

A Design Method for a Robust Stabilizing Multi-Period Repetitive Control System for Multiple-Input/Multiple-Output Plants with Specified Input-Output Frequency Characteristic Using the Parameterization

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ABSTRACT

The multi-period repetitive control system is a type of servomechanism for a periodic reference input. Recently, the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with uncertainty was obtained by Chen et al. However, using their method, it is complex to specify the low-pass filter in the internal model for the periodic reference input of which the role is to specify the input-output frequency characteristic. Because, the low-pass filter is related to four free parameters in the parameterization proposed by Chen et al. Chen et al. examined the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with specified input-output frequency characteristic such that the input-output frequency characteristic can be specified beforehand. However, they omitted complete proof on account of space limitations. This paper gives the complete proof and demonstrates the effectiveness of the parameterization. The control characteristics of the system are presented, along with a design procedure for a robust stabilizing multi-period repetitive controller with specified input-output frequency characteristic. A numerical example is presented to illustrate the effectiveness of the proposed method.

Keywords: Repetitive Control, Multi-Period Repetitive Controller, Uncertainty, Robust Stability, Parameterization, Multiple-Input/Multiple-Output Plant, Low-Pass Filter

1. INTRODUCTION

A modified repetitive control system is a type of servomechanism for a periodic reference input. In

other words, the repetitive control system follows a periodic reference input without steady state error, even when there exists a periodic disturbance or an uncertainty of a plant [1–6]. However, the modified repetitive control system has a bad effect on the disturbance attenuation characteristic [7], in that at certain frequencies, the sensitivity to disturbances of a control system with a conventional repetitive controller becomes twice as worse as that of a control system without a repetitive controller. Gotou et al. overcame this problem by proposing a multi-period repetitive control system [7]. However, the phase angle of the low-pass filter in a multi-period repetitive controller has a bad effect on the disturbance attenuation characteristic [8, 9]. Yamada et al. overcame this problem and proposed a design method for multi-period repetitive controllers to attenuate disturbances effectively [10, 11] using the time advance compensation described in [8, 9, 12]. Using this multi-period repetitive control structure, Steinbuch proposed a design method for repetitive control systems with uncertain period time [13].

On the other hand, there is an important control problem of finding all stabilizing controllers, named the parameterization problem [14–18]. The parameterization of all stabilizing multi-period repetitive controllers was solved in [19, 20].

When multi-period repetitive control design methods are applied to real systems, the influence of uncertainties in the plant must be considered. In some cases, uncertainties in the plant make the modified repetitive control system unstable, even though the controller was designed to stabilize the nominal plant. The stability problem with uncertainty is known as the robust stability problem [24]. Satoh et al. examined the parameterization of all robust stabilizing multi-period repetitive controllers for plants with uncertainties [21]. Chen et al. expanded the result in [21] and proposed the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants [22]. However, when employing the method in [22], it is complex to specify the low-pass filter in the internal

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model for the periodic reference input of which the role is to specify the input-output frequency characteristic. Because, the low-pass filter is related to four kinds of free parameters in the parameterization by Chen et al. When we design a robust stabilizing multi-period repetitive controller, if the low-pass filters in the internal model for the periodic reference input are settled beforehand, we can specify the input-output frequency characteristic more easily than the method in [22]. This is achieved by parameterizing all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with specified input-output frequency characteristic, which is the parameterization when the low-pass filters are settled beforehand. However, no paper has considered the problem to obtain the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with specified input-output frequency characteristic. In addition, the parameterization is useful to design stabilizing controllers [14–18]. From this viewpoint, Chen et al. examined the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with specified input-output frequency characteristic such that the input-output frequency characteristic can be specified beforehand [23]. If the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with specified input-output frequency characteristic is used, it is possible to easily design a robust stabilizing multi-period repetitive control system that has a desirable input-output frequency characteristic. However, Chen et al. omitted the complete proof of the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with specified input-output frequency characteristic on account of space limitations. In addition, control characteristics were not examined using the obtained parameterization in [23]. Furthermore, a design method for stabilizing a robust stabilizing multi-period repetitive control system with specified input-output frequency characteristic was not described. Therefore, we do not know whether the parameterization in [23] is effective.

In this paper, we give a complete proof of the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with specified input-output frequency characteristic, which was omitted in [23] and demonstrate the effectiveness of the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with specified input-output frequency characteristic. First, we give a complete proof of the theorem for the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with specified input-output frequency charac-

teristic, which was omitted in [23]. Next, we clarify control characteristics using the parameterization in [23]. In addition, a design procedure using the parameterization is presented. Finally, a numerical example is presented to illustrate the effectiveness of the proposed design method.

Notation

R	the set of real numbers.
R_+	$R \cup \{\infty\}$.
$R(s)$	the set of real rational functions with s .
RH_∞	the set of stable proper real rational functions.
H_∞	the set of stable causal functions.
D^\perp	orthogonal complement of D , i.e., $\begin{bmatrix} D & D^\perp \end{bmatrix}$ or $\begin{bmatrix} D \\ D^\perp \end{bmatrix}$ is unitary.
A^T	transpose of A .
A^\dagger	pseudo inverse of A .
$\rho(\{\cdot\})$	spectral radius of $\{\cdot\}$.
$\bar{\sigma}(\{\cdot\})$	largest singular value of $\{\cdot\}$.
$\ \{\cdot\}\ _\infty$	H_∞ norm of $\{\cdot\}$.
$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	represents the state space description $C(sI - A)^{-1}B + D$.
$\text{diag}(a_1, \dots, a_n)$	an $n \times n$ diagonal matrix with a_i as its i -th diagonal element.

2. PROBLEM FORMULATION

Consider the unity feedback control system in

$$\begin{cases} y &= G(s)u + d \\ u &= C(s)(r - y) \end{cases}, \quad (1)$$

where $G(s) \in R^{m \times p}(s)$ is the multiple-input/multiple-output plant, $G(s)$ is assumed to be stabilizable and detectable. $C(s)$ is the multi-period repetitive controller with m -th input and p -th output defined later, $u \in R^p$ is the control input, $d \in R^m$ is the disturbance, $y \in R^m$ is the output and $r \in R^m$ is the periodic reference input with period $T > 0$ satisfying

$$r(t + T) = r(t) \quad (\forall t \geq 0). \quad (2)$$

It is assumed that $m \leq p$ and $\text{rank } G(s) = m$. The nominal plant of $G(s)$ is denoted by $G_m(s) \in R^{m \times p}(s)$. Both $G(s)$ and $G_m(s)$ are assumed to have no zero or pole on the imaginary axis. In addition, it is assumed that the number of poles of $G(s)$ in the closed right half plane is equal to that of $G_m(s)$. The relation between the plant $G(s)$ and the nominal plant $G_m(s)$ is written as

$$G(s) = (I + \Delta(s))G_m(s), \quad (3)$$

where $\Delta(s)$ is an uncertainty. The set of $\Delta(s)$ is all rational functions satisfying

$$\bar{\sigma}\{\Delta(j\omega)\} < |W_T(j\omega)| \quad (\forall \omega \in R_+), \quad (4)$$

where $W_T(s) \in R(s)$ is a stable rational function.

