The parameterization of all two-degree-of-freedom strongly stabilizing controllers

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ABSTRACT

When a plant can be stabilized by using a stable controller, the controller is said to be a strongly stabilizing controller. The importance of strong stabilizations is to solve some problems occurred by using unstable stabilizing controllers, for example, feedback control systems become high sensitive for disturbances. Parameterizations of all strongly stabilizable plants and of all stable stabilizing controllers have already proposed. However, stable stabilizing controllers designed by using their parameterization cannot specify the input-output characteristic and the feedback characteristic separately. One of the ways to specify these characteristics separately is to use a two-degree-of-freedom control system. However, the parameterization of all two-degree-of-freedom strongly stabilizing controllers has not been examined.

The purpose of this paper is to propose the parameterization of all two-degree-of-freedom strongly stabilizing controllers for strongly stabilizable plants.

Keywords: Strong Stabilization, Two-Degree-of-Freedom Control

1. INTRODUCTION

In this paper, we examine the parameterization of all two-degree-of-freedom strongly stabilizing controllers for strongly stabilizable plants. The parameterization in the control theory is to express the necessary and sufficient condition to exist a stabilizing controller [1–8] or a stabilizable plant [9] by parameter. Since this parameterization can successfully search for all proper stabilizing controllers or all stabilizable plants, it is used as a tool for many control problems.

The strong stabilization is a control method to make control systems stable by stable controllers [4, 11–18]. The importance of strong stabilizations is to solve some problems occurred by using unstable stabilizing controllers as follows. Using unstable stabilizing controllers, unstable poles of stabilizing controller make the closed-loop transfer function have zeros in

right half plane. This occurs to make closed-loop systems very sensitive to disturbances and reduce the tracking performance to reference inputs [4, 10, 11]. In addition, if the feedback connection of feedback control systems is cut by breakdown, that is, control systems become feed-forward control systems, unstable poles of stabilizing controllers become unstable poles of control systems. Thus, control systems become unstable even if plants are stable. From above reasons, it is desirable in practice that control systems are stabilized by stable stabilizing controllers [11].

It is obvious that any plant can be stabilized by stable controllers. On the condition of strongly stabilizable plants, Youla et al. clarified the necessary and sufficient condition that a given plant can be stabilized by stable controllers [4, 12]. This condition is called p.i.p. (parity interlacing property) and used for the tool to confirm whether a given plant is strongly stabilizable or not. In addition, Youla et al. proposed a method to find strongly stabilizing controllers using Nevanlinna-Pick interpolation [4, 12]. In the method proposed by Youla et al., there exists a problem that the resulting controller may become high-order and irrational function [13, 14]. In order to design controllers and to tune parameters easily, it is desirable that stabilizing controllers are low-order and rational functions. To overcome this problem, Dorato et al. [14], Ganesh and Pearson [15], and Ito et al. [16] proposed the way to find low-order and rational strongly stabilizing controllers for single-input/single-output systems using Nevanlinna-Pick interpolation. In addition, Saif et al. proposed that way for multiple-input/multiple-output systems using Nevanlinna-Pick interpolation[17].

In this way, the study on strongly stabilizing controllers is considered, but the study on strongly stabilizable plants is almost nothing. For example, the strongly stabilizable plants is not parameterized. If this is done, it becomes easily to find all strongly stabilizing controllers for strongly stabilizable plants. From this viewpoint, Hoshikawa et al. clarified the parameterization of all strongly stabilizable plants, and showed that strongly stabilizable plants can be represented by a particular feedback control structure [18]. In addition, they proposed the parameterization of all strongly stabilizing controllers for above plants [18]. From their result, we can easily find whether a given plant can be stabilized by stable controllers

or not, and easily design stable controllers for given plant. However, using stable controllers, the response for step reference input has a steady state error, because stable controllers have no pole at the origin. In addition, stable stabilizing controllers designed by using their parameterization cannot specify the input-output characteristic and the feedback characteristic separately. That is, when we specify one characteristic, the other characteristic is also decided. From the practical point of view, it is desirable that the input-output characteristic and the feedback characteristic are specified separately. One of the ways to specify these characteristics separately is to use a two-degree-of-freedom control system. In addition, the two-degree-of-freedom control system has a possibility to realize no steady state error. However, the parameterization of all two-degree-of-freedom strongly stabilizing controllers has not been examined. From the fact that the parameterization is useful to design stabilizing controllers [1–8], in order to design a two-degree-of-freedom control system easily, the problem to clarify the parameterization of all two-degree-of-freedom strongly stabilizing controllers is important to solve.

The purpose of this paper is to propose the parameterization of all two-degree-of-freedom strongly stabilizing controllers for strongly stabilizable plants. First, a two-degree-of-freedom strongly stabilizing controller is defined. Next, the parameterization of all two-degree-of-freedom strongly stabilizing controllers for strongly stabilizable plants is clarified. In addition, a design method to suppose the breakdown that the feedback connection is cut is presented. Finally, numerical examples are illustrated to show the effectiveness of the proposed method.

