A Design Method for Robust Stabilizing Modified Repetitive Controllers for Multiple-Input/Multiple-Output Time-Delay Plants with Specified Input-Output Characteristic

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ABSTRACT

The modified repetitive control system is a type of servomechanism for a periodic reference input. When modified repetitive control design methods are applied to real systems, the influence of uncertainties in the plant must be considered. In some cases, uncertainties in the plant make the control system unstable, even though the controller was designed to stabilize the nominal plant. Recently, Chen et al. propose the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants. However, using their method, it is complex to specify the low-pass filter in the internal model for the periodic reference input of which the role is to specify the input-output characteristic. Because, the low-pass filter is related to four free parameters in the parameterization. To specify the input-output characteristic easily, this paper proposes the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic such that the input-output characteristic can be specified beforehand.

Keywords: Repetitive Control, Modified Repetitive Controller, Uncertainty, Robust Stability, Parameterization, Multiple-input/Multiple-output Time-delay Plant, Low-pass Filter

1. INTRODUCTION

A repetitive control system is a type of servomechanism for a periodic reference input. In other words, the repetitive control system follows a periodic reference input without steady-state error, even if a periodic disturbance or uncertainty exists in the plant [1–11]. It is difficult to design stabilizing controllers for the strictly proper plant, because the repetitive control system that follows any periodic reference input without steady-state error is a neutral type of time-delay control system [11]. To design a repetitive control system that follows any periodic reference input without steady-state error, the plant needs to be biproper [3–11]. In practice, the plant is strictly proper. Many design methods for repetitive control systems for strictly proper plants have been given [3–11]. These systems are divided into two types. One type uses a low-pass filter [3–10] and the other type uses an attenuator [11]. The latter type of system is difficult to design because it uses a state-variable time delay in the repetitive controller [11]. The former has a simple structure and is easily designed. Therefore, the former type of repetitive control system is called the modified repetitive control system [3–10].

On the other hand, there exists an important control problem to find all stabilizing controllers named the parameterization problem [12–16]. Yamada et al. give the parameterization of all stabilizing modified repetitive controllers for non-minimum phase systems [17].

When modified repetitive control design methods are applied to real systems, the influence of uncertainties in the plant must be considered. In some cases, uncertainties in the plant make the modified repetitive control system unstable, even though the controller was designed to stabilize the nominal plant. The stability problem with uncertainty is known as the robust stability problem [18]. Yamada and Satoh propose the parameterization of all robust stabilizing modified repetitive controllers [19]. However, the method in [19] cannot guarantee the stability of control system for time-delay plants with uncertainties. Yamada et al. propose the parameterization of all robust stabilizing modified repetitive controllers for time-delay plants [20, 21]. Chen et al. expand the result in [20, 21] and propose the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants [22]. However, using the method in [22], it is complex to specify the low-pass filter in the internal model for the periodic reference input of which the role is to specify the input-output characteristic, because the low-pass filter is related to four free parameters in

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the parameterization by Chen et al. When we design a robust stabilizing modified repetitive controller, if the low-pass filter in the internal model for the periodic reference input is set beforehand, we can specify the input-output characteristic more easily than in the method employed in [22]. This is achieved by parameterizing all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic, which is the parameterization when the low-pass filter is set beforehand. However, no paper has considered the problem of obtaining the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic. In addition, the parameterization is useful to design stabilizing controllers [12–16]. Therefore, the problem of obtaining the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic is important to solve.

In this paper, we propose the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic such that the low-pass filter in the internal model for the periodic reference input is settled beforehand. This paper is organized as follows. In Section 2., the problem considered in this paper is described. In Section 3., the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic is clarified. In Section 4., control characteristics of a robust stabilizing modified repetitive control system are described. In Section 5., we present a design procedure of robust stabilizing modified repetitive control system. In Section 6., we show a numerical example to illustrate the effectiveness of the proposed method. Section 7. gives concluding remarks.

Notation

R	the set of real numbers.
R_+	$R \cup \{\infty\}$.
$R(s)$	the set of real rational functions with s.
RH_∞	the set of stable proper real rational functions.
H_∞	the set of stable causal functions.
$D^\perp$	orthogonal complement of D, i.e., $\begin{bmatrix} D & D^\perp \end{bmatrix}$ or $\begin{bmatrix} D \\ D^\perp \end{bmatrix}$ is unitary.
A^T	transpose of A.
$A^\dagger$	pseudo inverse of A.
$\rho(\{\cdot\})$	spectral radius of $\{\cdot\}$.
$\bar{\sigma}(\{\cdot\})$	largest singular value of $\{\cdot\}$.
$\|\{\cdot\}\|_\infty$	H_∞ norm of $\{\cdot\}$.
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	represents the state space description $C(sI-A)^{-1}B+D$.
$\mathcal{L}\{\cdot\}$	the Laplace transformation of $\{\cdot\}$.
$\mathcal{L}^{-1}\{\cdot\}$	the inverse Laplace transformation of $\{\cdot\}$.
diag $(a_1, \cdots, a_n)$	an $n \times n$ diagonal matrix with a_i as its i-th diagonal element.

2. PROBLEM FORMULATION

Consider the unity feedback control system in

$$\begin{cases} y &= G(s)e^{-sL}u + d \\ u &= C(s)(r-y) \end{cases}, \tag{1}$$

where $G(s)e^{-sL}$ is the multiple-input/multiple-output time-delay plant, $L > 0$ is the time-delay, $G(s) \in R^{m \times p}(s)$ is assumed to be stabilizable and detectable. $C(s)$ is the modified repetitive controller with m-th input and p-th output defined later, $u \in R^p$ is the control input, $d \in R^m$ is the disturbance, $y \in R^m$ is the output and $r \in R^m$ is the periodic reference input with period $T > 0$ satisfying

$$r(t+T) = r(t) \ \ (\forall t \geq 0). \tag{2}$$

It is assumed that $m \leq p$ and rank $G(s) = m$. The nominal plant of $G(s)e^{-sL}$ is denoted by $G_m(s)e^{-sL_m}$, where $G_m(s) \in R^{m \times p}(s)$. Both $G(s)$ and $G_m(s)$ are assumed to have no zero or pole on the imaginary axis. In addition, it is assumed that the number of poles of $G(s)$ in the closed right half plane is equal to that of $G_m(s)$. The relation between the plant $G(s)e^{-sL}$ and the nominal plant $G_m(s)e^{-sL_m}$ is written as

$$G(s)e^{-sL} = (e^{-sL_m}I + \Delta(s))G_m(s), \tag{3}$$

where $\Delta(s)$ is an uncertainty. The set of $\Delta(s)$ is all functions satisfying

$$\bar{\sigma}\{\Delta(j\omega)\} < |W_T(j\omega)| \quad (\forall \omega \in R_+), \tag{4}$$

where $W_T(s) \in R(s)$ is a stable rational function.

