



Research Article

PERMEABILITY OF WIRE-NET POROUS MEDIA DETERMINED BY A SIMPLE DARCY-FORCHHEIMER EQUATION

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ABSTRACT:

The permeability (K) of porous media is the most significant parameter for describing the fluid flow mechanism within porous matrix. A simple experiment to investigate the K of wire-net porous media based on Darcy – Forchheimer principle is proposed. The stainless ASUS304 with four PPIs (Pores per inch), i.e., $PPI = 4, 8, 10$ and 12 , are examined and reported in the form of porosity (ε) yielding as $0.943, 0.898, 0.822$ and 0.794 respectively. Five thicknesses (H) consisting of $1.2, 2.4, 3.6, 4.8$ and 6.0 mm, are tested. Regarding to the Darcy – Forchheimer method, the equation of relation between pressure drop (ΔP) and velocity (u) is in the 2nd polynomial form: $\Delta P = au + bu^2$. To simplify this equation, the linear form is discussed: $\Delta P = a + bu$. The ΔP as measured by U-tube manometer is conducted in varying the velocity (u) from 0.225 m/s to 1.578 m/s. From the experiment, it is found that the value of K is depended on ε (PPI) and H . Thus, the equation of K estimated by multi-regression method can be reported by $K = (89.401\varepsilon - 66.412)/H \times 10^{-7}$ which determination coefficient (R^2) has 0.889 . To validate the present regression, three available models consisting of Kozeny-Carman correlation, Gebart equation and a nonlinear equation of Koponen are compared. Good agreement is obtained in comparison. Therefore, it can be said that the present regression form is highly believable and it is easily used.

Keywords: Permeability, Darcy – Forchheimer equation, Wire-net porous media, Pores per inch

1. INTRODUCTION

Investigation on the fluid flow mechanism within porous media become significant due to it has wide applications in both science and engineering such as petroleum engineering [1, 2], groundwater hydrology [3, 4], micromachining technology [5, 6] and fuel cells [7, 8]. Commonly, fluid flows through porous media are based on Darcy law which is a linear relation between the average (seepage) velocity and the pressure gradient. Unfortunately, the accuracy of this linear relationship is found at condition of very small pressure gradient or flow velocity. As the Reynolds number (Re) increases up to a critical value, nonlinear of the relationship is yielded. From this reason and to generate a universal relation including this nonlinear effect, an empirical formula is proposed by Forchheimer [9]: $-\nabla P = au + bu^2$. Regarding to Forchheimer equation, u is the average velocity, the coefficient a is defined by K/μ where K and μ are the permeability and the dynamic viscosity respectively, and b is constant related to the pore size, shape and porosity (ε).

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The Forchheimer equation is considered as a correction to Darcy law by adding a nonlinear term. It is meant that the pressure lost is attributed to viscous contribution and inertial contribution corresponding to linear and nonlinear terms of Forchheimer principle. Also, it is clear that the Forchheimer equation will be rearranged relating with Darcy law when the average velocity u is sufficiently small. A modified Forchheimer equation or Darcy-Forchheimer method gives a good depiction in a high velocity flow [10].

Permeability (K) estimated from Darcy-Forchheimer equation is a key of the transmitted ability of structure. For macroscopic phenomena of fluid flow through the porous matrix, the quantity of permeability (K) has perhaps the most important property to describe and to deeply understand the flow phenomena. Usually, the void structure in porous matrix as presented by the porosity (ε) is complex resulting to a complicated flow pattern exist within the void and across porous structure [11]. The most classical equation discussed in the relation between permeability (K) and porosity (ε) is Kozeny-Carman equation [12, 13] as given by:

$$K = \frac{\varepsilon^3}{C_{KC} \tau^2 S}, \quad (1)$$

where C_{KC} is Kozeny-Carman constant depended on porosity and microstructure, τ is tortuosity and S is the specific surface area. Consequently, in a last few decade, many experimental studies [14-16] and several numerical analysis particular obeyed with lattice Boltzmann method [17-19] are conducted. The permeability (K) controlled by porosity (ε) of fibrous materials or wire-net porous media is also proposed by many researches. Gebart [15] investigated the permeability (K) with a combination method of theoretical, numerical and experimental studies and the functional form of $K(\varepsilon)$ is yielded as:

$$\frac{K}{r^2} = C_G \left(\sqrt{\frac{1-\varepsilon_G}{1-\varepsilon}} - 1 \right)^{5/2}. \quad (2)$$