The robust stability condition for the plant $G(s)$ with uncertainty $\Delta(s)$ satisfying (4) is given by

$$\|T(s)W_T(s)\|_\infty < 1, \quad (5)$$

where $T(s)$ is the complementary sensitivity function given by

$$T(s) = (I + G_m(s)C(s))^{-1} G_m(s)C(s). \quad (6)$$

According to [7, 10, 11, 19, 20], the general form of multi-period repetitive controller $C(s)$ which makes the output y to follow the periodic reference input r with period T in (1) with small steady state error, is written by

$$C(s) = C_0(s) + \sum_{i=1}^N C_i(s)q_i(s)e^{-sT_i} \left(I - \sum_{i=1}^N q_i(s)e^{-sT_i} \right)^{-1}, \quad (7)$$

where N is an arbitrary positive integer, $T_i > 0 \in R$ ($i = 1, \dots, N$), $C_0(s) \in R^{p \times m}(s)$, $C_i(s) \in R^{p \times m}(s)$ ($i = 1, \dots, N$) satisfying $\text{rank } C_i(s) = m$ ($i = 1, \dots, N$), and $q_i(s) \in R^{m \times m}(s)$ ($i = 1, \dots, N$) are low-pass filters satisfying $\sum_{i=1}^N q_i(0) = I$. In the following, $e^{-sT_i}(I - \sum_{i=1}^N q_i(s)e^{-sT_i})^{-1}$ defines the internal model for the periodic signal with period T . Gotou et al. [7] proposed the design method for multi-period repetitive controller as

$$T_i = T \cdot i \quad (i = 1, \dots, N). \quad (8)$$

On the other hand, Yamada et al. [11] proposed the design method for multi-period repetitive controller such that T_i ($i = 1, \dots, N$) do not necessarily satisfy (8). Therefore, in this paper, we attach importance to the generality and assume that T_i ($i = 1, \dots, N$) do not necessarily satisfy (8).

According to [7, 10, 11, 19, 20], if low-pass filters $q_i(s)$ ($i = 1, \dots, N$) satisfy

$$\bar{\sigma} \left\{ I - \sum_{i=1}^N q_i(j\omega_k)e^{-j\omega_k T_i} \right\} \simeq 0 \quad (\forall k = 0, 1, \dots, n), \quad (9)$$

where ω_k are frequency components of the periodic reference input r written by

$$\omega_k = \frac{2\pi}{T}k \quad (k = 0, 1, \dots, n) \quad (10)$$

and ω_n is the maximum frequency component of the periodic reference input r , then the output y in (1) follows the periodic reference input r with small steady state error. That is, the low-pass filters $q_i(s)$ ($i = 1, \dots, N$) have the role to specify the input-output frequency characteristic of the multi-period

repetitive control system. Using the result in [22], the low-pass filter $q_i(s)$ ($i = 1, \dots, N$) takes the form in

$$q_i(s) = - \left(Z_{21}(s)\hat{Q}_{ni}(s) + Z_{22}(s)\hat{Q}_{di}(s) \right) \left(Z_{21}(s)\hat{Q}_{n0}(s) + Z_{22}(s)\hat{Q}_{d0}(s) \right)^{-1} \quad (i = 1, \dots, N), \quad (11)$$

where $Z_{21}(s)$ and $Z_{22}(s)$ are fixed functions, and $\hat{Q}_{n0}(s) \in RH_\infty^{p \times m}$, $\hat{Q}_{ni}(s) \in RH_\infty^{p \times m}$ ($i = 1, \dots, N$), $\hat{Q}_{d0}(s) \in RH_\infty^{m \times m}$ and $\hat{Q}_{di}(s) \in RH_\infty^{m \times m}$ ($i = 1, \dots, N$) are any functions. In order for the output y to follow the periodic reference input r with small steady state error, we must design $\hat{Q}_{n0}(s) \in RH_\infty^{p \times m}$, $\hat{Q}_{ni}(s) \in RH_\infty^{p \times m}$ ($i = 1, \dots, N$), $\hat{Q}_{d0}(s) \in RH_\infty^{m \times m}$ and $\hat{Q}_{di}(s) \in RH_\infty^{m \times m}$ ($i = 1, \dots, N$) to satisfy (9). In addition, in order for $q_i(s)$ ($i = 1, \dots, N$) to satisfy (9) in wide frequency range, $q_i(s)$ ($i = 1, \dots, N$) must be stable and of minimum phase. Therefore, it is complex to specify the low-pass filter. If we obtain the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants such that $q_i(s)$ ($i = 1, \dots, N$) in (7) are settled beforehand, we can design the robust stabilizing multi-period repetitive controller satisfying (9) more easily than the method in [22].

From the above practical requirement, Chen et al. examined the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with specified input-output frequency characteristic [23]. That is, when $q_i(s) \in RH_\infty^{m \times m}$ ($i = 1, \dots, N$) are settled beforehand, Chen et al. [23] obtained the parameterization of all controllers $C(s)$ in (7) satisfying (5) for multiple-input/multiple-output plants in (3) with any uncertainty $\Delta(s)$ satisfying (4). However, they omitted a complete proof on account of space limitations. The problem considered in this paper is to give the complete proof of the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with specified input-output frequency characteristic in [23] and to propose a design method for a control system using the parameterization in [23].

3. THE PARAMETERIZATION

In this section, we clarify the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with specified input-output frequency characteristic.

In order to obtain the parameterization of all robust stabilizing multi-period repetitive controllers with specified input-output frequency characteristic, we must see that controllers $C(s)$ satisfying (5). The problem of obtaining the controller $C(s)$, which is not

necessarily a multi-period repetitive controller, satisfying (5) is equivalent to the following H_∞ control problem. In order to obtain the controller $C(s)$ satisfying (5), we consider the control system shown in Fig. 1. $P(s)$ is selected such that the transfer func-

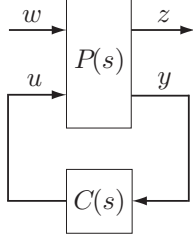


Fig.1: Block diagram of H_∞ control problem

tion from w to z in Fig. 1 is equal to $T(s)W_T(s)$. The state space description of $P(s)$ is, in general,

$$\begin{cases} \dot{x}(t) = Ax(t) + B_1w(t) + B_2u(t) \\ z(t) = C_1x(t) + D_{12}u(t) \\ y(t) = C_2x(t) + D_{21}w(t) \end{cases}, \quad (12)$$

where $A \in R^{n \times n}$, $B_1 \in R^{n \times m}$, $B_2 \in R^{n \times p}$, $C_1 \in R^{m \times n}$, $C_2 \in R^{p \times n}$, $D_{12} \in R^{m \times p}$, $D_{21} \in R^{p \times m}$, $x(t) \in R^n$, $w(t) \in R^m$, $z(t) \in R^m$, $u(t) \in R^p$ and $y(t) \in R^p$. $P(s)$ is called the generalized plant. $P(s)$ is assumed to satisfy following assumptions [24]:

1. (C_2, A) is detectable, (A, B_2) is stabilizable.
2. D_{12} has full column rank, and D_{21} has full row rank.
3. $\text{rank} \begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} = n + p \quad (\forall \omega \in R_+)$,
 $\text{rank} \begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} = n + m \quad (\forall \omega \in R_+)$.

