A Design Method for Robust Stabilizing Modified Repetitive Controllers for Multiple-Input/Multiple-Output Time-Delay Plants

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ABSTRACT

The modified repetitive control system is a type of servomechanism for a periodic reference input. When modified repetitive control design methods are applied to real systems, the influence of uncertainties in the plant must be considered. In some cases, uncertainties in the plant make the modified repetitive control system unstable, even though the controller was designed to stabilize the nominal plant. Recently, the parameterization of all robust stabilizing modified repetitive controllers was obtained by Yamada et al. In addition, Yamada et al. proposed the parameterization of all robust stabilizing modified repetitive controllers for time-delay plants. However, no paper has proposed the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants. In this paper, we expand the result by Yamada et al. and propose the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants.

Keywords: Repetitive Control, Modified Repetitive Controller, Uncertainty, Robust Stability, Parameterization, Multiple-Input/Multiple-Output Time-Delay Plant

1. INTRODUCTION

A repetitive control system is a type of servomechanism for the periodic reference input. That is, the repetitive control system follows the periodic reference input without steady state error, even if a periodic disturbance or an uncertainty exists in the plant [1–13]. It is difficult to design stabilizing controllers for the strictly proper plant, because the repetitive control system that follows any periodic reference input without steady state error is a neutral type of time-delay control system [11]. To design a repetitive control system that follows any periodic reference input without steady state error, the plant needs to be biproper [3–11]. In practice, the plant is strictly proper. Many design methods for repetitive control systems for strictly proper plants have been given [3–11]. These studies are divided into two types. One uses a low-pass filter [3–10] and the other uses an attenuator [11]. The latter is difficult to design because it uses a state variable time-delay in the repetitive controller [11]. The former has a simple structure and is easily designed. Therefore, the former type of repetitive control system is called the modified repetitive control system [3–10].

On the other hand, there exists an important control problem to find all stabilizing controllers named the parameterization problem [17–21]. At first, the parameterization of all stabilizing modified repetitive controllers was studied by Hara and Yamamoto [5]. In [5], since the stability sufficient condition of a modified repetitive control system is defined as an H_∞ norm problem, the parameterization of all stabilizing modified repetitive controllers is given by resolving the interpolation problem of Nevanlinna-Pick. Katoh and Funahashi gave the parameterization of all stabilizing modified repetitive controllers for minimum phase systems by solving exactly Bezout equation [13]. In [13], since the parameterization is not given based on stability sufficient condition that the modified repetitive control system is internally stable, this result is important in the sense that the class of modified repetitive controllers is extensive than a class of modified repetitive controllers given in [5]. However, in [13], the plant is assumed to be stable or be stabilized by local feedback control. This implies that [13] gave a parameterization of all stabilizing modified repetitive controllers for stable and minimum phase plants. That is [13] did not give the exact parameterization of all stabilizing modified repetitive controllers for minimum phase plants. In addition, in [13], it is assumed that the relative degree of low-pass filter in modified repetitive controller is equal to that of the plant. This implies that [13] gave the parameterization for special case of stabilizing modified repetitive controllers. Yamada and Okuyama overcame this problem and gave the parameteriza-

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tion of all stabilizing modified repetitive controllers for minimum phase systems those are not necessarily stable [14]. Yamada et al. [15] expanded the result in [14] and gave the parameterization of all stabilizing modified repetitive controllers for a certain class of non-minimum phase systems using the idea of parallel compensation technique and the solution of Bezout equation. Yamada et al. gave the parameterization of all stabilizing modified repetitive controllers for non-minimum phase systems [16]. In this way, the parameterization of all stabilizing modified repetitive controllers for non-minimum phase plants have been studied.

When modified repetitive control design methods are applied to real systems, the influence of uncertainties in the plant must be considered. In some cases, uncertainties in the plant make the modified repetitive control system unstable, even though the controller was designed to stabilize the nominal plant. The stability problem with uncertainty is known as the robust stability problem [26]. Yamada and Satoh proposed the parameterization of all robust stabilizing modified repetitive controllers [22]. Yamada et al. proposed the parameterization of all robust stabilizing modified repetitive controllers for time-delay plants [23, 24]. However, the method by Yamada et al. [23, 24] cannot be applied to multiple-input/multiple-output plants. Because, this method uses the characteristic of single-input/single-output systems. Many real plants have multiple-input and multiple-output, and include uncertainties and time-delays. In addition, the parameterization is useful to design stabilizing controllers [17–21]. Therefore, the problem of obtaining the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants is important.

In this paper, we propose the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants. This paper is organized as follows. In Section 2., the problem considered in this paper is described. In Section 3., the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants is clarified. In Section 4., control characteristics of a robust stabilizing modified repetitive control system are described. In Section 5., we present a design procedure of robust stabilizing modified repetitive control system. In Section 6., we show a numerical example to illustrate the effectiveness of the proposed method. Section 7. gives concluding remarks.

Notation

R	the set of real numbers.
R_+	$R \cup \{\infty\}$.
$R(s)$	the set of real rational functions with s.
RH_∞	the set of stable proper real rational functions.
H_∞	the set of stable causal functions.
$D^\perp$	orthogonal complement of D, i.e., $\left[\begin{array}{cc} D & D^\perp \end{array}\right]$ or $\left[\begin{array}{c} D \\ D^\perp \end{array}\right]$ is unitary.
A^T	transpose of A.
$A^\dagger$	pseudo inverse of A.
$\rho(\{\cdot\})$	spectral radius of $\{\cdot\}$.
$\bar{\sigma}(\{\cdot\})$	largest singular value of $\{\cdot\}$.
$\\|\{\cdot\}\\|_\infty$	H_∞ norm of $\{\cdot\}$.
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	represents the state space description $C(sI-A)^{-1}B+D$.
$\mathcal{L}\{\cdot\}$	the Laplace transformation of $\{\cdot\}$.
$\mathcal{L}^{-1}\{\cdot\}$	the inverse Laplace transformation of $\{\cdot\}$.
diag $(a_1, \cdots, a_n)$	an $n \times n$ diagonal matrix with a_i as its i-th diagonal element.