Here, r , C_G and ε_G are the fibre or wire diameter, Gebart constant and Gebart porosity respectively. A good application of his function is obtained for porosity (ε) is over 0.65. Koponen et al [17] applied the lattice Boltzmann method to establish an empirical relationship for porosities (ε) in the range of $0.4 \leq \varepsilon \leq 0.95$. Their empirical equation is given by:

$$\frac{K}{r^2} = \frac{5.55}{e^{10.1(1-\varepsilon)} - 1}. \quad (3)$$

Although above-mentioned works are accuracy, it is still difficult way to determine the permeability (K). Recently, the second authors and his co-worker [20] proposed a simple method in determination of the permeability (K) under the Darcy-Forchheimer equation. However, their work did not validate or compare to other studies.

Therefore, the present study is to propose a simple experiment for investigation the K of a wire-net porous media and to extend the previous work [20]. Several porosities (ε) governed by pores per inch (PPI) and porous thickness (H) are variable studies. Finally, the correlation between K and ε (PPI) & H is reported and the comparison results of a present correlation with available functions consisting of Kozeny-Carman equation [12, 13], Gebart [15] and Koponen et al [17] are discussed.

2. EXPERIMENTAL SETUP

2.1 Experimental diagram

A testing pipe with inner-diameter of 90 mm and length of 500 mm made of ASUS304 stainless steel is shown in Fig 1. The wire-net 304 stainless steel with diameter of 1.4 mm is employed as porous media and it is provided as six bundles to install inside the pipe. The distance between two porous bundles (L) is kept at 80 mm. In the present study, there are two interesting parameters: pores per inch (PPI), here reported as porosity (ε), and porous thickness (H). Four porosities (ε), i.e., 0.943 ($PPI = 4$), 0.898 ($PPI = 8$), 0.822 ($PPI = 10$) and 0.794 ($PPI = 12$), and Five H

consisting of 1.2, 2.4, 3.6, 4.8 and 6.0 mm, are examined. The reason in selection these parameters (ε and H) is that they are used in many our works and applications [21-23]. Twenty tested configurations of wire-net porous media are conducted and the physical properties of porous media is presented in Table 1.

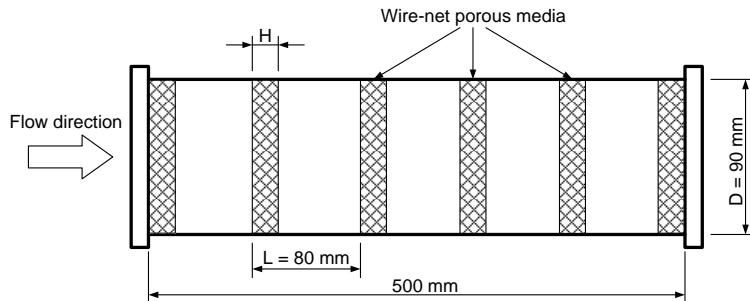


Fig. 1. Alignment of wire-net porous media in test section.

Table 1: Testing Configuration and Physical Properties of Wire-net Porous Media.

Case	PPI	Porosity (ε)	H (mm)	Case	PPI	Porosity (ε)	H (mm)
1	4	0.943	1.2	11	10	0.822	1.2
2	4	0.943	2.4	12	10	0.822	2.4
3	4	0.943	3.6	13	10	0.822	3.6
4	4	0.943	4.8	14	10	0.822	4.8
5	4	0.943	6.0	15	10	0.822	6.0
6	8	0.898	1.2	16	12	0.794	1.2
7	8	0.898	2.4	17	12	0.794	2.4
8	8	0.898	3.6	18	12	0.794	3.6
9	8	0.898	4.8	19	12	0.794	4.8
10	8	0.898	6.0	20	12	0.794	6.0

Figure 2 shows a schematic diagram of the experiment to determine the permeability (K) of wire-net porous media attached inside the circular pipe. This experimental set-up is general method to monitor the pressure drop (ΔP) across the pipe because it is a simple process in estimating the permeability (K) [24, 25].