Under these assumptions, according to [24], following lemma holds true.

Lemma 1: If controllers satisfying (5) exist, both

$$\begin{aligned} & X \left(A - B_2 D_{12}^\dagger C_1 \right) + \left(A - B_2 D_{12}^\dagger C_1 \right)^T X \\ & + X \left\{ B_1 B_1^T - B_2 (D_{12}^T D_{12})^{-1} B_2^T \right\} X \\ & + (D_{12}^\dagger C_1)^T D_{12}^\dagger C_1 = 0 \end{aligned} \quad (13)$$

and

$$\begin{aligned} & Y \left(A - B_1 D_{21}^\dagger C_2 \right)^T + \left(A - B_1 D_{21}^\dagger C_2 \right) Y \\ & + Y \left\{ C_1^T C_1 - C_2^T (D_{21} D_{21}^T)^{-1} C_2 \right\} Y \\ & + B_1 D_{21}^\dagger (B_1 D_{21}^\dagger)^T = 0 \end{aligned} \quad (14)$$

have solutions $X \geq 0$ and $Y \geq 0$ such that

$$\rho(XY) < 1 \quad (15)$$

and both

$$\begin{aligned} & A - B_2 D_{12}^\dagger C_1 \\ & + \left\{ B_1 B_1^T - B_2 (D_{12}^T D_{12})^{-1} B_2^T \right\} X \end{aligned} \quad (16)$$

and

$$\begin{aligned} & A - B_1 D_{21}^\dagger C_2 \\ & + Y \left\{ C_1^T C_1 - C_2^T (D_{21} D_{21}^T)^{-1} C_2 \right\} \end{aligned} \quad (17)$$

have no eigenvalue in the closed right half plane. Using X and Y , the parameterization of all controllers satisfying (5) is given by

$$\begin{aligned} C(s) &= C_{11}(s) + C_{12}(s)Q(s)(I - C_{22}(s)Q(s))^{-1}C_{21}(s), \end{aligned} \quad (18)$$

where

$$\begin{bmatrix} C_{11}(s) & C_{12}(s) \\ C_{21}(s) & C_{22}(s) \end{bmatrix} = \begin{bmatrix} A_c & B_{c1} & B_{c2} \\ C_{c1} & D_{c11} & D_{c12} \\ C_{c2} & D_{c21} & D_{c22} \end{bmatrix}, \quad (19)$$

$$\begin{aligned} A_c &= A + B_1 B_1^T X - B_2 (D_{12}^\dagger C_1 + E_{12}^{-1} B_2^T X) \\ &\quad - (I - YX)^{-1} (B_1 D_{21}^\dagger + Y C_2^T E_{21}^{-1}) \\ &\quad (C_2 + D_{21} B_1^T X), \end{aligned}$$

$$B_{c1} = (I - YX)^{-1} (B_1 D_{21}^\dagger + Y C_2^T E_{21}^{-1}),$$

$$B_{c2} = (I - YX)^{-1} (B_2 + Y C_1^T D_{12}) E_{12}^{-1/2},$$

$$C_{c1} = -D_{12}^\dagger C_1 - E_{12}^{-1} B_2^T X,$$

$$C_{c2} = -E_{21}^{-1/2} (C_2 + D_{21} B_1^T X),$$

$$D_{c11} = 0, \quad D_{c12} = E_{12}^{-1/2}, \quad D_{c21} = E_{21}^{-1/2}, \quad D_{c22} = 0,$$

$$E_{12} = D_{12}^T D_{12}, \quad E_{21} = D_{21} D_{21}^T$$

and $Q(s) \in H_\infty^{p \times m}$ is any function satisfying $\|Q(s)\|_\infty < 1$.

$C(s)$ in (18) is written using Linear Fractional Transformation(LFT). Using homogeneous transformation, (18) is rewritten by

$$\begin{aligned} C(s) &= (Z_{11}(s)Q(s) + Z_{12}(s))(Z_{21}(s)Q(s) + Z_{22}(s))^{-1} \\ &= \left(Q(s)\tilde{Z}_{21}(s) + \tilde{Z}_{22}(s) \right)^{-1} \\ &\quad \left(Q(s)\tilde{Z}_{11}(s) + \tilde{Z}_{12}(s) \right), \end{aligned} \quad (20)$$

where $Z_{ij}(s) (i = 1, 2; j = 1, 2)$ and $\tilde{Z}_{ij}(s) (i = 1, 2; j = 1, 2)$ are defined by

$$\begin{aligned} & \begin{bmatrix} Z_{11}(s) & Z_{12}(s) \\ Z_{21}(s) & Z_{22}(s) \end{bmatrix} \\ &= \begin{bmatrix} C_{12}(s) - C_{11}(s)C_{21}^{-1}(s)C_{22}(s) & \\ & -C_{21}^{-1}(s)C_{22}(s) \\ C_{11}(s)C_{21}^{-1}(s) & \\ & C_{21}^{-1}(s) \end{bmatrix} \end{aligned} \quad (21)$$

and

$$\begin{aligned} & \begin{bmatrix} \tilde{Z}_{11}(s) & \tilde{Z}_{12}(s) \\ \tilde{Z}_{21}(s) & \tilde{Z}_{22}(s) \end{bmatrix} \\ &= \begin{bmatrix} C_{21}(s) - C_{22}(s)C_{12}^{-1}(s)C_{11}(s) & \\ -C_{22}(s)C_{12}^{-1}(s) & \\ C_{12}^{-1}(s)C_{11}(s) & \\ C_{12}^{-1}(s) & \end{bmatrix} \end{aligned} \quad (22)$$

and satisfying

$$\begin{aligned} & \begin{bmatrix} \tilde{Z}_{22}(s) & \tilde{Z}_{12}(s) \\ \tilde{Z}_{21}(s) & \tilde{Z}_{11}(s) \end{bmatrix} \begin{bmatrix} Z_{11}(s) & -Z_{12}(s) \\ -Z_{21}(s) & Z_{22}(s) \end{bmatrix} = I \\ &= \begin{bmatrix} Z_{11}(s) & -Z_{12}(s) \\ -Z_{21}(s) & Z_{22}(s) \end{bmatrix} \begin{bmatrix} \tilde{Z}_{22}(s) & \tilde{Z}_{12}(s) \\ \tilde{Z}_{21}(s) & \tilde{Z}_{11}(s) \end{bmatrix}. \end{aligned} \quad (23)$$

Using Lemma 1 and Remark 3, according to [23], the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with specified input-output frequency characteristic is summarized in the following theorem.