Notation

R	The set of real numbers.
$R(s)$	The set of real rational functions with s.
RH_∞	The set of real rational functions, whose poles are in the open left half plane of complex plane and the relative degree is zero or more numbers.
$\mathcal{U}$	The set of unimodular functions on RH_∞. That is, $U(s) \in \mathcal{U}$ implies both $U(s) \in RH_\infty$ and $U^{-1}(s) \in RH_\infty$.

2. TWO-DEGREE-OF-FREEDOM STRONG STABILIZATION AND PROBLEM FORMULATION

Consider the two-degree-of-freedom control system shown in Fig. 1 that can specify the input-output characteristic and the feedback characteristic separately. Here, $G(s) \in R(s)$ is the plant, $C(s)$ is the two-degree-of-freedom controller written by

$$C(s) = \begin{bmatrix} C_1(s) & -C_2(s) \end{bmatrix}, \quad (1)$$
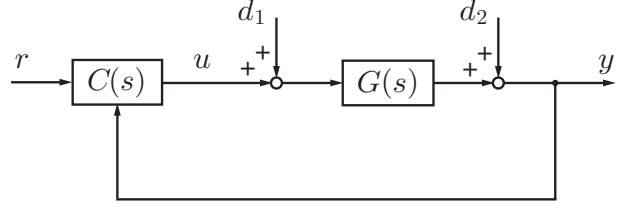
Fig.1: *Two-degree-of-freedom control system*

$u(s)$ is the control input and written by

$$
\begin{aligned}
u(s) &= C(s) \begin{bmatrix} r(s) \\ y(s) \end{bmatrix} = \begin{bmatrix} C_1(s) & -C_2(s) \end{bmatrix} \begin{bmatrix} r(s) \\ y(s) \end{bmatrix},
\end{aligned}
\quad (2)
$$

$r(s)$ is the reference input, $d_1(s)$ and $d_2(s)$ are disturbances and $y(s)$ is the output. In the following, we call $C_1(s) \in R(s)$ the feed-forward controller and $C_2(s) \in R(s)$ the feedback controller. From the definition of internal stability [4], when all transfer functions $V_i(s)(i = 1, \ldots, 6)$ written by

$$
\begin{bmatrix} u(s) \\ y(s) \end{bmatrix} = \begin{bmatrix} V_1(s) & V_2(s) & V_{c1}(s) \\ V_4(s) & V_5(s) & V_6(s) \end{bmatrix} \begin{bmatrix} r(s) \\ d_1(s) \\ d_2(s) \end{bmatrix} \quad (3)
$$

are stable, the two-degree-of-freedom control system in Fig. 1 is stable.

The strong stabilization is a control method that makes a given plant stable by using stable controller [4, 11–18]. According to the result of Hoshikawa et al. [18], $G(s)$ is a strongly stabilizable plant if and only if $G(s)$ is written by the form of

$$G(s) = \frac{Q_1(s)}{1 - Q_1(s)Q_2(s)}, \quad (4)$$

where $Q_1(s) \in RH_\infty$ and $Q_2(s) \in RH_\infty$ are any functions. In addition, Hoshikawa et al. gave the parameterization of all strongly stabilizing controllers for strongly stabilizable plants in (4) [18]. However, using stable controllers, the response for step reference input has a steady state error, because stable controllers have no pole at the origin. In addition, stable stabilizing controllers designed by using the parameterization in [18] cannot specify the input-output characteristic and the feedback characteristic separately, because one stabilizing controller specifies them. That is, when we specify one characteristic, the other characteristic is also decided. From the practical point of view, it is desirable that the input-output characteristic and the feedback characteristic are specified separately. One of the ways to specify these characteristics separately is to use a two-degree-of-freedom control system. However, the parameterization of all two-degree-of-freedom strongly stabilizing controllers has not been examined.

From this point of view, we propose the concept of a two-degree-of-freedom strongly stabilizing controller as follows:

Definition 1: (two-degree-of-freedom strongly stabilizing controller)

We call the controller $C(s)$ in (1) a "two-degree-of-freedom strongly stabilizing controller", if following expressions hold true:

1. The feed-forward controller $C_1(s)$ and the feedback controller $C_2(s)$ in (1) are stable.

2. The two-degree-of-freedom control system in Fig. 1 is stable. That is, all transfer functions $V_i(s)(i = 1, \ldots, 6)$ in (3) are all stable.

The problem considered in this paper is to obtain the parameterization of all two-degree-of-freedom strongly stabilizing controllers $C(s)$ defined in Definition 1.

3. THE   PARAMETERIZATION   OF   ALL TWO-DEGREE-OF-FREEDOM STRONGLY STABILIZING CONTROLLERS

In this section, we propose the parameterization of all two-degree-of-freedom strongly stabilizing controllers $C(s)$ for strongly stabilizable plants $G(s)$ written by the form of (4).

