Modified CIC Filter for Rational Sample Rate Conversion

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ABSTRACT

The modification of the conventional CIC (cascadedintegrator-comb) filter for rational sample rate conversion (SRC) in software defined radio (SWR) systems is presented here. The conversion factor is a ratio of two mutually prime numbers, where the decimation factor M can be expressed as a product of two integers. The overall filter realization is based on a stepped triangular form of the CIC impulse response, the corresponding expanded cosine filter, and sine-based compensation filter. This filter performs sampling rate conversion efficiently by using only additions/subtractions making it attractive for software defined radio (SWR) applications.

Keywords: CIC filter, Rational sample rate conversion, Software defined radio.

1. INTRODUCTION

The transfer function of the cascaded-integrator-comb (CIC) filter for sampling rate conversion by a factor M/L, shown in Fig. 1, is given by [1]

$$H(z) = \frac{\left(1 - z^{-DM}\right)^{K_1}\left(1 - z^{-DL}\right)^{K_2}}{\left(1 - z^{-1}\right)^{K}}. \tag{1}$$

where D is the delay of each comb stage and $K = K_1 + K_2$.

This filter is very simple and uses only additions/subtractions. However, it has a limited number of tuning parameters, and has a high passband droop, and does not provide enough attenuation in the region of interest in the stopband. Various methods have been proposed to improve the characteristics of the conventional CIC filter to make them suitable for software defined radio (SWR) applications, [2]-[8]. In [2] is proposed a modified CIC filter of the form

$$H_m(Z) = \frac{\left(1 - z^{-D_1}\right)\left(1 - z^{-D_2}\right)...\left(1 - z^{-D_N}\right)}{\left(1 - z^{-1}\right)^{K}}. \tag{2}$$

where $D = D_1, D_2, ..., D_N$ is a set of comb delays. It has been shown that the modified CIC filter provides higher image attenuation than the conventional CIC filter. Additionally, the modified CIC filter provides higher SNR, where the SNR is defined as the power ratio after lowpass filtering of the lowest power level in the desired signal to the highest power level in the images [2]. In [8] is presented one alternative modification of the CIC filter based on stepped triangular form of the CIC impulse response and expanded cosine filters. The filter exhibits lower complexity and better performances than the modified CIC filter introduced in [2]. The main objective of this work is to propose an alternate modification of the CIC filter from [8] in order to achieve an improved SNR than in [8]. Specially, we propose to cascade the simple second order compensation multiplierless filter to the modified CIC filter from [8]. The rest of the paper is organized in the following way. Next sections introduce the stepped triangular (ST) and ST-cosine based CIC filters. The section 4 presents the new compensation filter. The proposed structure is given in Section 5 and illustrated with one example.

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2. STEPPED TRIANGULAR CIC FILTER

We develop next the relation between the stepped triangular impulse response filter and the CIC filter of the order M

$$G(z) = \frac{1}{M}\frac{\left(1 - z^{-M}\right)}{1 - z^{-1}}. \tag{3}$$

In general, the transfer function of ST CIC filter is given by

$$H_{ST}(z) = \frac{1}{M}\frac{1 - z^{-M}}{1 - z^{-1}}\frac{1}{N_2}\frac{1 - z^{-N_1N_2}}{1 - z^{-N_1}}; N_1N_2 = N_{1,2}. \tag{4}$$

Note that $N_{1,2}$ can be can be either equal to or different than M. Considering the decimation factor M is a product of two integers, i.e.,

$$M = M_1 M_2. \tag{5}$$

eq. (4) can be rewritten as

$$H_{ST}(z) = \frac{1}{M_2}\frac{1 - z^{-M}}{1 - z^{-M_1}}\frac{1}{M_1}\frac{1 - z^{-M_1}}{1 - z^{-1}}\frac{1}{N_2}\frac{1 - z^{-N_1N_2}}{1 - z^{-N_1}}. \tag{6}$$