2. PROBLEM FORMULATION

Consider the unity feedback control system in

$$\left\{ \begin{array}{rcl} y & = & G(s)e^{-sL}u + d \\ u & = & C(s)(r-y) \end{array} \right. , \tag{1}$$

where $G(s)e^{-sL}$ is the multiple-input/multiple-output time-delay plant, $L > 0$ is the time-delay, $G(s) \in R^{m \times p}(s)$ is assumed to be stabilizable and detectable. $C(s)$ is the modified repetitive controller with m-th input and p-th output defined later, $u \in R^p$ is the control input, $d \in R^m$ is the disturbance, $y \in R^m$ is the output and $r \in R^m$ is the periodic reference input with period $T > 0$ satisfying

$$r(t+T) = r(t) \quad (\forall t \geq 0). \tag{2}$$

It is assumed that $m \leq p$ and $\operatorname{rank} G(s) = m$. The nominal plant of $G(s)e^{-sL}$ is denoted by $G_m(s)e^{-sL_m}$, where $G_m(s) \in R^{m \times p}(s)$. Both $G(s)$ and $G_m(s)$ are assumed to have no zero or pole on the imaginary axis. In addition, it is assumed that the number of poles of $G(s)$ in the closed right half plane is equal to that of $G_m(s)$. The relation between the plant $G(s)e^{-sL}$ and the nominal plant $G_m(s)e^{-sL_m}$ is written as

$$G(s)e^{-sL} = (e^{-sL_m}I + \Delta(s))G_m(s), \tag{3}$$

where $\Delta(s)$ is an uncertainty. The set of $\Delta(s)$ is all functions satisfying

$$\bar{\sigma}\{\Delta(j\omega)\} < |W_T(j\omega)| \quad (\forall \omega \in R_+), \tag{4}$$

where $W_T(s) \in R(s)$ is a stable rational function.

The robust stability condition for the plant $G(s)$ with uncertainty $\Delta(s)$ satisfying (4) is given by

$$\|T(s)W_T(s)\|_\infty < 1, \qquad (5)$$

where $T(s)$ is given by

$$T(s) = \left(I + G_m(s)e^{-sL_m}C(s)\right)^{-1} G_m(s)C(s). \qquad (6)$$

According to [3–10], the general form of modified repetitive controller $C(s)$ which makes the output y to follow the periodic reference input r with period T in (1) with small steady state error, is written by

$$C(s) \quad = \quad C_1(s) + C_2(s)C_r(s), \qquad (7)$$

where $C_1(s) \in R^{p\times m}(s)$, $C_2(s) \in R^{p\times m}(s)$ satisfying rank $C_2(s) = m$, $C_r(s)$ is the internal model for the periodic signal with period T written as

$$C_r(s) \quad = \quad e^{-sT}\left(I - q(s)e^{-sT}\right)^{-1}, \qquad (8)$$

where $q(s) \in R^{m\times m}(s)$ is a proper low-pass filter satisfying $q(0) = I$. From [3–10], if the low-pass filter $q(s)$ satisfy

$$\bar{\sigma}\left\{I - q(j\omega_i)\right\} \simeq 0 \quad (i = 0, 1, \dots, n), \qquad (9)$$

where $\omega_i (i = 0, 1, \dots, n)$ is the frequency component of the periodic reference input r written by

$$\omega_i = \frac{2\pi}{T} i \quad (i = 0, 1, \dots, n) \qquad (10)$$

and ω_n is the maximum frequency component of the periodic reference input r, then the output y in (1) follows the periodic reference input r with small steady state error.

The problem considered in this paper is to propose the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants. That is, we obtain the parameterization of all controllers $C(s)$ in (7) satisfying (5) for multiple-input/multiple-output time-delay plants $G(s)e^{-sL}$ in (3) with any uncertainty $\Delta(s)$ satisfying (4).

3. THE PARAMETERIZATION

In this section, we clarify the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants.

In order to obtain the parameterization of all robust stabilizing modified repetitive controllers for time-delay plants, we must see that controllers $C(s)$ satisfying (5). The problem of obtaining the controller $C(s)$, which is not necessarily a modified repetitive controller, satisfying (5) is equivalent to the following H_∞ control problem. In order to obtain the controller $C(s)$ satisfying (5), we consider the control system shown in Fig. 1. $P(s)$ is selected such that the transfer function from w to z in Fig. 1 is equal to

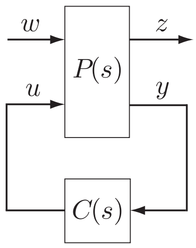

Fig.1: *Block diagram of H_∞ control problem*

$T(s)W_T(s)$. The state space description of $P(s)$ is, in general,

$$\begin{cases} \dot{x}(t) &= Ax(t) + B_1w(t) + B_2u(t - L_m) \\ z(t) &= C_1x(t) + D_{12}u(t) \\ y(t) &= C_2x(t) + D_{21}w(t) \end{cases}, \qquad (11)$$

where $A \in R^{n\times n}$, $B_1 \in R^{n\times m}$, $B_2 \in R^{n\times p}$, $C_1 \in R^{m\times n}$, $C_2 \in R^{m\times n}$, $D_{12} \in R^{m\times p}$, $D_{21} \in R^{m\times m}$, $x(t) \in R^n$, $w(t) \in R^m$, $z(t) \in R^m$, $u(t) \in R^p$ and $y(t) \in R^m$. $P(s)$ is called the generalized plant. $P(s)$ is assumed to satisfy following assumptions:

1. (C_2, A) is detectable, (A, B_2) is stabilizable.
2. D_{12} has full column rank, and D_{21} has full row rank.
3. rank $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} = n + p \quad (\forall\omega \in R_+)$,
rank $\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} = n + m \quad (\forall\omega \in R_+)$.
4. $C_1A^iB_2 = 0 \quad (i = 0, 1, 2, \dots)$.

Under these assumptions, from [25], following lemma holds true.

Lemma 1: There exists an H_∞ controller $C(s)$ for the generalized plant $P(s)$ in (11) if and only if there exists an H_∞ controller $C(s)$ for the generalized plant $\tilde{P}(s)$ written by

$$\begin{cases} \dot{q}(t) &= Aq(t) + B_1w(t) + \tilde{B}_2u(t) \\ \tilde{z}(t) &= C_1q(t) + D_{12}u(t) \\ \tilde{y}(t) &= C_2q(t) + D_{21}w(t) \end{cases}, \qquad (12)$$

where $\tilde{B}_2 = e^{-AL_m}B_2$. When $u(s) = C(s)\tilde{y}(s)$ is an H_∞ control input for the generalized plant $\tilde{P}(s)$ in (12),

$$u(t) \quad = \quad \mathcal{L}^{-1}\left\{C(s)\tilde{y}(s)\right\} \qquad (13)$$

is an H_∞ control input for the generalized plant $P(s)$ in (11), where

$$\tilde{y}(s) = \mathcal{L}\left\{y(t) + C_2\int_{-L_m}^{0} e^{-A(\tau+L_m)}B_2u(t+\tau)d\tau\right\}. \qquad (14)$$

From Lemma 1 and [26], the following lemma holds true.

