A Fuzzy Backstepping Control of a Mobile Manipulator with Torque Limits Avoidance

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ABSTRACT

In this paper, we present a dynamic redundancy resolution technique for mobile manipulator subject to joint torque limits. First, the dynamic model of the mobile manipulator in feasible motion space is given. Next, a control algorithm is proposed which completely decouples the motion of the system into the end-effector motion in the task space and an internal motion in the null space and controls them in prioritized basis with priority given to the primary task space and enables the selection of characteristics in both subspaces separately. A special attention is given to the joints torque limits avoidance where a new weighted pseudo-inverse of the Jacobian that accounts for both inertia and torque limits is proposed to solve problems inherent to torque limits of the system. Simulation results are given to illustrate the coordination of two subsystems in executing the desired trajectory without violating the joint torque limits.

Keywords: Mobile Manipulators, Dynamic Redundancy Resolution, Torque Limits, Task Space, Null Space

1. INTRODUCTION

A mobile manipulator refers to the mobile system that has a mobile platform carrying a robotic manipulator. Such systems combine the advantages of mobile platforms and robotic arms to reduce their drawbacks. For instance, the mobile platform extends the arm workspace, whereas an arm offers much operational functionality. Applications for such systems could be found in mining, construction, forestry, planetary exploration, and people assistance [1, 2, 3].

The combined system introduces new issues that are not present in the analysis of each subsystem considered separately. First, the dynamics of the combined system are much more complicated because they include dynamic interactions between mobile platform and manipulator. Second, due to complex structure of the mobile manipulator, the constraints which are valid only for one subsystem will also hold for the whole mobile manipulator. Third, combining the mobility of the base platform and the manipulator creates redundancy since the combined system typically possesses more degrees of freedom than necessary.

Most of the redundancy resolution methods in the literature have a principal underlying theme of optimizing a measure performance based on the kinematics of the system. Several of these results have been extended and applied to mobile manipulators. For example, Seraji [4] proposes an approach to motion control of mobile manipulators which incorporates the nonholonomic base constraints directly into the task formulation as kinematic constraints and thus treats the base nonholonomy and kinematic redundancy in a unified manner. Yamamoto and Yun [5] decompose the motions of the mobile manipulator into decoupled base and manipulator subsystems. The base is then controlled so as to bring the manipulator to a preferred configuration (using criteria such as the manipulability measure) as the end-effector performs a variety of unknown manipulation tasks.

Some extensions of the kinematic redundancy resolution have also been pursued for dynamic redundancy resolution. However, Yamamoto and Yun [6] examined the effect of the dynamic interaction between the manipulator and the mobile platform on the task performance. Khatib [7] proposed a method of controlling redundant serial-chain systems by projecting the system dynamics into the task-space to realize an end-effector dynamic model together with a dynamically consistent actuation that provides decoupled control of joint motions in the null space. This was subsequently extended for mobile manipulator systems with holonomic bases and fully actuated manipulators [8]. Similarly Tan et al. [9] controlled a similar holonomic mobile manipulator to manipulate a passive nonholonomic cart along straight lines, corners or sinusoidal trajectories. Rajankumar [10] consider the alternate partitioning of the dynamics of the mobile manipulator into a task/end-effector motion space and an internal/null motion space, under an appropriately defined metric.

Most previous approaches do not consider torque limits for both components of the system, manipulator and mobile platform which make the proposed schemes inappropriate for realistic applications. In

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this paper, a dynamic redundancy resolution for mobile manipulators is proposed with torque limits avoidance. This paper is organized as follows. Section 2 is devoted to mathematical description of the mobile manipulator with nonholonomic constraints. Section 3 presents the design of the control strategy for resolution of the redundancy for nonholonomic mobile manipulator with consideration of torque limits. Section 4 presents computer simulation results to illustrate the effectiveness of the proposed theory. Conclusions are formulated in Section 5.

