A Decision Making Approach for Multi-Objective Optimization Considering A Trade-Off Method

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ABSTRACT

In multi-objective optimization problem, a set of optimal solutions is obtained from an optimization algorithm. There are many trade-off optimal solutions. However, in practice, a decision maker or user only needs one or very few solutions for implementation. Moreover, these solutions are difficult to determine from a set of optimal solutions of complex system. Therefore, a trade-off method for multi-objective optimization is proposed for identifying the preferred solutions according to the decision maker's preference. The preference is expressed by using the trade-off between any two objectives where the decision maker is willing to worsen in one objective value in order to gain improvement in the other objective value. The trade-off method is demonstrated by using well-known two-objective and three-objective benchmark problems. Furthermore, a system design problem with component allocation is also considered to illustrate the applicability of the proposed method. The results show that the trade-off method can be applied for solving practical problems to identify the final solution(s) and easy to use even when the decision maker lacks some knowledge or not an expert in the problem solving. The decision maker only gives his/her preference information. Then, the corresponding optimal solutions will be obtained, accordingly.

Keywords: Multi-Objective Optimization, Decision Making Approach, Preference-Based Method, Trade-Off Method

1. INTRODUCTION

Most real-life optimization problems have several factors or objectives to be optimized simultaneously. These problems can be expressed as multi-objective optimization problems. In the problems, the objective functions may be conflicting with each other. For example, the objectives are to maximize system performance but minimize system cost simultaneously. Due to the conflicting objective functions, the optimal solutions have trade-off among the objectives which means a losing value in one objective while gaining value in other objective in return. Therefore, it is difficult for a Decision Maker (DM) or user to select one final solution. In other words, the problem solving becomes challenging because it contains many trade-off optimal solutions but only one solution will be selected for implementation. Hence, the user needs a solving method for helping him/her makes a decision and gets the most desirable or outstanding solution(s).

For solving multi-objective optimization problems, most multi-objective optimization researches concentrate on searching for the optimal solutions set. However, the DM still needs to choose the final solution for implementation as has been described. Therefore, we are interested in identifying methods for the most preferred solution. The preference-based method becomes more important. Many researches [1-5] showed that the preference-based optimization methods are effective to guide the optimization algorithm toward the preferred regions and identify the most preferred solution. There are many ways to express the DM's preferences including the reference point methods [6-8], a light beam search method [9-10] and a dynamic polar-based region [11]. However, we focus on compromising trade-off between any two objectives. The objective trade-offs concept is presented in [12]. The trade-off concept represents the effect of changing between two alternative solutions. J. Branke et al. [13] presented a method that modified the definition of dominance according to linear maximum and minimum trade-off functions for each pair of objectives. Tilahun and Ong [14] proposed the fuzzy preferences incorporated with genetic algorithm for multiple DMs. This method collected preferences as fuzzy conditional trade-offs. However, it is not easy for the DM to specify trade-off for every alternative. Therefore, we developed the trade-off preference-based method in order to help identifying preferred solution(s) even when the DM is not an expert in the problem solving.

In this research, the trade-off method according to the extreme solution is proposed. It is a preference-based method to identify the most preferred/outstanding solution(s) among multiple opti-

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mal solutions. The DM's demand can easily be expressed and maintained.

This paper is organized as follows. The next section describes multi-objective optimization problem. Then, the preference-based methods are presented in Section 3. Section 4 proposes a trade-off method according to the extreme solution. The benchmark problems are presented in Section 5., followed by the application problem, component allocation problem, in Section 6. The simulation results and discussion are presented in Section 7. Lastly, Section 8 gives conclusion.

2. MULTI-OBJECTIVE OPTIMIZATION

Multi-objective optimization is determining the decision variable(s) that optimize multiple objectives simultaneously under the constraints. For example, the objectives of system design are maximizing system performance while minimizing system cost and weight subject to system constraints. Decision variable(s) is represented by a n dimensional vector, $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T$ where n is the number of decision variable(s). The objective functions are $f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_M(\mathbf{x})$ where M is the number of objective functions. The objective function can be minimized or maximized. The mathematical formulation of the MOP in general format (1) is following [15]:
Maximize/Minimize $f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_M(\mathbf{x})$

$$\text{Subject to} \quad \begin{aligned} & g_j(\mathbf{x}) = 0, j = 1, 2, \ldots, J; \\ & h_k(\mathbf{x}) = 0, k = 1, 2, \ldots, K; \\ & x_i^{(L)} \leq x_i \leq x_i^{(U)}, \\ & i = 1, 2, \ldots, n. \end{aligned} \tag{1}$$

where the $g_j(\mathbf{x})$ and $h_k(\mathbf{x})$ functions are the constraint functions which have J inequality constraint functions and K equality constraint functions. The variable boundary is the last set of constraints ($x_i^{(L)} \leq x_i \leq x_i^{(U)}$), where the decision variable, x_i has a lower bound, $x_i^{(L)}$ and an upper bound, $x_i^{(U)}$ The objective function may be in conflict with each other and causes multiple trade-off solutions.

Multi-objective optimization problem can be solved efficiently by using well-known multi-objective evolutionary algorithms such as SPEA2 [16], Non-dominated Sorting Genetic Algorithm (NSGA-II) [17] and MOEA/D [18]. These algorithms can be used in order to obtain an approximation of Pareto-optimal solutions. Then, the preferred solution is identified/selected by using the proposed method, as the final solution for implementation. The well-known optimization method, NSGA-II [17] is used in this research according to its effectiveness.

3. PREFERENCE-BASED METHOD

The preference-based methods utilize the DM's preferences to select the most preferred solution(s) from the optimal solutions as shown in Fig 1. There are several methods that handle the DM's preferences for identifying the preferred solutions. In this research, the preference-based methods are grouped into three groups.