The robust stability condition for the plant $G(s)$ with uncertainty $\Delta(s)$ satisfying (4) is given by

$$\|T(s)W_T(s)\|_\infty < 1, \tag{5}$$

where $T(s)$ is given by

$$T(s) = \left(I + G_m(s)e^{-sL_m}C(s)\right)^{-1} G_m(s)C(s). \tag{6}$$

According to [3–10], the general form of modified repetitive controller $C(s)$ which makes the output y to follow the periodic reference input r with period T in (1) with small steady state error, is written by

$$C(s) = C_1(s) + C_2(s)C_r(s), \quad (7)$$

where $C_1(s) \in R^{p\times m}(s)$, $C_2(s) \in R^{p\times m}(s)$ satisfying rank $C_2(s) = m$, $C_r(s)$ is the internal model for the periodic signal with period T written as

$$C_r(s) = q(s)e^{-sT}\left(I - q(s)e^{-sT}\right)^{-1}, \quad (8)$$

where $q(s) \in R^{m\times m}(s)$ is a proper low-pass filter satisfying $q(0) = I$ and rank $q(s) = m$.

According to [3–10], if the low-pass filter $q(s)$ satisfy

$$\bar{\sigma}\{I - q(j\omega_i)\} \simeq 0 \quad (i = 0, 1, \dots, n), \quad (9)$$

where $\omega_i (i = 0, 1, \dots, n)$ is the frequency component of the periodic reference input r written by

$$\omega_i = \frac{2\pi}{T} i \quad (i = 0, 1, \dots, n) \quad (10)$$

and ω_n is the maximum frequency component of the periodic reference input r, then the output y in (1) follows the periodic reference input r with small steady state error. Using the result in [22], in order for $q(s)$ to satisfy (9) in wide frequency range, we must design $q(s)$ to be stable and of minimum phase. If we obtain the parameterization of all robust stabilizing modified repetitive controllers such that $q(s)$ in (7) is settled beforehand, we can design the robust stabilizing modified repetitive controller satisfying (9) more easily than the method in [22].

The problem considered in this paper is to propose the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic. That is, when $q(s) \in RH_\infty^{m\times m}$ is set beforehand, we obtain the parameterization of all controllers $C(s)$ in (7) satisfying (5) for multiple-input/multiple-output time-delay plants $G(s)e^{-sL}$ in (3) with any uncertainty $\Delta(s)$ satisfying (4).

3. THE PARAMETERIZATION

In this section, we clarify the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic.

In order to obtain the parameterization of all robust stabilizing modified repetitive controllers for time-delay plants with specified input-output characteristic, we must see that controllers $C(s)$ satisfying (5). The problem of obtaining the controller $C(s)$, which is not necessarily a modified repetitive controller, satisfying (5) is equivalent to the following H_∞ control problem. In order to obtain the controller $C(s)$ satisfying (5), we consider the control system shown in Fig. 1. $P(s)$ is selected such that

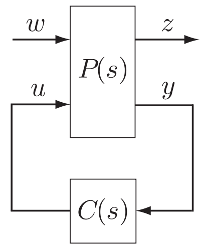

Fig.1: *Block diagram of H_∞ control problem*

the transfer function from w to z in Fig. 1 is equal to $T(s)W_T(s)$. The state space description of $P(s)$ is, in general,

$$\begin{cases} \dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t - L_m) \\ z(t) = C_1 x(t) + D_{12} u(t) \\ y(t) = C_2 x(t) + D_{21} w(t) \end{cases}, \quad (11)$$

where $A \in R^{n\times n}$, $B_1 \in R^{n\times m}$, $B_2 \in R^{n\times p}$, $C_1 \in R^{m\times n}$, $C_2 \in R^{m\times n}$, $D_{12} \in R^{m\times p}$, $D_{21} \in R^{m\times m}$, $x(t) \in R^n$, $w(t) \in R^m$, $z(t) \in R^m$, $u(t) \in R^p$ and $y(t) \in R^m$. $P(s)$ is called the generalized plant. $P(s)$ is assumed to satisfy following assumptions:

1. (C_2, A) is detectable, (A, B_2) is stabilizable.
2. D_{12} has full column rank, and D_{21} has full row rank.
3. rank $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} = n + p \quad (\forall \omega \in R_+)$,
   rank $\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} = n + m \quad (\forall \omega \in R_+)$.
4. $C_1 A^i B_2 = 0 \quad (i = 0, 1, 2, \dots)$.

Under these assumptions, from [23], following lemma holds true.

Lemma 1: There exists an H_∞ controller $C(s)$ for the generalized plant $P(s)$ in (11) if and only if there exists an H_∞ controller $C(s)$ for the generalized plant $\tilde{P}(s)$ written by

$$\begin{cases} \dot{q}(t) = Aq(t) + B_1 w(t) + \tilde{B}_2 u(t) \\ \tilde{z}(t) = C_1 q(t) + D_{12} u(t) \\ \tilde{y}(t) = C_2 q(t) + D_{21} w(t) \end{cases}, \quad (12)$$

where $\tilde{B}_2 = e^{-AL_m}B_2$. When $u(s) = C(s)\tilde{y}(s)$ is an H_∞ control input for the generalized plant $\tilde{P}(s)$ in (12),

$$u(t) = \mathcal{L}^{-1}\{C(s)\tilde{y}(s)\} \quad (13)$$

is an H_∞ control input for the generalized plant $P(s)$ in (11), where

$$\tilde{y}(s) = \mathcal{L}\left\{ y(t) + C_2 \int_{-L_m}^{0} e^{-A(\tau+L_m)} B_2 u(t+\tau) d\tau \right\}. \quad (14)$$

From Lemma 1 and [18], the following lemma holds true.

