	DE (DE/best/2/bin)	PBIL	JADE
Number of initial population	50	50	50
Crossover probability	0.7	-	Self-adaptive
Mutation Probability	-	0.05	-
Mutation shift	-	0.2	-
Learning rate	-	0.5	-
Scaling factor	0.7	-	Self-adaptive
Crossover rate	0.8	-	Self-adaptive
Generation number	250	250	250

5. Design Results

The results of case-1 obtained from the three optimizers are showed in Table 2. Fig. 3 shows the best linkage and coupler points and the path traced by the coupler point at the final solution. In this case the number of target points are 20 points. It is found that JADE gives the best result (error = 9.3936). For the second case with a circle sector and the number of target points of 20, the results obtained from using the various MHs are shown in Table 3. The best linkage and coupler curve is shown in Fig. 4. In this case, JADE is also the best performer (error = 10.8508) and DE is the second best.

Table 2: Comparative results for case study-1

Parameters	DE	PBIL	JADE
x_{O2} (mm)	15.9466	-1.4812	8.2351
y_{O2} (mm)	-10.0000	-9.2022	5.2943
r_1(mm)	270.0000	270.0000	265.6523
r_2 (mm)	63.1981	65.6100	63.1108
r_3 (mm)	208.3173	221.1962	211.7677
r_4 (mm)	143.9896	135.5958	151.7076
L_1 (mm)	170.9116	208.9570	155.8772
L_2 (mm)	70.0000	70.0114	70.6699
θ_1 (degree)	29.5294	20.4991	28.7442
error	14.3003	16.6126	9.3936

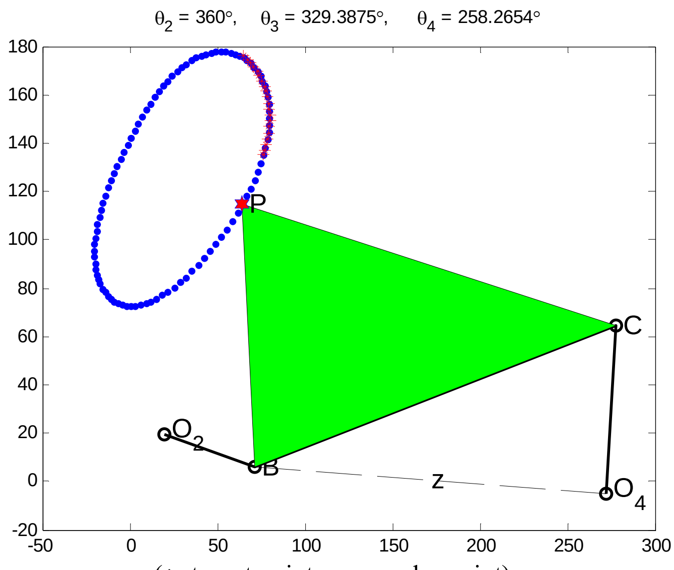


(* = target point, • = coupler point)

Fig. 3. Best linkage and coupler curve in Case study-1 obtained by JADE.

Table 3: Comparative results for case study-2

Parameters	DE	PBIL	JADE
x_{O2} (mm)	19.9290	19.6158	19.9818
y_{O2} (mm)	19.9932	2.4997	19.5933
r_1(mm)	270.0000	234.6493	260.9055
r_2 (mm)	50.0000	59.7032	53.5267
r_3 (mm)	250.0000	248.8322	214.5872
r_4 (mm)	248.8988	198.9138	111.6065
L_1 (mm)	111.8460	121.2654	108.8978
L_2 (mm)	220.0000	220.0000	219.8704
θ_1 (degree)	-30.0000	-15.1335	-14.7473
error	12.2089	16.6474	10.8508

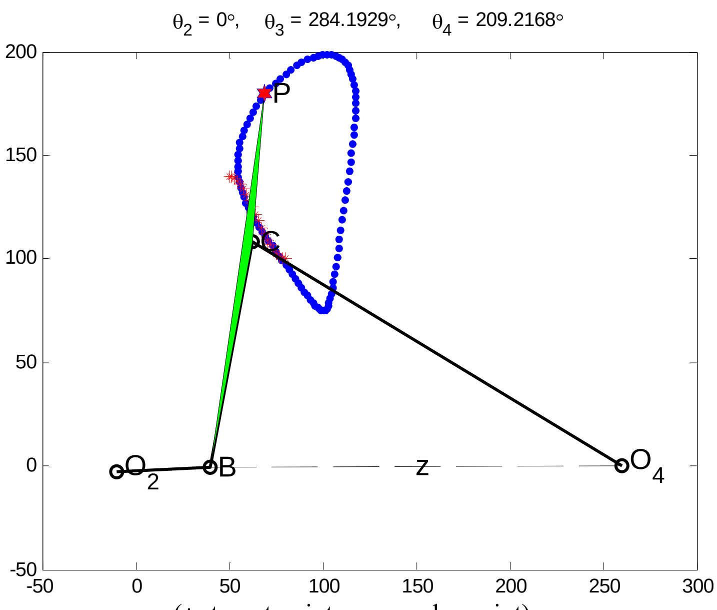


(∗=target point,• = coupler point).