First, the goal or reference value model simply uses the DM's goals or reference value for identifying the preferred solutions. The DM sets his/her desirable values of achievement. Then, the solutions closest to the preferred value are the preferred solutions. A number of models have been proposed such as the reference point method, the light beam search method, and goal programming method. Reference point method [6-8] extracts the DM's preference as a reference point of the aspiration values for each objective function. Reference-Point-Based NSGA-II (R-NSGA-II) [6] used the reference points to guide NSGA-II optimization algorithm. The objectives are equally emphasizes and evaluated by using the closest Euclidean distance to any reference points. Reference Direction Based NSGA-II (RD-NSGA-II) [19] expresses the DM's preference as a reference point. A reference direction is defined by the difference of a reference point and a starting objective vector. Light Beam Search based EMO [9] is the DM provides a reference direction and a threshold vector, which is used to find possibly interesting neighbouring objective vectors around the point defined by the reference direction. Preference Based Evolutionary Algorithm (PBEA) [20] is proposed using an achievement scalarizing function and the reference point(s).

Second, the ranking model ranks the alternative solutions over another or the priority of the objectives according to the DM's preferences. Then the best ranked alternatives are the preferred solutions.

Third, the value measurement and trade-offs models evaluate the solutions and give the scores for each alternative solution. The guided multi-objective evolutionary algorithm (G-MOEA) [13] expresses the DM's preferences in terms of acceptable trade-offs.

In this paper, we propose a preference-based method called the trade-off method according to extreme solution. The DM's preferences are expressed by the two trade-off objectives. The DM no needs to analyse his/her preferences mathematically. The main purpose of this method is to identify a practical solution among multiple optimal solutions.

4. TRADE-OFF METHOD ACCORDING TO THE EXTREME SOLUTION

In order to help the DM identify the preferred solution among multiple optimal solutions, the trade-off method according to extreme solution is proposed. This method is modified and extended from [2, 21] as a preference-based method. The DM's preference is expressed by using the trade-off between any two objectives. The DM is willing to loss in one objective value in order to gain improvement in other objec-

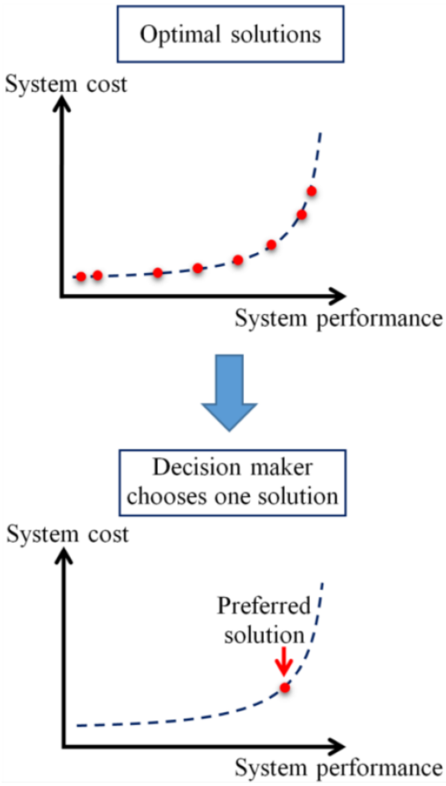

Fig.1: *The concept of a preference-based method and a preferred solution.*

tive value. For an example, sacrificing or paying 1 unit of cost so that some units of product can be gained. An optimization method finds the approximation of Pareto front before the proposed method identifies the preferred solutions. In this research, we use NSGA-II [17] which is a well-known multi-objective evolutionary optimization method. After obtaining an approximation of the optimal solutions from the optimization method, the proposed method identifies the preferred solution set and presents it to the DM. The flowchart of trade-off method is shown in Fig 2. The process of the trade-off method by using extreme solution as reference solution is as follows.

Step 1: The DM specifies preference information.
The DM specifies gaining objective value (hopefully to get the better value) and sacrificing the other objective (willing to pay more or get worst). In this research, the sacrifice in system cost is related to the gain in system reliability which means the user is willing to accept higher system cost (sacrificing objective) in order to increase system reliability (gaining objective).

Step 2: Normalize the objective values
All objective functions are scaled to be in the range from 0.0 to 1.0. The normalized objective value is given by:

$$Normalized(f_i(\mathbf{x})) = \frac{f_i(\mathbf{x}) - f_i^{min}}{f_i^{max} - f_i^{min}} \qquad (2)$$

$$i = 1, 2, \ldots, b.$$

where $\mathbf{x}$ is a set of the decision variables. The number of objective functions is b. In the approximation of optimal solutions, f_i^{max} and f_i^{min} are the maximum and the minimum values for the i^{th} objective function, $f_i(\mathbf{x})$, respectively.

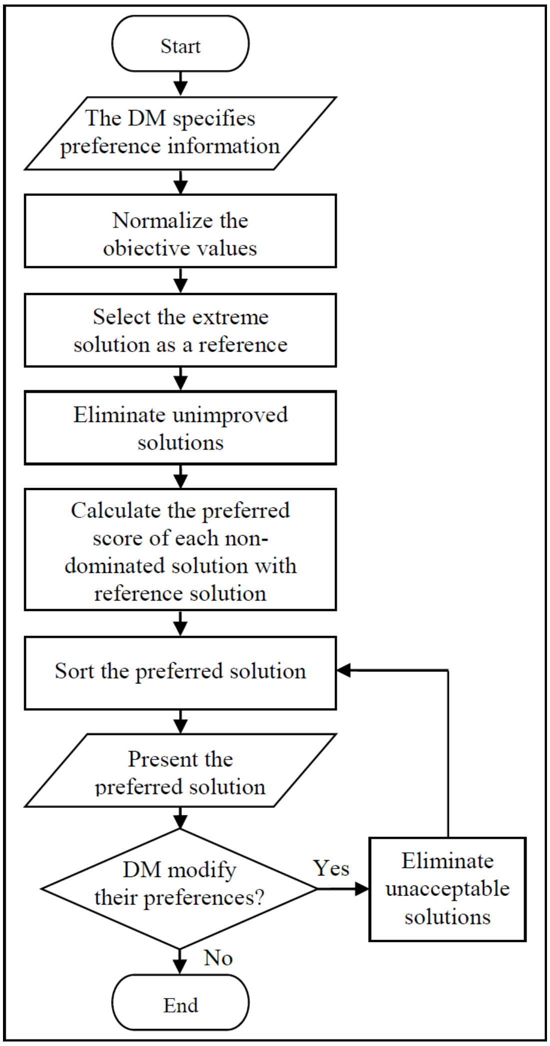

Fig.2: *Flowchart of Trade-off method according to the extreme solution.*

Step 3: Select the extreme solution as a reference.
The solution that has extremely good value in sacrificing objective (system cost) is selected as a reference solution. If there is more than one solution, choose the solution among them that has the best value in gaining objective (system reliability).