We relate N_1 and N_2 with M_1 and M_2 so that the subfilters in Eq. (6) can be moved to a lower rate using multirate identity [9]. Consequently, from Eq. (6) we arrive at

$$H_{ST}(z)=\begin{cases}\dfrac{1}{M_2}\dfrac{1\text{-}z^{-M}}{1-z^{-M_1}}\left[\dfrac{1}{M_1}\dfrac{1-z^{-M1}}{1-z^{-1}}\right]^2 & \textit{for } N_2=M_1 \\ & \quad N_1=1 \\ \left[\dfrac{1}{M_2}\dfrac{1\text{-}z^{-M}}{1-z^{-M_1}}\right]^2\dfrac{1}{M_1}\dfrac{1-z^{-M1}}{1-z^{-1}} & \textit{for } N_2=M_2 \\ & \quad N_1=M_1\end{cases} \tag{7}$$

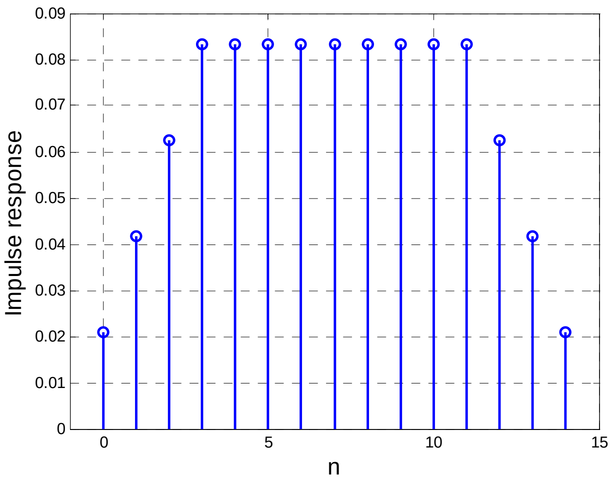
Note that N_2 in Eq. (7) is the number of levels in ST impulse response. More levels result in better magnitude characteristics. To this end we choose

$$H_{ST}(z)=\begin{cases}\dfrac{1}{M_2}\dfrac{1\text{-}z^{-M}}{1-z^{-M_1}}\left[\dfrac{1}{M_1}\dfrac{1-z^{-M1}}{1-z^{-1}}\right]^2 & \textit{for } M_1>M_2 \\ \left[\dfrac{1}{M_2}\dfrac{1\text{-}z^{-M}}{1-z^{-M_1}}\right]^2\dfrac{1}{M_1}\dfrac{1-z^{-M1}}{1-z^{-1}} & \textit{for } M_1<M_2\end{cases} \tag{8}$$

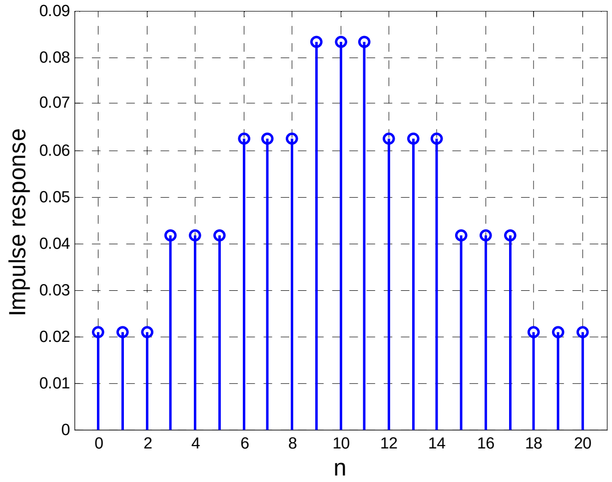
Example 1:
We consider ST CIC filter for M=12.
From Eq. (8) we have

$$H(z)=\begin{cases}\dfrac{1}{3}\cdot\dfrac{1-z^{-12}}{1-z^{-4}}\left[\dfrac{1}{4}\cdot\dfrac{1-z^{-4}}{1-z^{-1}}\right]^2 & \text{for } M_1=4, M_2=3, \\ \left[\dfrac{1}{4}\cdot\dfrac{1-z^{-12}}{1-z^{-4}}\right]^2\cdot\dfrac{1}{3}\cdot\dfrac{1-z^{-3}}{1-z^{-1}} & \text{for } M_1=3, M_2=4.\end{cases}$$

