A New Scheduling Algorithm for Shortening Response Time of Static Priority Task

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ABSTRACT
In scheduling algorithms based on the Rate Monotonic (RM) method which are widely used in development of real-time systems, tasks with shorter periods have higher priorities. In contrast, ones with longer periods are likely to suffer from increased response times and jitter due to their lower priorities. We propose an Execution Right Delegation (ERD) method based on RM where a high-priority server for particular (or important) task is introduced to shorten response time and jitter of the task. In the evaluation, it is confirmed that response times and jitter of a particular (important) task are reduced. We also show Response Time Analysis (RTA), which assures worst-case response time of the task. This paper shows the algorithm and RTA of ERD and evaluates it by comparing it to a Deadline Monotonic method. The evaluation by simulation shows that ERD can reduce the average worst-case response time by 13.45% at maximum compared to the Deadline Monotonic scheduling. In addition, we confirm that the RTA provides a worst-case response time close to the simulation results.

1. INTRODUCTION
The main purpose of real-time scheduling algorithms is to achieve optimal scheduling for tasks which have periodic execution times and/or deadlines. It is important for the algorithms to not only meet the deadline of each task but also to shorten its response time and jitter. For periodic task sets, there are two categories in scheduling algorithms; one is Rate Monotonic (RM) for static priority and the other is Earliest Deadline First (EDF) for dynamic priority [1]. EDF has the advantage of high CPU utilization, while RM causes less runtime overhead and has predictable behavior. In addition, RM is easy to realize on static-priority-based real-time operating systems (RTOSs) such as ITRON [2], by associating tasks’ priorities with their periods. As a result, scheduling algorithms based on the RM method are widely used in development of real-time systems.

In RM, tasks with shorter periods have higher priorities. In contrast, ones with longer periods are likely to suffer from increased response times and jitter due to their lower priorities. In this paper, we propose an Execution Right Delegation (ERD) method based on RM where a high-priority server for a particular (or important) task3 is introduced to shorten response time and jitter. We also show Response Time Analysis (RTA), which gives the worst case response time (WCRT) of ERD.

This paper consists of six sections. Section 2 describes related work in terms of real-time scheduling algorithms and response time analysis. Section 3 proves several theorems and lemmas and shows the algorithm of our ERD method. Section 4 proposes RTA of ERD. An evaluation of the proposed method is shown in Section 5. Finally, Section 6 concludes the paper.

2. RELATED WORK
2.1 Scheduling Algorithms
In the RM model, task \( \tau_i \) \( (1 \leq i \leq n) \) releases an infinite sequence of Jobs. Once a Job is released, it runs during the defined time \( C_i \). A Job is released ev-
every period \( T_i \). Notation of a task is \( \tau_i = (C_i, T_i) \).
A set of tasks is denoted as \( \Gamma = \{\tau_1, \tau_2, \ldots, \tau_n\} \), where the smaller the subscript figure a task has, the shorter the period and the higher priority it has (i.e. \( T_1 \leq T_2 \leq \ldots \leq T_n \)). A deadline miss occurs if a Job does not finish by the next task release.

The Deadline Monotonic (DM) [3] model includes relative deadline \( D_i \) in addition to period \( T_i \) and execution time \( C_i \). It is another scheduling algorithm. The shorter the relative deadline of a task is, the higher its priority is. As priority is determined by deadline independent of a period, the response time and jitter of a task with a longer period can be shortened by assigning a short deadline.

There are several fixed-priority server algorithms for shortening response times of particular aperiodic tasks. Defearable Server (DS) [4] and Priority Exchange (PE) [4] are representatives. Servers in these algorithms have a period and a capacity which corresponds to execution time. The servers are scheduled along with a set of periodic tasks while their capacity is consumed for execution of aperiodic tasks. In these algorithms, the server period and capacity are decided according to processor utilization of the whole periodic task set, where the importance of a particular periodic task is not considered. On the other hand, the method proposed in this paper obtains and uses the period and the capacity in favor of a particular (important) periodic task, although the server algorithm is basically PE.