Lemma 2: If controllers satisfying (5) exist, both

$$X\left(A-\tilde{B}_2D_{12}^{\dagger}C_1\right)+\left(A-\tilde{B}_2D_{12}^{\dagger}C_1\right)^TX$$
$$+X\left\{B_1B_1^T-\tilde{B}_2\left(D_{12}^TD_{12}\right)^{-1}\tilde{B}_2^T\right\}X$$
$$+\left(D_{12}^{\perp}C_1\right)^TD_{12}^{\perp}C_1=0 \quad (15)$$

and

$$Y\left(A-B_1D_{21}^{\dagger}C_2\right)^T+\left(A-B_1D_{21}^{\dagger}C_2\right)Y$$
$$+Y\left\{C_1^TC_1-C_2^T\left(D_{21}D_{21}^T\right)^{-1}C_2\right\}Y$$
$$+B_1D_{21}^{\perp}\left(B_1D_{21}^{\perp}\right)^T=0 \quad (16)$$

have solutions $X\geq 0$ and $Y\geq 0$ such that

$$\rho(XY)<1 \quad (17)$$

and both

$$A-\tilde{B}_2D_{12}^{\dagger}C_1+\left\{B_1B_1^T-\tilde{B}_2\left(D_{12}^TD_{12}\right)^{-1}\tilde{B}_2^T\right\}X \quad (18)$$

and

$$A-B_1D_{21}^{\dagger}C_2+Y\left\{C_1^TC_1-C_2^T\left(D_{21}D_{21}^T\right)^{-1}C_2\right\} \quad (19)$$

have no eigenvalue in the closed right half plane. Using X and Y, the parameterization of all controllers satisfying (5) is given by

$$C(s) = C_{11}(s)+C_{12}(s)Q(s)(I-C_{22}(s)Q(s))^{-1}C_{21}(s), \quad (20)$$

where

$$\begin{bmatrix} C_{11}(s) & C_{12}(s)\\ C_{21}(s) & C_{22}(s)\end{bmatrix}=\left[\begin{array}{c|cc} A_c & B_{c1} & B_{c2}\\ \hline C_{c1} & D_{c11} & D_{c12}\\ C_{c2} & D_{c21} & D_{c22}\end{array}\right], \quad (21)$$

$$\begin{aligned}A_c &= A+B_1B_1^TX-\tilde{B}_2\left(D_{12}^{\dagger}C_1+E_{12}^{-1}\tilde{B}_2^TX\right)\\ &\quad -(I-YX)^{-1}\left(B_1D_{21}^{\dagger}+YC_2^TE_{21}^{-1}\right)\\ &\quad \left(C_2+D_{21}B_1^TX\right),\end{aligned}$$

$$\begin{aligned}B_{c1} &= (I-YX)^{-1}\left(B_1D_{21}^{\dagger}+YC_2^TE_{21}^{-1}\right),\\ B_{c2} &= (I-YX)^{-1}\left(\tilde{B}_2+YC_1^TD_{12}\right)E_{12}^{-1/2},\end{aligned}$$

$$\begin{aligned}C_{c1} &= -D_{12}^{\dagger}C_1-E_{12}^{-1}\tilde{B}_2^TX,\\ C_{c2} &= -E_{21}^{-1/2}\left(C_2+D_{21}B_1^TX\right),\end{aligned}$$

$$D_{c11}=0,\ \ D_{c12}=E_{12}^{-1/2},\ \ D_{c21}=E_{21}^{-1/2},\ \ D_{c22}=0,$$

$$E_{12}=D_{12}^TD_{12},\ \ E_{21}=D_{21}D_{21}^T$$

and $Q(s)\in H_{\infty}^{p\times m}$ is any function satisfying $\|Q(s)\|_{\infty}<1$.

$C(s)$ in (20) is written using Linear Fractional Transformation (LFT). Using homogeneous transformation, (20) is rewritten by

$$\begin{aligned}C(s) &= (Z_{11}(s)Q(s)+Z_{12}(s))\\ &\quad (Z_{21}(s)Q(s)+Z_{22}(s))^{-1}\\ &= \left(Q(s)\tilde{Z}_{21}(s)+\tilde{Z}_{22}(s)\right)^{-1}\\ &\quad \left(Q(s)\tilde{Z}_{11}(s)+\tilde{Z}_{12}(s)\right),\end{aligned} \quad (22)$$

where $Z_{ij}(s)(i=1,2;j=1,2)$ and $\tilde{Z}_{ij}(s)(i=1,2;j=1,2)$ are defined by

$$\begin{bmatrix} Z_{11}(s) & Z_{12}(s)\\ Z_{21}(s) & Z_{22}(s)\end{bmatrix} = \begin{bmatrix} C_{12}(s)-C_{11}(s)C_{21}^{-1}(s)C_{22}(s) & C_{11}(s)C_{21}^{-1}(s)\\ -C_{21}^{-1}(s)C_{22}(s) & C_{21}^{-1}(s)\end{bmatrix} \quad (23)$$

and

$$\begin{bmatrix} \tilde{Z}_{11}(s) & \tilde{Z}_{12}(s)\\ \tilde{Z}_{21}(s) & \tilde{Z}_{22}(s)\end{bmatrix} = \begin{bmatrix} C_{21}(s)-C_{22}(s)C_{12}^{-1}(s)C_{11}(s) & -C_{22}(s)C_{12}^{-1}(s)\\ C_{12}^{-1}(s)C_{11}(s) & C_{12}^{-1}(s)\end{bmatrix} \quad (24)$$

and satisfying

$$\begin{bmatrix} \tilde{Z}_{22}(s) & \tilde{Z}_{12}(s)\\ \tilde{Z}_{21}(s) & \tilde{Z}_{11}(s)\end{bmatrix}\begin{bmatrix} Z_{11}(s) & -Z_{12}(s)\\ -Z_{21}(s) & Z_{22}(s)\end{bmatrix}=I$$
$$=\begin{bmatrix} Z_{11}(s) & -Z_{12}(s)\\ -Z_{21}(s) & Z_{22}(s)\end{bmatrix}\begin{bmatrix} \tilde{Z}_{22}(s) & \tilde{Z}_{12}(s)\\ \tilde{Z}_{21}(s) & \tilde{Z}_{11}(s)\end{bmatrix}. \quad (25)$$

Using Lemma 1, Lemma 2 and Remark 3, the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic is given by following theorem.