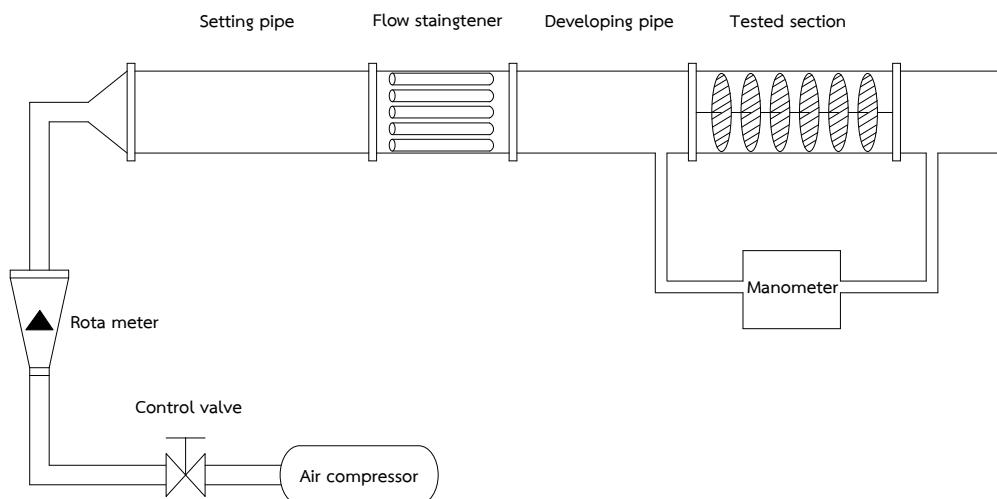


Fig. 2. Schematic diagram of the experimental setup.

As from the Fig. 2, an experimental procedure is explained following as: 1) the fresh air at the ambient condition is supplied by compressor into the system, 2) the air flow rate (Q) is controlled by valve and it is observed by Rota meter, 3) the Q is varied in the range of $0.00167 \text{ m}^3/\text{s}$ (100 LPM) to $0.01167 \text{ m}^3/\text{s}$ (700 LPM), 4) the fresh air is flow passing the setting pipe to provide the uniform stream line leading to record an accurate results, 5) The uniform flow is obtained by passing the straightened pipe and, in order to ensure the full developing flow in pipe, the pipe length of 900 mm ($10D$) [26] is aligned horizontally at front of a testing pipe, 6) The pressure drop (ΔP) across the testing pipe is measured by digital manometer.

2.2 Estimated Equation of Permeability (K)

In estimation of the permeability (K) from the data accumulated by the experimental procedure of Fig. 2, the Darcy-Forchheimer equation [9] is given by:

$$\Delta P = au + bu^2, \quad (4)$$

$$a = \frac{\mu L_0}{K}, \quad (5)$$

$$b = \frac{\rho F L_0}{\sqrt{K}}, \quad (6)$$

where μ , ρ , L_0 and F are air viscosity, air density, testing length and inertia coefficient (or Ergun coefficient) [14], respectively. Here, u is air velocity with the range of 0.262 m/s to 1.834 m/s which is calculated by $u = Q/A$ (A is cross section area of a testing pipe).

From Eq. (4), the experimental results are fitted by the second polynomial of regression method [27]. However, this is not the simplest method and it may has some difficult in prediction. Therefore, in order to propose a very simple calculation, Eq. (4) is transformed in a linear equation following as:

$$\frac{\Delta P}{u} = a + bu, \quad (7)$$

where variable a is the slope of the line and b is the y -axis ($\Delta P/u$) intercept. Such two variables are easiest to obtained but the K ($= \mu/a$) is the same value with Eq. (4) resulting to the present equation is very simple method.

