Equalization of Chrominance Gain Distortion Using Bernstein Polynomials

Kanok Janchitrapongvej[1] Chaipichit Cumpim[1] Pongpan Rattanathanawan[1]
and Virote Pirajnanchai[2]

Abstract

This paper describes the design of linear chrominance gain slope equalizer for correcting the distortions in the video signal as they pass through the system under test. By the used of Fourier series, it is shown that the linear gain slop equalizer can be used to correct the linear chrominance gain inequality of the color subcarrier frequency at 4.43 MHz. As the results, it is seen that the proposed gain slope equalizer with linear phase characteristic whose amplitude characteristic has a linear rise or a steep drop in the color video frequency region. The approximated transfer function of linear gain slope is based on Bernstein polynomials. In addition, the composite modulated 20T sine squared pulse was chosen to investigate the performance of the proposed gain slope equalizer. Moreover, Mikhailov's criteria for stability test is considered.

Keywords: Bernstein Polynomials, Linear Chrominance Gain, Modulated 20T Sine-Squared Pulse, Mikhailov's criteria.

[1] Department of Industrial Management Technology, Faculty of Science and Technology, Southeast Bangkok College. Bangkok

[2] Department of Electronics and Telecommunication, Faculty of Engineering, Rajamangala University of Technology Thanyaburi. Pathumthani

1. Introduction

The linear distortion in a color video waveform transmission is best investigated by considering the distortion of the composite modulated 20T sine-squared pulse [1,2]. As it is known, the performance of transmission systems of a color television picture quality is usually evaluated by means of amplitude and phase or group delay characteristics. In case of distorted transmission through the system under test, observation using oscilloscope can be used to measure the lower baseline of modulated 20T sine squared pulse are sufficient to determine the two distortions both gain and delay distortions.

In this paper, we focus on the equalization of linear chrominance gain distortion only. In the previous literature [3], a uniform distributed RC line (URC) was applied to analog amplitude equalizer. The used of URC has low sensitivity characteristic and can be fabricated in large scale integrated circuit. Unfortunately, the drawback of URC is its nonlinear phase characteristic which will cause the deteriorate of video signal. Hence, the insertion of group delay equalizer is needed.

In this paper, the design of linear chrominance gain slope equalizer based on Bernstein polynomial [4] is introduced. The proposed method can adjust both low gain and high inequalities at color sub-carrier 4.43 MHz without degrading the phase or group delay characteristic. Moreover, the paper introduced Mikhailov's criteria [5] for stability consideration.

The paper is organized as follows. In section 2, we describe the frequency response of chrominance gain slope and the calculated modulated sine squared response using Fourier series is presented in section 3. Section 4 describes the approximation of gain slope equalizer based on Bernstein polynomials. Section 5 describes the stability test and finally, the conclusion is given in section 6.

2. Frequency Response of Chrominance Gain Slope

In this section, the linear chrominance gain slope can be used to enhanced or compressed at color subcarrier 4.43 MHz in PAL system.

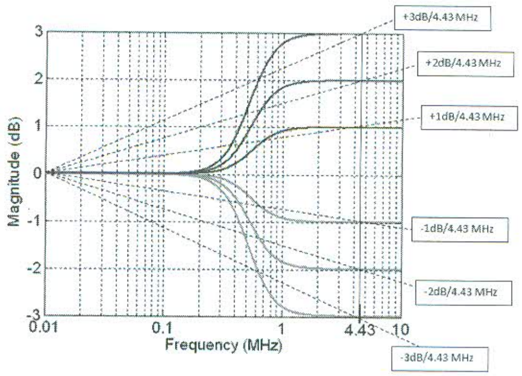

Fig.1. Amplitude distortion of transmission system.

Fig 1. shows the amplitude distortions occurred in the transmission system. Herein, the solid line and dash line shows the ideal and approximated linear gain slope of the high gain and low gain distortions of the chrominance signal respectively.

The amplitude gain slope equalizer is best suited in the vicinity within 3dB at 4.43 MHz.

3. Calculated modulated sine squared response of gain slope equalizer

The standard of composite modulated 20T sine-squared test pulse is given in Eq. (1)-(3) and it is illustrated in Fig. 2.