2. MODELING OF A MOBILE MANIPULATOR

In this section, we consider the mobile manipulator consisting of two subsystems, namely a nonholonomic wheeled mobile platform and holonomic rigid manipulator whose schematic top view is shown in Figure 1. We assume that the mobile platform has two coaxial wheels driven by two independent DC motors and a two-link manipulator mounted on top of the platform.

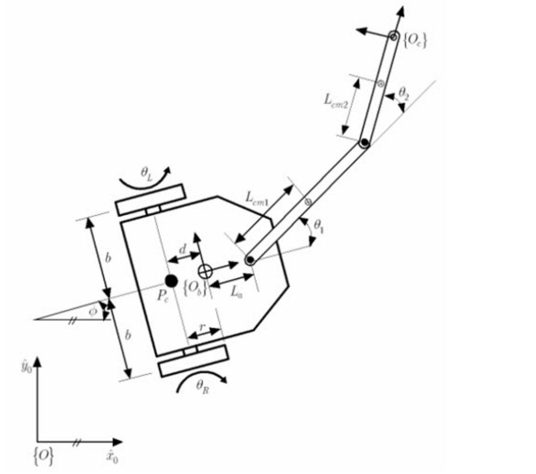

Fig.1: *Schematic of a Mobile Platform with 2 DOF Manipulator[10]*

The dynamics of a mobile manipulator subject to nonholonomic constraints can be obtained using the Lagrangian approach in the following form:

$$H(q)\ddot{q} + V(q,\dot{q}) + A^T(q)\lambda = E(q)T \tag{1}$$

The m nonholomonic constraints can be expressed as:

$$A(q)\dot{q} = 0 \tag{2}$$

where $q \in \mathfrak{R}^{n\times 1}$ denote the n generalized coordinates, $H(q) \in \mathfrak{R}^{n\times n}$is a symmetric and positive definite inertia matrix, $V(q,\dot{q}) \in \mathfrak{R}^{n\times 1}$ represents the centripetal and Coriolis vector,$A(q) \in \mathfrak{R}^{m\times n}$ is the constraint matrix,$A(q) \in \mathfrak{R}^{m\times n}$ is the Lagrange multiplier which denotes the vector of constraint forces,$E(q) \in \mathfrak{R}^{n\times(n-m)}$ is the input transformation matrix and $T \in \mathfrak{R}^{(n-m)\times 1}$ is the torque input vector.

We can now find an appropriate null space matrix S that satisfies $A.S = 0$. The set of feasible velocities may be expressed in terms of a suitable vector of $h = n - m$ independent velocities,$\dot{z} = \left[\dot{\theta}_R\ \dot{\theta}_L\ \dot{\theta}_1\ \dot{\theta}_2\right]^T$ as:

$$\dot{q} = S(q)\dot{z} \tag{3}$$

where $\dot{\theta}_R$ and $\dot{\theta}_L$ are the angular velocities of the left and right wheels respectively, $\dot{\theta}_1$ and $\dot{\theta}_2$ are the angular velocities of the arm 1 and the arm 2 of the manipulator respectively.

We consider the task space to consist of the xy-position of the end-effector. Hence, for subsequent analysis we also determine the Jacobian that relates the extended joint rates $\dot{q}$, to the task space $\dot{X}$, as:

$$\dot{X} = J(q)\cdot\dot{q} \tag{4}$$

Differentiating Equation (4) with respect to time yields the corresponding relationship for accelerations:

$$\ddot{X} = J(q)\ddot{q} + \dot{J}(q)\dot{q} \tag{5}$$

The null space matrix of the constraint as defined by equation (3) projects all the configuration spaces motions onto the feasible motion space so that they reflect only the feasible motions allowable by satisfying the constraints, we obtain:

$$M(q)\ddot{z} + C(q) + G(q) = \tau(q) \tag{6}$$

where $M(q) = S^THS$, $C(q) = S^TH\dot{S}\dot{z}$, $G(q) = S^TV\tau(q) = S^TET$, are the inertia matrix, centripetal and Coriolis vector and torque vector, all described in the feasible motion space.