Theorem 1: If multi-period repetitive controllers satisfying (5) exist, both (13) and (14) have solutions $X \geq 0$ and $Y \geq 0$ such that (15) and both (16) and (17) have no eigenvalue in the closed right half plane. Using X and Y , the parameterization of all robust stabilizing multi-period repetitive controllers with specified input-output frequency characteristic satisfying (5) is given by

$$\begin{aligned} C(s) &= (Z_{11}(s)Q(s) + Z_{12}(s))(Z_{21}(s)Q(s) + Z_{22}(s))^{-1} \\ &= \left(Q(s)\tilde{Z}_{21}(s) + \tilde{Z}_{22}(s) \right)^{-1} \\ &\quad \left(Q(s)\tilde{Z}_{11}(s) + \tilde{Z}_{12}(s) \right), \end{aligned} \quad (24)$$

where $Z_{ij}(s) (i = 1, 2; j = 1, 2)$ and $\tilde{Z}_{ij}(s) (i = 1, 2; j = 1, 2)$ are defined by (21) and (22) and satisfying (23), $C_{ij}(s) (i = 1, 2; j = 1, 2)$ are given by (19) and $Q(s) \in RH_{\infty}^{p \times m}$ is any function satisfying $\|Q(s)\|_{\infty} < 1$ and written by

$$\begin{aligned} Q(s) &= \left(Q_{n0}(s) + \sum_{i=1}^N Q_{ni}(s)q_i(s)e^{-sT_i} \right) \\ &\quad \left(Q_{d0}(s) + \sum_{i=1}^N Q_{di}(s)q_i(s)e^{-sT_i} \right)^{-1}, \end{aligned} \quad (25)$$

$Q_{n0}(s) \in RH_{\infty}^{p \times m}$, $Q_{d0}(s) \in RH_{\infty}^{m \times m}$, $Q_{ni}(s) \in RH_{\infty}^{p \times m}$ ($i = 1, \dots, N$) and $Q_{di}(s) \in RH_{\infty}^{m \times m}$ ($i = 1, \dots, N$) are any functions satisfying

$$\begin{aligned} & (Z_{21}(s)Q_{ni}(s) + Z_{22}(s)Q_{di}(s)) \\ & \quad (Z_{21}(s)Q_{n0}(s) + Z_{22}(s)Q_{d0}(s))^{-1} \\ &= -I \quad (i = 1, \dots, N) \end{aligned} \quad (26)$$

and

$$\text{rank} \{ Z_{11}(s)(Q_{n0}(s) + Q_{ni}(s)) + Z_{12}(s)(Q_{d0}(s) + Q_{di}(s)) \} = m \quad (i = 1, \dots, N). \quad (27)$$

Proof: First, the necessity is shown. That is, we show that if the multi-period repetitive controller $C(s)$ in (7) stabilizes the control system in (1) robustly and $q_i(s)$ ($i = 1, \dots, N$) are settled beforehand, then $C(s)$ is written by (24) and (25), respectively. From Lemma 1 and Remark 3, the parameterization of all robust stabilizing controllers $C(s)$ is written by (24), where $\|Q(s)\|_{\infty} < 1$. In order to prove the necessity, we will show that if $C(s)$ written by (7) stabilizes the control system in (1), then $Q(s)$ in (24) is written by (25). Substituting $C(s)$ in (7) for (24), we have (25), where

$$Q_{n0}(s) = -N_{0n}(s)N_d(s), \quad (28)$$

$$Q_{ni}(s) = -N_{in}(s) + N_{0n}(s)N_d(s) \quad (i = 1, \dots, N), \quad (29)$$

$$Q_{d0}(s) = D_{0n}(s)D_d(s)N_{0d}(s)N_d(s) \quad (30)$$

and

$$Q_{di}(s) = (D_{in}(s) - D_{0n}(s)D_d(s))N_{0d}(s)N_d(s) \quad (i = 1, \dots, N). \quad (31)$$

Here, $N_{0n}(s) \in RH_{\infty}^{p \times m}$, $N_{in}(s) \in RH_{\infty}^{p \times m}$ ($i = 1, \dots, N$), $N_{0d}(s) \in RH_{\infty}^{m \times m}$, $N_d(s) \in RH_{\infty}^{m \times m}$, $D_{0n}(s) \in RH_{\infty}^{m \times m}$, $D_{in}(s) \in RH_{\infty}^{m \times m}$ ($i = 1, \dots, N$), $D_{0d}(s) \in RH_{\infty}^{m \times m}$ and $D_d(s) \in RH_{\infty}^{m \times m}$ are coprime factors satisfying

$$\tilde{Z}_{21}(s)C_0(s) - \tilde{Z}_{11}(s) = D_{0n}(s)D_{0d}^{-1}(s), \quad (32)$$

$$\begin{aligned} & \tilde{Z}_{21}(s)C_i(s)D_{0d}(s) \\ &= D_{in}(s)D_d^{-1}(s) \quad (i = 1, \dots, N), \end{aligned} \quad (33)$$

$$\begin{aligned} & (\tilde{Z}_{22}(s)C_0(s) - \tilde{Z}_{12}(s))D_{0d}(s)D_d(s) \\ &= N_{0n}(s)N_{0d}^{-1}(s) \end{aligned} \quad (34)$$

and

$$\begin{aligned} & \tilde{Z}_{22}(s)C_i(s)D_{0d}(s)D_d(s)N_{0d}(s) \\ &= N_{in}(s)N_d^{-1}(s) \quad (i = 1, \dots, N). \end{aligned} \quad (35)$$