Extended Priority Exchange (EPE) [5] does not involve a server. Instead, execution time of each task is overestimated. The unused part of the overestimation is exploited as capacity for aperiodic tasks. It can be regarded as padding a bandwidth for aperiodic tasks into the other tasks’ execution time. Dual Priority (DP) [6] is another approach. In contrast to RM, this algorithm assumes that each task is assigned two priorities. A periodic task has two priorities, low and high, and an aperiodic task has middle priority. After a periodic task is released, it starts to run at the low priority for a while. The aim of DP is to treat aperiodic tasks preferentially by delaying periodic tasks as late as possible while meeting all the deadlines. Zero Slack Scheduling [7] is for a model of mixed-criticality systems. In this model, one system contains tasks with different degrees of importance. A task has criticality as well as a conventional priority. A task runs in a normal mode unless it is urgent. However, it switches to a critical mode at some point to keep schedulability.

The aim of ERD is to shorten response time of an important task regardless of its period. In order to assign priority to the task regardless of the period, Leung and Whitehead generalized that DM priority assignment is optimal for synchronous periodic task sets (without phases) with constrained deadlines [8]. In some different system models (e.g., tasks with phases, tasks with arbitrary deadlines, non-preemptive scheduling), it is known that DM is not optimal [9].

### 2.3 Response Time Analysis

There is a classical method to judge whether a task set is schedulable or not. Liu et al. [1] proposed a schedulability test by calculating processor utilization by all tasks and comparing it with the least upper bound for the task set. This test provides only a sufficient condition, which leaves the estimate pessimistic. Bini et al. proposed a polynomial-time and less-pessimistic way [10].

Response Time Analysis (RTA) [11] provides a necessary and sufficient condition of schedulability for RM, which is defined as follows:

**Definition 1** (Response Time Analysis (RTA) [11] of RM). In fixed-priority scheduling, the longest response time, \( R_i \), of task \( \tau_i \) is computed as: \(^4\)

\[
R_i = C_i + \sum_{j=1}^{i-1} \frac{R_j}{T_j} C_j. \tag{1}
\]

\( R_i \) appears on both sides. Thus, \( R_i \) is calculated by setting an appropriate value (e.g., \( C_i \)) as the initial value to \( R_i \) and increasing \( R_i \) until both sides become equal.

Several RTA methods for multiprocessor system are proposed. Bertogna et al. proposed RTA for global fixed-priority multiprocessor scheduling [13]. Compared with single processor RTA, multiprocessor RTA provides only a sufficient condition with its pessimism. In the method, interference time, which is a total amount of time that higher priority tasks prevent a certain task from working, is introduced. Guan et al. improved Bertogna’s method by limiting carry-in load, a part of interference time, of higher priority tasks (RTALC) [14]. Sun et al. revised RTA_LC to reduce its pessimism. The method provides a more accurate result [15]. They applied schedulability test for a multiprocessor partitioned schedule by the RTA [16]. Zhou et al. showed RTA for a model where a task has a non-preemptive point under multiprocessor global scheduling [17]. As mentioned above, there are various RTAs dedicated to specific models or different systems.

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\(^4\) If the first jobs of all tasks are released simultaneously at the instant \( t = 0 \), response time of each task becomes the worst-case for the corresponding task (Critical Instant [1]).

The smaller the subscript a task has, the shorter the period and the higher priority it has. That is, for \( \tau_i \) and \( \tau_j \), if \( i < j \), \( \tau_i \) has higher priority than \( \tau_j \).
Coutinho et al. showed another approach for the model of asynchronous periodic and sporadic tasks for a single processor\textsuperscript{5}[12]. The approaches mentioned above are calculating maximum interference time from higher priority tasks under a pessimistic situation. Coutinho’s approach is calculating interference time between the job’s release time and the job’s response time, in the same way as multiprocessor RTA. This checking is done until hyperperiod \((\text{LCM}(T_1,T_2,...,T_{n-1}))\). They reduce pessimism of the algorithm while calculation easily explodes due to it’s requirement of LCM of tasks’ periods.

2.4 Reducing Jitter

In a real-time system, especially in closed-loop control systems which operate sampling/sensing and actuating, it is required to reduce not only response time but also jitter [18]. In order to reduce jitter, several methods were studied [19] [20] and evaluated [21]. In this paper, relative finishing jitter (RFJ) and absolute finishing jitter (AFJ) are evaluated by using the following definitions:

\[
RFJ_i = \max_k (f_{i,k} - r_{i,k}) - (f_{i,k-1} - r_{i,k-1}) \tag{2}
\]

\[
AFJ_i = \max_k (f_{i,k} - r_{i,k}) - \min_k (f_{i,k} - r_{i,k}) \tag{3}
\]

where \(r_{i,k}\) is the release time of \(\tau_i’s k\text{th job and } f_{i,k}\) is the finishing time of \(\tau_i’s k\text{th job.}

3. EXECUTION RIGHT DELEGATION (ERD)

3.1 System Model and Theorems

The target system model of this study is single processor fixed task priority. ERD assumes that each task has periodic execution time, and that the deadline is equal to its period. A task does not have a phase, which means that the first job is released at \(t = 0\). The scheduling rule follows RM except for a particular (target) task to which the proposed virtual server (VS) is applied.