Lemma 2: If controllers satisfying (5) exist, both

$$X\left(A-\tilde{B}_2D_{12}^{\dagger}C_1\right)+\left(A-\tilde{B}_2D_{12}^{\dagger}C_1\right)^TX + X\left\{B_1B_1^T-\tilde{B}_2\left(D_{12}^TD_{12}\right)^{-1}\tilde{B}_2^T\right\}X + \left(D_{12}^{\perp}C_1\right)^TD_{12}^{\perp}C_1=0 \quad (15)$$

and

$$Y\left(A-B_1D_{21}^{\dagger}C_2\right)^T+\left(A-B_1D_{21}^{\dagger}C_2\right)Y + Y\left\{C_1^TC_1-C_2^T\left(D_{21}D_{21}^T\right)^{-1}C_2\right\}Y + B_1D_{21}^{\perp}\left(B_1D_{21}^{\perp}\right)^T=0 \quad (16)$$

have solutions $X \geq 0$ and $Y \geq 0$ such that

$$\rho(XY)<1 \quad (17)$$

and both

$$A-\tilde{B}_2D_{12}^{\dagger}C_1+\left\{B_1B_1^T-\tilde{B}_2\left(D_{12}^TD_{12}\right)^{-1}\tilde{B}_2^T\right\}X \quad (18)$$

and

$$A-B_1D_{21}^{\dagger}C_2+Y\left\{C_1^TC_1-C_2^T\left(D_{21}D_{21}^T\right)^{-1}C_2\right\} \quad (19)$$

have no eigenvalue in the closed right half plane. Using X and Y, the parameterization of all controllers satisfying (5) is given by

$$C(s) = C_{11}(s)+C_{12}(s)Q(s)(I-C_{22}(s)Q(s))^{-1}C_{21}(s), \quad (20)$$

where

$$\begin{bmatrix} C_{11}(s) & C_{12}(s) \\ C_{21}(s) & C_{22}(s)\end{bmatrix} = \left[\begin{array}{c|cc} A_c & B_{c1} & B_{c2} \\ \hline C_{c1} & D_{c11} & D_{c12} \\ C_{c2} & D_{c21} & D_{c22}\end{array}\right], \quad (21)$$

$$A_c = A+B_1B_1^TX-\tilde{B}_2\left(D_{12}^{\dagger}C_1+E_{12}^{-1}\tilde{B}_2^TX\right) - (I-YX)^{-1}\left(B_1D_{21}^{\dagger}+YC_2^TE_{21}^{-1}\right)\left(C_2+D_{21}B_1^TX\right),$$

$$B_{c1} = (I-YX)^{-1}\left(B_1D_{21}^{\dagger}+YC_2^TE_{21}^{-1}\right),$$
$$B_{c2} = (I-YX)^{-1}\left(\tilde{B}_2+YC_1^TD_{12}\right)E_{12}^{-1/2},$$

$$C_{c1} = -D_{12}^{\dagger}C_1-E_{12}^{-1}\tilde{B}_2^TX,$$
$$C_{c2} = -E_{21}^{-1/2}\left(C_2+D_{21}B_1^TX\right),$$

$$D_{c11}=0,\ D_{c12}=E_{12}^{-1/2},\ D_{c21}=E_{21}^{-1/2},\ D_{c22}=0,$$

$$E_{12}=D_{12}^TD_{12},\ E_{21}=D_{21}D_{21}^T$$

and $Q(s) \in H_\infty^{p\times m}$ is any function satisfying $\|Q(s)\|_\infty < 1$.
$C(s)$ in (20) is written using Linear Fractional Transformation (LFT). Using homogeneous transformation, (20) is rewritten by

$$C(s) = (Z_{11}(s)Q(s)+Z_{12}(s))(Z_{21}(s)Q(s)+Z_{22}(s))^{-1} = \left(Q(s)\tilde{Z}_{21}(s)+\tilde{Z}_{22}(s)\right)^{-1}\left(Q(s)\tilde{Z}_{11}(s)+\tilde{Z}_{12}(s)\right), \quad (22)$$

where $Z_{ij}(s)(i=1,2;j=1,2)$ and $\tilde{Z}_{ij}(s)(i=1,2;j=1,2)$ are defined by

$$\begin{bmatrix} Z_{11}(s) & Z_{12}(s) \\ Z_{21}(s) & Z_{22}(s)\end{bmatrix} = \begin{bmatrix} C_{12}(s)-C_{11}(s)C_{21}^{-1}(s)C_{22}(s) & C_{11}(s)C_{21}^{-1}(s) \\ -C_{21}^{-1}(s)C_{22}(s) & C_{21}^{-1}(s)\end{bmatrix} \quad (23)$$

and

$$\begin{bmatrix} \tilde{Z}_{11}(s) & \tilde{Z}_{12}(s) \\ \tilde{Z}_{21}(s) & \tilde{Z}_{22}(s)\end{bmatrix} = \begin{bmatrix} C_{21}(s)-C_{22}(s)C_{12}^{-1}(s)C_{11}(s) & C_{12}^{-1}(s)C_{11}(s) \\ -C_{22}(s)C_{12}^{-1}(s) & C_{12}^{-1}(s)\end{bmatrix} \quad (24)$$

and satisfying

$$\begin{bmatrix} \tilde{Z}_{22}(s) & \tilde{Z}_{12}(s) \\ \tilde{Z}_{21}(s) & \tilde{Z}_{11}(s)\end{bmatrix}\begin{bmatrix} Z_{11}(s) & -Z_{12}(s) \\ -Z_{21}(s) & Z_{22}(s)\end{bmatrix} = I = \begin{bmatrix} Z_{11}(s) & -Z_{12}(s) \\ -Z_{21}(s) & Z_{22}(s)\end{bmatrix}\begin{bmatrix} \tilde{Z}_{22}(s) & \tilde{Z}_{12}(s) \\ \tilde{Z}_{21}(s) & \tilde{Z}_{11}(s)\end{bmatrix}. \quad (25)$$

Using Lemma 1, Lemma 2 and Remark 3, the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants is given by following theorem.