From (28) ~ (31), all of $Q_{n0}(s)$, $Q_{ni}(s)$ ($i = 1, \dots, N$), $Q_{d0}(s)$ and $Q_{di}(s)$ ($i = 1, \dots, N$) are included in RH_{∞} . Thus, we have shown that if $C(s)$ written by (7) stabilizes the control system in (1) robustly and $q_i(s)$ ($i = 1, \dots, N$) are settled beforehand, $Q(s)$ in (24) is written by (25). From (29), (31), (33), (35) and (23), (26) is satisfied. In addition, from

the assumption of rank $C_i(s) = m$ ($i = 1, \dots, N$) and from (33) and (35),

$$\text{rank } D_{in}(s) = m \quad (i = 1, \dots, N) \quad (36)$$

and

$$\text{rank } N_{in}(s) = m \quad (i = 1, \dots, N) \quad (37)$$

hold true. From (36), (37), (29) and (31), (27) is satisfied. Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, it is shown that if $C(s)$ and $Q(s) \in H_\infty^{p \times m}$ are settled by (24) and (25), respectively, then the controller $C(s)$ is written by the form in (7) and rank $C_i(s) = m$ ($i = 1, \dots, N$) hold true. Substituting (25) into (24), we have (7), where $C_0(s)$ and $C_i(s)$ ($i = 1, \dots, N$) are denoted by

$$\begin{aligned} C_0(s) &= (Z_{11}(s)Q_{n0}(s) + Z_{12}(s)Q_{d0}(s)) \\ &\quad (Z_{21}(s)Q_{n0}(s) + Z_{22}(s)Q_{d0}(s))^{-1}, \end{aligned} \quad (38)$$

and

$$\begin{aligned} C_i(s) &= \{Z_{11}(s)(Q_{n0}(s) + Q_{ni}(s)) + Z_{12}(s)(Q_{d0}(s) \\ &\quad + Q_{di}(s))\} (Z_{21}(s)Q_{n0}(s) + Z_{22}(s)Q_{d0}(s))^{-1} \\ &\quad (i = 1, \dots, N) \end{aligned} \quad (39)$$

We find that if $C(s)$ and $Q(s)$ are settled by (24) and (25), respectively, then the controller $C(s)$ is written by the form in (7). From (27) and (39),

$$\text{rank } C_i(s) = m \quad (i = 1, \dots, N) \quad (40)$$

holds true. Thus, the sufficiency has been shown.

We have thus proved Theorem 1. ■

4. CONTROL CHARACTERISTICS

In this section, we describe control characteristics of the control system in (1) using the robust stabilizing multi-period repetitive controller $C(s)$ for multiple-input/multiple-output plants with specified input-output frequency characteristic in (24).

From Theorem 1, $Q(s)$ in (25) must be included in $H_\infty^{p \times m}$. Since $Q_{n0}(s) \in RH_\infty^{p \times m}$, $Q_{ni}(s) \in RH_\infty^{p \times m}$ and $q_i(s) \in RH_\infty^{m \times m}$ in (25), if

$$\left(Q_{d0}(s) + \sum_{i=1}^N Q_{di}(s)q_i(s)e^{-sT_i} \right)^{-1} \in H_\infty^{m \times m}, \quad (41)$$

then $Q(s)$ satisfies $Q(s) \in H_\infty^{p \times m}$. That is, the role of $Q_{d0}(s)$ and $Q_{di}(s)$ ($i = 1, \dots, N$) in (25) is to assure the stability of the control system in (1).

Next, we mention the input-output characteristic. The transfer function $S(s)$ from the periodic reference input r to the error $e = r - y$ is written by

$$S(s) = (I + G(s)C(s))^{-1} = S_n(s)S_d^{-1}(s), \quad (42)$$

where

$$\begin{aligned} S_n(s) &= \left(I - \sum_{i=1}^N q_i(s)e^{-sT_i} \right) C_{21}^{-1}(s) \\ &\quad (Q_{d0}(s) - C_{22}(s)Q_{n0}(s)) \end{aligned} \quad (43)$$

and

$$\begin{aligned} S_d(s) &= (Z_{22}(s) + G(s)Z_{12}(s)) \\ &\quad \left(Q_{d0}(s) + \sum_{i=1}^N Q_{di}(s)q_i(s)e^{-sT_i} \right) \\ &\quad + (Z_{21}(s) + G(s)Z_{11}(s)) \\ &\quad \left(Q_{n0}(s) + \sum_{i=1}^N Q_{ni}(s)q_i(s)e^{-sT_i} \right). \end{aligned} \quad (44)$$

From (42), for frequency components ω_k ($k = 0, 1, \dots, n$) in (10) of the periodic reference input r , since $q_i(s) \in RH_\infty$ ($i = 1, \dots, N$) are settled beforehand satisfying (9), the output y follows the periodic reference input r with small steady state error. That is, we find that by using the proposed parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with specified input-output frequency characteristic, the input-output frequency characteristic can be specified beforehand.

Finally, we mention the disturbance attenuation characteristic. The transfer function $S(s)$ from the disturbance d to the output y is written by (42), (43) and (44). From (42), for the disturbance d with same frequency components ω_k ($k = 0, 1, \dots, n$) in (10) of the periodic reference input r , since $S(s)$ satisfies $S(j\omega_k) \simeq 0$ ($\forall k = 0, 1, \dots, n$), the disturbance d is attenuated effectively. For the frequency component ω_d of the disturbance d that is different from that of the periodic reference input r (that is, $\omega_d \neq \omega_k$), even if

$$\bar{\sigma} \left\{ I - \sum_{i=1}^N q_i(j\omega_d) \right\} \simeq 0, \quad (45)$$

the disturbance d cannot be attenuated, because

$$e^{-j\omega_d T_i} \neq 1 \quad (46)$$

and

$$\bar{\sigma} \left\{ I - \sum_{i=1}^N q_i(j\omega_d)e^{-j\omega_d T_i} \right\} \neq 0. \quad (47)$$

To attenuate the frequency component ω_d of the disturbance $d(s)$ that is different from that of the periodic reference input $r(s)$, we need to set $Q_{n0}(s)$ and $Q_{d0}(s)$ satisfying

$$\bar{\sigma} \{ Q_{d0}(j\omega_d) - C_{22}(j\omega_d)Q_{n0}(j\omega_d) \} \simeq 0. \quad (48)$$

From above discussion, the role of $Q_{d0}(s)$ and $Q_{di}(s)$ ($i = 1, \dots, N$) is to assure the stability of the control system in (1) by satisfying $Q(s) \in H_\infty^{p \times m}$. The role of $q_i(s)$ ($i = 1, \dots, N$) is to specify the input-output frequency characteristic for the periodic reference input r and it can be specified beforehand. In addition, $q_i(s)$ ($i = 1, \dots, N$) specify the disturbance attenuation characteristic for the disturbance d with same frequency components ω_k ($k = 0, 1, \dots, n$) of the periodic reference input r . The role of $Q_{n0}(s)$ and $Q_{d0}(s)$ is to specify the disturbance attenuation characteristic for the frequency component of the disturbance $d(s)$ that is different from that of the periodic reference input $r(s)$.

5. DESIGN PROCEDURE

In this section, a design procedure for robust stabilizing multi-period repetitive controller for multiple-input/multiple-output plants with specified input-output frequency characteristic is presented.

A design procedure for robust stabilizing multi-period repetitive controllers $C(s)$ satisfying Theorem 1 is summarized as follows:

Procedure

Step 1) Obtain $C_{11}(s)$, $C_{12}(s)$, $C_{21}(s)$ and $C_{22}(s)$ by solving the robust stability problem using the Riccati equation based H_∞ control.