First, we describe theorems and lemmas as well as definitions underlying ERD. Based on these, a virtual server VS is introduced into a task set.

Theorem 1 (Schedulability Analysis [11]). For each task \(\tau_i\) in a task set \(\Gamma\), if \(R_i\) is equal to or less than \(T_i\), \(\Gamma\) is schedulable.

Definition 2 (Assigning Priority to a Task). In the proposed method, in addition to the priority assignment of RM, a particular (e.g. important) task can be assigned priority independent of its period.

Lemma 1 (Exchanging Priorities between Tasks).

Assume that a schedulable task set \(\Gamma\) includes \(\tau_p\) and \(\tau_h\) and that the priority of \(\tau_h\) is higher by one than that of \(\tau_p\). Let \(\Gamma’\) be a task set in which the priorities of \(\tau_p\) and \(\tau_h\) are exchanged while the other tasks have the same priority as in \(\Gamma\). It is schedulable if \(R_p\) is equal to or less than \(T_h\).

Proof. From Theorem 1, schedulability for \(\Gamma’\) is guaranteed by confirming that every task meets its deadline \((R_i’ \leq T_i)\). (From this point forward, symbols with a prime mark, like \(R_i’\), denote tasks in \(\Gamma’\).)

First, \(\Gamma\) is divided into three sets, \(\Gamma_{high}, \{\tau_h, \tau_p\}\), and \(\Gamma_{low}\), so that priorities of tasks in \(\Gamma_{high}\) are higher than that of \(\tau_h\) and those in \(\Gamma_{low}\) are lower than that of \(\tau_p\).

Response time of any task, \(\tau_{high}\), in \(\Gamma_{high}\) is not affected by exchanging the priorities of \(\tau_h\) and \(\tau_p\) since the priority exchange between tasks with lower priorities than \(\tau_{high}\) can change scheduling only after completion of \(\tau_{high}’\)’s first job. Similarly, for any task, \(\tau_{low}\), in \(\Gamma_{low}\), its response time is not affected, since the instant when the right of execution is given to \(\tau_{low}\) is not changed by exchanging the execution order of higher tasks.

The response time of \(\tau_p\) does not increase since its priority is higher than it was before the exchange. Since \(R_p \leq T_h, T_h \leq T_p\), and \(R_p’ < R_p, R_h’ \leq T_p\), the deadline constraint is met.

Next, consider \(\tau_p\) in two cases: \(\tau_p\) or \(\tau_h\) is blocked by jobs in \(\Gamma_{high}\) or they are not blocked. For the not-blocked case, let \(s_h\) and \(s_p\) be start times of Jobs of \(\tau_h\) and \(\tau_p\) (before changing priority), respectively. Then the response time of each task is given as:

\[
R_h = s_h + C_h, \quad R_p = s_p + C_p.
\]

Letting \(s_p’\) and \(s_h’\) be the start times of Jobs of \(\tau_p\) and \(\tau_h\), respectively. After the priority exchange, their response times, \(R_p’\) and \(R_h’\), become:

\[
R_p’ = s_p’ + C_p, \quad R_h’ = s_h’ + C_h.
\]

\(s_p\) is equal to \(R_h (s_p = R_h)\) since neither \(\tau_h\) nor \(\tau_p\) is blocked by Jobs in \(\Gamma_{high}\) and Job of \(\tau_p\) starts immediately when \(\tau_h\)’s Job finishes. This is the case after the priority exchange, that is, \(s_p’ = R_p’\). In addition, obviously, \(s_h’\) is the same as \(s_h\). Then, \(R_h’\) becomes equal to \(R_p’\) as follows:

\textsuperscript{5}If release time/offset/phase is introduced to the model, WCRT is unknown as is the case in multiprocessor WCRT.
\[ R'_h = \begin{align*} & s'_h + C_h \\ &= R'_{p} + C_h \\ &= s'_p + C_p + C_h \\ &= s_h + C_p + C_h \\ &= R_h + C_p \\ &= s_p + C_p \\ &= R_p \end{align*} \]

From the prerequisite \( R_p \leq T_h \), it turns out to be \( R'_h \leq T_h \). Thus, the deadline requirement is satisfied.