Theorem 1: If modified repetitive controllers satisfying (5) exist, both (15) and (16) have solutions $X \geq 0$ and $Y \geq 0$ such that (17) and both (18) and (19) have no eigenvalue in the closed right half plane. Using X and Y, the parameterization of all robust stabilizing modified repetitive control laws satisfying (5) is given by

$$u(t) = \mathcal{L}^{-1}\left\{C(s)\tilde{y}(s)\right\}, \quad (26)$$

where

$$\tilde{y}(s) = \mathcal{L}\left\{y(t)+C_2\int_{-L_m}^{0}e^{-A(\tau+L_m)}B_2u(t+\tau)d\tau\right\} \quad (27)$$

and

$$\begin{aligned} C(s) &= (Z_{11}(s)Q(s) + Z_{12}(s)) \\ &\quad (Z_{21}(s)Q(s) + Z_{22}(s))^{-1} \\ &= \left(Q(s)\tilde{Z}_{21}(s) + \tilde{Z}_{22}(s)\right)^{-1} \\ &\quad \left(Q(s)\tilde{Z}_{11}(s) + \tilde{Z}_{12}(s)\right), \end{aligned} \tag{28}$$

where $Z_{ij}(s)(i = 1,2; j = 1,2)$ and $\tilde{Z}_{ij}(s)(i = 1,2; j = 1,2)$ are defined by (23) and (24) and satisfying (25), $C_{ij}(s)(i = 1,2; j = 1,2)$ are given by (21) and $Q(s) \in H_\infty^{p\times m}$ is any function satisfying $\|Q(s)\|_\infty < 1$ and written by

$$\begin{aligned} Q(s) &= \left(Q_{n1}(s) + Q_{n2}(s)e^{-sT}\right) \\ &\quad \left(Q_{d1}(s) + Q_{d2}(s)e^{-sT}\right)^{-1}, \end{aligned} \tag{29}$$

$Q_{n1}(s) \in RH_\infty^{p\times m}$, $Q_{d1}(s) \in RH_\infty^{m\times m}$, $Q_{n2}(s) \in RH_\infty^{p\times m}$ and $Q_{d2}(s) \in RH_\infty^{m\times m}$ are any functions satisfying

$$\begin{aligned} &\bar{\sigma}\left\{Z_{22}(0)\left(Q_{d1}(0) + Q_{d2}(0)\right)\right. \\ &\quad \left.+Z_{21}(0)\left(Q_{n1}(0) + Q_{n2}(0)\right)\right\} = 0 \end{aligned} \tag{30}$$

and

$$\text{rank}\left(Q_{n2}(s) - Q_{n1}(s)Q_{d1}^{-1}(s)Q_{d2}(s)\right) = m. \tag{31}$$

Proof: First, the necessity is shown. That is, we show that if the modified repetitive controller $C(s)$ in (7) stabilizes the control system in (1) robustly, then $C(s)$ is written by (28) and (29), respectively. From Lemma 2 and Remark 3, the parameterization of all robust stabilizing controllers $C(s)$ for $G(s)e^{-sL}$ is written by (28), where $\|Q(s)\|_\infty < 1$. In order to prove the necessity, we will show that if $C(s)$ written by (28) works as a modified repetitive controller, then $Q(s)$ in (28) is written by (29). Substituting $C(s)$ in (7) for (28), we have (29), where

$$Q_{n1}(s) = N_{1n}(s)N_{2d}(s), \tag{32}$$

$$Q_{n2}(s) = N_{2n}(s), \tag{33}$$

$$Q_{d1}(s) = D_{1n}(s)D_{2d}(s)N_{1d}(s)N_{2d}(s) \tag{34}$$

and

$$Q_{d2}(s) = D_{2n}(s)N_{1d}(s)N_{2d}(s). \tag{35}$$

Here, $N_{1n}(s) \in RH_\infty^{p\times m}$, $N_{2n}(s) \in RH_\infty^{p\times m}$, $N_{1d}(s) \in RH_\infty^{m\times m}$, $N_{2d}(s) \in RH_\infty^{m\times m}$, $D_{1n}(s) \in RH_\infty^{m\times m}$, $D_{2n}(s) \in RH_\infty^{m\times m}$, $D_{1d}(s) \in RH_\infty^{m\times m}$ and $D_{2d}(s) \in RH_\infty^{m\times m}$ are coprime factors satisfying

$$\tilde{Z}_{21}(s)C_1(s) - \tilde{Z}_{11}(s) = D_{1n}(s)D_{1d}^{-1}(s), \tag{36}$$

$$\begin{aligned} &\left(\tilde{Z}_{21}(s)C_2(s) - \tilde{Z}_{21}(s)C_1(s)q(s) + \tilde{Z}_{11}(s)q(s)\right)D_{1d}(s) \\ &= D_{2n}(s)D_{2d}^{-1}(s), \end{aligned} \tag{37}$$

$$\begin{aligned} &\left(\tilde{Z}_{12}(s) - \tilde{Z}_{22}(s)C_1(s)\right)D_{1d}(s)D_{2d}(s) \\ &= N_{1n}(s)N_{1d}^{-1}(s) \end{aligned} \tag{38}$$

and

$$\begin{aligned} &-\left(\tilde{Z}_{22}(s)C_2(s) - \tilde{Z}_{22}(s)C_1(s)q(s) + \tilde{Z}_{12}(s)q(s)\right) \\ &D_{1d}(s)D_{2d}(s)N_{1d}(s) = N_{2n}(s)N_{2d}^{-1}(s). \end{aligned} \tag{39}$$

From (32) $\sim$ (35), all of $Q_{n1}(s)$, $Q_{n2}(s)$, $Q_{d1}(s)$ and $Q_{d2}(s)$ are included in RH_∞. Thus, we have shown that if $C(s)$ written by (7) stabilizes the control system in (1) robustly, $Q(s)$ in (28) is written by (29). Since $q(0) = I$, from (32)$\sim$(35) and (25), (30) holds true. In addition, from the assumption of rank $C_2(s) = m$ and from (37) and (39),

$$\text{rank } D_{2n}(s) = m \tag{40}$$

and

$$\text{rank } N_{2n}(s) = m \tag{41}$$

hold true. From (40), (41), (33) and (35), (31) is satisfied. Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, it is shown that if $C(s)$ and $Q(s) \in H_\infty^{p\times m}$ are settled by (28) and (29), respectively, then the controller $C(s)$ is written by the form in (7) and rank $C_2(s) = m$ hold true. Substituting (29) into (28), we have (7), where $C_1(s)$, $C_2(s)$ and $q(s)$ are denoted by