This parameterization is summarized in the following theorem.

Theorem 1: $G(s)$ in Fig. 1 is assumed to be strongly stabilizable, i.e. that is assumed to be written by (4). $C(s)$ is a two-degree-of-freedom strongly stabilizing controller for control system in Fig. 1 if and only if $C(s)$ is written by (1), where

$$C_1(s) = \frac{Q_{c1}(s)}{\hat{Q}(s)}, \tag{5}$$

$$C_2(s) = Q_2(s) + \frac{Q(s)}{1 - Q_1(s)Q(s)}, \tag{6}$$

$Q_{c1}(s) \in RH_\infty$ is any function, $Q(s)$ is given by

$$Q(s) = \frac{1 - \hat{Q}(s)}{Q_1(s)}, \tag{7}$$

$\hat{Q}(s) \in \mathcal{U}$ is any function to make $Q(s)$ proper and to satisfy

$$\frac{1}{(s - s_i)^{m_i - 1}} \left(1 - \hat{Q}(s)\right)\bigg|_{s=s_i} = 0 \ \ (\forall i = 1, \ldots, n), \tag{8}$$

$s_i(i = 1, \ldots, n)$ are unstable zeros of $Q_1(s)$ and multiplicities of $s_i(i = 1, \ldots, n)$ are denoted by $m_i(i = 1, \ldots, n)$.

Proof: First, the necessity is shown. That is, we show that if stable controllers $C_1(s)$ and $C_2(s)$ make the control system in Fig. 1 stable, that is, all transfer functions $V_i(s)(i = 1, \ldots, 6)$ in (3) are stable, then $C_1(s)$ and $C_2(s)$ are written by (5) and (6), $Q(s) \in$

RH_∞ is written by (7) and $\hat{Q}(s) \in \mathcal{U}$ satisfies (8). The transfer functions $V_i(s)(i = 1, \ldots, 6)$ in (3) are written as

$$V_1(s) = \frac{C_1(s)}{1 + C_2(s)G(s)}, \tag{9}$$

$$V_2(s) = -\frac{C_2(s)G(s)}{1 + C_2(s)G(s)}, \tag{10}$$

$$V_3(s) = -\frac{C_2(s)}{1 + C_2(s)G(s)}, \tag{11}$$

$$V_4(s) = \frac{C_1(s)G(s)}{1 + C_2(s)G(s)}, \tag{12}$$

$$V_5(s) = \frac{G(s)}{1 + C_2(s)G(s)} \tag{13}$$

and

$$V_6(s) = \frac{1}{1 + C_2(s)G(s)}. \tag{14}$$

First, we show that the feedback controller $C_2(s)$ is written by (6). From the assumption that transfer functions in (10), (11), (13) and (14) are stable, $C_2(s)$ makes $G(s)$ stable. This means that $C_2(s)$ is a stabilizing controller for $G(s)$. Using the parameterization of all stabilizing controllers in [1], $C_2(s)$ is then written by

$$C_2(s) = \frac{X(s) + D(s)Q(s)}{Y(s) - N(s)Q(s)}, \tag{15}$$

where $N(s)$ and $D(s)$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \frac{N(s)}{D(s)}, \tag{16}$$

$X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are any functions satisfying

$$N(s)X(s) + D(s)Y(s) = 1 \tag{17}$$

and $Q(s) \in RH_\infty$ is any function. Since strongly stabilizable plants $G(s)$ is written by the form of (4), $N(s)$ and $D(s)$ satisfying (16) are written by

$$N(s) = Q_1(s) \tag{18}$$

and

$$D(s) = 1 - Q_1(s)Q_2(s), \tag{19}$$

respectively. From (18) and (19), a pair of $X(s)$ and $Y(s)$ satisfying (17) is written by

$$X(s) = Q_2(s) \tag{20}$$

and

$$Y(s) = 1, \tag{21}$$

respectively. Substituting (18), (19), (20) and (21) for (15), we have (6). From $Q_2(s) \in RH_\infty$, $Q(s) \in RH_\infty$ and the assumption that $C_2(s) \in RH_\infty$, $1 - Q_1(s)Q(s) \in \mathcal{U}$ is satisfied. Let $\hat{Q}(s) \in \mathcal{U}$ as

$$\hat{Q}(s) = 1 - Q_1(s)Q(s). \tag{22}$$

From easy manipulation and (22), we have (7). Since $Q(s) \in RH_\infty$ and $\hat{Q}(s) \in \mathcal{U}$, if $Q(s)$ in (7) is unstable, then unstable poles of $Q(s)$ are equal to unstable zeros of $Q_1(s)$. Therefore, in order to make $Q(s)$ stable and proper, $\hat{Q}(s)$ must make $Q(s)$ proper and satisfy (8).

