# Synthesis of Linear Phase Sharp Transition FIR Digital Filter

**Joseph Rodrigues***, **K R Pai****, and **Lucy J. Gudino *****, Non-members

**ABSTRACT**

This paper proposes a new technique for synthesis of a sharp transition, equiripple passband, low arithmetic complexity, linear phase lowpass FIR filter. The frequency response of the filter with narrow transition width is modeled using trigonometric functions of frequency and its transfer function is evolved in frequency and time domain. The synthesized filter proves to be a good alternative to filters of the same class reported in the literature with added advantage of simplicity of design and ease of computation of the impulse response.

**Keywords**: Linear phase FIR filter, Sharp transition filter.

## 1. INTRODUCTION

It is well known fact that the filter length being inversely proportional to transition width the complexity becomes prohibitively high for sharp transition filters which causes severe implementation problems. Linear phase FIR digital filter have many advantages such as guaranteed stability, free of phase distortion and low coefficient sensitivity. Several methods have been proposed in the literature for reducing the complexity of sharp FIR filters. One of the most successful techniques for synthesis of very narrow transition width filter is the Frequency Response Masking (FRM) technique because of reduced arithmetic complexity involved [1]-[5]. Linear phase FIR filters have constant group delay in the entire frequency band, but for a filter with very narrow transition width, the group delay can be exceedingly large which is undesirable in many applications. Linear phase FRM filters are successful in reducing realization complexity but its group delay is even larger than that of direct form FIR filter with the same approximation accuracy. The optimal design of FRM filters with reduced passband group delay is dealt in [6] The major advantages of FRM approach is that the filter has a very sparse coefficient vector so its arithmetic complexity is very low, though its length and delays are slightly longer than those in the conventional implementations. These filters are suitable for VLSI implementations since hardware complexity is reduced.

In the FRM techniques, closed form expressions for impulse response coefficients were not obtained. We propose an analytical approach to the design of sharp transition filters with least arithmetic complexity. The approach is simple, analytical, without extensive computations and can be extended to design sharp cutoff highpass, bandpass, bandstop filters with arbitrary passband. Expressions for impulse response coefficients are derived, coefficients obtained and simulation of the filter is carried out.

## 2. FILTER MODEL

The proposed model for the pseudo-magnitude of the filter transfer function is formulated for equiripple passband and sharp transition using trigonometric functions of frequency. In the proposed model for a linear phase, equiripple passband, sharp transition, low pass FIR filter the various regions of the filter responses are formulated as follows.

In the passband region, the frequency response is

$$H_{pm}(\omega) = 1 + \frac{\delta}{2} cos k_p \omega \qquad 0 \leq \omega \leq \omega_p \qquad (1)$$

where $\omega$ the frequency variable, $H_{pm}(\omega)$ is the pseudo-magnitude of the filter response, $\delta$ is passband loss, $k_p$ an integer is a filter parameter in the passband and $\omega_p$ is the end of ripple channel frequency.

Transition region spans part of the passband $(\omega_p,\omega_c)$ as well as part of the stopband $(\omega_s,\omega_z)$ where $\omega_c$ is the cutoff frequency and $\omega_s$ is the stopband edge frequency.

In the transition region, the frequency response is

$$H_{pm}(\omega) = A \ \ cos k_t(\omega - \omega_0) \qquad \omega_p \leq \omega \leq \omega_z \qquad (2)$$

where $k_t$ an integer is a filter parameter in the transition region, $A$ is amplitude parameter and $\omega_0$ is chosen greater than 1, is the frequency at which $H_{pm}(\omega)$ equals $A$ , $\omega_z$ is the frequency at which $H_{pm}(\omega)$ is zero in the stopband region.

In the stopband region, the frequency response is

$$H_{pm}(\omega) = -\frac{\delta_s}{2} sin k_s(\omega - \omega_z) \qquad \omega_z \leq \omega \leq \pi \qquad (3)$$

where $\delta_s$ is the stopband loss, $k_s$ is the filter parameter in the stopband region.