$$\begin{aligned} &C_1(s) \\ &= (Z_{11}(s)Q_{n1}(s) + Z_{12}(s)Q_{d1}(s)) \\ &\quad (Z_{21}(s)Q_{n1}(s) + Z_{22}(s)Q_{d1}(s))^{-1}, \end{aligned} \tag{42}$$

$$\begin{aligned} C_2(s) &= \left(Q_{n1}(s)Q_{d1}^{-1}(s)\tilde{Z}_{21}(s) + \tilde{Z}_{22}(s)\right)^{-1} \\ &\quad \left(Q_{n2}(s) - Q_{n1}(s)Q_{d1}^{-1}(s)Q_{d2}(s)\right) \\ &\quad (Z_{21}(s)Q_{n1}(s) + Z_{22}(s)Q_{d1}(s))^{-1} \end{aligned} \tag{43}$$

and

$$\begin{aligned} q(s) &= -(Z_{21}(s)Q_{n2}(s) + Z_{22}(s)Q_{d2}(s)) \\ &\quad (Z_{21}(s)Q_{n1}(s) + Z_{22}(s)Q_{d1}(s))^{-1}. \end{aligned} \tag{44}$$

We find that if $C(s)$ and $Q(s)$ are settled by (28) and (29), respectively, then the controller $C(s)$ is written by the form in (7). Substituting (30) into (44), we have $q(0) = I$. From (31) and (43),

$$\text{rank } C_2(s) = m \tag{45}$$

holds true. Thus, the sufficiency has been shown.

We have thus proved Theorem 1. ■

4. CONTROL CHARACTERISTICS

In this section, we describe control characteristics of control system in (1) using robust stabilizing modified repetitive controllers $C(s)$ in (28).

From Theorem 1, $Q(s)$ in (29) must be included in H_∞. Since $Q_{n1}(s) \in RH_\infty^{p\times m}$ and $Q_{n2}(s) \in RH_\infty^{p\times m}$ in (29), if

$$\left(Q_{d1}(s) + Q_{d2}(s)e^{-sT}\right)^{-1} \in H_\infty^{m\times m}, \qquad (46)$$

then $Q(s)$ satisfies $Q(s) \in H_\infty^{p\times m}$.

Next, we mention the input-output characteristic. The transfer function $S(s)$ from the periodic reference input r to the error $e = r - y$ is written by

$$S(s) = (I + C(s)G(s))^{-1} = S_n(s)S_d^{-1}(s), \qquad (47)$$

where

$$\begin{aligned} S_n(s) &= C_{21}^{-1}(s)\left\{I + (Q_{d2}(s) - C_{22}(s)Q_{n2}(s)) (Q_{d1}(s) - C_{22}(s)Q_{n1}(s))^{-1} e^{-sT}\right\} \\ &\quad (Q_{d1}(s) - C_{22}(s)Q_{n1}(s)) \end{aligned} \qquad (48)$$

and

$$\begin{aligned} S_d(s) &= Z_{22}(s)Q_{d1}(s) + Z_{21}(s)Q_{n1}(s) \\ &\quad + G(s)\left(Z_{12}(s)Q_{d1}(s) + Z_{11}(s)Q_{n1}(s)\right)e^{-sL} \\ &\quad + \left(Z_{22}(s)Q_{d2}(s) + Z_{21}(s)Q_{n2}(s)\right)e^{-sT} \\ &\quad + G(s)\left(Z_{12}(s)Q_{d2}(s) + Z_{11}(s)Q_{n2}(s)\right)e^{-s(T+L)}. \end{aligned} \qquad (49)$$

From (47), for frequency components $\omega_i (i = 0, 1, \ldots, n)$ in (10) of the periodic reference input r, if

$$\begin{aligned} &\bar{\sigma}\left\{I + (Q_{d2}(j\omega_i) - C_{22}(j\omega_i)Q_{n2}(j\omega_i)) (Q_{d1}(j\omega_i) - C_{22}(j\omega_i)Q_{n1}(j\omega_i))^{-1}\right\} \simeq 0 \\ &(i = 0, 1, \ldots, n), \end{aligned} \qquad (50)$$

then the output y in (1) follows the periodic reference input r with small steady state error.

Finally, we mention the disturbance attenuation characteristic. The transfer function $S(s)$ from the disturbance d to the output y is written by (47), (48) and (49). From (47), for the disturbance d with same frequency components $\omega_i (i = 0, 1, \ldots, n)$ in (10) of the periodic reference input r, if

$$\begin{aligned} &\bar{\sigma}\left\{I + (Q_{d2}(j\omega_i) - C_{22}(j\omega_i)Q_{n2}(j\omega_i)) (Q_{d1}(j\omega_i) - C_{22}(j\omega_i)Q_{n1}(j\omega_i))^{-1}\right\} \simeq 0 \\ &(i = 0, 1, \ldots, n), \end{aligned} \qquad (51)$$

then the disturbance d is attenuated effectively. For the frequency component ω_d of the disturbance d that is different from that of the periodic reference input r, that is $\omega_d \neq \omega_i$, even if

$$\bar{\sigma}\left\{I + (Q_{d2}(j\omega_d) - C_{22}(j\omega_d)Q_{n2}(j\omega_d)) (Q_{d1}(j\omega_d) - C_{22}(j\omega_d)Q_{n1}(j\omega_d))^{-1}\right\} \simeq 0, \qquad (52)$$

the disturbance d cannot be attenuated, because

$$e^{-j\omega_d T} \neq 1 \qquad (53)$$

and

$$\bar{\sigma}\left\{I + (Q_{d2}(j\omega_d) - C_{22}(j\omega_d)Q_{n2}(j\omega_d)) (Q_{d1}(j\omega_d) - C_{22}(j\omega_d)Q_{n1}(j\omega_d))^{-1} e^{-j\omega_d T}\right\} \neq 0. \qquad (54)$$

In order to attenuate this frequency component, we need to settle $Q_{n1}(s)$ and $Q_{d1}(s)$ satisfying

$$\bar{\sigma}\left\{Q_{d1}(j\omega_d) - C_{22}(j\omega_d)Q_{n1}(j\omega_d)\right\} \simeq 0. \qquad (55)$$

From above discussion, the role of $Q_{d1}(s)$ and $Q_{d2}(s)$ is to assure the stability of the control system in (1) by satisfying $Q(s) \in H_\infty^{p\times m}$. The role of $Q_{n2}(s)$ and $Q_{d2}(s)$ is to specify the input-output characteristic for the periodic reference input r and to specify the disturbance attenuation characteristic for the disturbance d with same frequency components ω_i $(i = 0, 1, \ldots, n)$ of the periodic reference input r. The role of $Q_{n1}(s)$ and $Q_{d1}(s)$ is to specify the disturbance attenuation characteristic for the disturbance d with frequency components $\omega_d \neq \omega_i$ $(\forall i = 0, 1, \ldots, n)$.