3. RESULTS AND DISCUSSION

3.1 Effect of ε on K

Figure 3 (a) presents the relation between the ratio of pressure drop and air velocity ($\Delta P/u$) and air velocity (u) for the effect of porosity (ε or PPI) at the experimental condition of porous thickness (H) of 1.2 mm and inlet air temperature (T_{in}) of 27°C. From the experiment, it is found that the $\Delta P/u$ is increased as increasing u owing to the nature of fluid flow in pipe [28, 29]. As consideration at any u , the pressure drop ($\Delta P/u$) is reduced as increasing porosity (ε or PPI) due to the difficult flow is occurred at a lower porosity (ε) [29]. The solid lines based on regression analysis in this figure indicate the curves fitted through the experimental points (Symbols). All determination coefficients (R^2) between solid line and symbols are higher than 0.95 resulting to this regression is reliable [27]. The solid lines or linear equations are summarized in Table 2. By fitting a linear equation through these points, the coefficient a and b given in Eq. (7) can be determined. As replacing the values of a and b from this figure into Eqs. (5) and (6), one can obtain the corresponding values for permeability K and inertia coefficient F . Figure 3 (b) presents the relation between the permeability (K) and porosity (ε or PPI) at the experimental condition of $H = 1.2 \text{ mm}$ and $T_{in} = 27^\circ\text{C}$. The K is decreased as increasing porosity (ε) due to the effect of a raiser pressure drop (ΔP) similar with the reason as clarified in Fig. 3 (a).

3.2 Effect of H on K

Figure 4 (a) presents the relation between the $\Delta P/u$ and u at the experimental condition of porosity (ε) 0.943 ($PPI = 4$) and T_{in} of 27°C . The $\Delta P/u$ is increased with u which is the same trend in effect of ε (Fig. 3(a)). For consideration at any u , the trend of pressure drop ($\Delta P/u$) is raised as increasing H because it is the effect of a thicker of porous media leading to more difficult flow of air through the porous media is occurred. The curves fitted (The solid lines) using data from experiment (Symbols) give good agreement owing to the determination coefficients (R^2) between solid line and symbols are higher than 0.95 summarized in Table 3. Figure 4 (b) illustrates the relation between the K and H . The trend of K is decreased as increasing H which is the same reason to Fig. 4 (a).

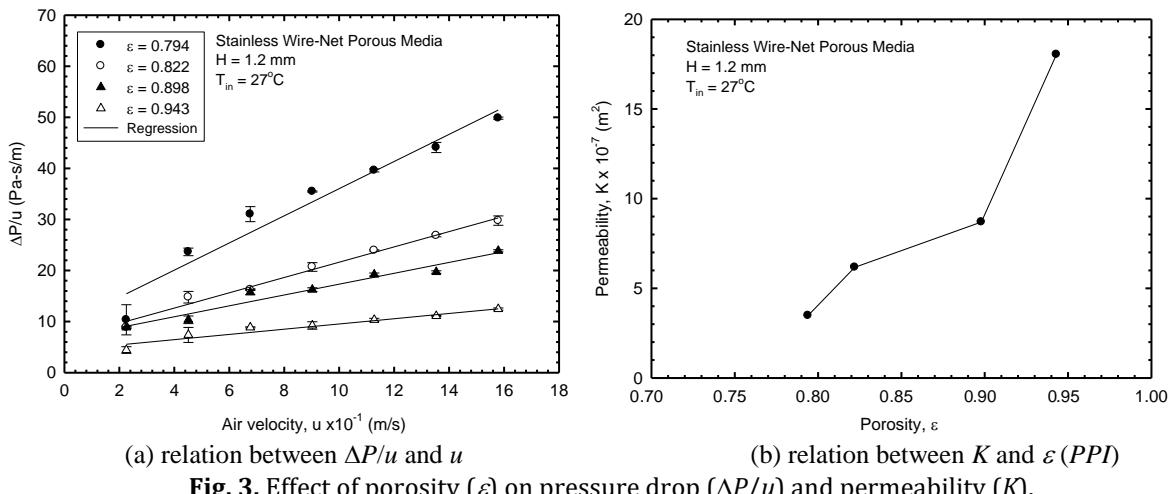


Fig. 3. Effect of porosity (ε) on pressure drop ($\Delta P/u$) and permeability (K).