**Fig 1** and **Fig 2** show examples of the case in which tasks in \( \Gamma_{high} \) do not block \( \tau_h \) or \( \tau_p \).

Next, consider the other case in which \( \tau_p \) or \( \tau_h \) is blocked by new Jobs of tasks in \( \Gamma_{high} \) which are released before \( T_h \). From the prerequisite that \( \tau_h \) and \( \tau_p \) finish their jobs by the instant \( T_h \), \( BT \), blocking time of \( \tau'_h \)'s and \( \tau_p \)'s job after \( s_h \), has the following upper bound:

\[ BT \leq T_h - s_h - (C_h + C_p) \quad (4) \]

\( R_p \) is given as:

\[ R_p = s_h + C_h + C_p + BT. \]

After exchanging priorities, \( R'_h \) is:

\[ R'_h = s'_p + C_p + C_h + BT. \]

With \( s'_p = s_h \) and by moving \( BT \) to the left side:

\[ BT = R'_h - s_h - C_p - C_h. \]

By expanding \( BT \) in (4):

\[ R'_h - s_h - C_p - C_h \leq T_h - s_h - (C_h + C_p) \]

\[ R'_h \leq T_h - s_h - (C_h + C_p) + s_h + C_p + C_h \]

\[ R'_h \leq T_h. \]

Thus, the deadline constraint is satisfied.

Examples of this case are shown in **Fig 3** and **Fig 4**.

**Theorem 2** (Changing Priorities of Tasks). Assume that a schedulable task set \( \Gamma \) includes \( \tau_p \) and \( \tau_h \) and that the priority of \( \tau_h \) is higher than that of \( \tau_p \). Let \( \Gamma' \) be a task set in which the priority of \( \tau_p \) is changed to be higher than \( \tau_h \) while the other tasks have the same priority as in \( \Gamma \). It is schedulable if \( R_p \) is equal to or less than \( T_h \).

**Proof.** For periods of tasks, \( \tau_h, \tau_{h+1}, \tau_{h+2}, \ldots, \) and \( \tau_{p-1} \), the following condition is satisfied:

\[ T_h \leq T_{h+1} \leq T_{h+2} \leq \ldots \leq T_{p-1} \]

From the prerequisite, \( R_p \leq T_h \), and the above condition, the following condition is satisfied.

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\(^6\) In Lemma 1, there are no other tasks between \( \tau_p \) and \( \tau_h \). This theorem relaxes the restriction.
\[ R_p \leq T_h \leq T_{h+1} \leq T_{h+2} \leq \ldots \leq T_{p-1} \]

Here, focusing on \( \tau_{p-1} \) whose priority is higher by one than \( \tau_p \), a task set in which priorities of \( \tau_p \) and \( \tau_{p-1} \) are exchanged is schedulable from Lemma 1. In the same way, priorities of \( \tau_p \) and \( \tau_{p-2} \) are exchangeable. This is repeated until the priority exchange is done between \( \tau_p \) and \( \tau_h \).

\[ \text{Definition 3 (Idle Time) Idle Time is a period in which any task’s job is not executed. idle}(t) \text{ gives the total amount of idle time between } 0 \text{ and the instant } t. \]

\[ \text{Example 1 (Idle Time). In Fig 5, idle}(4) = 1 \text{ and idle}(6) = 2. \]

\[ \text{Fig. 5: idle}(t). \]

\[ \text{Lemma 2 (Idle Time and Completion of Job). The first jobs of all tasks in } \Gamma \text{ finish by the instant } t \text{ if } idle(t) > 0. \]

\[ \text{Proof. In fixed-priority scheduling, the highest-priority job is necessarily scheduled if there exist released jobs in a ready state. Thus, it turns out that } \text{no job exists during an Idle Time. Therefore, the first job of each task has finished execution by } t \text{ (or by the first Idle Time).} \]

\[ \text{Lemma 3 (Adding an Idle Task). If there exists Idle Time between } 0 \text{ and } \tau_h \text{’s period } T_h, \text{ adding a new task } \tau_{idle}, \text{ whose execution time is } idle(T_h) \text{ and whose period is } T_h, \text{ to } \Gamma \text{ leaves the new task set schedulable.} \]