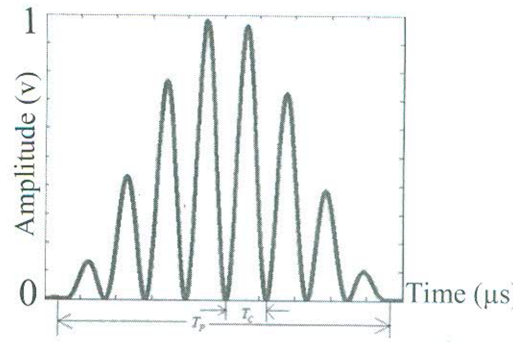

Fig.

Fig. 2. Detail of 20T sine squared test signal

$$f_i(t) = 0.5\left[g(t) + g(t)\sin(\omega_c t)\right] \quad (1)$$

Where $$g(t) = \sin^2\left(\frac{\pi t}{T_p}\right) \qquad |t| \le \frac{T_p}{2} \quad (2)$$

Substitute Eq. (2) into Eq. (1) yields,

$$0.25\left[\{1-\cos(at)\} + \{1-\cos(at)\}\sin(\omega_c t)\right] \quad (3)$$

where

$$a = \frac{2\pi}{T_p}, \quad c = \frac{2\pi}{T_c}$$

T_c = 0.2257 μs, T_p = 4 μs

The output response of gain slope equalizer $f_o(t)$ is given as follows.

$$f_o(t) = \frac{a_o F_o}{2} + \sum_{k=1}^{N_{max}} F_k\left[a_k\cos(k\omega t) + b_k\sin(k\omega t)\right] \quad (4)$$

$$a_k = \frac{2}{T_p}\int_o^{T_p} f_i(t)\cos(k\omega_p t)\,dt$$

$$b_k = \frac{2}{T_p}\int_o^{T_p} f_i(t)\sin(k\omega_p t)\,dt \quad (5)$$

where $N_{max} = \frac{4}{0.2257} \simeq 18$

F_k : discrete gain slope

where $F_k = 10^{2.257\times10^{-5}\alpha k}$

And α is the desired gain at 4.43 MHz. The Fourier coefficient is obtained as follows;

$$a_k = \frac{1}{2T_p}\left[\begin{array}{l} \frac{\sin(akT_p)}{ak} - \frac{[\cos\{(c+ak)T_p\}]-1}{2(c+ak)} \\ + \frac{[\cos\{(c+ak)T_p\}]-1}{2(c+ak)} - \frac{\sin(1+k)aT_p}{2(1+k)a} \\ - \frac{\sin(k-1)aT_p}{2(k-1)a} + \frac{[\cos\{(1+k)a+c\}T_p]-1}{4(1+k)a+c} \\ - \frac{[\cos\{(1+k)a-c\}T_p]-1}{4(1+k)a-c} \\ + \frac{[\cos\{(k-1)a+c\}T_p]-1}{4(k-1)a+c} \\ - \frac{[\cos\{(1-k)a+c\}T_p]-1}{4(1-k)a+c} \end{array}\right]$$

where $\frac{a_0}{2} = 0.25$

$$b_k = \frac{1}{2T_p}\left[\begin{array}{l} \frac{1-\cos(akT_p)}{ak} + \frac{[\cos\{(1+ak)T_p\}]-1}{2(c+ak)} \\ + \frac{[\cos\{(k+1)aT_p\}]-1}{2(k-1)a} - \frac{\sin(c+ka)T_p}{2(c+ak)} \\ + \frac{\sin\{(k+1)a-c\}T_p}{4\{(k+1)a-c\}} - \frac{[\sin\{(1-K)a+c\}T_p]}{4(1+k)a+c} \\ - \frac{[\sin\{(k+1)a+c\}T_p]-1}{4(1+k)a-c} \\ + \frac{[\cos\{(k-1)a+c\}T_p]-1}{4(k-1)a+c} \\ - \frac{[\cos\{(1-K)a+c\}T_p]}{4(k-1)a+c} \end{array}\right] \quad (6)$$

Substituting Eq. (6) into Eq. (4), thus the output response is plotted in Fig. 3.