Next, we show that the feed-forward controller $C_1(s)$ is written by (5). Using $C_2(s)$ in (6), the transfer function in (9) and (12) are written by

$$V_1(s) = C_1(s)\hat{Q}(s)\left(1 - Q_1(s)Q_2(s)\right) \tag{23}$$

and

$$V_4(s) = C_1(s)Q_1(s)\hat{Q}(s), \tag{24}$$

respectively. Since $C_1(s)$, $V_1(s)$ in (23) and $V_4(s)$ in (24) are stable, $C_1(s)$ is written by

$$C_1(s) = \frac{Q_{c1}(s)}{\hat{Q}(s)}. \tag{25}$$

Here, $Q_{c1}(s) \in RH_\infty$ is any function. Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, we show that if $C_1(s)$ and $C_2(s)$ are written by (5) and (6), $Q(s) \in RH_\infty$ is written by (7) and $\hat{Q}(s) \in \mathcal{U}$ satisfies (8), then $C_1(s) \in RH_\infty$, $C_2(s) \in RH_\infty$, and $C_1(s)$ and $C_2(s)$ make the control system in Fig. 1 stable. Since $Q_{c1}(s) \in RH_\infty$ and $\hat{Q}(s) \in \mathcal{U}$, $C_1(s)$ in (5) is stable. From (7), $C_2(s)$ in (6) is written by

$$C_2(s) = Q_2(s) + \frac{1 - \hat{Q}(s)}{Q_1(s)\hat{Q}(s)}. \tag{26}$$

Since $Q_2(s) \in RH_\infty$ and $\hat{Q}(s) \in \mathcal{U}$, if $C_2(s)$ in (26) is unstable, unstable poles of $C_2(s)$ in (26) are unstable zeros of $Q_1(s)$. From the assumption that $\hat{Q}(s) \in \mathcal{U}$ satisfies (8), unstable zeros of $Q_1(s)$ are not poles of $(1 - \hat{Q}(s))/(Q_1(s)\hat{Q}(s))$. Thus, $C_2(s)$ in (26) is stable.

Using $C_1(s)$ in (5) and $C_2(s)$ in (6), transfer functions $V_i(s)(i = 1, \ldots, 6)$ are written as

$$V_1(s) = Q_{c1}(s)\left(1 - Q_1(s)Q_2(s)\right), \tag{27}$$

$$V_2(s) = -Q_1(s)\left(Q_2(s)\hat{Q}(s) + Q(s)\right), \tag{28}$$

$$\begin{aligned} V_3(s) \\ = -\left(1 - Q_1(s)Q_2(s)\right)\left(Q_2(s)\hat{Q}(s) + Q(s)\right), \end{aligned} \tag{29}$$

$$V_4(s) = Q_1(s)Q_{c1}(s), \tag{30}$$

$$V_5(s) = Q_1(s)\hat{Q}(s) \tag{31}$$

and

$$V_6(s) = \hat{Q}(s)\left(1 - Q_1(s)Q_2(s)\right). \tag{32}$$

Since $Q_1(s) \in RH_\infty$, $Q_2(s) \in RH_\infty$, $Q_{c1}(s) \in RH_\infty$, $Q(s) \in RH_\infty$ and $\hat{Q}(s) \in \mathcal{U}$, (27), (28), (29), (30), (31) and (32) are all stable. Thus, the sufficiency has been shown.

We have thus proved Theorem 1. ∎

Next, we explain control characteristics of the control system in Fig. 1 using the parameterization of all two-degree-of-freedom strongly stabilizing controllers in (5) and (6). First, the input-output characteristic is shown. The transfer function from the reference input $r(s)$ to the output $y(s)$ is written by

$$\frac{y(s)}{r(s)} = Q_1(s)Q_{c1}(s). \tag{33}$$

In order for the output $y(s)$ to follow the step reference input $r(s) = 1/s$ without steady state error,

$$Q_1(0)Q_{c1}(0) = 1 \tag{34}$$

must be satisfied. Therefore, $Q_{c1}(s)$ is selected satisfying

$$Q_{c1}(0) = \frac{1}{Q_1(0)}. \tag{35}$$

From (35), we find that the one-degree-of-freedom control in [18] cannot realize no steady state error, but the two-degree-of-freedom control in this method can do that.

Next, the disturbance attenuation characteristic, which is one of the feedback characteristic is shown. Transfer functions from the disturbance $d_1(s)$ to the output $y(s)$ and from the disturbance $d_2(s)$ to the output $y(s)$ of the control system in Fig. 1 are written as

$$\frac{y(s)}{d_1(s)} = Q_1(s)\hat{Q}(s) \tag{36}$$

and

$$\frac{y(s)}{d_2(s)} = \left(1 - Q_1(s)Q_2(s)\right)\hat{Q}(s), \tag{37}$$

respectively. In order to attenuate step disturbances $d_1(s) = 1/s$ and $d_2(s) = 1/s$ effectively,

$$\hat{Q}(0) = 0 \tag{38}$$

must be satisfied. Since $\hat{Q}(s) \in \mathcal{U}$, $\hat{Q}(s)$ has no zero at the origin. This implies that we cannot design the feedback controller such that step disturbances

$d_1(s) = 1/s$ and $d_2(s) = 1/s$ are attenuated effectively. However, if $\hat{Q}(s)$ is chosen to satisfy

$$\hat{Q}(0) \simeq 0, \tag{39}$$

step disturbances $d_1(s) = 1/s$ and $d_2(s) = 1/s$ are attenuated. A design method for $\hat{Q}(s)$ satisfying (39) is described in [18].