Step 4: Eliminate unimproved solutions.
Eliminate solutions that have gaining objective value not as good as that of the reference solution.

Step 5: Calculate the preferred score
The preferred score is the trade-off value of the solution k and reference solution which is given by:

$$T_{ij}^{k} = \frac{|f_j^{\text{reference solution}}(\mathbf{x}) - f_j^k(\mathbf{x})|}{|f_i^{\text{reference solution}}(\mathbf{x}) - f_i^k(\mathbf{x})|} \quad (3)$$

where the decision variable vector is $\mathbf{x}$. The T_{ij}^k represents the amount of gaining in the objective j, f_j to sacrifice one unit in the objective i, f_i of solution k. There is no need to calculate score for the reference and eliminated solutions.

Step 6: Sort the preferred solution
Solution with the highest score is the most preferred solution. The alternative that has the highest preferred score means that it has the maximum amount to gain in one objective when compare to the extreme reference solution.

Step 7: Present the preferred solution
Present appropriate solutions to the DM. The process is completed if the DM satisfies them.

Step 8: If the DM does not satisfy with the preferred solution, he/she specifies an acceptable value.

Step 9: Eliminate unacceptable solutions
Eliminate solutions that gaining objective value is not as good as the acceptable value. Then repeat steps 6 to 9. Among the remaining solutions, solution that has the highest preferred score is the most preferred solution.

The main contribute in this research is to help the DM express the trade-off preference easily and more flexibility in order to find the final solution.

5. BENCHMARK PROBLEMS

The preference-based methods for pruning mechanism have different ways to express the DM's preference. Thus, it is difficult to compare the results directly among the different preference-based methods. The preference-based methods focus on the most desirable solutions. However, many multi-objective benchmark problems have been proposed to evaluate the algorithms. The benchmark problem is used to evaluate the ability and tests the effectiveness of a preference-based method with known non-dominated solutions.

The common benchmark problems are including Zitzler-Deb-Thiele test problems (ZDT) [22] test suite and Walking-Fish-Group (WFG) test problems [23]. ZDT test suite is the most popular used in the multi-objective optimization. ZDT problems consist of six bi-objective problems. The characteristics of test problems include concave, convex, continuous and discontinuous problems. WFG is scalable to any number of parameters and objectives and have more comprehensive challenges among other test suite [23]. The mathematical models are presented as the following.

5.1 Two-Objective Benchmark Problems

The ZDT test problems [22] consist of six problems including ZDT1 to ZDT6. ZDT test problem suite has two objectives to be minimized. This research considers all ZDT problems except ZDT4 because the ZDT4 has similar PF as ZDT1. The ZDT problem suite provides sufficient complexity to compare the preferred solution on various shape of Pareto front where the concave, convex, continuous and discontinuous shape of Pareto front are the problem features that causes difficulty for identifying the preferred solutions.

5.2 Three-Objective Benchmark Problems

For three-objective problem, we consider WFG2 3D from WFG test problems [23]. The characteristics of WFG2 3D test problem include convex, discontinuous and multimodal as shown in Fig 6 (a). The mathematical model is presented as the following [23, 24].

Minimize:

$$f_{m=1:m-1}(\vec{x}) = x_M + S_M convex_m(x_1, \ldots, x_{M-1})$$

$$f_m(\vec{x}) = x_M + S_M disc_m(x_1, \ldots, x_{M-1})$$

where

$y_{i=1:M-1} = \text{r\_sum}(\left[y'\frac{(i-1)k}{(M-1)}+1, \ldots, y'\frac{ik}{(M-1)}\right], [1, \ldots, 1])$

$y_m = \text{r\_sum}(\left[y'_{k+1}, \ldots, y'_{k+l/2}\right], [1, \ldots, 1])$

$y'_{i=1:k} = y''_i$

$y'_{i=k+1:k+l/2} = \text{r\_nonsep}(\left[y''_{k+2(i-k)-1}, y''_{k+2(i-k)-1}\right], 2)$

$y'_{i=1:k} = z_{i,[0,1]}$

$y''_{k+1:n} = \text{s\_linear}(z_{i,[0,1]}, 0.35)$

where M is the number of objectives and z are the decision variables. S_m is a constant to modify position and scale for the optimal solutions of objective m^{th}. i and m are the indices. k is a position-related parameter. l is a distance-related parameter.

Transformation function:
r_sum is the weighted sum reduction transformation function.
r_nonsep is the non-separable reduction transformation function.
s_linear is the linear shift transformation function.

Shape function:
convex is the convex shape function.
disc is the disconnected shape function.

6. APPLICATION PROBLEM: COMPONENT ALLOCATION PROBLEM

The component and redundancy allocation problem is to optimize a system design by allocating redundant components from the design alternatives. This research considers a series-parallel system, the independent alternative component and mixing of non-identical components. The system structure is shown in Fig 3.

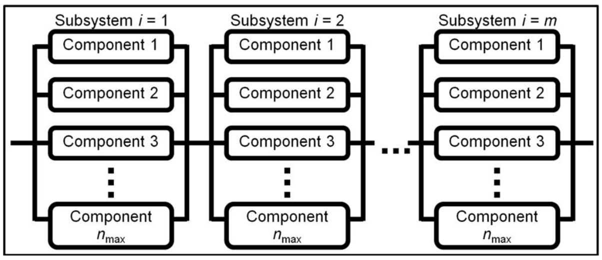

Fig.3: *General series-parallel redundancy system.*

The problem is a well-known NP-hard problem [25] which is difficult to solve within a reasonable time. The mixing of non-identical components is allowed in each subsystem causing many possible alternative solutions and also giving a large set of optimal solutions. Therefore, it is difficult to identify the preferred solutions. The system and components have only two states: work or fail while the component failure is statistically independent. The example solution for small system design is shown in Fig 4, where subsystem 1 has one component type A while subsystem 2 has one component type B, and subsystem 3 has one component type A together with one of type C.