3. DYNAMIC REDUNDANCY RESOLUTION ALGORITHM WITH TORQUE LIMITS AVOIDANCE

For a redundant system, the Jacobian matrix expressed in equation (4) is a non-square. The general solution for the kinematic redundancy resolution may be written as:

$$\dot{z} = J^{\#}\dot{X} + N\dot{z} \tag{7}$$

where $J^{\#} = J^T(JJ^T)^{-1}$ is the pseudo-inverse of the Jacobian matrix J and N is the null space of J defined as $N = I - J^{\#}J$.

A dynamic redundancy resolution technique based on weighted pseudo-inverse that accounts for inertia can be given as [7]:

$$J_M^{\#} = M^{-1}J^T(JM^{-1}J^T)^{-1} \tag{8}$$

If we assume symmetric torque limits such that:

$$-\tau_i^{\lim} \le \tau_i \le_i^{\lim}, \qquad i = (1,\ldots,4) \tag{9}$$

Then the normalized actuator torques, $\tilde{\tau}$ can be written as :

$$\tilde{\tau} = L^{-1}\tau \quad (10)$$

where: $L = diag\left(\tau_R^{\lim}, \tau_L^{\lim}, \tau_1^{\lim}, \tau_2^{\lim}\right)$.
The set of admissible torques can then be represented as inequalities equations written in the following compact form:

$$\|\tilde{\tau}\|_\infty \leq 1 \quad (11)$$

Substituting Equations 5, 6 and 8 in the above equation we obtain:

$$(\ddot{X} - \ddot{X}_b)^T (J^{-T} M L^{-2} M J^{-1})(\ddot{X} - \ddot{X}_b) \leq 1 \quad (12)$$

where $\ddot{X}_b = -JM^{-1}(C + G) + \dot{J}\dot{q}$
Following the recommendation by Chiacchio [11] we utilize the weighted pseudo-inverse of the Jacobian:

$$\bar{J} = J_Q^{\#} = Q^{-1} J^T (J Q^{-1} J^T)^{-1} \quad (13)$$

where $Q = ML^{-2}$ is a weight matrix that accounts for both inertia and torque limits. And the corresponding null space as: $\bar{N} = I - \bar{J}J$.
Now, using the following relationship between the generalized forces in task space F and the corresponding joint space torques τ:

$$\tau = J^T F + (I - J^T \bar{J}^T)\tau \quad (14)$$

And substituting (6) in the above equation and rearranging the terms yields:

$$J^T F + \bar{N}^T \tau = F_T + \tau_N + C_F \quad (15)$$

where $F_T = J^T \bar{J}^T (M\bar{J}(\ddot{X} - \dot{J}\ddot{z}) + C + G)$ corresponds to the forces in the task space.
$\tau_N = \bar{N}^T(M\bar{N}\ddot{z} + C + G)$corresponds to the torques in the null space.
$C_F = J^T \bar{J}^T M\bar{N}\ddot{z} + \bar{N}^T M\bar{J}(\ddot{X} - \dot{J}\ddot{z})$contains the coupling torques and forces.
The task space forces and the null space torques are fully decoupled if $C_F = 0$. Thus,

$$\bar{J}^T M \bar{N} = \bar{N}^T M \bar{J} = 0 \quad (16)$$

A control law can now be proposed that decouples the task spaces behavior and the null space behavior with torque limits consideration and controls them in prioritized basis with priority given to the primary task space behavior as [10]:

$$\tau = J^T W(u_T - \dot{J}\ddot{z}) + \bar{N}^T M(u_N + \dot{\bar{J}}\dot{X}) + C + G \quad (17)$$

where u_T and u_N are the control laws for the task-space and null-space, respectively and $W = (JM^{-1}J^T)^{-1}$.