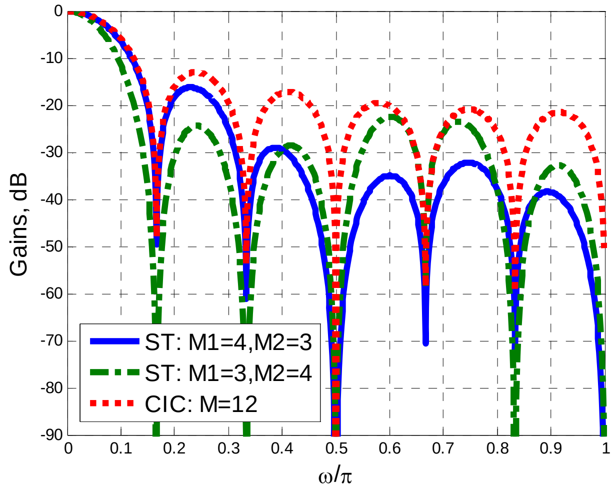
The corresponding impulse responses are shown in Fig. 2(a) and (b), while the associated gain responses are presented in Fig. 2 (c), along with the gain response of the overall CIC filter.



(a) N_2= M_1=4, N_1=1, M_2=3.



(b) N_2=M_2=4, N_1=M_1=3.



(c) Gain responses.

Fig.2: *Example 1.*

3. ST-COSINE CIC FILTER

Consider sampling rate conversion by a factor L/M where $M > L$ and the decimation factor can be expressed as in Eq. (3). We consider a decimation filter which provides enough attenuation of the aliasing caused by down-sampling and also enough attenuation of images introduced by interpolation. It is assumed that the signal occupies 3/4 of the available band. Additionally, we propose to cascade an expanded cosine filter $H_{cos}(z)$ with ST CIC filter of Eq. (8) to introduce an additional zero in the frequency band where the worst case of aliasing occurs so as to increase the SNR. The expanded filter is given by

$$H_{cos}(z)=\left(1+z^{-R}\right)/2. \tag{9}$$

where

$$R_{min} \leq R \leq M. \quad (10)$$

The value of R which is equal to R_{min} from Eq. (10) usually results in higher aliasing attenuation. This value is computed as using

$$R_{min} = \texttt{int}\,[1/\omega_1]\,. \quad (11)$$

where int[.] means integer part of [.] and

$$\omega_1 = \frac{2}{M} - \omega_p\ ;\ \omega_p = \frac{3}{4}\frac{1}{M}. \quad (12)$$

The magnitude characteristic of $H_{cos}(z)$ is given as

$$H_{cos}\,(\omega) = cos\,(R\omega/2)\,. \quad (13)$$

Using Eqs. (8) and (9) we arrive at the transfer function of the ST-Cosine CIC decimation filter

$$H(z) = \left[\frac{1}{M_2}\frac{1-z^{-M}}{1-z^{-M_1}}\right]^{k_1}\left[\frac{1}{M_1}\frac{1-z^{-M_1}}{1-z^{-1}}\right]^{k_2}\left[\frac{(1+z^{-R}}{2}\right]^{k_3} \quad (14)$$

where k_1, k_2 and k_3 denote the corresponding stages and

$$k_2 \geq 2k_1\ for\ M_1 > M_2;$$
$$k_1 \geq 2k_2\ for\ M_1 < M_2. \quad (15)$$

Making use of the multirate identity [9] we have the final structure which consists of three stages. In general, the first stage is a cascade of k_3 combs with a delay R and k_2 integrators. The next stage is a cascade of $(k_2 - k_1)$ combs or integrators as indicated in Eq. (15). Finally, the last stage is a cascade of k_1 combs. Note that for R=M, k_3 combs from first stage can be moved to the last stage with a unity delay. Similarly for $R = rM_1$, where r is an integer, the cascade of combs with delay R can be moved from the first stage to the second stage with a delay of r. The complexity of the proposed filter, presented in terms of their memory requirements and number of additions (or subtractions) per output sample (APOS), is given in Table 1.

Table 1: *Complexity of structure [8]*

R	**Memory req.**	**APOS**		
R	$2k_2$+Rk_3	k_1+	k_2-k_1	M_2+$(k_3$+$k_2)M$
R=rM_1	$2k_2$+rk_3	k_1+	k_2-k_1	M_2+k_3M_2+ k_2M
R=M	$2k_2$+k_3	k_1+k_3+	k_2-k_1	M_2+ k_2M

In the following example we compare the ST-cosine CIC structure with the one proposed in [2].