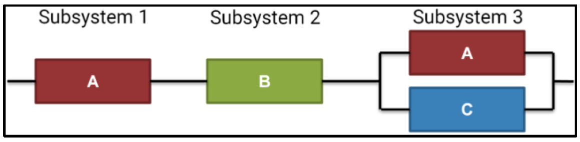

Fig.4: *Example of small system design problem.*

The objective functions of system design problem are as follows [21, 25, 26].
Objective 1: Maximize system reliability (R_{sys})

$$\max R_{sys}(\mathbf{x}) = \prod_{i=1}^{m} R_i(\mathbf{x}_i) \quad (4)$$

Objective 2: Minimize system cost (C_{sys})

$$\min C_{sys}(\mathbf{x}) = \sum_{i=1}^{m}\sum_{j=1}^{t_i} c_{ij}x_{ij} \quad (5)$$

Objective 3: Minimize system weight (W_{sys})

$$\min W_{sys}(\mathbf{x}) = \sum_{i=1}^{m}\sum_{j=1}^{t_i} w_{ij}x_{ij} \quad (6)$$

The system constraints are as follows.

$$1 \le \sum_{j=1}^{t_i} x_{ij} \le n_{max}$$

$$R_i(\mathbf{x}) = 1 - \prod_{j=1}^{t_i} (1 - R_{ij}(\mathbf{x}))^{x_{ij}}$$

$$0 \le R_{ij}(\mathbf{x}) \le 1$$

$x_{ij} \in \{0, 1, 2, \ldots, n_{max}\}$, i = 1, 2, ..., m and j = 1, 2, ..., t_i

where the number of components of type j in subsystem i is x_{ij} while x, x_i are the vector of x_{ij}. R_i is the reliability of subsystem i. The reliability, cost and weight of the j^{th} component in subsystem i represented by r_{ij}, c_{ij} and w_{ij}, respectively are given as input data for component allocation alternatives. The designed system is composed of m subsystems connected in series. The number of allocated components connected in parallel in subsystem i is t_i. The maximum number of components in subsystem i is n_{max} which is a predefined parameter. As a result, ti cannot exceed the n_{max} value.

7. SIMULATION RESULTS AND DISCUSSION

The trade-off method is evaluated the effectiveness by using the well-known benchmark problems and an application problem. The benchmark problems are including two-objective problems, ZDT test suite [22] and three-objective problem, WFG2 3D [23] in which all objectives are minimized. The details of ZDT test suite are in the appendix.

7.1 Two-objective problem

The results of five different ZDT problems are shown in Table 1 and Fig 5. There are ZDT1, ZDT2, ZDT3, ZDT5 and ZDT6 as described in Section 5.1. The red squares represent the preferred solutions of "the sacrifice in f_2 related to the gain in f_1" preference while the green stars represent the preferred solutions of "the sacrifice in f_1 related to the gain in f_2" preference.

Table 1: *The preferred solutions of ZDT problems.*

Problem	Preferred solutions	
	"the sacrifice in f_2 related to the gain in f_1" (shown as red square)	**"the sacrifice in f_1 related to the gain in f_2"** (shown as green star)
ZDT1	[0.999, 0.0005]	[0.001, 0.968377]
ZDT2	[0, 1]	[1, 0]
ZDT3	[0.85, -0.77]	[0.00, 0.97]
ZDT5	[30, 0.333333]	[2, 5]
ZDT6	[0.999999, 1.03E-06]	[0.999999, 1.03E-06]

In ZDT1, it is observed that the preferred solution, [0.999, 0.0005] obtained in "the sacrifice in f_2 related to the gain in f_1" preference are the worth solution among the optimal solutions according to the reference solution, [1, 0] which has the extremely good value in sacrificing objective (f_2) as shown in Fig 5 (a) and represented by red square. In "the sacrifice in f1 related to the gain in f_2" preference, the preferred solution, [0.001, 0.968377] are the worth solution among the optimal solutions according to the reference solution, [0, 1] which has the extremely good value in sacrificing objective (f_1) as shown in Fig 5 (a) and represented by green star. As well as the ZDT2, 3, 5 and 6, the results in Fig 5 (b), (c), (d) and (e) show the most worth solution with their corresponding reference solutions.