Theorem 1: If modified repetitive controllers satisfying (5) exist, both (15) and (16) have solutions $X\geq 0$ and $Y\geq 0$ such that (17) and both (18) and (19) have no eigenvalue in the closed right half plane. Using X and Y, the parameterization of all robust stabilizing modified repetitive control laws with specified input-output characteristic satisfying (5) is given by

$$u(t) = \mathcal{L}^{-1}\left\{C(s)\tilde{y}(s)\right\}, \quad (26)$$

where

$$\tilde{y}(s) = \mathcal{L}\left\{y(t) + C_2\int_{-L_m}^{0} e^{-A(\tau+L_m)}B_2u(t+\tau)d\tau\right\} \quad (27)$$

and

$$C(s) = (Z_{11}(s)Q(s) + Z_{12}(s))(Z_{21}(s)Q(s) + Z_{22}(s))^{-1} = \left(Q(s)\tilde{Z}_{21}(s) + \tilde{Z}_{22}(s)\right)^{-1}\left(Q(s)\tilde{Z}_{11}(s) + \tilde{Z}_{12}(s)\right), \quad (28)$$

where $Z_{ij}(s)(i = 1,2; j = 1,2)$ and $\tilde{Z}_{ij}(s)(i = 1,2; j = 1,2)$ are defined by (23) and (24) and satisfying (25), $C_{ij}(s)(i = 1,2; j = 1,2)$ are given by (21) and $Q(s) \in H_\infty^{p\times m}$ is any function satisfying $\|Q(s)\|_\infty < 1$ and written by

$$Q(s) = \left(Q_{n1}(s) + Q_{n2}(s)q(s)e^{-sT}\right)\left(Q_{d1}(s) + Q_{d2}(s)q(s)e^{-sT}\right)^{-1}, \quad (29)$$

$Q_{n1}(s) \in RH_\infty^{p\times m}$, $Q_{n2}(s) \in RH_\infty^{p\times m}$, $Q_{d1}(s) \in RH_\infty^{m\times m}$ and $Q_{d2}(s) \in RH_\infty^{m\times m}$ are any functions satisfying

$$(Z_{21}(s)Q_{n2}(s) + Z_{22}(s)Q_{d2}(s))(Z_{21}(s)Q_{n1}(s) + Z_{22}(s)Q_{d1}(s))^{-1} = -I \quad (30)$$

and

$$\text{rank}\ \{Z_{11}(s)(Q_{n1}(s) + Q_{n2}(s)) + Z_{12}(s)(Q_{d1}(s) + Q_{d2}(s))\} = m. \quad (31)$$

Proof: First, the necessity is shown. That is, we show that if the modified repetitive controller $C(s)$ in (7) stabilizes the control system in (1) robustly and $q(s)$ is set beforehand, then $C(s)$ is written by (28) and (29), respectively. From Lemma 2 and Remark 3, the parameterization of all robust stabilizing controllers $C(s)$ for $G(s)e^{-sL}$ is written by (28), where $\|Q(s)\|_\infty < 1$. In order to prove the necessity, we will show that if $C(s)$ written by (7) stabilizes the control system in (1) robustly and $q(s)$ is set beforehand, then $Q(s)$ in (28) is written by (29). Substituting $C(s)$ in (7) for (28), we have (29), where

$$Q_{n1}(s) = -N_{1n}(s)N_{2d}(s), \quad (32)$$

$$Q_{n2}(s) = -N_{2n}(s) + N_{1n}(s)N_{2d}(s), \quad (33)$$

$$Q_{d1}(s) = D_{1n}(s)D_{2d}(s)N_{1d}(s)N_{2d}(s) \quad (34)$$

and

$$Q_{d2}(s) = (D_{2n}(s) - D_{1n}(s)D_{2d}(s))N_{1d}(s)N_{2d}(s). \quad (35)$$

Here, $N_{1n}(s) \in RH_\infty^{p\times m}$, $N_{2n}(s) \in RH_\infty^{p\times m}$, $N_{1d}(s) \in RH_\infty^{m\times m}$, $N_{2d}(s) \in RH_\infty^{m\times m}$, $D_{1n}(s) \in RH_\infty^{m\times m}$, $D_{2n}(s) \in RH_\infty^{m\times m}$, $D_{1d}(s) \in RH_\infty^{m\times m}$ and $D_{2d}(s) \in RH_\infty^{m\times m}$ are coprime factors satisfying

$$\tilde{Z}_{21}(s)C_1(s) - \tilde{Z}_{11}(s) = D_{1n}(s)D_{1d}^{-1}(s), \quad (36)$$

$$\tilde{Z}_{21}(s)C_2(s)D_{1d}(s) = D_{2n}(s)D_{2d}^{-1}(s), \quad (37)$$

$$\left(\tilde{Z}_{22}(s)C_1(s) - \tilde{Z}_{12}(s)\right)D_{1d}(s)D_{2d}(s) = N_{1n}(s)N_{1d}^{-1}(s) \quad (38)$$

and

$$\tilde{Z}_{22}(s)C_2(s)D_{1d}(s)D_{2d}(s)N_{1d}(s) = N_{2n}(s)N_{2d}^{-1}(s). \quad (39)$$