5. DESIGN PROCEDURE

In this section, a design procedure of robust stabilizing modified repetitive controllers C(s) for multiple-input/multiple-output time-delay plants is presented.

A design procedure of robust stabilizing modified repetitive controllers C(s) satisfying Theorem 1 is summarized as follows:

<u>Procedure</u>

Step 1) Obtain $C_{11}(s)$, $C_{12}(s)$, $C_{21}(s)$ and $C_{22}(s)$ by solving the robust stability problem using the Riccati equation based H_∞ control as Theorem 1.

Step 2) $Q_{d1}(s) \in RH_\infty^{m\times m}$ and $Q_{d2}(s) \in RH_\infty^{m\times m}$ are settled so that

$$\left(Q_{d1}(s) + Q_{d2}(s)e^{-sT}\right)^{-1} \in H_\infty^{m\times m} \qquad (56)$$

is satisfied.

Step 3) $Q_{n1}(s) \in RH_\infty^{p\times m}$ is settled so that for the frequency component ω_d of the disturbance d, $\bar{\sigma}(Q_{d1}(j\omega_d) - C_{22}(j\omega_d)Q_{n1}(j\omega_d)) \simeq 0$ is satisfied. To design $Q_{n1}(s)$ to satisfy $\bar{\sigma}(Q_{d1}(j\omega_d) -$

$C_{22}(j\omega_d)Q_{n1}(j\omega_d)) \simeq 0$, $Q_{n1}(s)$ is designed according to

$$Q_{n1}(s) = C_{22o}^{\dagger}(s)\bar{q}_d(s)Q_{d1}(s), \tag{57}$$

where $C_{22o}(s) \in RH_{\infty}^{m\times p}$ is an outer function of $C_{22}(s)$ satisfying

$$C_{22}(s) = C_{22i}(s)C_{22o}(s), \tag{58}$$

$C_{22i}(s) \in RH_{\infty}^{m\times m}$ is an inner function satisfying $C_{22i}(0) = I$, $\bar{q}_d(s)$ is a low-pass filter satisfying $\bar{q}_d(0) = I$, as

$$\begin{aligned}&\bar{q}_d(s)\\ &= \text{diag}\left\{\frac{1}{(1+s\tau_{d1})^{\alpha_{d1}}},\cdots,\frac{1}{(1+s\tau_{dm})^{\alpha_{dm}}}\right\}\end{aligned} \tag{59}$$

is valid, α_{di} $(i = 1,\ldots,m)$ are arbitrary positive integers to make $C_{22o}^{\dagger}(s)\bar{q}_d(s)$ proper and $\tau_{di} \in R$ $(i = 1,\ldots,m)$ are any positive real numbers satisfying

$$\bar{\sigma}\left[I - C_{22i}(j\omega_d)\text{diag}\left\{\frac{1}{(1+j\omega_d\tau_{d1})^{\alpha_{d1}}},\cdots,\frac{1}{(1+j\omega_d\tau_{dm})^{\alpha_{dm}}}\right\}\right] \simeq 0. \tag{60}$$

Step 4) $Q_{n2}(s) \in RH_{\infty}^{p\times m}$ is settled so that for the frequency component $\omega_i (i = 0,1,\ldots,n)$ of the periodic reference input r, $\bar{\sigma}\{I + (Q_{d2}(j\omega_i) - C_{22}(j\omega_i)Q_{n2}(j\omega_i))(Q_{d1}(j\omega_i) - C_{22}(j\omega_i)Q_{n1}(j\omega_i))^{-1}\} \simeq 0$ is satisfied. To design $Q_{n2}(s)$ to hold $\bar{\sigma}\{I + (Q_{d2}(j\omega_i) - C_{22}(j\omega_i)Q_{n2}(j\omega_i))(Q_{d1}(j\omega_i) - C_{22}(j\omega_i)Q_{n1}(j\omega_i))^{-1}\} \simeq 0$, $Q_{n2}(s)$ is designed according to

$$\begin{aligned}&Q_{n2}(s)\\ &= C_{22o}^{\dagger}(s)\left\{\bar{q}_r(s)\left(Q_{d1}(s) - C_{22}(s)Q_{n1}(s)\right)\right.\\ &\quad \left.+Q_{d2}(s)\right\}\end{aligned} \tag{61}$$

where $\bar{q}_r(s)$ is a low-pass filter satisfying $\bar{q}_r(0) = I$, as

$$\begin{aligned}&\bar{q}_r(s)\\ &= \text{diag}\left\{\frac{1}{(1+s\tau_{r1})^{\alpha_{r1}}},\cdots,\frac{1}{(1+s\tau_{rm})^{\alpha_{rm}}}\right\}\end{aligned} \tag{62}$$

is valid, α_{ri} $(i = 1,\ldots,m)$ are arbitrary positive integers to make $C_{22o}^{\dagger}(s)\bar{q}_r(s)$ proper and $\tau_{ri} \in R$ $(i = 1,\ldots,m)$ are any positive real numbers satisfying

$$\bar{\sigma}\left[I - C_{22i}(j\omega_i)\text{diag}\left\{\frac{1}{(1+j\omega_i\tau_{r1})^{\alpha_{r1}}},\cdots,\frac{1}{(1+j\omega_i\tau_{rm})^{\alpha_{rm}}}\right\}\right] \simeq 0 \ \ (i = 0,1,\ldots,n). \tag{63}$$

$Q_{d2}(s)$ is settled to make $C_{22o}^{\dagger}(s)Q_{d2}(s)$ proper.