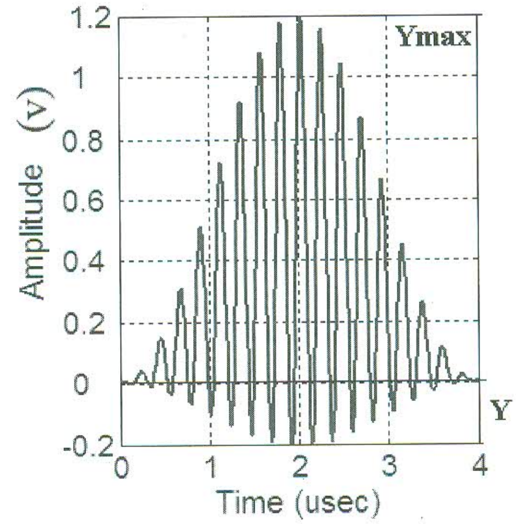

Fig. 3. The calculated output response with

α = +1dB

4. Chrominance Gain Slope Equalized

The Bernstein polynomial of nth order is given by [4].

$$(f;x) = \sum_{i=0}^{n} f\left(\frac{i}{n}\right)\binom{n}{i} x^i (1-x)^{n-i} \quad (7)$$

where f (x) is defined in the interval (0, 1). For where $\binom{n}{i} = \frac{n!}{i!(n-i)!}$

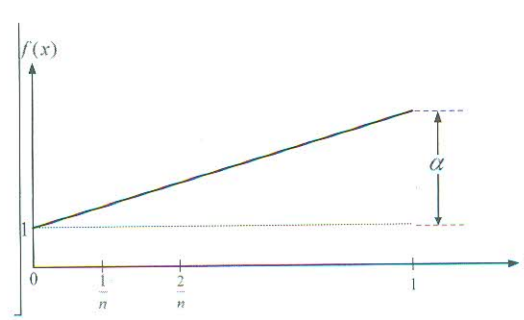

Fig. 4 Gain slop function

Considering the approximation of desired functions as shown in Fig. 4. The gain slope function can be approximated by using the Bernstein polynomials as follows

$$T_n(f;x) = \frac{1}{B_n(f;x)} = 1 + \alpha x \qquad (8)$$

From Eq. (7) by means of Bernstein polynomials. We get

$$B_n(f;x) = \sum_{i=0}^{n} f\left(1+\alpha\frac{i}{n}\right)^{-1}\binom{n}{i}x^i(1-x)^{n-i} \qquad (9)$$

From Eq. (9) using interval conversion [0, 1] transforms [0 ∞]to given as follows.

$$x = \frac{\Omega^2}{1+\Omega^2} \qquad (10)$$

Hence, Eq. (9) can be rewritten as

$$B_n(f;x) = \sum_{i=0}^{n} f\left(1+\alpha\frac{i}{n}\right)^{-1}\binom{n}{i}\frac{\Omega^{2i}}{\left(1+\Omega^2\right)^n} \qquad (11)$$

Herein, we choose the desired chrominance gain slope at +1dB where and the transfer function is 4th order. After some manipulation, the transfer function is obtained as follows

$$T(\Omega^2) = \frac{\Omega^8 + 4\Omega^6 + 6\Omega^4 + 4\Omega^2 + 1}{0.9\Omega^8 + 4.3\Omega^6 + 6.36\Omega^4 + 4.1\Omega^2 + 1} \qquad (12)$$

The magnitude squared gain slope function from Eq. (12) in terms of s-domain is given by

$$T(-s^2) = \frac{1.125s^8 - 4.44s^6 + 6.66s^4 - 4.44s^2 + 1.112}{s^8 - 4.13s^6 + 7.06s^4 - 4.33s^2 + 1.112} \qquad (13)$$

For the stability of the equalizer, we selected the poles on the left half side of s-plane. The gain slope equalizer transfer function is given by

$$T(s) = \frac{1.125\left(s^4 + 4s^3 + 6s^2 + 4s + 1\right)}{\left(s^2 + 0.832s + 1.966\right)\left(s^2 + 0.4402s + 0.5376\right)} \qquad (14)$$

From Eq. (14), the magnitude and phase characteristics of high gain of the gain slope response of +1dB are plotted in Fig. 4 and Fig. 5 respectively.