From (32) $\sim$ (35), all of $Q_{n1}(s)$, $Q_{n2}(s)$, $Q_{d1}(s)$ and $Q_{d2}(s)$ are included in RH_∞. Thus, we have shown that if $C(s)$ written by (7) stabilizes the control system in (1) robustly and $q(s)$ is set beforehand, $Q(s)$ in (28) is written by (29). From (33), (35), (37), (39) and (25), (30) is satisfied. In addition, from the assumption of rank $C_2(s) = m$ and from (37) and (39),

$$\text{rank}\ D_{2n}(s) = m \quad (40)$$

and

$$\text{rank}\ N_{2n}(s) = m \quad (41)$$

hold true. From (40), (41), (33) and (35), (31) is satisfied. Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, it is shown that if $C(s)$ and $Q(s) \in H_\infty^{p\times m}$ are settled by (28) and (29), respectively, then the controller $C(s)$ is written by the form in (7) and rank $C_2(s) = m$ hold true. Substituting (29) into (28), we have (7), where $C_1(s)$ and $C_2(s)$ are denoted by

$$C_1(s) = (Z_{11}(s)Q_{n1}(s) + Z_{12}(s)Q_{d1}(s))(Z_{21}(s)Q_{n1}(s) + Z_{22}(s)Q_{d1}(s))^{-1}, \quad (42)$$

and

$$\begin{aligned} & C_2(s) \\ = & \{Z_{11}(s)\left(Q_{n1}(s)+Q_{n2}(s)\right)+Z_{12}(s)\left(Q_{d1}(s)\right. \\ & \left.+Q_{d2}(s)\right)\}\left(Z_{21}(s)Q_{n1}(s)+Z_{22}(s)Q_{d1}(s)\right)^{-1}. \end{aligned} \tag{43}$$

We find that if $C(s)$ and $Q(s)$ are settled by (28) and (29), respectively, then the controller $C(s)$ is written by the form in (7). From (31) and (43),

$$\operatorname{rank} C_2(s) = m \tag{44}$$

holds true. Thus, the sufficiency has been shown.

We have thus proved Theorem 1. ■

4. CONTROL CHARACTERISTICS

In this section, we describe control characteristics of control system in (1) using robust stabilizing modified repetitive controllers $C(s)$ in (28).

From Theorem 1, $Q(s)$ in (29) must be included in H_∞. Since $Q_{n1}(s) \in RH_\infty^{p\times m}$, $Q_{n2}(s) \in RH_\infty^{p\times m}$ and $q(s) \in RH_\infty^{m\times m}$ in (29), if

$$\left(Q_{d1}(s)+Q_{d2}(s)q(s)e^{-sT}\right)^{-1} \in H_\infty^{m\times m}, \tag{45}$$

then $Q(s)$ satisfies $Q(s) \in H_\infty^{p\times m}$.

Next, we mention the input-output characteristic. The transfer function $S(s)$ from the periodic reference input r to the error $e = r - y$ is written by

$$S(s) = (I + C(s)G(s))^{-1} = S_n(s)S_d^{-1}(s), \tag{46}$$

where

$$\begin{aligned} S_n(s) = & \left(I - q(s)e^{-sT}\right)C_{21}^{-1}(s) \\ & \left(Q_{d1}(s) - C_{22}(s)Q_{n1}(s)\right) \end{aligned} \tag{47}$$

and

$$\begin{aligned} & S_d(s) \\ = & Z_{22}(s)Q_{d1}(s)+Z_{21}(s)Q_{n1}(s) \\ & +G(s)\left(Z_{12}(s)Q_{d1}(s)+Z_{11}(s)Q_{n1}(s)\right)e^{-sL} \\ & +\left(Z_{22}(s)Q_{d2}(s)+Z_{21}(s)Q_{n2}(s)\right)q(s)e^{-sT} \\ & +G(s)\left(Z_{12}(s)Q_{d2}(s)+Z_{11}(s)Q_{n2}(s)\right)q(s) \\ & e^{-s(T+L)}. \end{aligned} \tag{48}$$

From (46), for frequency components $\omega_i (i = 0, 1, \ldots, n)$ in (10) of the periodic reference input r, since $q(s) \in RH_\infty^{m\times m}$ is set beforehand satisfying (9), the output y follows the periodic reference input r with small steady state error. That is, we find that by using the proposed parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic, the input-output characteristic can be specified beforehand.

Finally, we mention the disturbance attenuation characteristic. The transfer function $S(s)$ from the disturbance d to the output y is written by (46), (47) and (48). From (46), for the disturbance d with same frequency components $\omega_i (i = 0, 1, \ldots, n)$ in (10) of the periodic reference input r, since $S(s)$ satisfies $S(j\omega_i) \simeq 0$ $(\forall i = 0, 1, \ldots, n)$, the disturbance d is attenuated effectively. For the frequency component ω_d of the disturbance d that is different from that of the periodic reference input r, that is $\omega_d \neq \omega_i$, even if

$$\bar{\sigma}\left\{I - q(j\omega_d)\right\} \simeq 0, \tag{49}$$

the disturbance d cannot be attenuated, because

$$e^{-j\omega_d T} \neq 1 \tag{50}$$

and

$$\bar{\sigma}\left\{I - q(j\omega_d)e^{-j\omega_d T}\right\} \neq 0. \tag{51}$$

To attenuate the frequency component ω_d of the disturbance $d(s)$ that is different from that of the periodic reference input $r(s)$, we need to set $Q_{n1}(s)$ and $Q_{d1}(s)$ satisfying

$$\bar{\sigma}\left\{Q_{d1}(j\omega_d) - C_{22}(j\omega_d)Q_{n1}(j\omega_d)\right\} \simeq 0. \tag{52}$$

Thus, the role of $Q_{d1}(s)$ and $Q_{d2}(s)$ is to assure the stability of the control system in (1) by satisfying $Q(s) \in H_\infty^{p\times m}$. The role of $q(s)$ is to specify the input-output frequency characteristic for the periodic reference input r and it can be specified beforehand. In addition, $q(s)$ specifies the disturbance attenuation characteristic for the disturbance d with same frequency components ω_i $(i = 0, 1, \ldots, n)$ of the periodic reference input r. The role of $Q_{n1}(s)$ and $Q_{d1}(s)$ is to specify the disturbance attenuation characteristic for the frequency component of the disturbance $d(s)$ that is different from that of the periodic reference input $r(s)$.

5. DESIGN PROCEDURE

In this section, a design procedure for robust stabilizing modified repetitive controller for multiple-input/multiple-output time-delay plants with specified input-output characteristic is presented.