Step 2) $q_i(s) \in RH_\infty^{m \times m}$ ($i = 1, \dots, N$) and T_i ($i = 1, \dots, N$) in (25) are set so that for the frequency components ω_k ($k = 0, 1, \dots, n$) of the periodic reference input $r(s)$,

$$\bar{\sigma} \left\{ I - \sum_{i=1}^N q_i(j\omega_k) e^{-j\omega_k T_i} \right\} \simeq 0 \quad (\forall k = 0, 1, \dots, n) \quad (49)$$

is satisfied.

Step 3) $Q_{ni}(s) \in RH_\infty^{p \times m}$ ($i = 1, \dots, N$) and $Q_{di}(s) \in RH_\infty^{m \times m}$ ($i = 1, \dots, N$) in (25) are set according to

$$Q_{ni}(s) = C_{22d}(s) \bar{Q}_i(s) - Q_{n0}(s) \quad (\forall i = 1, \dots, N) \quad (50)$$

and

$$Q_{di}(s) = C_{22n}(s) \bar{Q}_i(s) - Q_{d0}(s) \quad (\forall i = 1, \dots, N) \quad (51)$$

so that (26) is satisfied, where $C_{22n}(s) \in RH_\infty^{m \times p}$, $C_{22d}(s) \in RH_\infty^{p \times p}$ are coprime factors of $C_{22}(s)$ satisfying

$$C_{22}(s) = C_{22n}(s) C_{22d}^{-1}(s) \quad (52)$$

and $\bar{Q}_i(s) \in RH_\infty^{p \times m}$ ($i = 1, \dots, N$) is any function.

Step 4) In order to hold $Q(s) \in H_\infty$ in (25), $Q_{d0}(s) \in RH_\infty^{m \times m}$ in (25) and $\bar{Q}_i(s) \in RH_\infty^{p \times m}$ ($i = 1, \dots, N$) in (51) are set to satisfy

$$\left(Q_{d0}(s) + \sum_{i=1}^N Q_{di}(s) q_i(s) e^{-sT_i} \right)^{-1} \in H_\infty. \quad (53)$$

Step 5) $Q_{n0}(s) \in RH_\infty^{p \times m}$ is set so that for the frequency component ω_d of the disturbance d , $\bar{\sigma} \{Q_{d0}(j\omega_d) - C_{22}(j\omega_d) Q_{n0}(j\omega_d)\}$ is effectively small. To design $Q_{n0}(s)$ to make $\bar{\sigma} \{Q_{d0}(j\omega_d) - C_{22}(j\omega_d) Q_{n0}(j\omega_d)\}$ effectively small, $Q_{n0}(s)$ is set according to

$$Q_{n0}(s) = C_{22o}^\dagger(s) \bar{q}_d(s) Q_{d0}(s), \quad (54)$$

where $C_{22o}(s) \in RH_\infty^{m \times p}$ is an outer function of $C_{22}(s)$ satisfying

$$C_{22}(s) = C_{22i}(s) C_{22o}(s), \quad (55)$$

$C_{22i}(s) \in RH_\infty^{m \times m}$ is an inner function satisfying $C_{22i}(0) = I$, $\bar{q}_d(s) \in RH_\infty^{m \times m}$ is a low-pass filter satisfying $\bar{q}_d(0) = I$, as

$$\begin{aligned} \bar{q}_d(s) &= \text{diag} \left\{ \frac{1}{(1 + \tau_{d1}s)^{\alpha_{d1}}} \cdots \frac{1}{(1 + \tau_{dm}s)^{\alpha_{dm}}} \right\} \end{aligned} \quad (56)$$

is valid, α_{di} ($i = 1, \dots, m$) are arbitrary positive integers to make $C_{22o}^\dagger(s) \bar{q}_d(s)$ proper and $\tau_{di} \in R$ ($i = 1, \dots, m$) are any positive real numbers satisfying

$$\bar{\sigma} \{I - C_{22i}(j\omega_d) \bar{q}_d(j\omega_d)\} \simeq 0. \quad (57)$$

6. NUMERICAL EXAMPLE

In this section, a numerical example is shown to illustrate the effectiveness of the proposed parameterization.

Consider the problem to obtain the parameterization of all robust stabilizing multi-period repetitive controllers with the specified input-output frequency characteristic for plant $G(s)$ in (3), where the nominal plant $G_m(s)$ and the upper bound $W_T(s)$ of the set of $\Delta(s)$ are given by

$$G_m(s) = \begin{bmatrix} \frac{s+3}{(s-2)(s+9)} & \frac{2}{(s-2)(s+9)} \\ \frac{s+3}{(s-2)(s+9)} & \frac{s+4}{(s-2)(s+9)} \end{bmatrix} \quad (58)$$

and

$$W_T(s) = \frac{s+400}{550}. \quad (59)$$

The period T of the periodic reference input r in (2) is $T = 2[\text{sec}]$. Solving the robust stability problem using Riccati equation based H_∞ control as Theorem 1, the parameterization of all robust stabilizing multi-period

repetitive controllers with the specified input-output frequency characteristic is obtained as (24) and (25). Here, $C_{ij}(s)$ ($i = 1, 2; j = 1, 2$) are

$$C_{11}(s) = \begin{bmatrix} \frac{3.452 \cdot 10^3 (s+9)(s+4)}{(s+408.6)(s+3)(s+2)} \\ \frac{-3.452 \cdot 10^3 (s+9)}{(s+408.6)(s+2)} \\ \frac{-6.905 \cdot 10^3 (s+9)}{(s+408.6)(s+3)(s+2)} \\ \frac{3.452 \cdot 10^3 (s+9)}{(s+408.6)(s+2)} \end{bmatrix}, \quad (60)$$

$$C_{12}(s) = \begin{bmatrix} \frac{289.2(s+9)(s+0.7639)}{(s+408.6)(s+3)} \\ \frac{178.7(s+9)}{s+408.6} \\ \frac{-467.9(s+9)(s+5.236)}{(s+408.6)(s+3)} \\ \frac{757.0(s+9)}{s+408.6} \end{bmatrix}, \quad (61)$$

$$C_{21}(s) = \begin{bmatrix} \frac{s+400}{s+408.6} & 0 \\ 0 & \frac{s+400}{s+408.6} \end{bmatrix}, \quad (62)$$

$$C_{22}(s) = \begin{bmatrix} \frac{292.5}{s+408.6} & \frac{-473.2}{s+408.6} \\ \frac{473.2}{s+408.6} & \frac{292.5}{s+408.6} \end{bmatrix}. \quad (63)$$

N is selected as $N = 3$ and T_i ($i = 1, 2, 3$) are set as $T_i = T \cdot i$ ($i = 1, 2, 3$). The low-pass filter $q_i(s) \in RH_\infty^{2 \times 2}$ ($i = 1, 2, 3$) is settled by

$$q_i(s) = \frac{1}{3(0.01s+1)} I \quad (i = 1, 2, 3). \quad (64)$$

$Q_{ni}(s) \in RH_\infty^{2 \times 2}$ ($i = 1, 2, 3$) and $Q_{di}(s) \in RH_\infty^{2 \times 2}$ ($i = 1, 2, 3$) are set according to (50) and (51).