\[ \text{Proof. } \Gamma \text{ is divided into two sets, } \Gamma_{high} = \{ \tau_1, \tau_2, \ldots, \tau_{p-1} \} \text{ and } \Gamma_{low} = \{ \tau_h, \tau_{h+1}, \tau_{h+2}, \ldots, \tau_p \}. \text{ Any task in } \Gamma_{high} \text{ has a period equal to or shorter than } T_h. \text{ On the other hand, any task, except } \tau_h, \text{ in } \Gamma_{low} \text{ has a period equal to or longer than } T_h. \text{ For tasks in } \Gamma_{high}, \text{ through the same discussion as in the proof for lemma 1, their feasibility is not affected. For } \tau_{idle}, \text{ the deadline requirement is met since its job’s response time clearly becomes equal to or less than } T_h. \text{ Therefore, it is sufficient to confirm the schedulability for } \Gamma_{low}. \]

\[ \text{Focusing on the lowest-priority task } \tau_n \text{ in } \Gamma_{low}, \text{ the response time } R_n \text{ of the first job of } \tau_n \text{ and } T_h \text{ have the following relationship:} \]

\[ R_n + idle(T_h) + BT = T_h \]

\[ (5) \]

\[ \text{Here, } BT \text{ is the sum of time duration when the second or subsequent jobs of tasks in } \Gamma_{high} \text{ are scheduled from the instant } R_n \text{ to } T_h. \text{ Let } R'_n \text{ be the response time of } \tau_n \text{’s job after adding } \tau_{idle} = \{ idle(T_h), T_h \} \text{ to } \Gamma. \text{ } \tau_{idle} \text{ releases its job only once from } 0 \text{ to } T_h \text{ and interferes with } \tau_n \text{ by } idle(T_h). \]

\[ \text{From lemma 2, before adding } \tau_{idle} \text{ to } \Gamma, \text{ all jobs finish the execution by the instant } T_h. \text{ At the instant } T_h \text{ after adding } \tau_{idle}, \text{ tasks in } \Gamma_{high} \text{ have finished their first job since addition of } \tau_{idle} \text{ does not affect } \Gamma_{high}. \text{ Thus, the maximum interference in } \tau_n \text{ by adding } \tau_{idle} \text{ is } idle(T_h) \text{ and the total amount of execution time of the second or subsequent jobs of } \Gamma_{high} \text{’s tasks before } T_h \text{ is just } BT. \text{ Therefore, } R'_n \text{ has the following upper bound.} \]

\[ R'_n \leq R_n + idle(T_h) + BT \]

\[ (6) \]

\[ \text{From equations (5) and (6):} \]

\[ R'_n \leq R_n + idle(T_h) + BT \]

\[ = R_n + idle(T_h) + T_h - R_n - idle(T_h) \]

\[ = T_h. \]

\[ \text{From } T_h \leq T'_n, R'_n \leq T_h \leq T_n \text{ so that } \tau_n \text{ meets its deadline constraint.} \]

\[ \text{For tasks in } \Gamma_{low} \text{ after adding } \tau_{idle}, \text{ the response times and periods satisfy the following condition:} \]

\[ R'_h \leq R'_{h+1} \leq \ldots \leq R'_{n} \leq T_h \leq T_{h+1} \leq \ldots \leq T_n \]

\[ \text{Therefore, by theorem 1, all tasks in } \Gamma_{low} \text{ meet their deadlines after } \tau_{idle} \text{ is added.} \]

\[ \text{Example 2 (Addition of } \tau_{idle}). \text{ Fig 6 shows an example of adding } \tau_{idle} \text{ to Fig 5, where } \tau_h = \tau_2 \text{ and } T_h = T_2 = 6. \]

\[ \text{Footnotes:} \]

\footnote{In this case, a relative time duration, e.g., } T_h \text{ is regarded as a time instant (} t = T_h \text{).}

\footnote{The reason for ‘maximum’ is simple. Suppose } T_2 = T_3 = 7 \text{ and } \tau_n = \tau_3 \text{ in Fig 5. In that case, } BT = 2, \text{ since the second job of } \tau_1 \text{ and } \tau_2 \text{ are scheduled before the instant } T_h. \text{ After adding } \tau_{idle}, \text{ } \tau_n \text{ is not affected by } \tau_2 \text{’s second job, and finally } R'_n = 6(< 7) \text{ is given.}
3.2 Execution Right Delegation (ERD)

ERD is a method to shorten response time and jitter of a particular (or important) task, $\tau_p$, in a task set by using a high-priority virtual server, VS, which has capacity of $C_s$ and period of $T_s$ while meeting the deadline constraints of the task set. We assume $\tau_p$ has a relatively lower priority due to its longer period. By making the priority of VS high with short $T_s$, $\tau_p$ can be executed at the high priority while consuming $C_s$. The basic behavior of VS is from PE [4].