A design procedure of robust stabilizing modified repetitive controllers C(s) satisfying Theorem 1 is summarized as follows:

<u>Procedure</u>

Step-1) Obtain $C_{11}(s)$, $C_{12}(s)$, $C_{21}(s)$ and $C_{22}(s)$ by solving the robust stability problem using the Riccati equation based H_∞ control as Theorem 1.

Step-2) The low-pass filter $q(s) \in RH_\infty^{m\times m}$ is set so that for the frequency components ω_i $(i = 0, 1, \ldots, n)$ of the periodic reference input $r(s)$,

$$\bar{\sigma}\{I - q(j\omega_i)\} \simeq 0 \ (\forall i = 0, 1, \ldots, n) \quad (53)$$

is satisfied.

Step 3) $Q_{n2}(s) \in RH_\infty^{p\times m}$ and $Q_{d2}(s) \in RH_\infty^{m\times m}$ in (29) are set according to

$$Q_{n2}(s) = C_{22d}(s)\bar{Q}(s) - Q_{n1}(s) \quad (54)$$

and

$$Q_{d2}(s) = C_{22n}(s)\bar{Q}(s) - Q_{d1}(s) \quad (55)$$

so that (30) is satisfied, where $C_{22n}(s) \in RH_\infty^{m\times p}$, $C_{22d}(s) \in RH_\infty^{p\times p}$ are coprime factors of $C_{22}(s)$ satisfying

$$C_{22}(s) = C_{22n}(s)C_{22d}^{-1}(s) \quad (56)$$

and $\bar{Q}(s) \in RH_\infty^{p\times m}$ is any function.

Step 4) In order to hold $Q(s) \in H_\infty$ in (29), $Q_{d1}(s) \in RH_\infty^{m\times m}$ in (29) and $\bar{Q}(s) \in RH_\infty^{p\times m}$ in (55) are set to satisfy

$$\left(Q_{d1}(s) + Q_{d2}(s)q(s)e^{-sT}\right)^{-1} \in H_\infty. \quad (57)$$

Step 5) $Q_{n1}(s) \in RH_\infty^{p\times m}$ is set so that for the frequency component ω_d of the disturbance d, $\bar{\sigma}\{Q_{d1}(j\omega_d) - C_{22}(j\omega_d)Q_{n1}(j\omega_d)\} \simeq 0$ is satisfied. To design $Q_{n1}(s)$ to satisfy $\bar{\sigma}\{Q_{d1}(j\omega_d) - C_{22}(j\omega_d)Q_{n1}(j\omega_d)\} \simeq 0$, $Q_{n1}(s)$ is set according to

$$Q_{n1}(s) = C_{22o}^{\dagger}(s)\bar{q}_d(s)Q_{d1}(s), \quad (58)$$

where $C_{22o}(s) \in RH_\infty^{m\times p}$ is an outer function of $C_{22}(s)$ satisfying

$$C_{22}(s) = C_{22i}(s)C_{22o}(s), \quad (59)$$

$C_{22i}(s) \in RH_\infty^{m\times m}$ is an inner function satisfying $C_{22i}(0) = I$, $\bar{q}_d(s) \in RH_\infty^{m\times m}$ is a low-pass filter satisfying $\bar{q}_d(0) = I$, as

$$\bar{q}_d(s) = \text{diag}\left\{\frac{1}{(1+\tau_{d1}s)^{\alpha_{d1}}} \cdots \frac{1}{(1+\tau_{dm}s)^{\alpha_{dm}}}\right\} \quad (60)$$

is valid, $\alpha_{di}(i = 1, \ldots, m)$ are arbitrary positive integers to make $C_{22o}^{\dagger}(s)\bar{q}_d(s)$ proper and $\tau_{di} \in R$ $(i = 1, \ldots, m)$ are any positive real numbers satisfying

$$\bar{\sigma}\{I - C_{22i}(j\omega_d)\bar{q}_d(j\omega_d)\} \simeq 0. \quad (61)$$

6. NUMERICAL EXAMPLE

In this section, a numerical example is shown to illustrate the effectiveness of the proposed method.

Consider the problem to obtain the parameterization of all robust stabilizing modified repetitive controllers with the specified input-output characteristic for time-delay plant $G(s)e^{-sL}$ written by

$$G(s)e^{-sL} = (e^{-sL_m}I + \Delta(s))G_m(s). \quad (62)$$

The nominal time-delay plant of $G(s)e^{-sL}$ and the upper bound $W_T(s)$ of the set of $\Delta(s)$ are given by

$$G_m(s)e^{-sL_m} = \begin{bmatrix} \frac{s+3}{(s+2)(s+9)} & \frac{2}{(s+2)(s+9)} \\ \frac{s+3}{(s+2)(s+9)} & \frac{s+4}{(s+2)(s+9)} \end{bmatrix} e^{-0.3s}, \quad (63)$$

and

$$W_T(s) = \frac{3s+6}{s+10}, \quad (64)$$

where

$$G_m(s) = \begin{bmatrix} \frac{s+4}{(s+2)(s+9)} & \frac{2}{(s+2)(s+9)} \\ \frac{s+3}{(s+2)(s+9)} & \frac{s+4}{(s+2)(s+9)} \end{bmatrix} \quad (65)$$

and $L_m = 0.3$[sec]. The period T of the periodic reference input r in (2) is $T = 20$[sec]. Solving the robust stability problem using Riccati equation based H_∞ control as Theorem 1, the parameterization of all robust stabilizing modified repetitive controllers with the specified input-output characteristic is obtained as (28) and (29). The low-pass filter $q(s) \in RH_\infty^{2\times 2}$ is settled by

$$q(s) = \frac{1}{0.01s+1}I. \quad (66)$$

$Q_{n2}(s) \in RH_\infty^{2\times 2}$ and $Q_{d2}(s) \in RH_\infty^{2\times 2}$ are set according to (54) and (55).