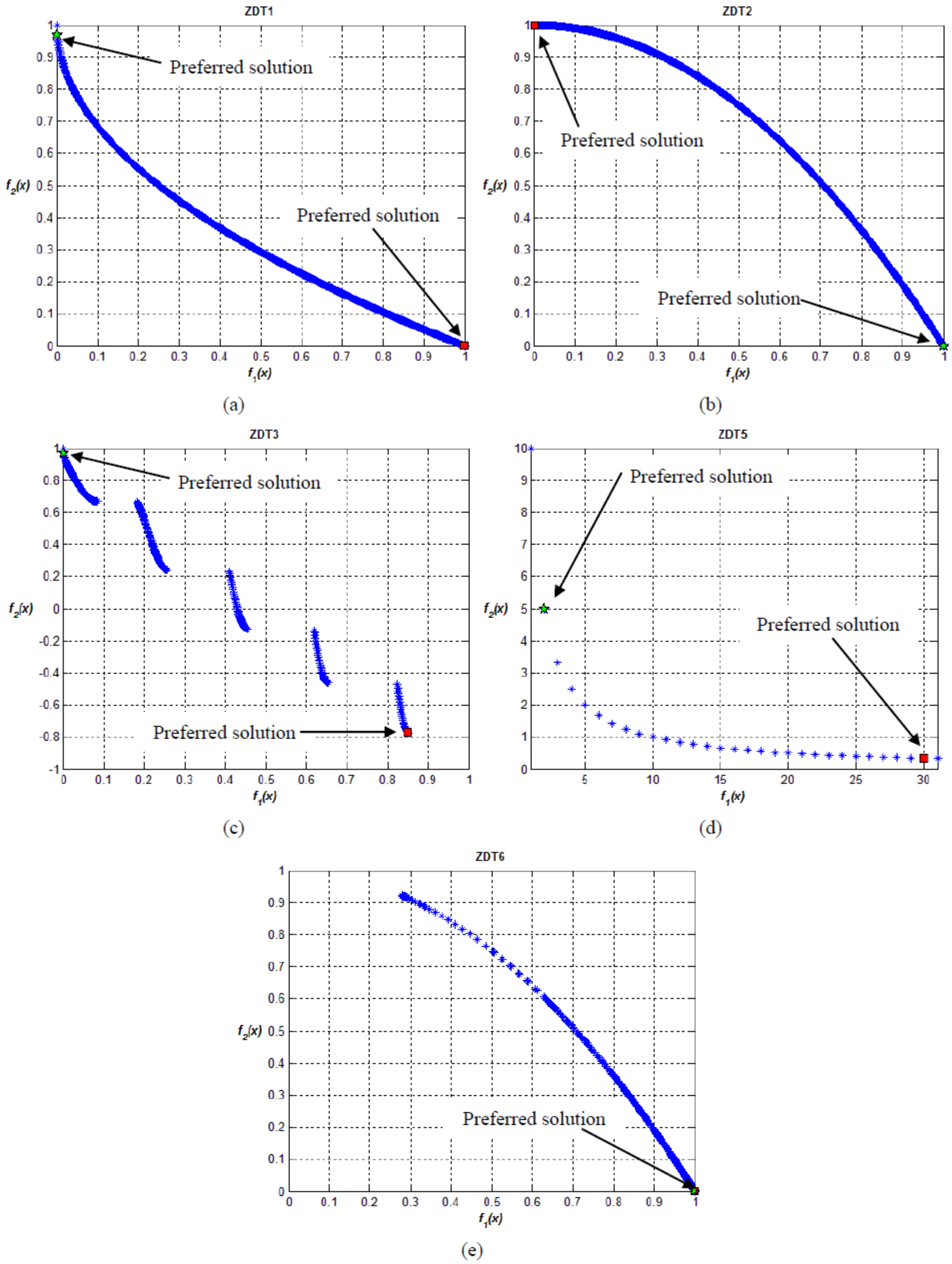

Fig.5: *The Pareto graphs and the preferred solutions of ZDT problem.*

7.2 Three-objective problem

For three-objective problem, the results of WFG2 3D are shown in Table 2, Figs 6 and 7. The preferences are "the sacrifice in f_3 related to the gain in f_2" and "the sacrifice in f_3 related to the gain in f_1". The blue dots represent Pareto-optimal solutions.

Table 2: *The preferred solutions of WFG2 3D problem.*

	Preferences	
	"the sacrifice in f_3 for the gain in f_2"	"the sacrifice in f_3 for the gain in f_1"
Preferred solution	[1.36, 3.41E-04, 1.2]	[0, 3.937, 0.205]
Preferred solution (normalized)	[0.702, 8.66E-05, 0.1716]	[0, 1, 0]

In Fig 6, "the sacrifice in f_3 related to the gain in f_2" preference, the green triangle represents an extreme solution, [1.937, 4.86E-04, 0.205] while the red circle represents a preferred solution, [1.36, 3.41E-04, 1.2]. We can see the eliminated solutions from step 3 and step 4 which are represented by the black dots. It shows that the trade-off method reduce the possible solutions by eliminated the solutions that not worth to sacrifice the objective f_3 in order to improve in the objective f_2. As well as the "the sacrifice in f_3 is related to the gain in f_1" preference, the results show the most worth solution with their corresponding reference solutions as shown in Fig 7.

The results show that the proposed method is able to identify appropriate solution set to the DM on the two-objective and three-objective benchmark problems. Moreover, it is able to identify the preferred solutions on different shapes of optimal solutions characteristics including the concave, convex, continuous and discontinuous.

7.3 Component and Redundancy Allocation Problem

For practicality, we consider the component information from Fyffe et al. [25], where the system consists of 14 subsystems and each subsystem has three or four different component alternatives. The component cost, weight, and reliability values are given as shown in Table 3. For all subsystems, the minimum number of components for the i^{th} subsystem, $k_i = 1$ and the maximum number of components arranged in parallel for each subsystem, $n_{max,i} = 8$ when considers 1-out-of-ni problem. The preference for the problem is that the DM expects better system reliability while he/she pays higher system cost. Fig 8 shows the approximated Pareto-optimal solutions [26] obtained from the most well-known algorithms, NSGA-II [17]. From the approximation of optimal solutions [26], the total solutions consist of more than 6000 solutions. From table 4, the maximum and minimum values of each objective show the variation of multiple optimal solutions.

Table 5 shows part of optimal solutions for the 3-objective problem considering system reliability is greater than 0.99. In this case, 1550 solutions satisfy this condition. Examples of preferred, reference and eliminated solutions are shown in Table 5, with corresponding system reliability (R_{sys}), system cost (C_{sys}) and system weight (W_{sys}).

Table 3: *Component input data Note: Sub = subsystem, Comp = component, R = reliability, C = cost, W = weight. The symbol "-" = the choice is not available. .*

Sub	Design Alternative (j)											
	Comp Type 1			Comp Type 2			Comp Type 3			Comp Type 4		
	R	C	W	R	C	W	R	C	W	R	C	W
1	0.90	1	3	0.93	1	4	0.91	2	2	0.95	4	5
2	0.95	4	8	0.94	2	10	0.93	1	9	-	-	-
3	0.85	2	7	0.90	3	5	0.87	1	6	0.92	4	4
4	0.83	3	5	0.87	4	6	0.85	5	4	-	-	-
5	0.94	2	4	0.93	2	3	0.95	5	5	0.94	2	4
6	0.99	6	5	0.98	4	4	0.97	2	5	0.96	2	4
7	0.91	4	7	0.92	4	8	0.94	5	9	-	-	-
8	0.81	3	4	0.9	5	7	0.91	6	6	-	-	-
9	0.97	2	8	0.99	3	9	0.96	4	7	0.91	3	8
10	0.83	4	6	0.85	4	5	0.9	5	6	-	-	-
11	0.94	3	5	0.95	4	6	0.96	5	6	-	-	-
12	0.79	2	4	0.82	3	5	0.85	4	6	0.9	5	7
13	0.98	2	5	0.99	3	5	0.97	2	6	-	-	-
14	0.9	4	6	0.92	4	7	0.95	5	6	0.99	6	9

Table 4: *The maximum and minimum values of the system reliability, system cost and system weight .*

Value	R_{sys}	C_{sys}	W_{sys}
MIN	0.214791745	34	68
MAX	0.999988616	251	414

From Table 5, solution ID 4902 has extremely good (low) system cost. Thus, it is the reference solution. The normalized system reliability values of solution IDs 2703 and 5226 are not as good as that of the reference solution (solution ID 4902). Thus, these two solutions are eliminated. The preferred score is calculated using the following formula.

$$T^k_{\text{cost,reliability}} = \frac{|f^{\text{reference solution}}_{\text{reliability}}(\mathbf{x}) - f^k_{\text{reliability}}(\mathbf{x})|}{f^{\text{reference solution}}_{\text{cost}}(\mathbf{x}) - f^k_{cost}(\mathbf{x})}$$

For the solution k, the $T^k_{\text{cost,reliability}}$ represents the gaining amount in the system reliability objective, $f^k_{\text{reliability}}(\mathbf{x})$ when one unit in the system cost is sacrificed, $f^k_{\text{cost}}(\mathbf{x})$. The trade-off values $T_(\text{cost,reliability})^k$ is shown in Table 5 while reference and eliminated solutions are ignored.