Fig. 4. Best linkage and coupler curve in Case study-2 obtained by JADE.

Table 4: Comparative results for case study-3

Parameters	DE	PBIL	JADE
x_{O2} (mm)	-10.0000	-4.3196	-8.6017
y_{O2} (mm)	-2.6346	7.2969	9.2389
r_1(mm)	270.0000	102.8745	137.1272
r_2 (mm)	50.0000	64.2040	68.1969
r_3 (mm)	111.1858	144.6188	183.8578
r_4 (mm)	220.8331	106.1747	115.0614
L_1 (mm)	182.6002	91.4598	85.6565
L_2 (mm)	71.6901	185.4967	219.8300
θ_1 (degree)	2.4811	-24.9771	-27.8648
error	33.0815	39.7500	38.0838



(∗=target point,• = coupler point).

Fig. 5. Best linkage and coupler curve in Case study-3 obtained by DE.

In the third case, the prescribed curve is a cosine curve with 20 points which results in the combination of convex and concave curves. The results obtained by those algorithms are shown in Table 4 and Fig. 5. From the results, it is found that DE gives the best result (error = 33.0815). Nevertheless, it can be seen from the figure that the path generated by the coupler cannot follow the prescribed cosine curve satisfactorily. This is the weak point of path synthesis by using a four-bar linkage which should be solved in our future work.

6. Conclusion

This paper presents a numerical approach to design the dimensions of a four-bar linkage, which can generate the coupler path with the given target points by using evolutionary algorithms. It is found that the propose technique is an alternative to synthesis a four-bar linkage and it is possible to extend using such methods with other kinds of mechanism synthesis. Furthermore, comparative performance of EAs show that the self-adaptive JADE is superior to the others. This implies that future work on path generation should focus on using self-adaptive meta-heuristics rather than methods requiring initial parameter settings. Our future work is to figure out how to generate a path to satisfactorily match the prescribed cosine curve, the curve with the combination of convex and concave shapes.

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References

[1] Ebrahimi, S. and Payvandy, P. Efficient constrained synthesis of path generating four-bar mechanisms based on the heuristic optimization algorithms, Mechanism and Machine Theory, Vol. 85, 2015, pp.189-204.

[2] Cabrera, J. A., Nadal, F., Muñoz, J.P. and Simon, A. Multiobjective constrained optimal synthesis of planar mechanisms using a new evolutionary algorithm, Mechanism and Machine Theory, Vol. 42(7), 2007, pp.791-806.

[3] Acharyya, S. K. and Mandal, M. Performance of EAs for four-bar linkage synthesis, Mechanism and Machine Theory, Vol. 44(9), 2009, pp.1784-1794.

[4] Al-Dwairi, A.F., Dweiri, F.T. and Ashour, O.M. A novice-centered decision-support system for type synthesis of function-generation mechanisms, Mechanism and Machine Theory, Vol. 45(9), 2010, pp. 1252-1268.

[5] Cabrera, J.A., Ortiz, A., Nadal, F. and Castillo, J.J. An evolutionary algorithm for path synthesis of mechanisms, Mechanism and Machine Theory, Vol. 46(2), 2011, pp. 127-141.

[6] Heo, J.C. and Yoon, G.H. Size and configuration syntheses of rigid-link mechanisms with multiple rotary actuators using the constraint force design method, Mechanism and Machine Theory, Vol. 64, 2013, pp.18-38.

[7] Chanekar, P.V. and Ghosal, A. Optimal synthesis of adjustable planar four-bar crank-rocker type mechanisms for approximate multi-path generation, Mechanism and Machine Theory, Vol. 69, 2013, pp. 263-277.

[8] Storn, R.M. and Price, K.V. Differential evolution-a simple and efficient heuristic for global optimization over continuous spaces, Journal of Global Optimization, Vol. 11, 1997, pp. 341-359.

[9] Zhang, J. and Sanderson, A.C. JADE: adaptive differential evolution with optional external archive, IEEE Transaction on Evolution Computation, Vol. 13(5), 2009, pp. 945-958.

[10] Baluja, S. Population-based incremental learning: a method for integrating genetic search based function optimization and competitive learning, Technical ReportCMU_CS_95_163, 1994, Carnegie Mellon University.

[11] Pholdee, N. and Bureerat, S. Hybridisation of real-code population-based incremental learning and differential evolution for multi-objective design of trusses, Information Science, Vol. 223 (2013), pp. 136-152.