Example 2:
We apply the design of a sampling rate converter for a conversion factor of 9/10 [2].
From Eqs. (11) and (12) we have

$$\omega_1 = \tfrac{2}{10} - \tfrac{3}{4} \times \tfrac{1}{10} = 0.125$$

and $R_{min} = 8$

We choose M_1=2 and M_2=5, and R=8.Using k_3=1, k_1=8 and, k_2=4 from Eqs. (13) and (14) we have

$$H_p(z) = \left[\frac{1}{5}\frac{1-z^{-10}}{1-z^{-2}}\right]^8\left[\frac{1}{2}\frac{1-z^{-2}}{1-z^{-1}}\right]^4\left[\frac{(1+z^{-8})}{2}\right]$$

The corresponding gain responses are shown in Fig. 3. The conventional CIC has two interpolation combs, two interpolation integrators, two decimation combs, and two decimation integrators. The modified CIC filter [2] of the same order N=4 has four combs with the delays 16, 14, 12 and 10, and four integrators, all working at the high input rate. The corresponding SNRs are 15 dB (Conventional CIC), 50 dB (Modified CIC [2]) and 58.9 dB, [8], respectively. The modified CIC filter requires: 16+14+12+10+4=56 memory elements, while the filter [8] requires 12 memory elements (Table 1). The filter [8] and the modified CIC filter [2] require 73 and 80 APOS, respectively. Therefore the filter [8] exhibits better performance while having a lower complexity. The corresponding structure is shown in Fig.4.

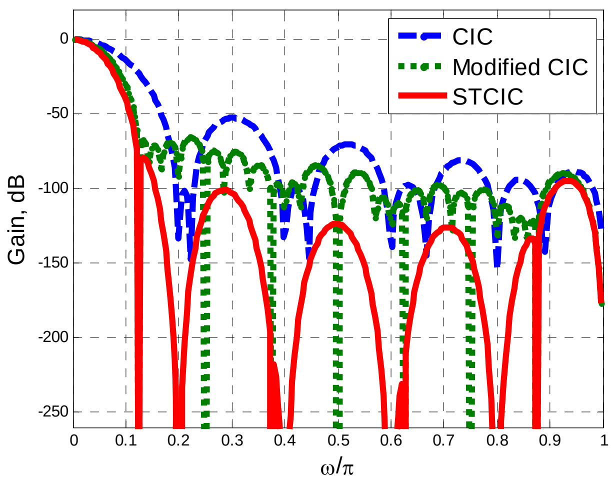


Fig.3: *Example 2.*

In the next section we introduce the simple compensator filter in order to further improve SNR.

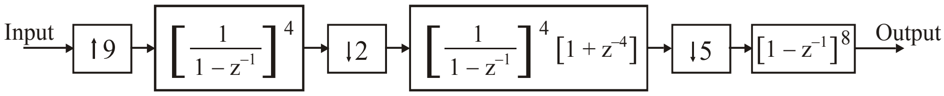


Fig.4: *Example 2: SRC 9/10, M1=2, M2=5.*

4. COMPENSATION FILTER

Consider the filter with the magnitude response

$$\left|G\left(e^{j\omega M}\right)\right| = \left|1 + 2^{-b}\ sin^2\left(\omega M/2\right)\right|. \qquad (16)$$

Using the well known relation

$$sin^2\alpha = \left(1 - cos\ 2\alpha\right)/2. \qquad (17)$$

the corresponding transfer function is given as

$$G\left(z^M\right) = B\left[1 + Az^{-M} + z^{-2M}\right]. \qquad (18)$$

where B is the scaling factor,

$$B = -2^{-(b+2)}. \qquad (19)$$

and

$$A = -\left[2^{(b+2)} + 2\right]. \qquad (20)$$

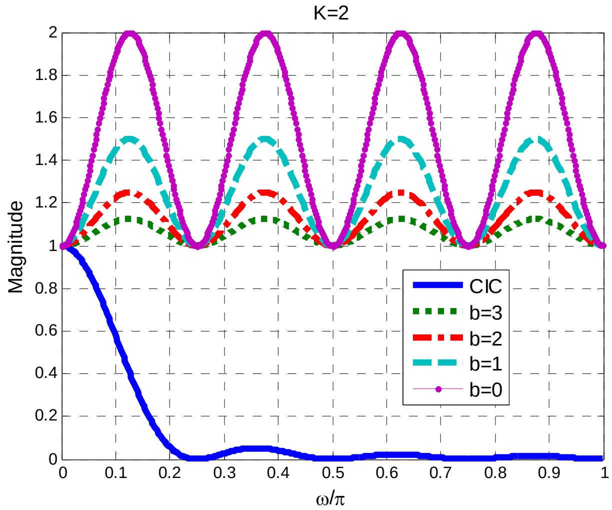
Two principal characteristics of this filter are:

• The transfer function is a function of z^{-M} and using the multirate identity, [9] the filter can be moved after the downsampling, and,

• The compensator filter is multiplier-free and has only one coefficient A which can be realized using only additions and shifts.