In order to hold $Q(s) \in H_\infty^{2\times 2}$ in (29), $Q_{d1}(s) \in RH_\infty$ in (29) and $\bar{Q}(s) \in RH_\infty^{2\times 2}$ in (54) and (55) are settled by

$$Q_{d1}(s) = \begin{bmatrix} \frac{s+713.2}{s+417.4} & \frac{-478.7}{s+417.4} \\ \frac{478.7}{s+417.4} & \frac{s+713.2}{s+417.4} \end{bmatrix} \quad (67)$$

and

$$\bar{Q}(s) = I. \quad (68)$$

When $Q_{d1}(s)$ and $\bar{Q}(s)$ are set as (67) and (68), the fact that $Q(s) \in H_\infty$ in (29) is confirmed as follows: Since $Q_{n1}(s) \in RH_\infty$, $Q_{n2}(s) \in RH_\infty$ and $q(s) \in RH_\infty$, if the Nyquist plot of $\det(Q_{d1}(s) + Q_{d2}(s)q(s)e^{-sT})$ does not encircle the origin, then $Q(s) \in H_\infty$ holds true. The Nyquist plot of $\det(Q_{d1}(s) + Q_{d2}(s)q(s)e^{-sT})$ is shown in Fig. 2. From Fig. 2, since the Nyquist plot of $\det(Q_{d1}(s) + Q_{d2}(s)q(s)e^{-sT})$ does not encircle the origin, we find that $Q(s) \in H_\infty$ holds true.

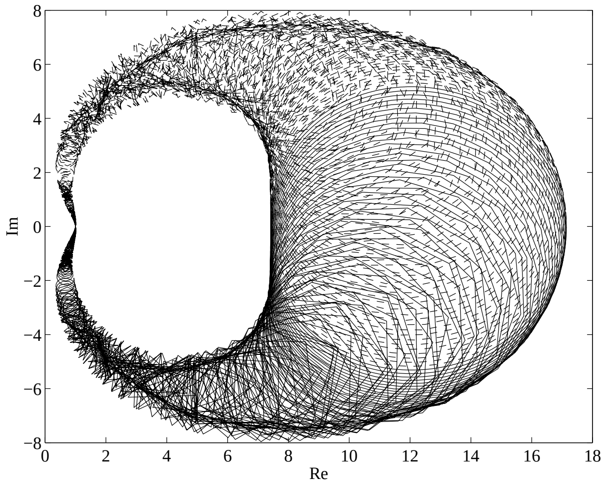

Fig.2: *The Nyquist plot of* $\det(Q_{d1}(s) + Q_{d2}(s)q(s)e^{-sT})$

In order for disturbance

$$d(t) = \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} = \begin{bmatrix} 2\sin(0.05\pi t) \\ \sin(0.05\pi t) \end{bmatrix} \quad (69)$$

of which the frequency component is different from that of the periodic reference input $r(t)$ to be attenuated effectively, $Q_{n1}(s)$ is settled by (58), where

$$C_{22o}(s) = C_{22}(s) \in RH_\infty^{2\times 2}, \quad (70)$$

and

$$\bar{q}_d(s) = \frac{1}{0.02s+1} I \in RH_\infty^{2\times 2}. \quad (71)$$

The largest singular value plot of $Q(s)$ is shown in Fig. 3. Figure 3 shows that the designed $Q(s)$

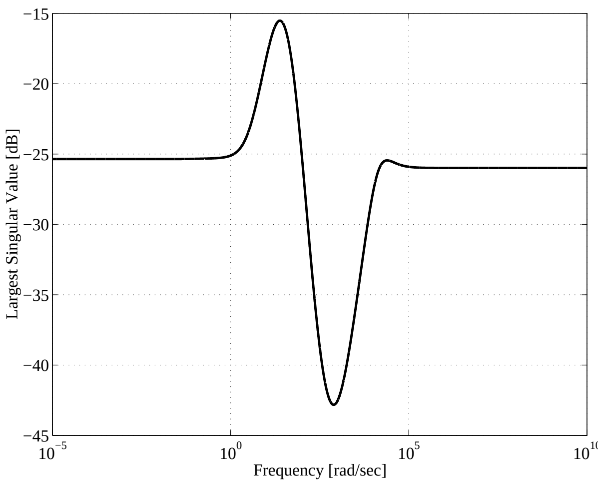

Fig.3: *The largest singular value plot of* $Q(s)$

satisfies $\|Q(s)\|_\infty < 1$.

When $\Delta(s)$ is given by

$$\Delta(s) = \begin{bmatrix} \dfrac{s-100}{s+500} & \dfrac{200}{s+600} \\ \dfrac{200}{s+500} & \dfrac{s-100}{s+600} \end{bmatrix}, \quad (72)$$

in order to confirm that $\Delta(s)$ satisfies (4), the largest singular value plot of $\Delta(s)$ and the gain plot of $W_T(s)$ are shown in Fig. 4. Here, the solid line shows the

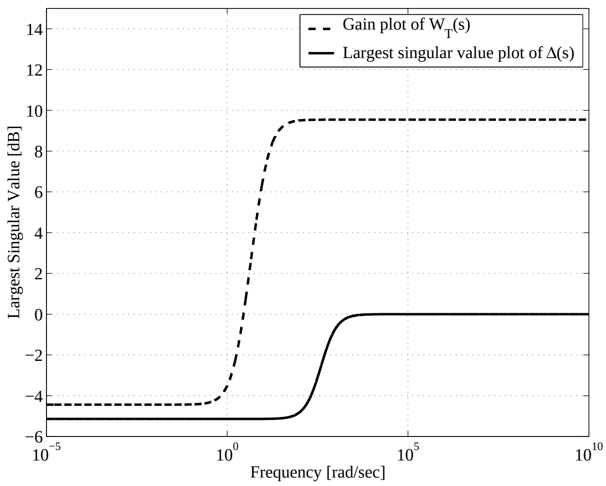

Fig.4: *The largest singular value plot of* $\Delta(s)$ *and the gain plot of* $W_T(s)$

largest singular value plot of $\Delta(s)$ and the broken line shows the gain plot of $W_T(s)$. Figure 4 shows that the uncertainty $\Delta(s)$ satisfies (4).