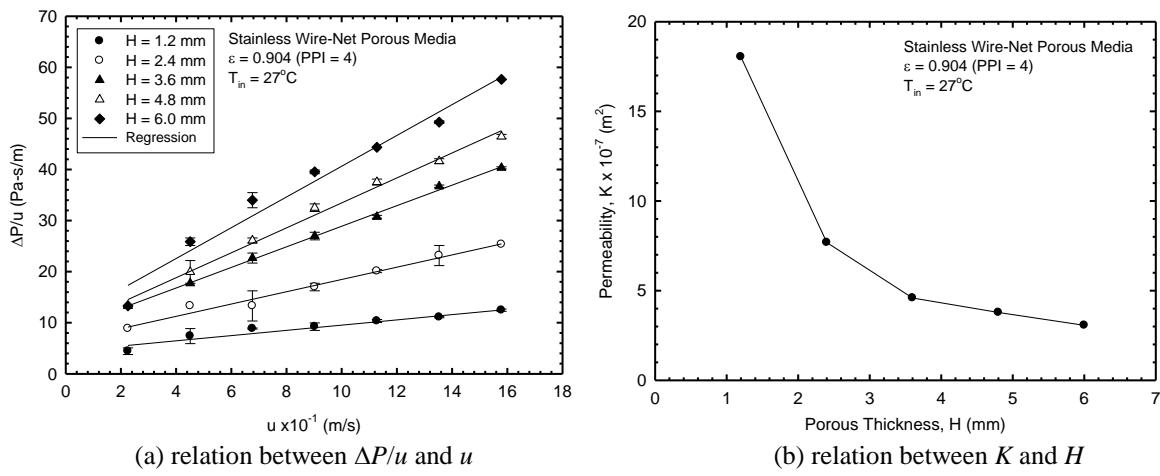


Fig. 4. Effect of porous thickness (H) on pressure drop ($\Delta P/u$) and permeability (K).

Table 2: Linear equation of function $\Delta P/u$ for effect of porosity (ε).

Porosity (ε)	Linear Equation	R^2
0.934 ($PPI = 12$)	$\Delta P/u = 4.411 + 5.215u$	0.953
0.898 ($PPI = 10$)	$\Delta P/u = 6.696 + 10.632u$	0.957
0.822 ($PPI = 8$)	$\Delta P/u = 6.661 + 14.971u$	0.986
0.794 ($PPI = 4$)	$\Delta P/u = 9.445 + 26.583u$	0.950

Table 3: Linear equation of function $\Delta P/u$ for effect of porous thickness (H).

Porous Thickness (H)	Linear Equation	R^2
1.2 mm	$\Delta P/u = 4.411 + 5.215u$	0.953
2.4 mm	$\Delta P/u = 6.448 + 12.022u$	0.980
3.6 mm	$\Delta P/u = 8.764 + 20.020u$	0.998
4.2 mm	$\Delta P/u = 9.038 + 24.421u$	0.992
6.0 mm	$\Delta P/u = 10.531 + 30.112u$	0.972

3.3 Regression analysis of K to ε (PPI) and H

Figure 5 proposes the comparison between the present regression curves as given in Eq. (8) having determination coefficient (R^2) of 0.889 and experimental data. From this comparison, it is noted that the present equation is good agreement with experimental data based on the range of $0.794 \leq \varepsilon \leq 0.934$ ($4 \leq PPI \leq 12$) and $1.2 \leq H \leq 6.0$ mm. However, the difference between our regression and experimental is a little in the case of 1.2 and 2.4 mm. It is due to the effect of thickness (H) become significant at a higher ($H > 3.6$) mm.

