Analysis of Heat Transfer Characteristics of The Annular Fin Under Partially Wet Surface Conditions

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Abstract

In the present study, theoretical results of the heat transfer characteristics and the performance of the annular fin under partially wet surface conditions are presented. The mathematical models based on the conservation equations of mass and energy are developed. The central finite difference method is employed for solving this model to obtain temperature distribution of tube wall and fin. Effects of inside Biot number, outside Biot number, relative humidity of air, and hot fluid temperature on the heat transfer characteristics of annular fin are considered. The temperature distribution are presented in the dimensionless form, $(T - T_h)/(T_c - T_h)$. It is found that the inlet of both working fluids conditions have significant effect on the heat transfer characteristics of annular fin. In addition, the position of wet-dry interface is affected by the working fluids conditions and the fin dimensions.

Keywords: Heat transfer characteristics, annular fin, partially wet-surface

1. Introduction

Annular finned-tube heat exchangers are widely employed in many industrial applications e.g. heat recovery process, refrigeration, and air conditioning. Heat and mass transfer occur simultaneously when the coil surface temperature is below the dew point temperature of the air being cooled. The overall performance of cooling coils with dehumidification conditions have been continuously presented on heat transfer and flow characteristics. Coney et al. [1] predicted the fin effectiveness, fin temperature distribution, and condensate film thickness in a laminar humid air cross flow. Mirth and Ramadhyani [2] used the dry-surface heat transfer correlations to predict the chilled-water cooling-coil performance under condensing conditions. Kaxeminejad [3] analyzed a one-dimensional conduction heat transfer model to study the performance of a cooling and dehumidifying fin assembly. The effects of the relative humidity, dry bulb temperature, and cold fluid temperature on the temperature distribution and also on the augmentation factor were considered. Srinivasan and Shah [4] identified the principle fin configurations in two-phase flow applications for determining the fin efficiency. Salah EI- Din [5] studied the effects of the cold fluid temperature, the air temperature and relative humidity and the fin thermal conductivity on the fin assembly performance and on the temperature distribution. Rosario and Rahman [6] studied effects of relative humidity, dry bulb temperature of air, and cold fluid temperature inside the coil on the performance of the heat exchanger under dehumidification conditions. Kundu [7] analyzed the performance and optimization of a cooling and dehumidifying straight taper longitudinal fin. Experiments for temperature distribution and heat transfer of the fully wet fins have been determined.

As mentioned above, the theoretical study of the heat transfer characteristics and performance of the annular fin under partially wet-surface conditions is still limited. The aim of this paper is to study heat transfer characteristics of fin under partially wet surface conditions. Effects of relevant parameters on the temperature distribution are considered.

2. Mathematical Modelling

As shown in Fig. 1, the heat transfer characteristics of annular finned tube assembly under partially wet-surface can be determined by the conservation equations energy for each element. The mathematical model is based on that of Rosario et al. [6] and Kraus et al. [8] with the following main assumptions:

- Thermal resistance of the condensate is not included.
- The thermal conductivity of the fin and wall are constant.
- The thermal contact resistance between the tube wall and the fin is neglected.
- The convective heat and mass transfer coefficients are constant.
- All properties of the condensate are constant.
- Condensation occurs when the coil surface temperature is below the dew point temperature of air.
- Flow of hot fluid and cold fluid are steady.

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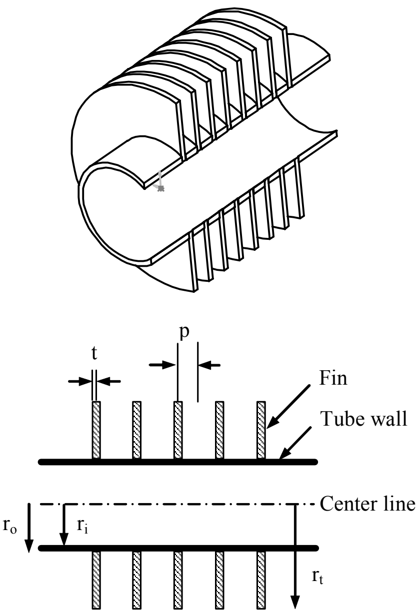