Using above-mentioned procedure, from (44), $q(s)$ in (8) satisfies

$$\begin{aligned}q(j\omega_i) &= C_{22i}(j\omega_i)\bar{q}_r(j\omega_i)\\ &= C_{22i}(j\omega_i)\text{diag}\left\{\frac{1}{(1+j\omega_i\tau_{r1})^{\alpha_{r1}}},\cdots\right.\\ &\quad \left.,\frac{1}{(1+j\omega_i\tau_{rm})^{\alpha_{rm}}}\right\}.\end{aligned} \tag{64}$$

Therefore, for ω_i $(i = 0,1,\ldots,n)$ in (10), those are frequency components of the periodic reference input r, if $\tau_{ri}(i = 1,\ldots,m)$ in (64) are settled satisfying

$$\begin{aligned}&\bar{\sigma}\left\{I - q(j\omega_i)\right\}\\ &= \bar{\sigma}\left[I - C_{22i}(j\omega_i)\text{diag}\left\{\frac{1}{(1+j\omega_i\tau_{r1})^{\alpha_{r1}}},\cdots\right.\right.\\ &\quad \left.\left.,\frac{1}{(1+j\omega_i\tau_{rm})^{\alpha_{rm}}}\right\}\right]\\ &\simeq 0 \quad (\forall i = 0,1,\ldots,n),\end{aligned} \tag{65}$$

then the output y follows the periodic reference input r with small steady state error.

6. NUMERICAL EXAMPLE

In this section, a numerical example simulated by using MATLAB is shown to illustrate the effectiveness of proposed method.

Consider the problem to obtain the parameterization of all robust stabilizing modified repetitive controllers for time-delay plant $G(s)e^{-sL}$ written by

$$G(s)e^{-sL} = (e^{-sL_m}I + \Delta(s))G_m(s). \tag{66}$$

The nominal time-delay plant of $G(s)e^{-sL}$ and the upper bound $W_T(s)$ of the set of $\Delta(s)$ are given by

$$\begin{aligned}&G_m(s)e^{-sL_m}\\ &= \begin{bmatrix}\frac{s+3}{(s+2)(s+9)} & \frac{2}{(s+2)(s+9)}\\ \frac{s+3}{(s+2)(s+9)} & \frac{s+4}{(s+2)(s+9)}\end{bmatrix}e^{-0.3s},\end{aligned} \tag{67}$$

and

$$W_T(s) = \frac{2s+7}{s+10}, \tag{68}$$

where

$$G_m(s) = \begin{bmatrix}\frac{s+3}{(s+2)(s+9)} & \frac{2}{(s+2)(s+9)}\\ \frac{s+3}{(s+2)(s+9)} & \frac{s+4}{(s+2)(s+9)}\end{bmatrix} \tag{69}$$

and $L_m = 0.3$[sec]. The period T of the periodic reference input r in (2) is $T = 20$[sec]. Solving the robust stability problem using Riccati equation based H_{∞} control as Theorem 1, the parameterization of all robust stabilizing modified repetitive controllers $C(s)$

is obtained as (28). In order to hold $Q(s) \in H_\infty$ in (29), $Q_{d1}(s) \in RH_\infty$ and $Q_{d2}(s) \in RH_\infty$ in (29) are settled by

$$Q_{d1}(s) = I \quad (70)$$

and

$$Q_{d2}(s) = 0. \quad (71)$$

When $Q_{d1}(s)$ and $Q_{d2}(s)$ are set as (70) and (71), (46) satisfies

$$\left(Q_{d1}(s) + Q_{d2}(s)e^{-sT}\right)^{-1} = I \in H_\infty. \quad (72)$$

That is, $Q(s) \in H_\infty$ holds true.

In order for disturbances both

$$d(t) = \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} = \begin{bmatrix} 2\sin(0.1\pi t) \\ \sin(0.1\pi t) \end{bmatrix} \quad (73)$$

and

$$d(t) = \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} = \begin{bmatrix} 2\sin(0.05\pi t) \\ \sin(0.05\pi t) \end{bmatrix} \quad (74)$$

to be attenuated effectively and for the output $y(t) = [y_1(t), y_2(t)]^T$ to follow the periodic reference input

$$r(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} = \begin{bmatrix} 2\sin(0.1\pi t) \\ \sin(0.1\pi t) \end{bmatrix} \quad (75)$$

with small steady state error, $Q_{n1}(s)$ and $Q_{n2}(s)$ are settled by (57) and (61), respectively, where

$$\bar{q}_r(s) = \begin{bmatrix} \frac{1}{0.02s+1} & 0 \\ 0 & \frac{1}{0.02s+1} \end{bmatrix}, \quad (76)$$

$$\bar{q}_d(s) = \begin{bmatrix} \frac{1}{0.02s+1} & 0 \\ 0 & \frac{1}{0.02s+1} \end{bmatrix}, \quad (77)$$

$$C_{22o}(s) = C_{22}(s) \quad (78)$$

and

$$C_{22i}(s) = I. \quad (79)$$

The largest singular value plot of $Q(s)$ is shown in Fig. 2. Figure 2 shows that the designed $Q(s)$ satisfies $\|Q(s)\|_\infty < 1$.

When $\Delta(s)$ is given by

$$\Delta(s) = \begin{bmatrix} \frac{s-100}{s+500} & \frac{200}{s+600} \\ \frac{200}{s+500} & \frac{s-100}{s+600} \end{bmatrix}, \quad (80)$$

the largest singular value plot of $\Delta(s)$ and the gain plot of $W_T(s)$ are shown in Fig. 3. Here, the dotted line shows the gain plot of $W_T(s)$ and the solid line

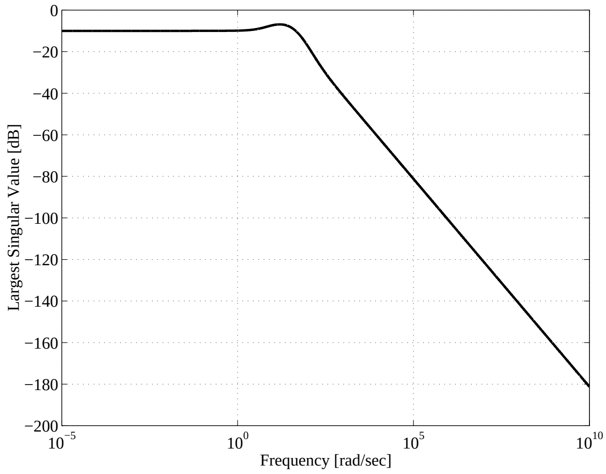

Fig.2: *The largest singular value plot of $Q(s)$*

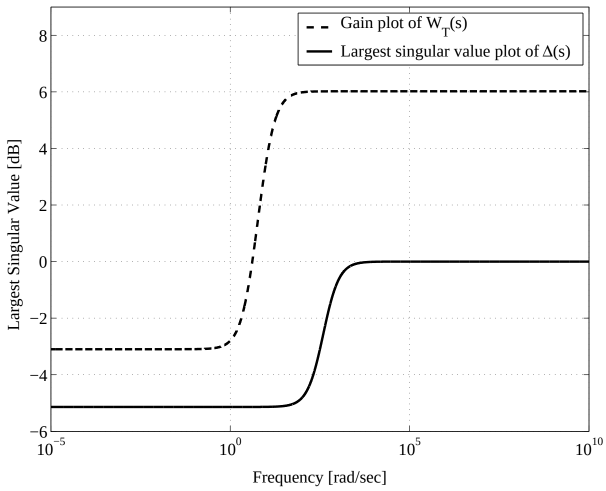

Fig.3: *The largest singular value plot of $\Delta(s)$ and the gain plot of $W_T(s)$*

shows the largest singular value plot of $\Delta(s)$. Figure 3 shows that the uncertainty $\Delta(s)$ satisfies (4).