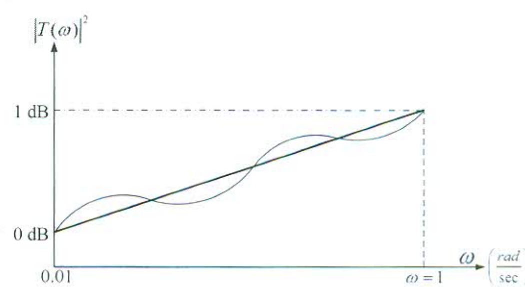

Fig. 5. Magnitude characteristics of the gain slope

In short, the proposed chrominance gain slope equalizer is efficient to correct the chrominance gain distortion without degrade its phase characteristics. It is seen that its phase difference has small partial phase variation in the specific frequency range.

Fig. 6. Phase characteristics of the gain slope equalizer.

5. Stability consideration

It is easily seen that Bernstein polynomial given Eq. (15):

$$B_{k,n}(x)=\binom{n}{k}x^k(1-x)^{n-k} \qquad (15)$$

This function is non-negative over and interval $x\in(0,1)$. A recursive form of Bernstein polynomials can be written as

$$B_{k,n}(x)=\{(1-x)B_{k,n-1}(x)\}+\{(x)B_{k-1,n-1}(x)\} \qquad (16)$$

By induction, all Bernstein polynomials are non-negative for $0\le x\le 1$. Hence, the poles of the proposed transfer function are lied on the left half of s-plane. The system is stable.

Other test conditions for checking the stability is the Mikhailov's criterion. Mikhailov stability criterion stable that an n[th] order polynomials be defined as

$$D(s)=a_0+a_1s+a_2s^2+\ldots+a_ks^k+a_ns^n \qquad (17)$$

where a_k are all positive real number. Substituting s = jω and separating into real and imaginary parts. Thus, Eq. (17) can be rewritten as

$$D(j\omega)=\left(a_0+a_2\omega^2+a_4\omega^4+\ldots\right)+j\left(a_1\omega-a_3\omega^3+\ldots\right)$$
$$D(j\omega)=u(\omega)+jv(\omega) \qquad (18)$$

The curve obtained as ω varies from 0 to ∞ with u (ω) as abscissa and are ordinate is called Mikhailov frequency characteristic of the polynomials D(jω). If D(jω) is stable, according to Mikhailov frequency characteristic starts from the real at ω = 0 and intersects the imaginary and real axis alternatively in the anticlockwise direction as ω is increased. Thus, the contour is encircle around the origin of D(jω). For example, the design gain slope equalizer is +1dB the denominator of the 4[th] order transfer function is given as

$$D(s)=s^4+1.2722s^3+2.8698s^2+1.3126s+1.0569 \qquad (19)$$

and $$D(j\omega)=u(\omega)+jv(\omega) \qquad (20)$$

where $u(\omega)=1.0569-2.8698\omega^2+\omega^4$
$v(\omega)=1.3126\omega-1.2722\omega^3$

From Eq. (19) Mikhailov's hodograph is plotted in Fig. 7. Thus the proposed equalizer is stable.

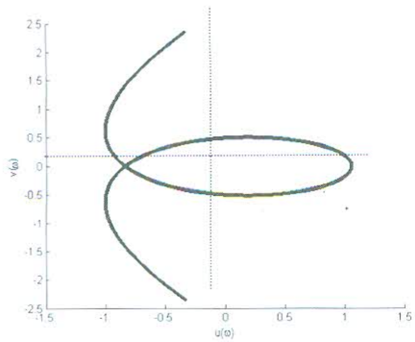

Fig. 7. Stability test of the given chrominance gain slope equalizer.

6. Conclusions

In section 3 shown that the linear chrominance gain can be applied to equalize the low gain and high gain at color sub-carrier 4.43 MHz in the vicinity with +1dB. Herein, the composite modulated 20T sine squared test signal in use for testing the performance by means of Fourier series technique.

The proposed gain slope chrominance equalizer is applied to enhance and compress of linear chrominance gain distortion by using Bernstein polynomials without degrading the delay characteristic. A realization of the proposed equalizer is stable, according to Mikhailov's criterion. The Mikhailov's criterion detail is given in section 5.

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