Figure 1 Schematic diagram of annular fin assembly

2.1 Tube wall region

$$\frac{d^2T_w}{dr_w^2}+\frac{1}{r_w}\frac{dT_w}{dr_w} = 0 \tag{1}$$

Boundary conditions
- Inner tube wall, (r_i)

$$-k_w\frac{dT_w}{dr_w} = h_i\left(T_c-T_w\right) \tag{2}$$

- Interface between the tube wall and fin, (r_o)

$$T_w = T_f \tag{3}$$

and

$$-k_w\cdot p\cdot\frac{dT_w}{dr_w} = (p-t)\left\langle h_o\left(T_w-T_h\right)+h_{fg}\cdot h_D\left(\omega_f-\omega_h\right)\right\rangle - k_f\cdot t\cdot\frac{dT_f}{dr_f} \tag{4}$$

2.2. Wet fin region, $\left(r_o \langle r_f \langle r_{wd}\right)$

$$\frac{d^2T_f}{dr_f^2}+\frac{1}{r_f}\frac{dT_f}{dr_f}-\frac{1}{k_f\cdot t}\left\langle h_o\left(T_f-T_h\right)+h_{fg}\cdot h_D\left(\omega_f-\omega_h\right)\right\rangle = 0 \tag{5}$$

2.3 Dry fin region, $\left(r_{wd} \langle r_f \langle r_t\right)$

$$\frac{d^2T_f}{dr_f^2}+\frac{1}{r_f}\frac{dT_f}{dr_f}-\frac{h_o}{k_f\cdot t}\left(T_f-T_h\right) = 0 \tag{6}$$

Boundary conditions
- Interface between wet fin region and dry fin region, $\left(r_f = r_{wd}\right)$

$$T_{f,wet} = T_{f,dry} \tag{7}$$

$$\frac{dT_{f,wet}}{dr_f} = \frac{dT_{f,dry}}{dr_f} \tag{8}$$

- Fin tip, (r_t)

$$-k_f\frac{dT_f}{dr_f} = h_o\left(T_f-T_h\right) \tag{9}$$

where ω_h is the humidity ratio of hot fluid, h_D is the mass transfer coefficient, h_{fg} is the latent heat of condensation. The saturated humidity ratio at the outer tube wall, ω_f, can be calculated from the correlation given by Laing [9]:

$$\omega_f = \left(3.7444+0.3078\,T_f+0.0046\,T_f^2+0.0004\,T_f^3\right)\times 10^{-3}$$
$$0 < T_f < 30^o C \tag{10}$$

where T_w is the tube wall temperature, T_c is the cold fluid temperature, T_f is the annular fin temperature, T_h is the hot fluid temperature, r_w is the radius within tube wall region, r_f is the radius within annular fin region, k_w is the thermal conductivity of tube wall, k_f is the thermal conductivity of annular fin, h_i is the inside convective heat transfer coefficient, h_o is the outside convective heat transfer coefficient, p is the half fin pitch, t is the half fin thickness.

These equations as mentioned above can be expressed in a dimensionless form as follows:

Wall region

$$\frac{d^2\theta_w}{dR_w^2}+\frac{1}{R_w}\frac{d\theta_w}{dR_w} = 0 \tag{11}$$

Boundary conditions
- Inner tube wall, $\left(R_w = r_i/p\right)$

$$\frac{d\theta_w}{dR_w} = -Bi_i\left(1-\theta_w\right) \tag{12}$$

- Interface between the tube wall and the fin, $\left(R_w = r_o/p\right)$

$$\theta_w = \theta_f \tag{13}$$

and

Wet fin region, $\left(R_o \langle R_f \langle R_{wd}\right)$

$$\frac{d\theta_w}{dR_w} = -Bi_o \cdot (1-P)\left\langle \theta_b + \frac{h_{fg} \cdot h_D}{h_o}\frac{(\omega_f - \omega_h)}{(T_c - T_h)}\right\rangle + K \cdot P\frac{d\theta_f}{dR_f} \quad (14)$$