In this way, we find that the input-output characteristic is specified by $Q_{c1}(s)$ in (5) and the disturbance attenuation characteristic is specified by $\hat{Q}(s)$ in (7). That is, this method can specify the input-output characteristic and the disturbance attenuation characteristic separately.

4. A DESIGN METHOD TO SUPPOSE THE BREAKDOWN

In this section, a design method to converge to the specified value after the breakdown that the feedback connection is cut is presented. When the feedback connection is cut by breakdown, that is, the control system in Fig. 1 becomes feed-forward control system, the transfer function from the reference input $r(s)$ to the output $y(s)$ is written by

$$\frac{y(s)}{r(s)} = \frac{Q_1(s)Q_{c1}(s)}{\hat{Q}(s)\left(1 - Q_1(s)Q_2(s)\right)}. \tag{40}$$

Therefore, when we make the final value of (40) converge to that of (33) times $l \in R$ for the step reference input,

$$\frac{Q_1(0)Q_{c1}(0)}{\hat{Q}(0)\left(1 - Q_1(0)Q_2(0)\right)} = lQ_1(0)Q_{c1}(0) \tag{41}$$

must be satisfied. From (41), $\hat{Q}(s)$ is designed to satisfy

$$\hat{Q}(0) = \frac{1}{l\left(1 - Q_1(0)Q_2(0)\right)}. \tag{42}$$

In this way, when we design $\hat{Q}(s)$ to satisfy (42), we can make the final value of (40) converge to that of (33) times $l \in R$ for the step reference input.

5. NUMERICAL EXAMPLE

In this section, two numerical examples are illustrated to show the effectiveness of the proposed method.

5.1 Two-degree-of-freedom strongly stabilizing controller design

In this subsection, a numerical example is illustrated to show that the plant written by the form of (4) can be stabilized by using stable controllers.

Consider the problem to design a two-degree-of-freedom control system for unstable plant $G(s)$ written by

$$G(s) = \frac{s + 10}{(s - 2)(s + 5)}. \tag{43}$$

Since $G(s)$ in (43) is rewritten by the form of (4), $G(s)$ in (43) is strongly stabilizable, where

$$Q_1(s) = \frac{s + 10}{(s + 2)(s + 2.5)} \tag{44}$$

and

$$Q_2(s) = 1.5. \tag{45}$$

In order for the output $y(s)$ to follow the step reference input $r(s) = 1/s$ without steady state error, $Q_{c1}(s)$ is designed to satisfy (35) as

$$Q_{c1}(s) = \frac{0.1(2.9s + 5)}{0.01s + 1}. \tag{46}$$

In addition, from the result in [18], in order to attenuate step disturbances, $\hat{Q}(s)$ is designed as

$$\hat{Q}(s) = 1 - Q_1(s)\bar{Q}(s), \tag{47}$$

where $\bar{Q}(s) \in RH_\infty$ is written by

$$\bar{Q}(s) = \frac{k}{Q_{1o}(s)(\tau s + 1)^\alpha}, \tag{48}$$

$\tau \in R$, $Q_{1o}(s) \in RH_\infty$ is an outer function of $Q_1(s)$, α is an arbitrary positive integer to make $\hat{Q}(s)$ proper and $k \in R$ is a real number satisfying $k < 1$. Therefore, we settle $\tau = 0.1$, $\alpha = 1$, $k = 0.999$ and $Q_{1o}(s) = Q_1(s)$ in (48). Then we have $\bar{Q}(s)$ as

$$\bar{Q}(s) = \frac{0.999(s + 2)(s + 2.5)}{(s + 10)(0.1s + 1)}. \tag{49}$$

Using mentioned parameters, we have a two-degree-of-freedom strongly stabilizing controller $C(s)$ written by (1), where the feed-forward controller $C_1(s)$ and the feedback controller $C_2(s)$ are written by

$$C_1(s) = \frac{29(s + 10)(s + 1.667)}{(s + 0.01)(s + 100)} \tag{50}$$

and

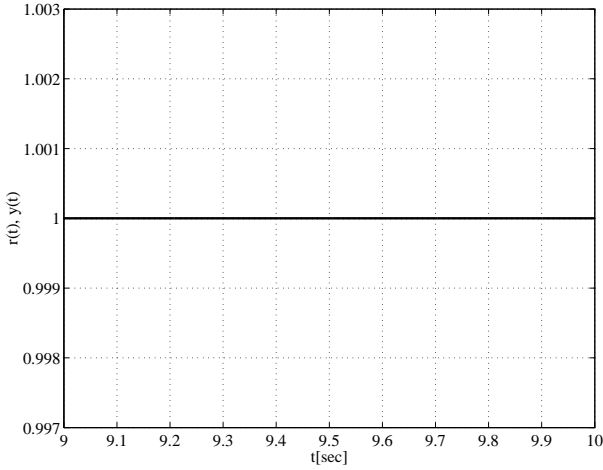$$C_2(s) = \frac{11.49(s + 1.044)(s + 4.175)}{(s + 0.01)(s + 10)}, \tag{51}$$

respectively.