3.1 Task space controller

After substituting equation (17) into (15) and pre-multiplying both the sides by JM^{-1}, and simplifying, the final equivalent controlled closed loop dynamics in the independent task space may be written as:

$$u_T - \ddot{X} = 0 \quad (18)$$

It is important to note that the controlled, closed-loop task-space motion described by equation (18) is completely independent from the motion of the null space. Therefore, the task space motion is guaranteed to be satisfied at all times and is not affected in anyway by inputs that control the null space.

Hence for the task space, we choose to control the position coordinate of the end-effector with a fuzzy logic controller that has two inputs: the position error , the velocity error and one output which is the torque for motor, tuned by fuzzy inference during the system's running period.

Input and output fuzzy members functions are symmetric. Trapezoid and triangle member functions were used in membership functions. The position error is partitioned into five fuzzy sets (Figure 2): Big Negative (BN), Small Negative (SN), Zero (Z), Small Positive (SP) and Big Positive (BP).

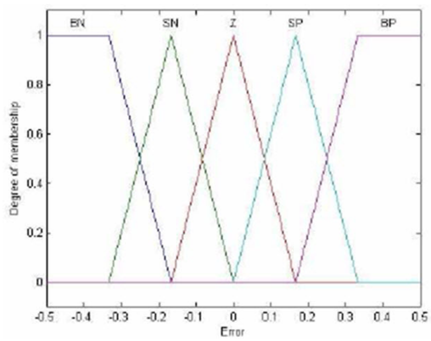

Fig.2: *Membership functions for position error input*

The velocity error consists of five fuzzy sets (Figure 3): Far Left (FL), Medium Left (ML), Centre (C), Medium Right (MR) and Far Right (FR).

The torque output for DC motors is partitioned into five fuzzy sets (Figure 4): Left Big (LB), Left Small (LS), Zero (Z), Right Small (RS) and Right Big (RB).

Using the above fuzzy sets of the input and output variables, fuzzy control rules are listed in table 1 as follows [12]:
The rule set is established and shown in surface in Figure. 5.

3.2 Null space controller

After substituting equation (17) into (15) and pre-multiplying both the sides by $\bar{N}M^{-1}$, and simplifying, the final equivalent controlled closed loop dy-

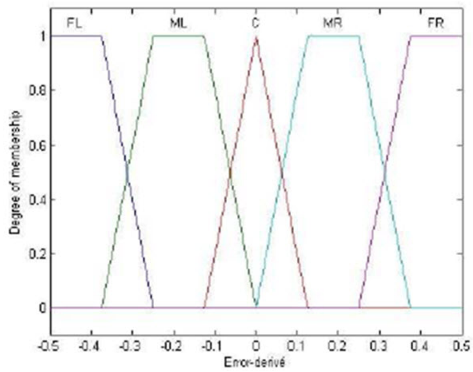

Fig.3: *Membership functions for velocity error input*

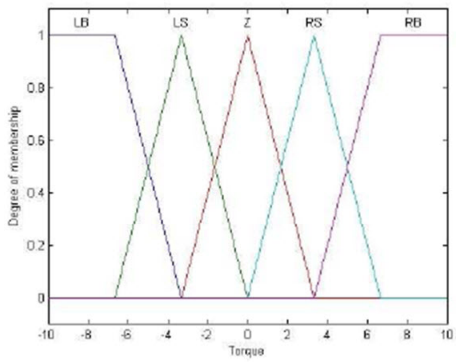

Fig.4: *Membership functions for torque output*

Table 1: *The fuzzy control rules*

Torque u_T		**Position error** e				
		FL	**ML**	**C**	**MR**	**FR**
Velocity error $\dot{e}$	**BN**	LB	LB	LB	Z	RS
	SN	LB	LS	LS	Z	RS
	Z	LB	LS	Z	RS	RB
	SP	LS	Z	RS	RS	RB
	BP	LS	Z	RB	RB	RB