Without the specified acceptable value, the most preferred solution is solution ID 2784 as shown with the red dot in Fig 9. The detail value of preferred solution from Fig 9 is shown in Table 5. From Fig 10, the preferred, reference and eliminated solutions are represented by star, circle and x shapes, respectively. The results show that from the reference solution, the DM pays less system cost for the preferred solution

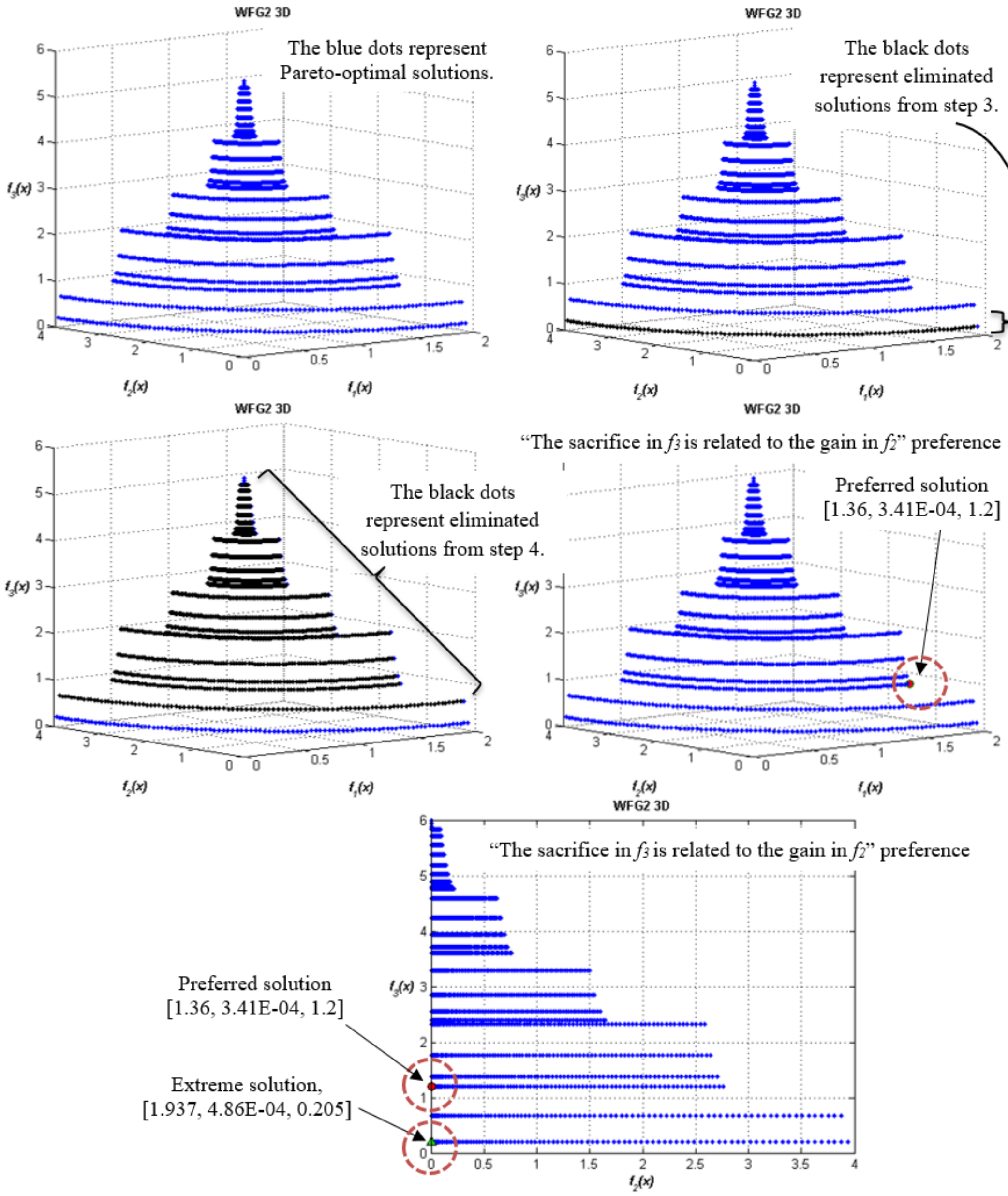

Fig.6: *The Pareto optimal solution graph and the preferred solutions for WFG2 3D problem with "the sacrifice in f_3 for the gain in f_2" preference .*

than that of the eliminated solution, while gaining more system reliability than that of the eliminated solution. Thus, the preferred solution is the most cost-effective solution.

With specified acceptable value, the DM prefers the gaining in system reliability of at least 0.005 from the reference solution that has system reliability 0.990271. The solutions that do not meet with this condition are eliminated. Therefore, the solution IDs 2784, 4930 and 4927 with system reliability less than 0.990271+0.005 = 0.995271 are eliminated. Now, the most preferred solution is changed to solution ID 5528.

The experimental results show that the trade-off method is possible to identify the preferred solution by using extreme solution as reference solution. As a result, the approximation of Pareto front is reduced given the objective preference. Besides, the most preferred solution is presented to the DM for implementation.