Using designed controller $C(s)$, the response of the output $y(t)$ of the control system in Fig. 1 for the step reference input $r(t) = 1$ is shown in Fig. 2. The solid line shows the response of the output $y(t)$ and the broken line shows that of the step reference input $r(t) = 1$. Figure 2 shows that the control system in Fig. 1 is stable. In addition, in order to confirm that the response has no steady state error, an enlarged view from 9[sec] to 10[sec] of Fig. 2 is shown in Fig. 3. Figure 3 shows that the output $y(t)$ follows the step reference input $r(t) = 1$ without steady state error.
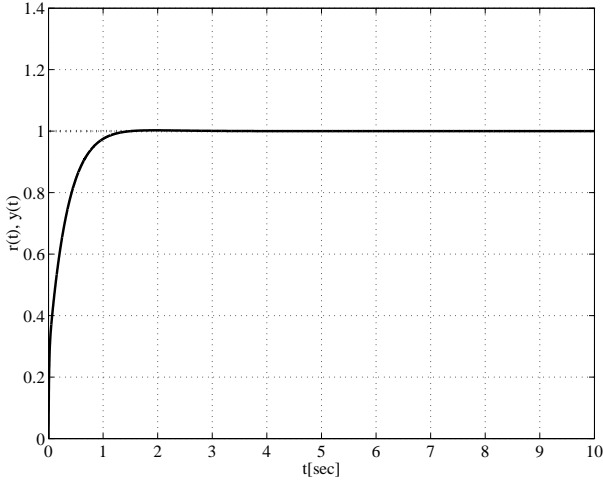
Fig.2: *Response of the output $y(t)$ of the control system in Fig. 1 for the step reference input $r(t) = 1$*
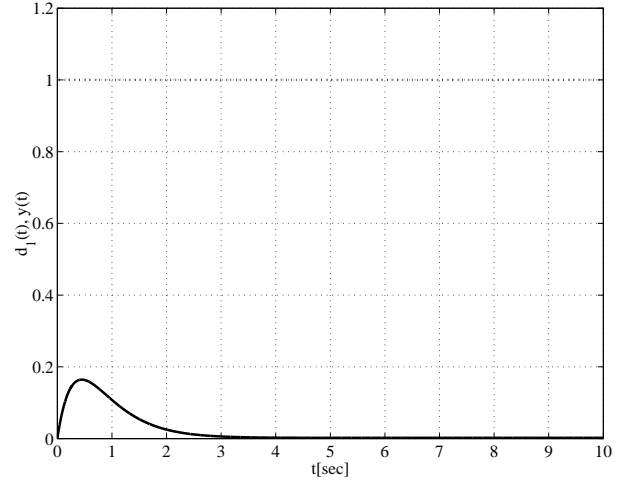
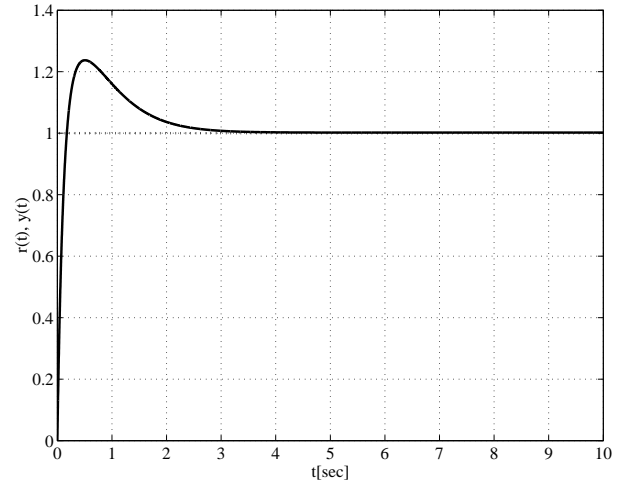**Fig.4:** *Response of the output $y(t)$ of the control system in Fig. 1 for the step disturbance $d_1(t) = 1$*

Fig.3: *Enlarged view from $9[\sec]$ to $10[\sec]$ of Fig. 2*

Fig.5: *Response of the output $y(t)$ of the control system in (52) for step reference input $r(t) = 1$*

On the other hand, when the step disturbance $d_1(t) = 1$ exists, the response of the output $y(t)$ of the control system in Fig. 1 is shown in Fig. 4. The solid line shows the response of the output $y(t)$ and the broken line shows that of the step disturbance $d_1(t) = 1$. Figure 4 show that the step disturbance $d_1(t)$ is attenuated effectively.