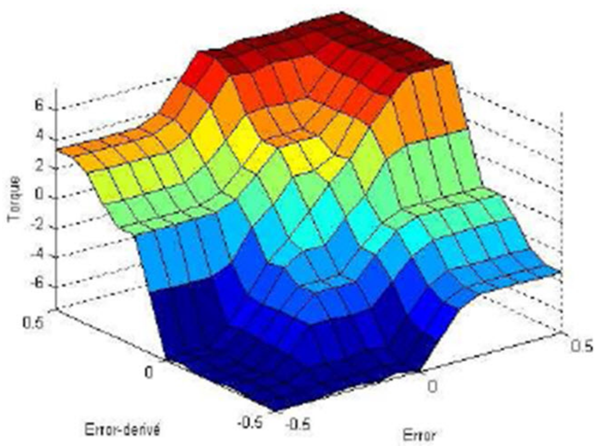

Fig.5: *Fuzzy control rule in surface*

namics in the independent null space may be written as:

$$\bar{N}(\ddot{z} - v - \dot{\bar{J}}\ddot{x}) = 0 \tag{19}$$

The closed-loop null-space motion described by equation (19) is completely independent from the motion of the task space. Therefore, the null space motion is not affected in anyway by inputs that control the task space. Hence for the null space, we choose to control the position coordinate of the platform in (x-y) plane. Controlling two such null space outputs, in addition to two task space outputs, effectively eliminates all redundancy within the system.

For the platform, the problem of the trajectory tracking in task space is resolved using the following backstepping technique as [13]:

$$\begin{pmatrix} v_c \\ w_c \end{pmatrix} = \begin{pmatrix} v_d \cos(e_3) + k_1 e_1 \\ w_d + k_2 v_d e_2 + k_3 v_d \sin(e_3) \end{pmatrix} \tag{20}$$

where v_c and w_c are the controller linear and angular velocities, respectively. k_1, k_2 and k_3 are positive parameters. v_d and w_d are the reference linear and angular velocities, respectively. e_1, e_2 and e_3 describe the difference of position and orientation of the reference platform from the real platform and are given as:

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_p^d - x_p \\ y_p^d - y_p \\ \varphi^d - \varphi \end{pmatrix} \tag{21}$$

where x_p^d, y_p^d and φ^d are the desired position and orientation of the platform and x_p, y_p and φ are the actual position and orientation of the platform.

The controlled linear and angular velocities can be transformed to the controlled wheel velocities, and adding a damping terms to stabilize the velocities, we obtain:

$$u_N = \begin{bmatrix} v_R \\ v_L \end{bmatrix} = R \begin{bmatrix} K_{pv}(v_c - v) \\ K_{pw}(w_c - w) \end{bmatrix} \tag{22}$$

where R is a transformation matrix between the linear/angular velocity space and the wheel velocity space, K_{pv} and K_{pw} are the gains. Suitable gains were selected for the platform motion to provide a stable/damped convergence with minimal oscillation in both the linear and angular velocities while the arm-control inputs are set to zero.

4. SIMULATION

The experiments were developed under the MATLAB simulation environment. The aim is to verify the proposed dynamic redundancy resolution. The parameters for the mobile manipulator (Figure.1) are shown in Table 2.