Wet fin region, $(R_o \langle R_f \langle R_{wd})$

$$\frac{d^2\theta_f}{dR_f^2} + \frac{1}{R_f}\frac{d\theta_f}{dR_f} - \frac{Bi_a}{\theta(f) \cdot P^2}\theta_f = 0 \quad (15)$$

Where

$$\theta(f) = \left\langle 1 + \frac{h_{fg} \cdot h_D}{h_o \cdot \theta_f}\frac{(\omega_b - \omega_h)}{(T_c - T_h)}\right\rangle^{-1} \quad (16)$$

Dry fin region, $(R_{wd} \langle R_f \langle R_t)$

$$\frac{d^2\theta_f}{dR_f^2} + \frac{1}{R_f}\frac{d\theta_f}{dR_f} - \frac{Bi_a}{P^2}\theta_f = 0 \quad (17)$$

Boundary conditions

- Interface between the wet fin region and the dry fin region, $(R_f = R_{wd})$

$$\theta_{f,wet} = \theta_{f,dry} \quad (18)$$

$$\frac{d\theta_{f,wet}}{dR_f} = \frac{d\theta_{f,dry}}{dR_f} \quad (19)$$

- Fin tip, $(R_f = R_t)$

$$\frac{d\theta_f}{dR_f} = -\frac{Bi_a}{P}\theta_f \quad (20)$$

3. Solution Method

An annular fin as shown in Fig. 2 is divided into various segments along the radial direction. Eqs. (1)-(10) are written in the dimensionless form as in Eqs. (11)-(20). These equations are solved simultaneously by using the implicit central finite different method to determine the temperature distribution along the fin. The calculation begins at the innermost segment of the tube wall and then is done segment by segment along the tube wall and the fin length. In order to solve the model, the fin dimensions and properties of working fluids, as well as the operating conditions, are needed.

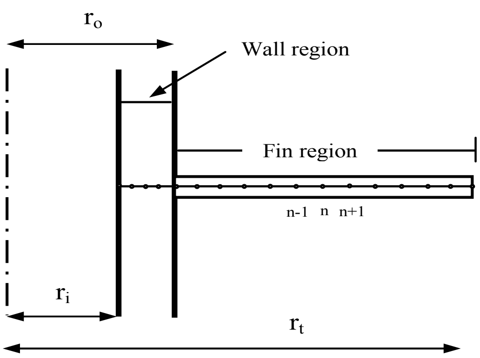

Figure 2 Schematic diagram of simulation approach

4. Results and Discussion

In the following sections, results of the effects of flow conditions on the temperature distribution of tube wall and fin are presented. In this study the simulation were carried out in the partially wet-surface conditions in which the condensation occurs on some region of fin. The numerical results are based on the same pipe diameter of 10 mm, tube thickness of 2.5 mm, and fin height of 7.5 mm. A large number of graphs can be drawn from the results of the study but because of the space limitations, only typical results are shown.