In addition, in order to compare responses, we show responses of control system written by

$$\begin{cases} y(s) & = & G(s)u(s) + d(s) \\ u(s) & = & C_2(s)\left(r(s) - y(s)\right) \end{cases}, \quad (52)$$

so-called one-degree-of-freedom control system. Figure 5 shows the response of the output $y(t)$ of one-degree-of-freedom control system in (52) for step reference input $r(t) = 1$ and Fig. 6 shows that for step disturbance $d_1(t) = 1$. By comparing Fig. 2 and Fig. 5, we find that the response of two-degree-of-freedom control system has no overshoot and the

settling time of two-degree-of-freedom control system is shorter than that of one-degree-of-freedom control system. On the other hand, by comparing Fig. 4 and Fig. 6, we find that both control systems have same disturbance attenuation characteristic. In this way, the input-output characteristic of two-degree-of-freedom control system is different from that of one-degree-of-freedom control system, and the disturbance attenuation characteristic of two-degree-of-freedom control system is same to that of one-degree-of-freedom control system.

In this way, we find that we can easily design two-degree-of-freedom strongly stabilizing controllers using the proposed method.
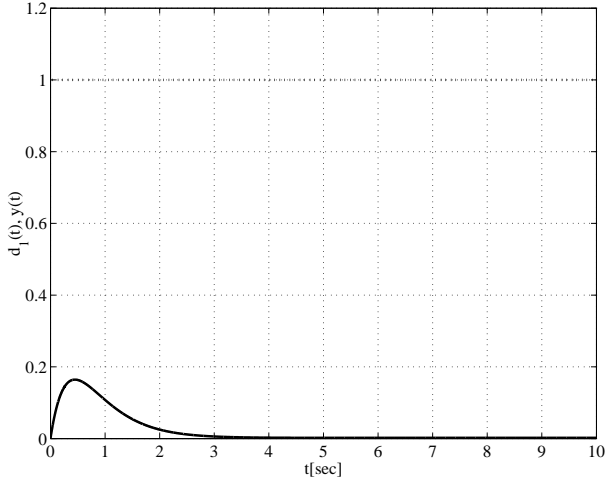
Fig.6: *Response of the output $y(t)$ of the control system in (52) for step disturbance $d_1(t) = 1$*

5.2 Controller design to suppose the breakdown

In this subsection, we consider to converge to the specified value after the breakdown that the feedback connection is cut.

Consider the problem to design a stable controller $C(s)$ to stabilize $G(s)$ written by

$$G(s) = \frac{s + 10}{(s + 4)(s + 5)}. \tag{53}$$

$G(s)$ in (53) is rewritten by the form of (4), $G(s)$ in (43) is strongly stabilizable, where

$$Q_1(s) = \frac{s + 10}{s^2 + 10s + 30} \tag{54}$$

and

$$Q_2(s) = 1. \tag{55}$$

In order for the output $y(s)$ to follow the step reference input $r(s) = 1/s$ without steady state error, $Q_{c1}(s)$ is designed to satisfy (35) as

$$Q_{c1}(s) = \frac{3}{0.1s + 1}. \tag{56}$$

In addition, in order to make the final value after the breakdown converge to the final value before the breakdown times 1.1, $\hat{Q}(s) \in \mathcal{U}$ must satisfy

$$\hat{Q}(0) = \frac{1}{1.1\left(1 - Q_1(0)Q_2(0)\right)}. \tag{57}$$

Therefore, $\hat{Q}(s)$ is designed as

$$\hat{Q}(s) = \frac{s + 6.818}{s + 5}. \tag{58}$$

Using mentioned parameters, we have a two-degree-of-freedom strongly stabilizing controller $C(s)$ written by (1), where the feed-forward controller $C_1(s)$

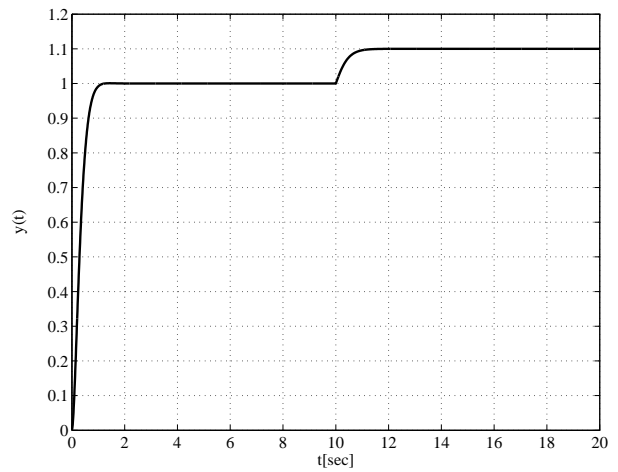and the feedback controller $C_2(s)$ are written by

$$C_1(s) = \frac{30(s + 5)}{(s + 6.818)(s + 10)} \tag{59}$$

and

$$C_2(s) = -\frac{0.8182(s - 3.333)(s + 5)}{(s + 6.818)(s + 10)}, \tag{60}$$

respectively.