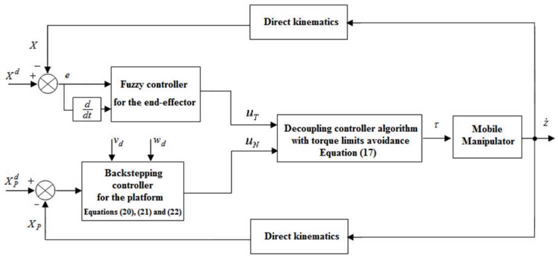

Fig.6: *The proposed control scheme*

In this Section, we show the redundancy resolution results with considering joints torque limits. The desired trajectory for the mobile manipulator has been chosen as follows:

$$\begin{pmatrix} X^d \\ X_P^d \end{pmatrix} = \begin{pmatrix} x^d \\ y^d \\ x_P^d \\ y_P^d \end{pmatrix} = \begin{pmatrix} 0.2t + 0.3 \\ 0.5 + 0.25\sin(0.2\pi t) \\ 0.2t \\ 0 \end{pmatrix} \quad (23)$$

It consists of a sinusoidal path for the end-effector and a straight line for the platform where (x^d, y^d) are the desired position coordinate for the end-effector and (x_P^d, y_P^d) are the desired position coordinate for the platform.

The time of the motion is 20 seconds and the initial position of the platform was equal to: $(x_P(0), y_P(0), \varphi(0)) = (0, 0, \frac{\pi}{2})$ and the initial position of the manipulator was equal to $(\theta_1(0), \theta_2(0)) = (\frac{\pi}{4}, -\frac{\pi}{2})$.

Table 2: *Parameters of the mobile manipulator*

Parameters (Units)	Platform	Manipulator
Dimension (m)	$b = 0.182$ $r = 0.0508, l_a = 0.1$	$l_1 = 0.514, l_2 = 0.362$ $l_{cm1} = 0.252, l_{cm2} = 0.243$
Mass (kg)	$m_c = 17.25$ $M_w = 0.159$	$m_1 = 2.56$ $m_2 = 1.07$
Inertia $(kg.m^2)$	$I_c = 0.297$ $I_w = 0.0002$	$I_1 = 0.148$ $I_2 = 0.0228$
torque limits $(N.m)$	$\tau_R^{\lim} = 10$ $\tau_L^{\lim} = 10$	$\tau_1^{\lim} = 10$ $\tau_2^{\lim} = 10$

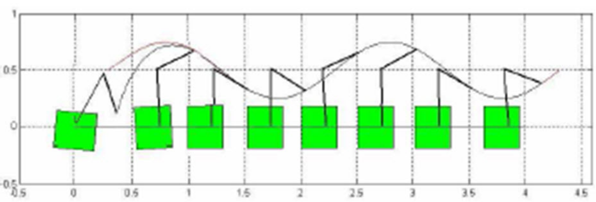

Fig.7: *Mobile Manipulator tracking the prescribed trajectories*

Figure. 6 illustrates motion trajectories of the mobile manipulator when its end-effector is moving along a sinusoidal path and the platform is moving along a straight line. We can see that the mobile manipulator track the desired trajectories with a perfect coordination between the arm and the platform which demonstrate the effectiveness of the proposed dynamic redundancy resolution algorithm.

We notice that all the trajectories tracking errors (Figure 7 and 8) asymptotically tend to zero which demonstrate that the goal control performance was reached and validate the effectiveness of the proposed algorithm to drive simultaneously the end-effector and the platform and to realize a complex trajectories in the plane.

It is also important to notice that the proposed weighted pseudo-inverse of the Jacobian that accounts for both inertia and torque limits can achieve the desired trajectories without exceeding the joints torque limits. However, at the beginning the torque in joint 1 and 2 are about 32 $N.m$ and 15 $N.m$ (Figure 9) for a controller with a weighted pseudo-inverse of the Jacobian that accounts only for inertia. Whereas, the torque in joint 1 and 2 was successfully suppressed to less than 10 $N.m$ (Figure 10) which is the limit torque with the pseudo-inverse of the Jacobian that accounts for both inertia and torque limits simultaneously and thus the solution becomes applicable for physical system.