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From (1),

$$cos k_p \omega_p = 0 \tag{4}$$

$$k_p = \frac{L\pi}{2\omega_p} \tag{5}$$

where $L$ is odd ,i.e., 1,3,5... to give negative slope due to roll off.

$$H_{pm}(\omega_z) = 0 = A cos k_t \ (\omega_z - \omega_0) \tag{6}$$

$$k_t = \frac{\pi}{2(\omega_z - \omega_0)} \tag{7}$$

$$H_{pm}(\pi) = -\frac{\delta_s}{2} = -\frac{\delta_s}{2} sin k_s (\pi - \omega_z) \tag{8}$$

$$k_s = \frac{\pi}{2(\pi - \omega_z)} \tag{9}$$

from (2), $H_{pm}(\omega_p) = 1$ , we obtain

$$A = \frac{1}{cos k_t (\omega_p - \omega_0)} \tag{10}$$

Also,

$$\omega_0 = \omega_p - \left(\frac{1}{k_t}\right) cos^{-1}\left(\frac{1}{A}\right) \tag{11}$$

Cut-off frequency

$$\omega_c = \omega_0 + \frac{1}{k_t} cos^{-1}\left(\frac{1-\delta/2}{A}\right) \tag{12}$$

Stopband edge frequency

$$\omega_s = \omega_0 + \frac{1}{k_t} cos^{-1}\left(\frac{\delta_s}{2A}\right) \tag{13}$$

Transition region width

$$(\omega_s - \omega_c) = \frac{1}{k_t}\left[cos^{-1}\left(\frac{\delta_s}{2A}\right) cos^{-1}\left(\frac{1-\delta/2}{A}\right)\right] \tag{14}$$

In the stop band,

$$\omega_z = \omega_0 + \frac{(4k_z+1)\pi}{2k_t} \tag{15}$$

where $k_z = 0,1,2,...$ Choose $k_z = 0$ for narrowest transition band of the lowpass filter.

The magnitude response $H_{pm}(\omega)$ is as shown in fig. 1.

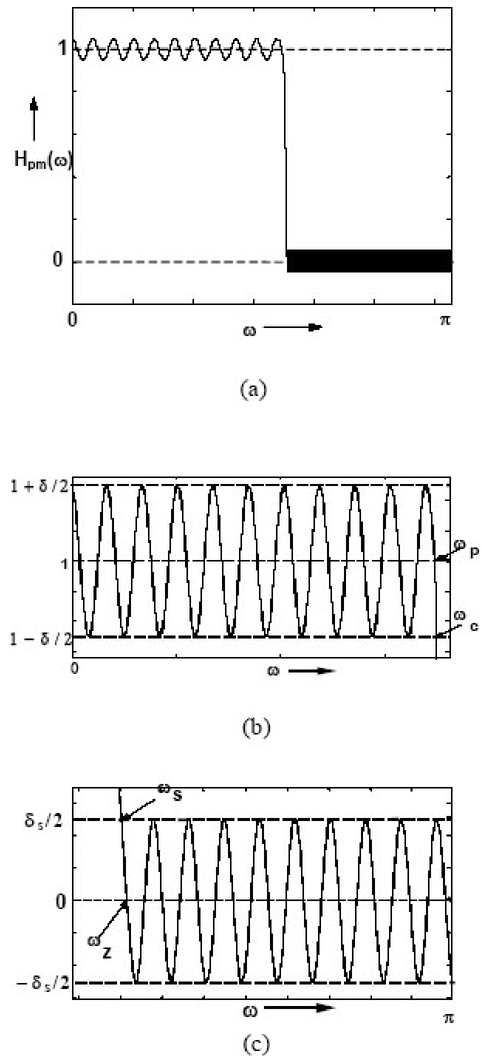



***Fig.1:*** *(a) Magnitude response $H_{pm}(\omega)$ of the proposed model for lowpass filter, (b) Magnified view of the passband of $H_{pm}(\omega)$ , (c) Magnified view of the stopband of $H_{pm}(\omega)$.*

## 2.1 Impulse Response Coefficients

Let $h(n)$, $0 \le n \le N-1$, be the impulse response of an N-point linear phase FIR digital filter. The linear phase condition implies that the impulse response satisfies the symmetry condition [7],

$$h(n) = h(N-1-n), n = 0, 1, 2, \ldots, N-1 \tag{16}$$

The frequency response for a linear-phase FIR filters for odd N is given by

$$H(e^{j\omega}) = e^{-j(\frac{N-1}{2})\omega} H_{pm}(\omega) \tag{17}$$

where the pseudo magnitude response $H_{pm}(\omega)$ is

$$H_{pm}(\omega) = h\left(\frac{N-1}{2}\right) + 2\sum_{n-1}^{\left(\frac{N-1}{2}\right)} h\left(\frac{N-1}{2} - n\right) cosn\omega \qquad (18)$$