Using designed controller $C(s)$, the response if the output $y(t)$ of the control system in Fig. 1 for the step reference input $r(t) = 1$ in the case that the feedback connection is cut by breakdown at 10[sec] is shown in Fig. 7. Figure 7 shows that the final value after

Fig.7: *Response of the output $y(t)$ of the control system in Fig. 1 in the case that the feedback connection is cut at 10[sec]*

the breakdown converges to the final value before the breakdown times 1.1.

In this way, using the presented method, we find that we can design a two-degree-of-freedom strongly stabilizing controller $C(s)$ to converge to the specified value after the breakdown that the feedback connection is cut.

6. CONCLUSION

In this paper, we clarified the parameterization of all two-degree-of-freedom strongly stabilizing controllers for strongly stabilizable plants. First, the two-degree-of-freedom strongly stabilizing controller was defined. Next, the parameterization of all two-degree-of-freedom strongly stabilizing controllers for strongly stabilizable plants was clarified. In addition, a design method to converge to the specified value after the breakdown that the feedback connection is cut was presented. Finally, numerical examples were illustrated to show the effectiveness of the proposed method.

References

[1] D.C. Youla, J.J. Bongiorno and H.A. Jabr, "Modern Wiener–Hopf design of optimal controllers Part I: The single-input-output case", *IEEE Transactions on Automatic Control*, Vol.21, pp.3-13, 1976.

[2] C.A. Desoer, R.W. Liu, J. Murray and R. Saeks, "Feedback system design-The fractional representation approach to analysis and synthesis", *IEEE Transactions on Automatic Control*, Vol.25, pp.399-412, 1980.

[3] G. Zames, "Feedback and optimal sensitivity: model reference transformations, multiplicative seminorms and approximate inverses, *IEEE Transactions on Automatic Control*, Vol.26, pp.301-320. 1981.

[4] M. Vidyasagar, *Control System Synthesis -A factorization approach-*, MIT Press, 1985.

[5] M. Morari and E. Zafiriou, *Robust Process Control*, Prentice Hall, 1989.

[6] J.J. Glaria and G. C. Goodwin, "A parametrization for the class of all stabilizing controllers for linear minimum phase plants", *IEEE Transactions on Automatic Control*, Vol.39, pp.433-434, 1994.

[7] K. Zhou, J.C. Doyle and K. Glover, *Robust and optimal control*, Prentice Hall, 1996.

[8] K. Yamada, "A parametrization for the class of all proper stabilizing controllers for linear minimum phase systems, *Preprints of the 9th IFAC/IFORS/IMACS/IFIP/ Symposium on Large Scale Systems: Theory and Applications*, pp.578-583, 2001.

[9] T. Hagiwara and K. Yamada and T. Sakanushi and S. Aoyama and A. C. Hoang, The parameterization of all plants stabilized by proportional controller, *The 25th International Technical Conference on Circuit/Systems Computers an Communications CD-ROM*, pp.76-78, 2010.

[10] L. Shaw, "Pole placement: stability and sensitivity of dynamic compensators", *IEEE Transactions on Automatic Control*, Vol.16, pp.210, 1971.

[11] Y.S. Chou, T.Z. Wu and J.L. Leu, On strong stabilization and H_∞ strong stabilization problems, *42nd IEEE Conference on Decision and Control*, Vol.5, pp.5155-5160, 2003.

[12] D.C. Youla, J.J. Bongiorno Jr. and C.N. Lu, "Single-loop feedback-stabilization of linear multivariable dynamical plants", *Automatica*, Vol.10, pp.159-173, 1974.

[13] M.C. Smith and K.P. Sondergeld, "On the order of stable compensators", *Automatica*, Vol.22, pp.127-129, 1986.

[14] P. Dorato, H.B. Park and Y. Li, "An algorithm for interpolation with units in H_∞ with applications to feedback stabilization", *Automatica*, Vol.25, No.3, pp.427-430, 1989.

[15] C. Ganesh and J.B. Pearson, "H_2-optimization with stable controllers", *Automatica*, Vol.25, No.4, pp.629-634, 1989.

[16] H. Ito, H. Ohmori and A. Sano, "Design of stable controllers attaining low H_∞ weighted sensitivity", *IEEE Transactions on Automatic Control*, Vol.38, No.3, pp.485-488, 1993.

[17] A. Saif, D.W. Gu and I. Postlethwaite, "Strong stabilization of MIMO systems via unimodular interpolation in H_∞", *Int. J. Contr*, Vol.69, pp.797-818, 1998.

[18] T. Hoshikawa, K. Yamada and Y. Tatsumi, "Parameterizations of all strongly stabilizable plants and of all stable stabilizing controllers", submitted for publication.

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