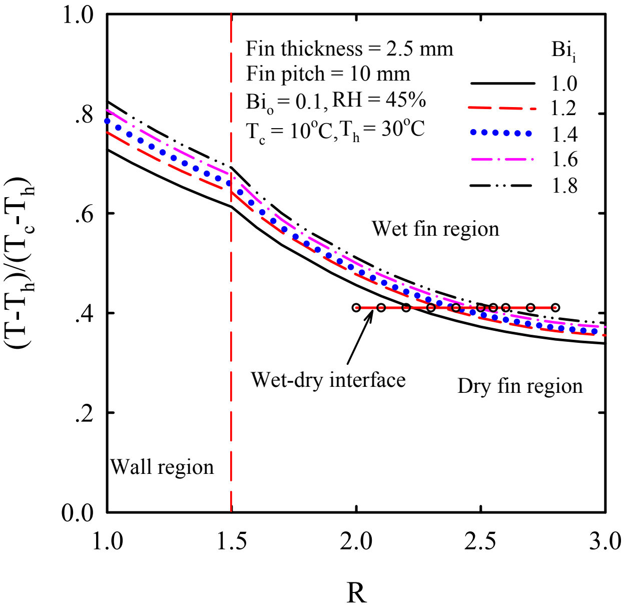

Figure 3 Effect of inside Biot number on the temperature distribution

Figure 3 shows the variation of dimensionless temperature with the dimensionless radius for different inside Biot numbers. In this case, the heat transferred from the outside fluid to the inside fluid. It can be clearly seen from figure that the dimensionless temperature decreases with increasing radius. Notice that the temperature distribution can be separated into three regions. Firstly, the temperature distribution occurs in the tube wall region. For the tube wall region, due to the pure conduction heat transfer, the temperature distribution is nearly linear. Secondly, the temperature distribution occurs in the wet fin region. In this region, the fin temperature is below dew-point temperature, moisture condenses on the fin surface. For the wet fin region, due to the condensation convective heat transfer, the slope of curves increases causing higher heat transfer rate. Thirdly, the temperature distribution occurs in the dry fin region. The fin temperature is higher than dew-point temperature. The slopes of curves tend to decreases causing lower heat transfer rate. The wet-dry interface is the point that the temperature of fin equal to the dew-point temperature of air. For a given inlet air relative humidity, cold and hot fluid temperatures, the dew-point temperature of air is kept constant. The dimensionless temperature increases with increasing inside Biot number. This is because the cooling capacity depends on the inside Biot number. Therefore, the dimensionless temperature also increases as the inside Biot number increases across the range of radius. In addition, the position of the wet-dry interface shifts to the fin tip as the inside Biot number increases. Effect of outside Biot number on the temperature distribution is shown in Figure 4. It can be seen from figure that the dimensionless temperature decreases with increasing outside Biot number. This is because the heating capacity depends directly on the outside Biot number. But the cooling capacity is kept constant. Therefore, increase outside Biot number results in lower dimensionless temperature.