Using above-mentioned parameters, we have a robust stabilizing modified repetitive controller. When the designed robust stabilizing modified repetitive controller $C(s)$ is used, the response of the output $y(t) = [y_1(t), y_2(t)]^T$ in (1) for the periodic reference input r in (75) is shown in Fig. 4. Here, the thick broken line shows the response of the periodic reference input r_1, the thin broken line shows that of the periodic reference input r_2, the thick solid line shows that of the output y_1 and the thin solid line shows that of the output y_2. Figure 4 shows that the output y follows the periodic reference input r in (75) with small steady state error, even if the plant has uncertainty $\Delta(s)$.

Next, using the designed robust stabilizing modified repetitive controller $C(s)$, the disturbance attenuation characteristic is shown. The response of the output y for the disturbance d in (73) of which the

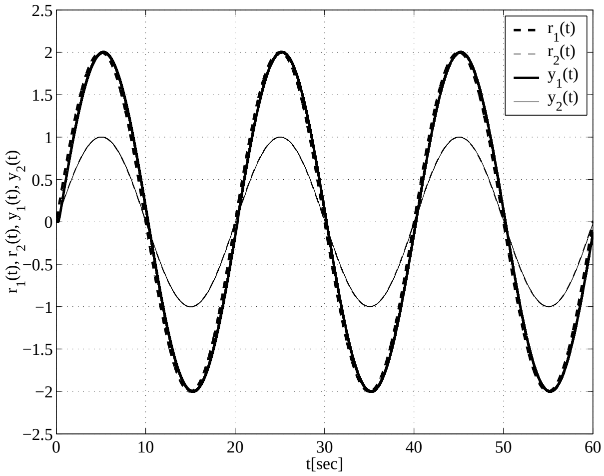

Fig.4: *The response of the output $y(t)$ for the periodic reference input $r(t)$ in (75)*

frequency component is equivalent to that of the periodic reference input r is shown in Fig. 5. Here,

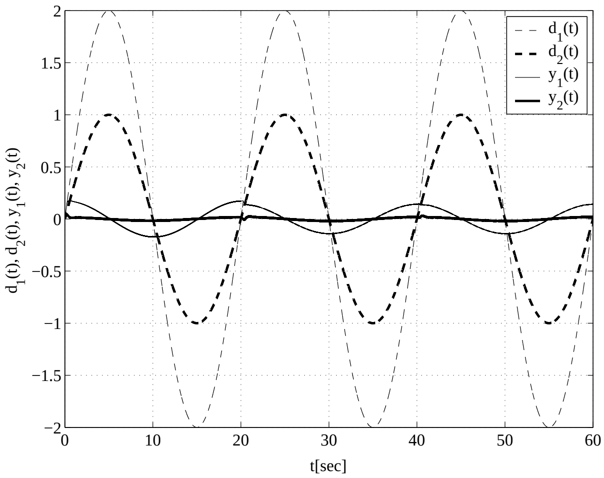

Fig.5: *The response of the output $y(t)$ for the disturbance $d(t)$ in (73)*

the thin broken line shows the response of the disturbance d_1, the thick broken line shows that of the disturbance d_2, the thin solid line shows that of the output y_1 and the thick solid line shows that of the output y_2. Figure 5 shows that the disturbance d in (73) is attenuated effectively. Finally, the response of the output y for the disturbance d in (74) of which the frequency component is different from that of the periodic reference input r is shown in Fig. 6. Here, the thin broken line shows the response of the disturbance d_1, the thick broken line shows that of the disturbance d_2, the thin solid line shows that of the output y_1 and the thick solid line shows that of the output y_2. Figure 6 shows that the disturbance d in (74) is attenuated effectively.

In this way, we find that we can easily design a

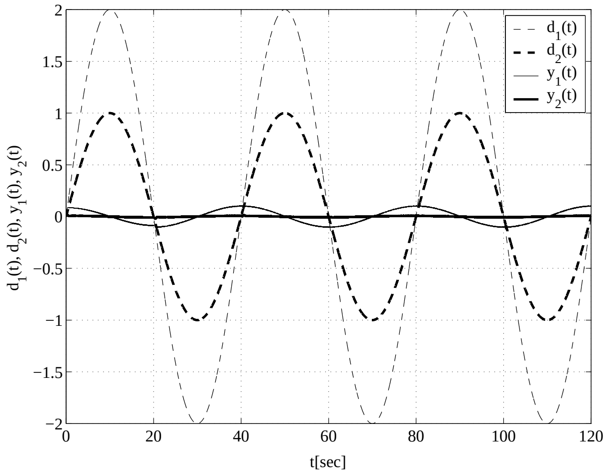

Fig.6: *The response of the output $y(t)$ for the disturbance $d(t)$ in (74)*

robust stabilizing modified repetitive controller using Theorem 1.

7. CONCLUSIONS

In this paper, we proposed the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output time-delay plants with uncertainties. That is, we found out the parameterization of all robust stabilizing modified repetitive controllers $C(s)$ written as the form in (7) such that the control system in (1) is robustly stable and the output y follows the periodic reference input r with small steady state error even in the presence of uncertainty $\Delta(s)$. Control characteristics of a robust stabilizing modified repetitive control system are presented, as well as a design procedure for a robust stabilizing modified repetitive controller. A numerical example was shown to illustrate the effectiveness of the proposed parameterization. Because the modified repetitive control system using the proposed parameterization has merits, such as the robust stability of the time-delay system with uncertainty is guaranteed and the control system can be easily designed, practical applications for real time-delay system, for example, remote control and temperature control, are expected.

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