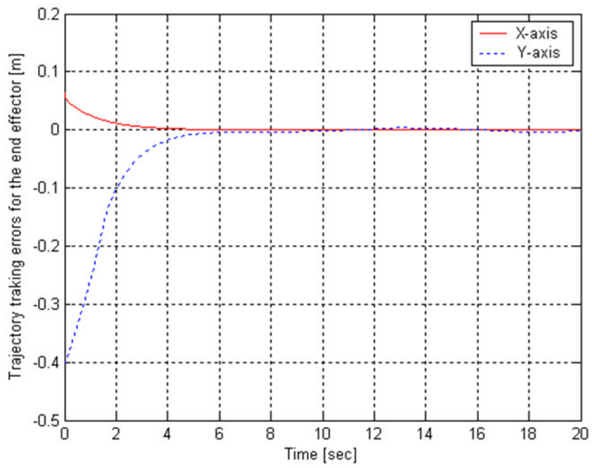

Fig.8: *Trajectory tracking errors for the end-effector in x-axis and y-axis.*

5. CONCLUSION

This paper describes a dynamic redundancy resolution control strategy for mobile manipulators. The proposed dynamic control low decomposes the system dynamics into decoupled task space (end- effector motion) and a dynamically consistent null space (internal motion) component. This simplifies the subsequent development of a prioritized task space con-

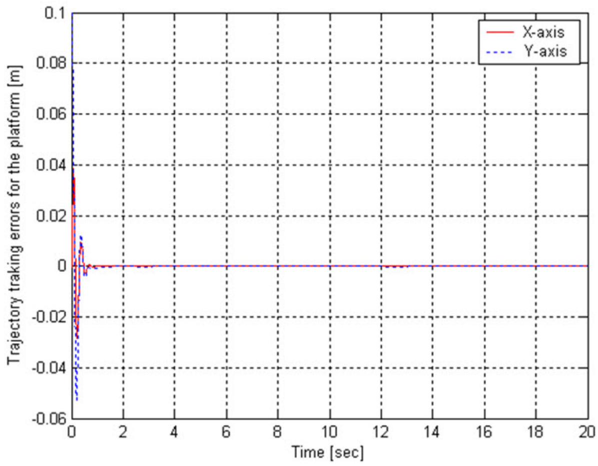

Fig.9: *Trajectory tracking errors for the platform in x-axis and y-axis.*

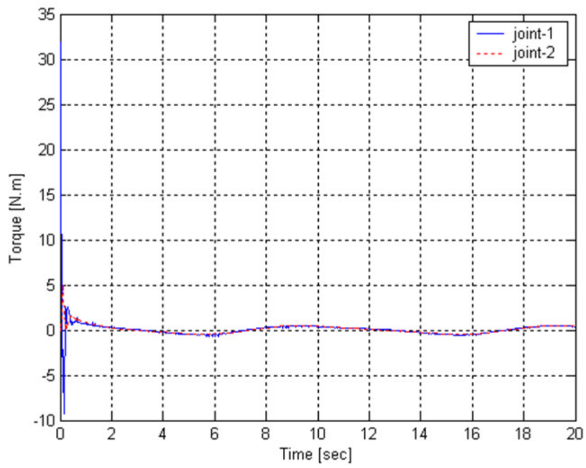

Fig.10: *Torque in joint 1 and joint 2 with a pseudo-inverse of the Jacobian that accounts for inertia only*

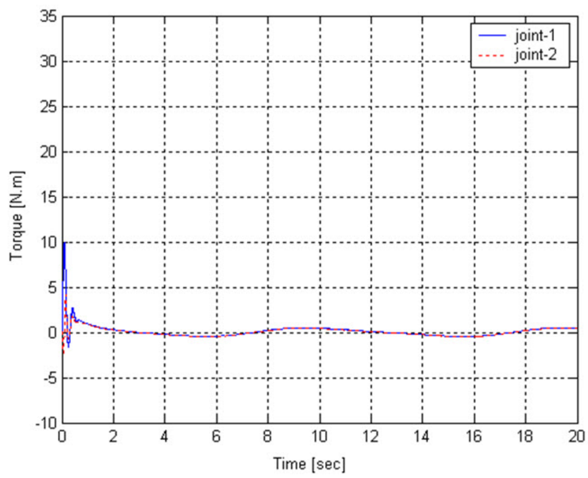

Fig.11: *Torque in joint 1 and joint 2 with a pseudo-inverse of the Jacobian that accounts for inertia and torque limits*