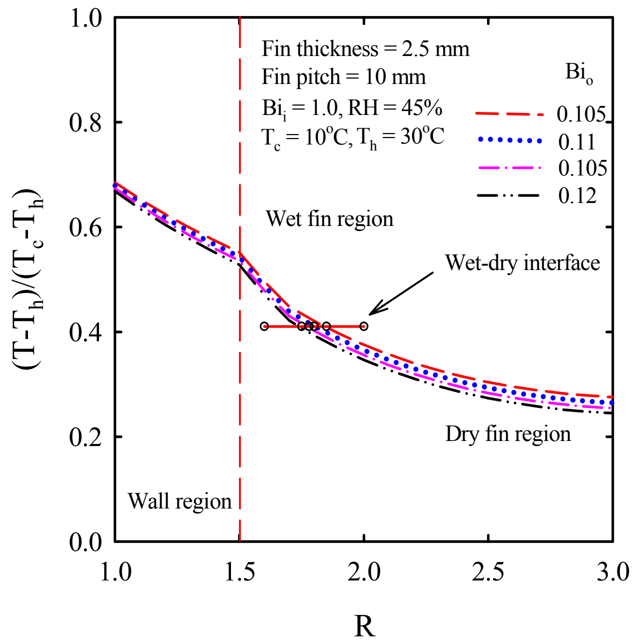

Figure 4 Effect of outside Biot number on the temperature distribution

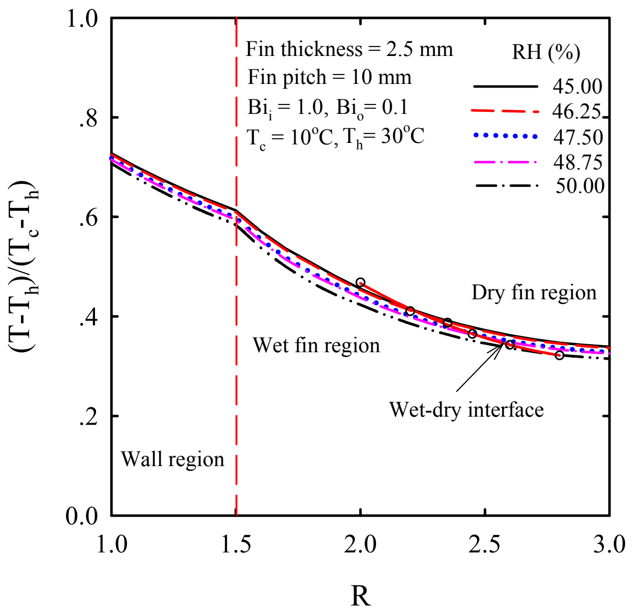

Figure 5 Effect of relative humidity on the temperature distribution

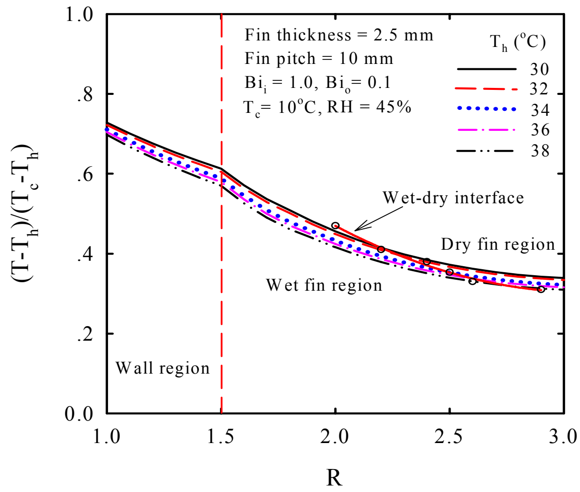

Figure 6 Effect of hot fluid temperature on the temperature distribution

Figure 5 shows the effect of inlet relative humidity of air on the temperature distribution. For a given inlet air temperature, the dew-point temperature of air constantly increases as relative humidity constantly increases across the range of radius. The width of wet-surface region increases causing higher heat transfer rate from the hot fluid to the cold fluid. Therefore, the dimensionless temperature at higher relative humidity is lower than that of lower relative humidity. Effect of hot fluid temperature on the temperature distribution is shown in Figure 6. As expected, the temperature is directly proportional to the hot fluid temperature. For a given relative humidity, the dew-point temperature of air increases as the hot fluid temperature increases. These mean that the wet fin region increases as the hot fluid temperature increases causing higher heat transfer rate.

5. Conclusion

The heat transfer characteristics of the rectangular annular finned-tube were investigated. It has been found that the inlet of both working fluids conditions have significant effect on the heat transfer characteristics as follows:

- Inside and outside Biot numbers have significant effect on temperature distribution.
- The dimensionless temperature decreases with increasing relative humidity.
- The hot water temperature has significant effect on the temperature distribution.
- In addition, the both working fluids conditions and the fin dimensions have significant effect on the position of wet-dry interface.

Nomenclature

Bi_a	Biot number, $(h_o \cdot t / k_f)$
Bi_i	inside Biot number, $(h_i \cdot p / k_w)$
Bi_o	outside Biot number, $(h_o \cdot p / k_w)$
h_D	mass transfer coefficient, $(kg / m^2 s)$
h_{fg}	latent heat of condensation, (J / kg)
k	thermal conductivity, $(W / m\,^oC)$
K	thermal conductivity ratio, (k_f / k_w)
Le	Lewis number
p	half fin pitch, m
P	aspect ratio, (t / p)
r_i	internal radius of tube, m
r_o	external radius of tube, m
r_t	external radius of fin, m
r_f	radius in fin region, m
r_w	redius in tube wall region, m
R_f	dimensionless radius within fin, r_f / p
R_w	dimensionless radius within wall, r_w / p
t	half fin thickness, m
T_c	cold fluid temperature, ^{o}C
T_f	fin temperature, ^oC
T_h	hot fluid temperature, ^oC
T_w	wall temperature, ^oC
ω	humidity ratio
η	fin efficiency
θ	dimensionless temperature, $(T - T_h)/(T_c - T_h)$
RH	relative humidity

Subscript

c	cold fluid
h	hot fluid
b	fin base
f	fin
i	inside
o	outside
t	fin tip
w	wall
dry	dry region
wet	wet region
wd	wet-dry interface

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