Using above-mentioned parameters, we have a robust stabilizing modified repetitive controller with specified input-output characteristic. When the designed robust stabilizing modified repetitive controller $C(s)$ is used, the response of the output $y(t) = [y_1(t), y_2(t)]^T$ in (1) for the periodic reference input $r(t) = [r_1(t), r_2(t)]^T = [\sin(0.1\pi t), 2\sin(0.1\pi t)]^T$ is shown in Fig. 5. Here, the thick broken line shows the response of the periodic reference input r_1, the thin broken line shows that of the periodic reference input r_2, the thick solid line shows that of the output y_1 and the thin solid line shows that of the output y_2. Figure 5 shows that the output y follows the periodic reference input $r(t) = [r_1(t), r_2(t)]^T = [\sin(0.1\pi t), 2\sin(0.1\pi t)]^T$ with small steady state error, even if the plant has uncertainty $\Delta(s)$.

Next, using the designed robust stabilizing modified repetitive controller $C(s)$ with specified input-output characteristic, the disturbance attenuation characteristic is shown. The response of the output $y(t) = [y_1(t), y_2(t)]^T$ for the disturbance $d(t) = [d_1(t), d_2(t)]^T = [\sin(0.2\pi t), 2\sin(0.2\pi t)]^T$ of which the frequency component is equivalent to that of the periodic reference input $r(t)$ is shown in Fig. 6. Here, the thin broken line shows the response of the disturbance d_1, the thick broken line shows that of the disturbance d_2, the thin solid line shows that of the

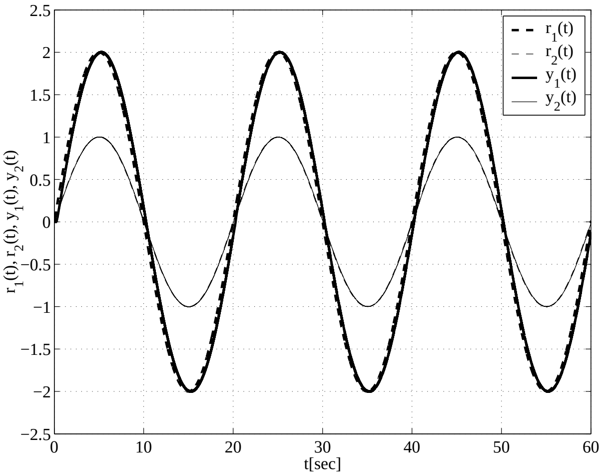

Fig.5: *The response of the output* $y(t) = [y_1(t), y_2(t)]^T$ *for the periodic reference input* $r(t) = [r_1(t), r_2(t)]^T = [\sin(0.1\pi t), 2\sin(0.1\pi t)]^T$

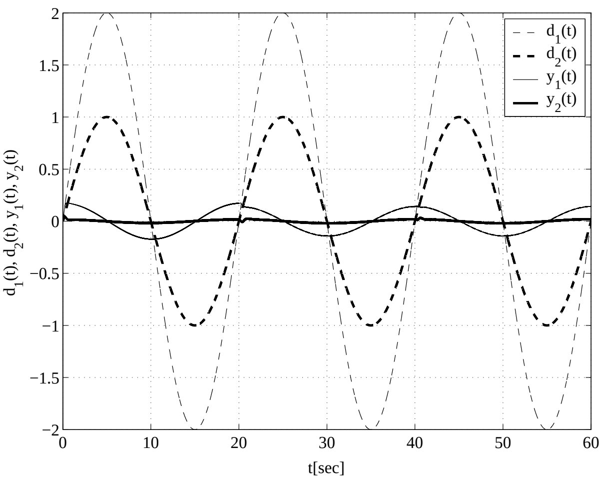

Fig.6: *The response of the output* $y(t) = [y_1(t), y_2(t)]^T$ *for the disturbance* $d(t) = [d_1(t), d_2(t)]^T = [\sin(0.2\pi t), 2\sin(0.2\pi t)]^T$

output y_1 and the thick solid line shows that of the output y_2. Figure 6 shows that the disturbance d in (68) is attenuated effectively. Finally, the response of the output $y(t) = [y_1(t), y_2(t)]^T$ for the disturbance $d(t)$ in (69) of which the frequency component is different from that of the periodic reference input r is shown in Fig. 7. Here, the thin broken line shows the response of the disturbance d_1, the thick broken line shows that of the disturbance d_2, the thin solid line shows that of the output y_1 and the thick solid line shows that of the output y_2. Figure 7 shows that the disturbance d in (69) is attenuated effectively.

Therefore, using the method shown here, a robust stabilizing modified repetitive controller with specified input-output characteristic can be easily designed.

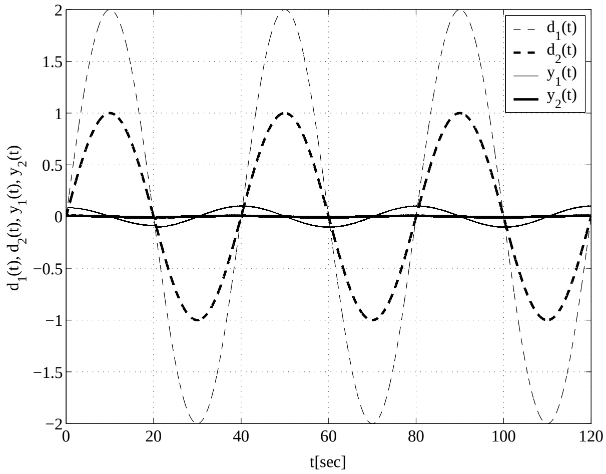

Fig.7: *The response of the output* $y(t)$ *for the disturbance* $d(t)$ *in (69)*

7. CONCLUSIONS

In this paper, we proposed the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic such that low-pass filter in the internal model for the periodic reference input can be set beforehand. The control characteristics of a robust stabilizing modified repetitive control system were presented, along with a design procedure for a robust stabilizing modified repetitive controller with specified input-output characteristic. Finally, a numerical example was shown to illustrate the effectiveness of the proposed parameterization. Advantages of the modified repetitive control system using the proposed parameterization are that its input-output characteristic is easily specified and the system to guarantee the robust stability is easy to design. This control system is expected to have practical applications in, for example, engines, electrical motors and generators, converters, and other machines that perform cyclic tasks.

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