trol (of end-effector interactions) and a decoupled but secondary null space control (of internal motion) in a hierarchical mobile manipulator controller. A new weighted pseudo-inverse of the Jacobian that accounts for both inertia and torque limits is proposed to solve the problems of joints torque limits of the system. The proposed controller is capable to accomplish a good tracking performance without violating the joints torque limits of the mobile manipulator. However, further experimentation needs to be carried out to explore the maximum potentials of the scheme when other different tasks, parameters or operating and loading conditions are considered. The practical issues related to the physical mobile manipulator should also be investigated.

6. ACKNOWLEDGMENTS

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References

[1] O. Khatib, "Mobile manipulation: The robotic assistant. Robot," *Auton.Syst*, vol. 26, no. 2/3, pp. 175-183, 1999.

[2] Y. Das. K Russell. N. Kircanski and A. Goldenberg., "An articulated robotic scanner for mine detection-a novel approach to vehicle mounted systems," *Proceedings of the SPIE Conference*, Orlando. Florida, pp. 5-9, April 1999.

[3] *Mars Exploration Rover Launches.* NASA Press Kit , pp.38-43, June 2003.

[4] H. Seraji, "A Unified Approach to Motion Control of Mobile Manipulators," *The International Journal of Robotics Research*, Vol.,17, no.2, Feb.,1998, pp. 107-118.

[5] Y. Yamamoto and X. Yun, "Coordinating locomotion and manipulation of a mobile manipulator," *IEEE Transactions on Automatic Control*, 39(6), 1326-1332, 1994.

[6] Y. Yamamoto and X. Yun, "Effect of the dynamic interaction on coordinated control of mobile manipulators," *IEEE Transactions on Robotics and Automation*, 12(5), 816-824, 1996.

[7] O.Khatib, "A unified approach to motion and force control of robot manipulators: The operational space formulation," *IEEE Journal on Robotics and Automation*, Vol. RA-3, no, pp. 43-53, 1987.

[8] O.Khatib. K. Yokoi. K. Chang, D. Ruspini. R. Holmberg, and A.Casal, "Vehicle/arm coordination and multiple mobile manipulator decentralized cooperation," in *Proc. 1996 IEEE/RSJ International Conference on Intelligent Robots and Systems*, Osaka, Japan, 1996.

[9] J.Tan, N. Xi and Y. wang, "Integrated task planning and control for mobile manipulators," *The International Journal of Robotics Research*, vol. 22, no.5, pp.337-354, 2003.

[10] Rajankumar M. Bhatt, *Towards Modular Cooperation Between Multiple Nonholonomic Mobile Manipulators*, Ph.D.dissertation, Univ. of New York at Buffalo, USA 2007.

[11] P. Chiacchio, "A new dynamic manipulability ellipsoid for redundant manipulators," *Robotica*, vol. 18, no. 4, pp. 381-387, 2000.

[12] M. Nil. U. Yuzgek and M. Sonmez, "Fuzzy Neural Network Based Intelligent Controller for 3 DOF Manipulators," *Proceedings of 5th International Symposium on Intelligent Manufacturing Systems*, May 29-31, 2006.

[13] R. Fierro and F. Lewis, "Control of a Nonholonomic Mobile Robot: Backstepping Kinematics into Dynamics," *Journal of Robotic Systems*, 14(3), 149-163, 1997.

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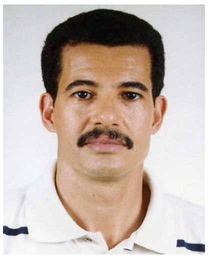

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