8. CONCLUSIONS

So far, most multi-objective optimization techniques have focused on searching for a set of optimal solutions. We consider beyond that challenge, to identify the most preferred solution among the set of optimal solutions. This is because, in practice, one

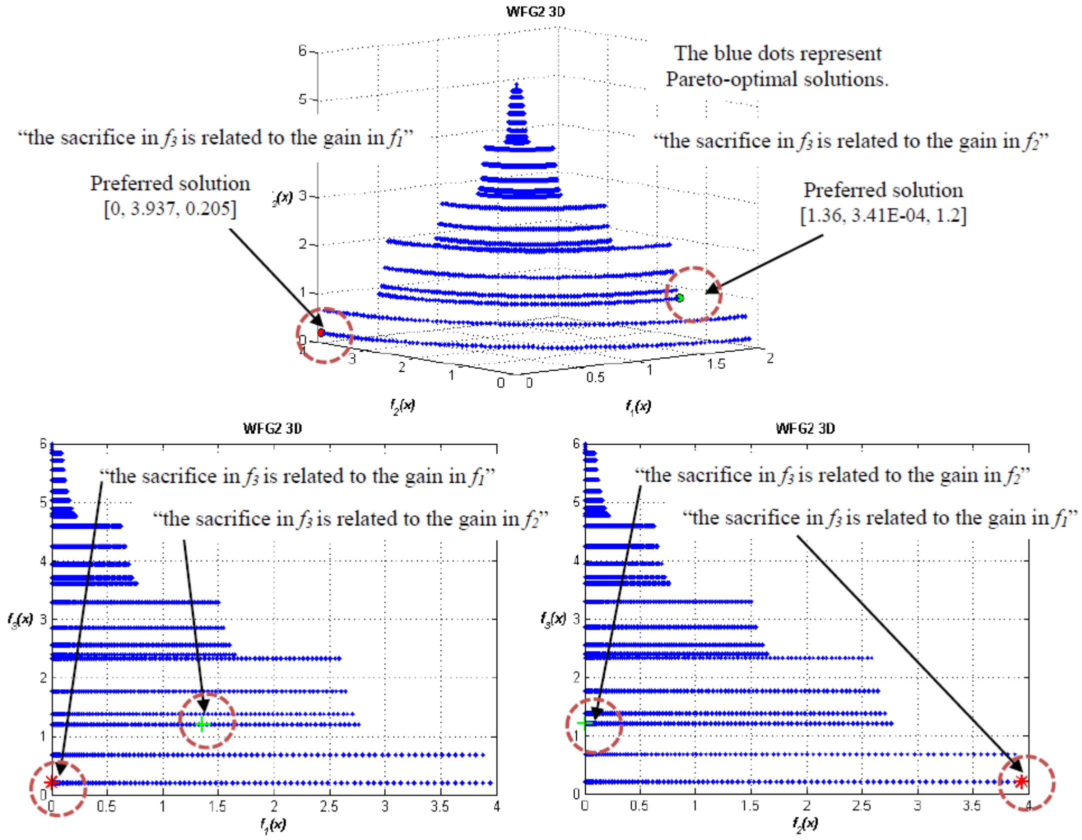

Fig.7: *The Pareto optimal solution graph and the preferred solutions of WFG2 3D problem for "the sacrifice in f_3 for the gain in f_1" and "the sacrifice in f_3 for the gain in f_2"preferences.*

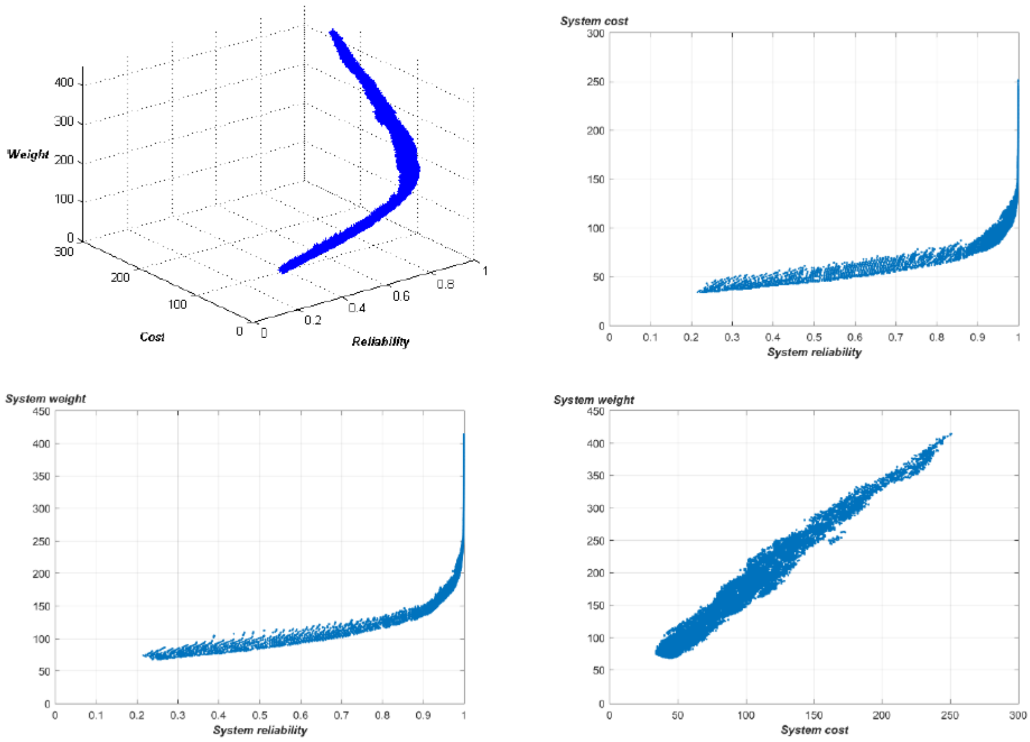

Fig.8: *The approximation of Pareto-optimal solutions in four views. a) three-objective plane, b) System reliability and system cost in two-objective plane, c) System reliability and system weight in two-objective plane and d) System cost and system weight in two-objective plane.*

Table 5: *Obtained solutions when system cost is sacrificed in order to gain improvement in system reliability.*