Figure 5 illustrates the magnitude responses of the compensator filters for four different values of b along with the magnitude responses of the CIC filter with $M = 8$, and K=1 and 2.

Note that for the compensation of the CIC filter in the narrow passband a good choice would be to use b=3 for K=1, and b=2 for K=2. Similarly, for the compensation in the wideband interval, a good choice would be b=2, for K=1, and b=1 for K=2.
The same is confirmed in Figure 6 which shows the gain responses of the compensated CIC filters along with that of the CIC filter in the passband.
Different simulations show that the parameter b mainly depends on the chosen value of k_1, (for $M_1 < M_2$), and k_2 (for $M_1 > M_2$), as given in the Table 2.



(b) K=2.

Fig.5: *CIC and compensation filters.*

Table 2: *Typical parameters*

Parameter $k_1\ (k_2)$	***Parameter*** b
8, 9	-3
7	-2
5	0
4	1

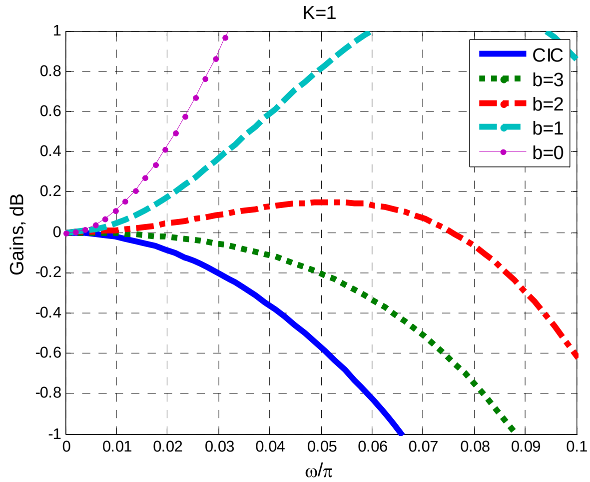
5. PROPOSED FILTER

We propose to cascade the STCIC filter from Eq.(14) with the compensated filter (18). Using Eqs (14), (18) we have the proposed filter

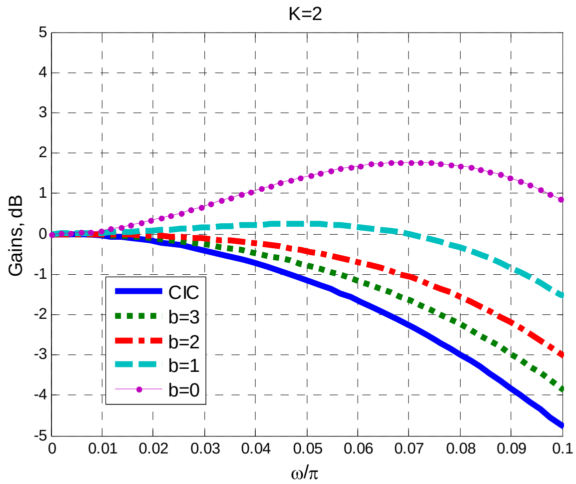
$$H(z) = H_p(z)\, G\left(z^M\right). \qquad (21)$$

or

$$H(z) = \left[\frac{1}{M_2}\frac{1 - z^{-M}}{1 - z^{-M_1}}\right]^{k_1}$$



(a) K=1.



(b) K=2.