In order to hold $Q(s) \in H_\infty^{2 \times 2}$ in (25), $Q_{d0}(s) \in RH_\infty^{2 \times 2}$ in (25) and $\bar{Q}_i(s) \in RH_\infty^{2 \times 2}$ ($i = 1, 2, 3$) in (50) and (51) are settled by

$$Q_{d0}(s) = \begin{bmatrix} \frac{s+993.5}{s+408.6} & \frac{-946.4}{s+408.6} \\ \frac{946.4}{s+408.6} & \frac{s+993.5}{s+408.6} \end{bmatrix} \quad (65)$$

and

$$\bar{Q}_i(s) = 2I \quad (i = 1, 2, 3). \quad (66)$$

When $Q_{d0}(s)$ and $\bar{Q}_i(s)$ ($i = 1, 2, 3$) are set as (65) and (66), the fact that $Q(s) \in H_\infty$ in (25) is confirmed as follows: Since $Q_{n0}(s) \in RH_\infty$, $Q_{ni}(s) \in RH_\infty$ ($i = 1, 2, 3$) and $q_i(s) \in RH_\infty$ ($i = 1, 2, 3$), if the Nyquist plot of $\det(Q_{d0}(s) + \sum_{i=1}^3 Q_{di}(s)q_i(s)e^{-sT_i})$ does not encircle the origin, then $Q(s) \in H_\infty$ holds true. The Nyquist plot of $\det(Q_{d0}(s) + \sum_{i=1}^3 Q_{di}(s)q_i(s)e^{-sT_i})$ is shown in Fig. 2. From Fig. 2, since the Nyquist plot of $\det(Q_{d0}(s) + \sum_{i=1}^3 Q_{di}(s)q_i(s)e^{-sT_i})$ does not

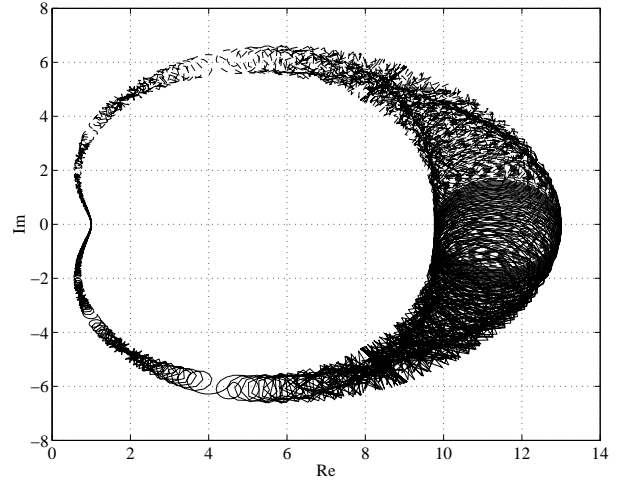


Fig.2: The Nyquist plot of $\det(Q_{d0}(s) + \sum_{i=1}^3 Q_{di}(s)q_i(s)e^{-sT_i})$

encircle the origin, we find that $Q(s) \in H_\infty$ holds true.

In order for disturbance

$$d(t) = \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} = \begin{bmatrix} \sin(0.5\pi t) \\ 2\sin(0.5\pi t) \end{bmatrix} \quad (67)$$

of which the frequency component is different from that of the periodic reference input $r(t)$ to be attenuated effectively, $Q_{n0}(s)$ is settled by (54), where

$$C_{22o}(s) = C_{22}(s) \in RH_\infty^{2 \times 2}, \quad (68)$$

and

$$\bar{q}_d(s) = \frac{1}{0.02s+1} I \in RH_\infty^{2 \times 2}. \quad (69)$$

The largest singular value plot of $Q(s)$ is shown in Fig. 3. Figure 3 shows that the designed $Q(s)$

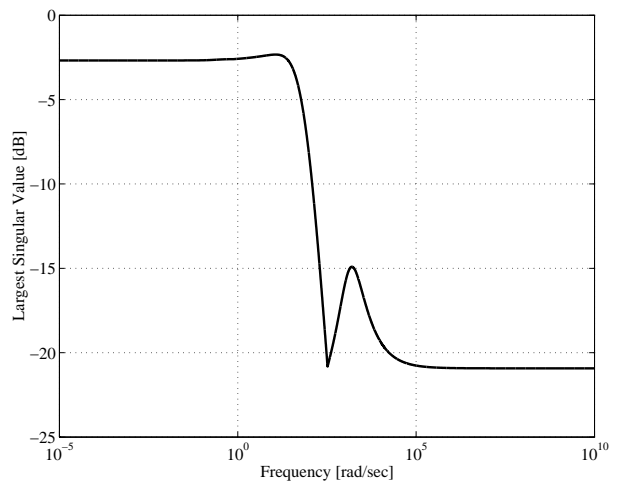


Fig.3: The largest singular value plot of $Q(s)$

satisfies $\|Q(s)\|_\infty < 1$.

When $\Delta(s)$ is given by

$$\Delta(s) = \begin{bmatrix} \frac{s-100}{s+500} & \frac{200}{s+600} \\ \frac{200}{s+500} & \frac{s-100}{s+600} \end{bmatrix}, \quad (70)$$

in order to confirm that $\Delta(s)$ satisfies (4), the largest singular value plot of $\Delta(s)$ and the gain plot of $W_T(s)$ are shown in Fig. 4. Here, the solid line shows the

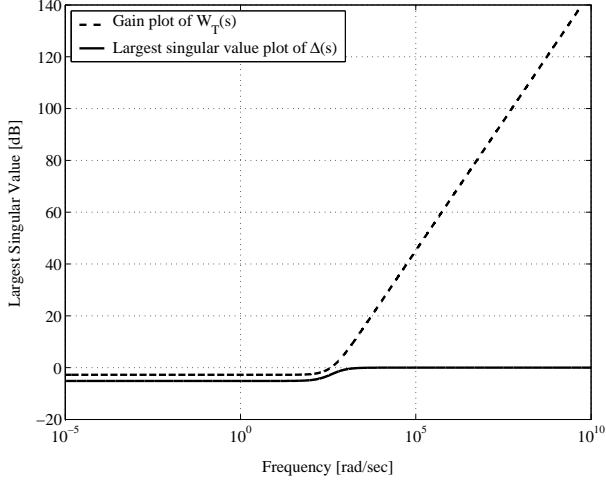


Fig.4: The largest singular value plot of $\Delta(s)$ and the gain plot of $W_T(s)$

largest singular value plot of $\Delta(s)$ and the broken line shows the gain plot of $W_T(s)$. Figure 4 shows that the uncertainty $\Delta(s)$ satisfies (4).