Input = Optimal solutions							Output		
								Status	
ID	R_{sys}	C_{sys}	W_{sys}	Norm. R_{sys}	Norm. C_{sys}	Norm. W_{sys}	$T^k_{\text{cost,reliability}}$	Without specified acceptable value	Specified acceptable value
2703	0.990002	118	215	0	0.094	0.124	-	Eliminate	Eliminate
5226	0.990256	115	223	0.026	0.054	0.206	-	Eliminate	Eliminate
4902	0.990271	111	230	0.028	0	0.278	-	Reference	Reference
2784	0.991766	112	236	0.184	0.014	0.340	11.548	Most preferred	Eliminate
4930	0.991736	112	235	0.181	0.014	0.329	11.318	2nd preferred	Eliminate
4927	0.991647	112	233	0.172	0.014	0.309	10.625	3rd preferred	Eliminate
5528	0.995519	122	250	0.576	0.149	0.484	3.686	21st preferred	Most preferred
5531	0.995484	122	249	0.572	0.149	0.474	3.660	22nd preferred	2nd preferred
5533	0.995455	122	248	0.569	0.149	0.464	3.640	23rd preferred	3rd preferred

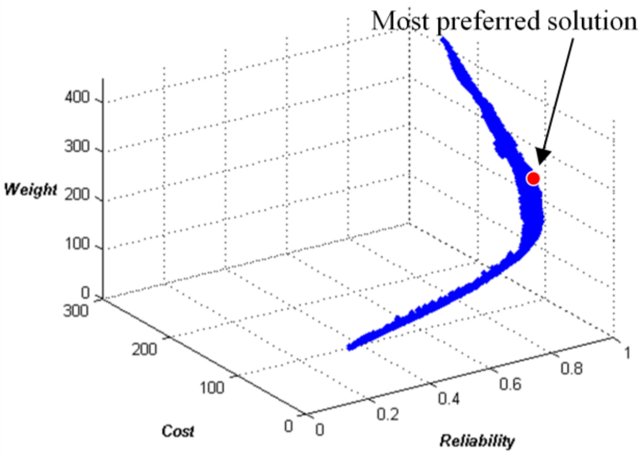

Fig.9: *An approximated optimal solutions (blue) of the problem and the preferred solution (red) when system reliability > 0.99 is requested.*

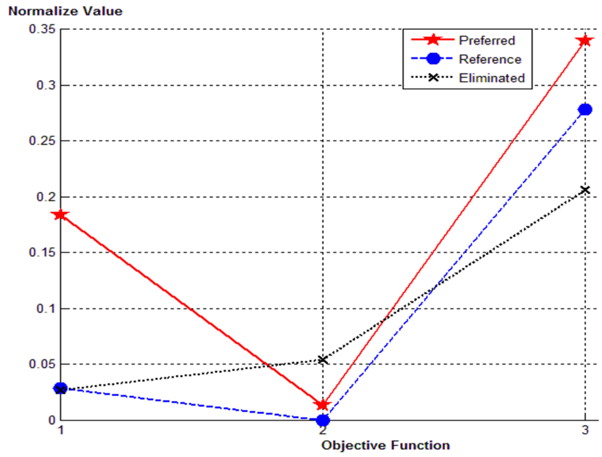

Fig.10: *The normalized values of objectives for preferred, reference and eliminated solutions where objective 1 is system reliability, objective 2 is system cost and objective 3 is system weight.*

or very few solutions will be considered as final solutions for implementation. The proposed method is designed to be flexible and easy for the decision maker to use, so that he/she does not have to be an expert in the problem solving.

The main contribution of this paper is a preference-based optimization method to identify the most preferred solution. The well-known benchmark problems as well as an application problem are considered to evaluate the effectiveness of the proposed method.

As part of future work, this proposed technique should be seamlessly integrated with a multi-objective optimization algorithm for searching toward the preferred region, so that the most preferred/desirable solution will be able to identify at once.

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APPENDIX A. APPENDIX OF FUNCTIONS

Description of the benchmark problems is given as follows, where the number of decision variables is n and $\vec{x}$ is a vector of decision variables.

ZDT1 :

$$\begin{aligned} \text{Minimize:} \quad & f_1(\vec{x}) = x_1 \\ & f_2(\vec{x}) = g(\vec{x})[1 - \sqrt{x_1/g(\vec{x})}] \\ \text{where} \quad & g(\vec{x}) = 1 + \tfrac{9}{n-1}(\textstyle\sum_{i=2}^{n} x_i) \\ \text{Subject to:} \quad & 0 \leq x_i \leq 1 \\ & n = 30 \end{aligned}$$

ZDT2 :

$$\begin{aligned} \text{Minimize:} \quad & f_1(\vec{x}) = x_1 \\ & f_2(\vec{x}) = g(\vec{x})[1 - (x_1/g(\vec{x}))^2] \\ \text{where} \quad & g(\vec{x}) = 1 + \tfrac{9}{n-1}(\textstyle\sum_{i=2}^{n} x_i) \\ \text{Subject to:} \quad & 0 \leq x_i \leq 1 \\ & n = 30 \end{aligned}$$

ZDT3 :

Minimize: $f_1(\vec{x}) = x_1$

$$f_2(\vec{x}) = g(\vec{x}) \left[1 - \sqrt{\frac{x_1}{g(\vec{x})}} - (\frac{x_1}{g(\vec{x})}) \sin(10\pi x_1) \right]$$

where $g(\vec{x}) = 1 + \frac{9}{n-1}(\sum_{i=2}^{n} x_i)$

Subject to: $0 \leq x_i \leq 1$

$n = 30$

ZDT5 :

Minimize: $f_1(\vec{x}) = 1 + u(x_1)$

$f_2(\vec{x}) = g(\vec{x})[\frac{1}{x_1}]$

where $g(\vec{x}) = (\sum_{i=2}^{n} x_i) v(u(x_i))$

$$v(u(x_i)) = \left\{ \begin{array}{ll} 2 + u(x_i) & if\ u(x_i) < 5 \\ 1 & if\ u(x_i) = 5 \end{array} \right\}$$ Subject to:

$x_1 \in \{0, 1\}^{30}$

$x_2, \ldots, x_n \in \{0, 1\}^5$

$n = 11$

ZDT6 :

Minimize: $f_1(\vec{x}) = 1 - \exp(-4x_1) \sin^6(6\pi x_1)$

$$f_2(\vec{x}) = g(\vec{x}) \left[1 - (\frac{f_1}{g(\vec{x})})^2 \right]$$

where $g(\vec{x}) = 1 + 9(\frac{\sum_{i=2}^{n} x_i}{n-1})^{0.25}$

Subject to: $0 \leq x_i \leq 1$

$n = 10$

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