Fig.6: *CIC and compensated CIC filters.*

$$\times\left[\frac{1}{M_1}\frac{1-z^{-M_1}}{1-z^{-1}}\right]^{k_2}\left[\frac{1+z^{-R_1}}{2}\right]^{k_3}$$
$$\times B\left[1+Az^{-M}+z^{-2M}\right]. \qquad (22)$$

Denoting

$$H_{p1}(z)=\left(\frac{1}{1-z^{-1}}\right)^{k_2}\left(1+z^{-1}\right)^{k_3}.$$
$$H_{p2}=\left(1-z^{-1}\right)^{k_2-k_1}.$$
$$H_{p3}=\left(1-z_{-1}\right)^{k_1}\left[1-\left(2^{b+2}+2\right)Az^{-1}+z^{-2}\right].(23)$$

and making use of the multirate identity [9] we have the final structure shown in Fig. 7 The structure consists of three stages. In general, the first stage is a cascade of k3 combs with the delays R_1, and k_2 integrators. The next stage is a cascade of (k_2-k_1) combs or integrators as indicated in Eq. (15). Finally, the last stage is a cascade of k_1 combs and a second order filter. The complexity of the proposed filter, presented in terms of their memory requirements and number of additions (or subtractions) per output sample (APOS), is given in Table 3.

Example 3:
We consider the design of a sampling rate converter for a conversion factor of 9/10 from Example 2.

Choosing the same parameters as in Example 2 and additionally $k_3=2$, and $b=$ -3 we have

$$H(z)=\left[\frac{1}{5}\frac{1-z^{-10}}{1-z^{-2}}\right]^8\left[\frac{1}{2}\frac{1-z^{-2}}{1-z^{-1}}\right]^4\left[\frac{1-z^{-8}}{2}\right]^2$$
$$\times\left[1-\left(2^{-1}+2\right)z^{-1}+z^{-2}\right.$$

The corresponding gain response is shown in Fig. 8 along with that of the filter from [8]. The proposed filter has SNR of 73.16 dB, while the filters from [8] and [2] have SNRs of 58.9dB, and 50dB, respectively.

The proposed filter requires 14 memory elements, comparing with the number of 12 for the filter [8] and 56 for the filter [2]. The number of APOS for the proposed, and filters from [8], and [2] are 81, 73 and 80, respectively.

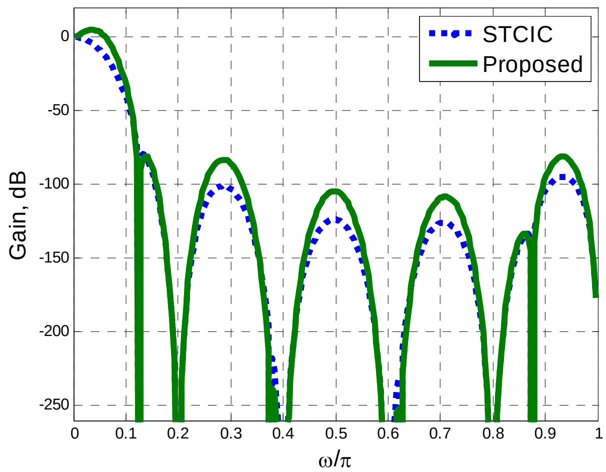


Fig.8: *Example 3.*

Table 3: *Complexity of proposed structure*

R	**Memory req.**	**APOS**
R	$2k_2+Rk_3+2$	$k_1+\lvert k_2-k_1\rvert M_2+(k_3+k_2)M+3$
$R=rM_1$	$2k_2+rk_3+2$	$k_1+\lvert k_2-k_1\rvert M_2+k_3M_2+k_2M+3$
$R=M$	$2k_2+k_3+2$	$k_1+k_3+\lvert k_2-k_1\rvert M_2+k_2M+3$

6. CONCLUSIONS

This paper has presented a new multiplierless decimation filter for a rational conversion factor where interpolation and decimation factors are mutually prime numbers and the decimation factor is the product of two simple factors M_1 and M_2. The proposed

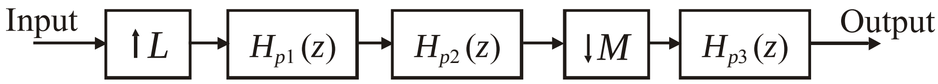


Fig.7: *General structure.*

filter is based on the stepped triangular CIC filter, expanded cosine filters and the sine-based compensation filter. The compensation filter is a simple second order filter with one multiplier per output sample which can be realized by shift and add operations. The proposed filter has an improved SNR. The expense of the frequency improvement is the slight increase of memory elements and APOS.

7. ACKNOWLEDGEMENT

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