Using above-mentioned parameters, we have a robust stabilizing multi-period repetitive controller with specified input-output frequency characteristic. When the designed robust stabilizing multi-period repetitive controller $C(s)$ is used, the response of the error $e(t) = r(t) - y(t) = [e_1(t), e_2(t)]^T$ in (1) for the periodic reference input $r(t) = [r_1(t), r_2(t)]^T = [\sin(\pi t), 2\sin(\pi t)]^T$ is shown in Fig. 5. Here, the broken line shows the response of the periodic reference input $r_1(t)$, the dotted line shows that of the periodic reference input $r_2(t)$, the solid line shows that of the error $e_1(t)$, and the dotted and broken line shows that of the error $e_2(t)$. Figure 5 shows that the output $y(t)$ follows the periodic reference input $r(t)$ with small steady state error, even if the plant has uncertainty $\Delta(s)$.

Next, using the designed robust stabilizing multi-period repetitive controller with specified input-output frequency characteristic, the disturbance attenuation characteristic is shown. The response of the output $y(t) = [y_1(t), y_2(t)]^T$ for the disturbance $d(t) = [d_1(t), d_2(t)]^T = [\sin(2\pi t), 2\sin(2\pi t)]^T$ of which the frequency component is equivalent to that of the periodic reference input $r(t)$ is shown in Fig. 6. Here, the broken line shows the response of the disturbance $d_1(t)$, the dotted line shows that of the disturbance $d_2(t)$, the solid line shows that of the

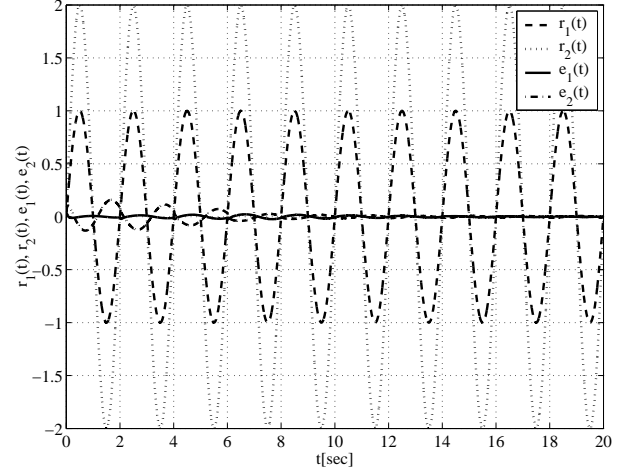


Fig.5: The response of the error $e(t) = r(t) - y(t) = [e_1(t), e_2(t)]^T$ for the periodic reference input $r(t) = [r_1(t), r_2(t)]^T = [\sin(\pi t), 2\sin(\pi t)]^T$

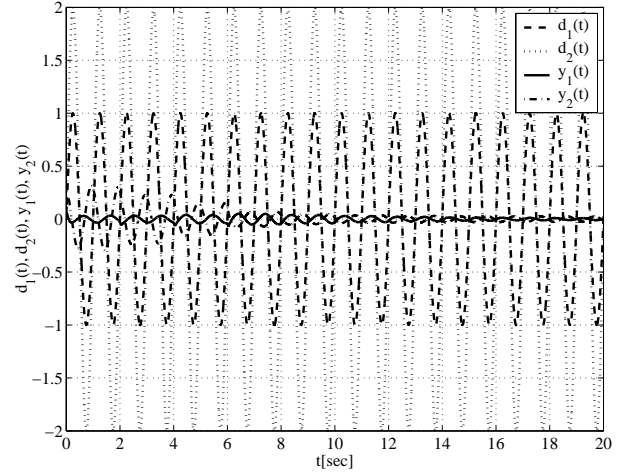


Fig.6: The response of the output $y(t) = [y_1(t), y_2(t)]^T$ for the disturbance $d(t) = [d_1(t), d_2(t)]^T = [\sin(2\pi t), 2\sin(2\pi t)]^T$

output $y_1(t)$ and the dotted and broken line shows that of the output $y_2(t)$. Figure 6 shows that the disturbance $d(t)$ is attenuated effectively. Finally, the response of the output $y(t) = [y_1(t), y_2(t)]^T$ for the disturbance $d(t)$ in (67) of which the frequency component is different from that of the periodic reference input $r(t)$ is shown in Fig. 7. Here, the broken line shows the response of the disturbance $d_1(t)$, the dotted line shows that of the disturbance $d_2(t)$, the solid line shows that of the output $y_1(t)$ and the dotted and broken line shows that of the output $y_2(t)$. Figure 7 shows that the disturbance $d(t)$ in (67) is attenuated effectively.

In this way, we find that we can easily design a robust stabilizing multi-period repetitive controller with specified input-output frequency characteristic using Theorem 1.

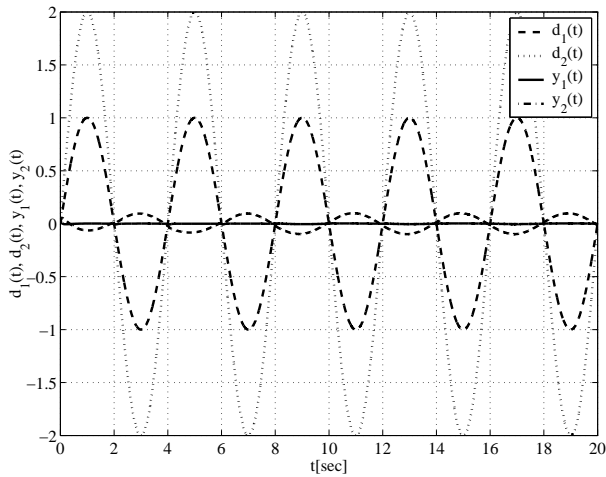


Fig. 7: The response of the output $y(t) = [y_1(t), y_2(t)]^T$ for the disturbance $d(t) = [d_1(t), d_2(t)]^T = [\sin(0.5\pi t), 2\sin(0.5\pi t)]^T$

7. CONCLUSIONS

In this paper, we gave a complete proof of the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with specified input-output frequency characteristic such that low-pass filters in the internal model for the periodic reference input are settled beforehand. In addition, we demonstrated the effectiveness of the parameterization of all robust stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with specified input-output frequency characteristic. Control characteristics of a robust stabilizing multi-period repetitive control system are presented, as well as a design procedure for a robust stabilizing multi-period repetitive controller with specified input-output frequency characteristic. A numerical example was shown to illustrate the effectiveness of the proposed parameterization.

Advantages of the multi-period repetitive control system using the proposed design method are that its input-output frequency characteristic is easily specified than in the method employed in [22]. This control system is expected to have practical applications in, for example, engines, electrical motors and generators, converters, and other machines that perform cyclic tasks.

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