The impulse response sequence determined by this frequency response is obtained from

$$h_d(n) = \frac{1}{2\pi}\int_{\pi}^{-\pi} H_{pm}(\omega)\, d\omega \qquad (19)$$

The impulse response coefficients $h(n)$ for the resultant filter are obtained by evaluating the integral (19) using equations (1), (2) and (3) modeling the pseudo-magnitude response $H_{pm}(\omega)$, in the passband region, transition region and the stopband region respectively. Accordingly, we obtain the impulse response coefficients as

$$h_{(}\frac{n-1}{2}) = \frac{\omega_p}{\pi} + \frac{\delta}{2\pi k_p} + \frac{A - \sqrt{A^2-1}}{\pi k_t} - \frac{\delta_s}{2\pi k_s} cosk_s\omega_z \qquad (20)$$

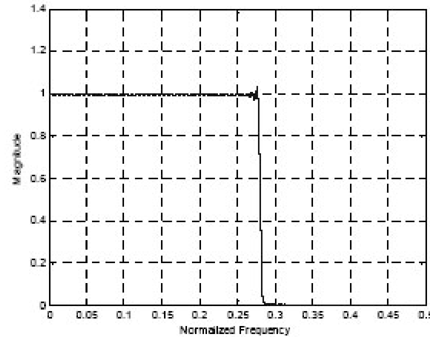
$$h_{(}\frac{N-1}{2} - k) = \frac{sink\omega_p}{k\pi} + \frac{\delta k_p \ cos(k\omega_p)}{2(k_p^2 - k^2)} \frac{A}{2\pi(k_t^2 - k^2)}\{2k_t\left(cosk\omega_z - \frac{\sqrt{A^2-1}}{A}cosk\omega_p\right) + \frac{2}{kA}sink\omega_p\} + \frac{\delta_s k_s}{2\pi(k_s^2 - k^2)}[(-1)^{k+k_s}cosk_s\omega_z - cosk\omega_z] \qquad (21)$$

where $k = 1, 2, \ldots, \frac{N-1}{2}$

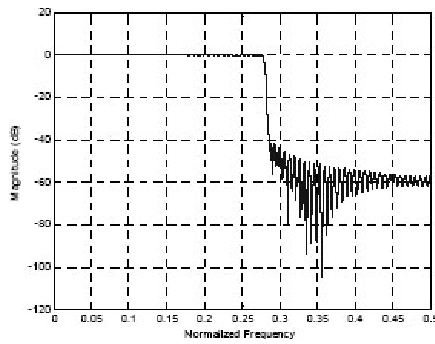
## 3. RESULTS

A lowpass linear phase FIR filter with passband edge specified at $0.556\pi$, stopband edge at $0.566\pi$ , maximum passband ripple of $\pm0.01$dB and minimum stopband attenuation of 40dB was designed using our approach.

The design and numerical computation was done using MATLAB [8]. Results approximate to the given filter specifications closely i.e. maximum passband ripple of $\pm0.015$dB, minimum stopband attenuation of 36dB and transition bandwidth of $0.01\pi$ was obtained. The filter order required for realization was found to be 285. The magnitude response of the proposed filter is shown in Fig. 2.




(a)




(b)

***Fig.2:*** *Magnitude response of the proposed lowpass filter (a) Linear plot (b) dB plot.*

***Table 1:*** *Variation of lowpass filter order with passband loss and stopband attenuation for constant transition width of $0.011\pi$ and passband width of $0.333\pi$.*

| Filter Order | Passband loss in dB | Stopband attenuation in dB |
|---|---|---|
| 201 | 0.487 | 26.06 |
| 301 | 0.174 | 31.60 |
| 401 | 0.136 | 33.96 |
| 501 | 0.074 | 35.45 |
| 601 | 0.126 | 37.79 |

## 4. CONCLUSIONS

We have proposed a model for a sharp transition, equiripple passband, low arithmetic complexity, linear phase lowpass FIR filter. Various regions of the filter are approximated with trigonometric functions of frequency, making it convenient to evaluate the impulse response coefficients in closed form. In our filters the group delay is reduced though filter realization complexity is more compared to FRM approach. This approach can be extended to sharp cutoff highpass, bandpass, and bandstop filters with arbitrary passband.

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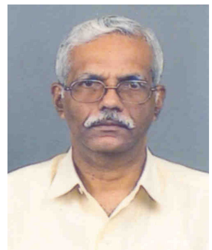

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