$$K = \left(\frac{89.401\varepsilon - 66.412}{H} \right) \times 10^{-7}. \quad (8)$$

3.4 Comparison results

Figure 6 shows the comparison results of our equation (Eq. (8)) to available empirical equation already discussed in section 1 (Introduction) consisting of Kozeny-Carman equation [12, 13], Gebart [15] and Koponen et al [17]. Good agreement is appeared particular in porosity (ε) not over 0.90. It is said that the accuracy of the proposed correlation is in the range of $0.80 \leq \varepsilon \leq 0.90$.

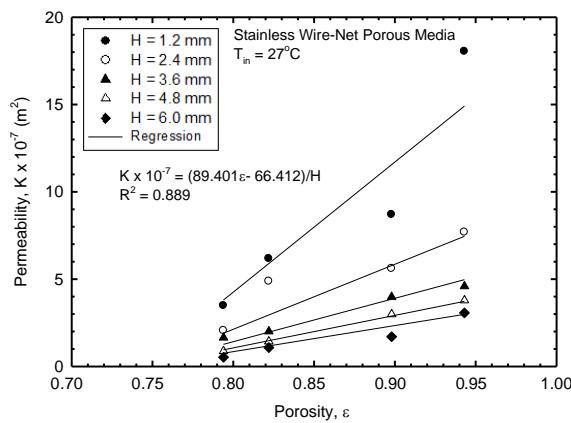


Fig. 5. Relation between K and ε (PPI) & H .

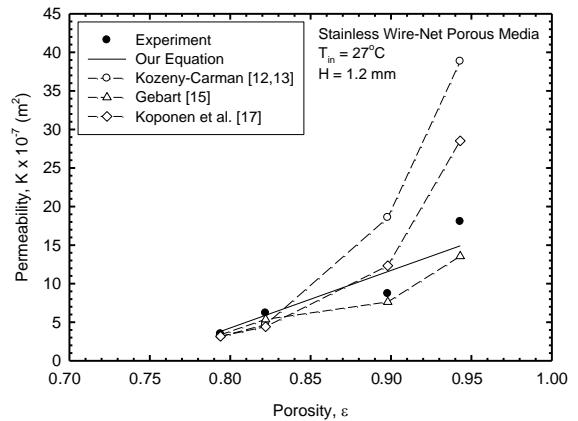


Fig. 6. Comparison of K between the present function with available works.

4. CONCLUSION

A simple experiment and analytical studies on investigation of the K of wire-net porous media based on Darcy – Forchheimer principle is proposed. The stainless ASUS304 with different porosity (ε or PPI) and thicknesses (H) are examined. The second polynomial form of Darcy – Forchheimer ($\Delta P = au+bu^2$) is rearranged as a linear form: $\Delta P = a+bu$. From investigation, equation of K estimated by multi-regression method can be reported by $K = \left(\frac{89.401\varepsilon - 66.412}{H} \right) \times 10^{-7}$ which the accurate of $4 \leq PPI \leq 12$ and $1.2 \leq H \leq 6.0$ mm. Good agreement between our equation and three available works [13, 15, 17] is obtained particular in the range of $0.80 \leq \varepsilon \leq 0.90$.

NOMENCLATURE

<i>a</i>	the slope of the line
<i>b</i>	the <i>y</i> -axis ($\Delta P/u$) intercept
<i>F</i>	inertia coefficient
<i>H</i>	porous thickness, m
<i>K</i>	permeability, m ²
<i>L₀</i>	testing length, m
<i>PPI</i>	pores per inch
<i>Q</i>	volumetric flow rate, m ³ /s
<i>r</i>	material diameter, m
<i>R²</i>	determination coefficient
<i>T</i>	temperature, °C
<i>u</i>	air velocity, m/s
ΔP	pressure drop across the testing pipe, N/m ²
ε	porosity
ρ	fluid density, kg/m ³
μ	fluid viscosity, Pa-s
<i>Subscripts</i>	
in	inlet

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