**Definition 4 (Delegation of Execution Right).** In the ERD method, VS is scheduled based on an RM rule. When VS is given the execution right, a particular periodic task, $\tau_p$, is executed instead of VS. This situation is called Delegation of Execution Right. If job of $\tau_p$ has already finished when the execution right is available, the behavior follows PE rule [4] where jobs of the other tasks are executed while the server capacity is accumulated at the priority level of the running job.

Next, the following definition gives the algorithm for finding $C_s$ and $T_s$.

**Definition 5 (Candidates of VS).** Let $\Gamma$ be a schedulable task set and $\tau_p$ in $\Gamma$ be a particular task whose response time and jitter should be shortened. Then, RTA in Definition 1 is applied to $\tau_1$, ..., and $\tau_p$. From the relationship between $R_p$ and $T_1$, ..., $T_{p-1}$, $C_s$ and $T_s$ for VS are obtained as follows:

$$C_s = \begin{cases} C_p, & \text{if } R_p \leq T_{p-1} \\ \text{idle}'(T_s), & \text{otherwise} \end{cases} \quad \text{(7)}$$

$$T_s = \begin{cases} T_{h}, & \text{if } R_p \leq T_{p-1} \\ t \in \Psi, & \text{otherwise} \end{cases} \quad \text{(9)}$$

where

$$\Psi = \{T_1, T_2, ..., T_{p-1}\}$$

$$T_h = \min(\{t | t \in \Psi, R_p \leq t\})$$

$$\text{idle}'(t) = t - \sum_{j=1}^{p-1} \left\lceil \frac{t}{T_j} \right\rceil C_j \quad (t \in \Psi)$$

VS derived by the equations (7) and (9) makes it possible for $\tau_p$ to be executed at the same priority as a higher priority task. **Fig 7** shows an example. Since $R_p \not\leq T_1$ but $R_p \leq T_2$, VS $(C_p, T_2) = (1, 6)$.

**Theorem 3 (Schedulability with VS (Exchange of Priority)).** Let $\Gamma$ be a schedulable task set and $\Gamma'$ be the new task set which is derived by applying equations (7) and (9) to change the priorities of $\tau_p$ and $\tau_h$ which has higher priority than $\tau_p$ in $\Gamma$. Then, $\Gamma'$ is schedulable.

**Proof.** Since $R_p \leq T_h$, theorem 2 guarantees that $\Gamma'$ is schedulable. $\square$

**Example 3 (ERD method - 1).** With $\Gamma = \{(2, 4), (3, 12), (3, 14)\}$ and $\tau_p = \tau_3$, RTA gives response times of the three tasks as $R_1 = 2$, $R_2 = 7$, and $R_3 = 12$. Since $R_3 \leq T_2 = 12$, equations (7) and (9) offer $VS = (3, 12)$.

Scheduling results by RM and ERD are shown in **Fig 8 and Fig 9**, respectively. At the instants $t = 2, 3, \text{ and } 6$, delegation of execution right can be confirmed. In this example, ERD improves response time of $\tau_3$ from 12 to 7\(^9\).

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\(^9\) Strictly speaking, ERD does not improve response time of $\tau_3$ but $\tau_3$'s first job. We discuss this problem in the next section.
As for VS provided by the equations (8) and (10), more than one candidate can exist. \( \Psi = \{T_1, T_2, \ldots, T_{p-1}\} \) is a set of periods which are shorter than \( T_p \), and \( idle'(T_i) \) gives the length during which \( \tau_p \) is executed. (Note that \( idle'(\cdot) \) in Definition 3, and \( idle'(\cdot) \) in Definition 3 are different. While the former gives the sum of all idle times before any instant \( t \), the latter means, in each period in \( \Psi \), the sum of time slots during which any task in \( \tau_1, \ldots, \tau_{p-1} \) is not executed but \( \tau_p \) is executed due to \( R_p > T_{p-1} \).) \textbf{Fig 10} shows an example. With \( \Psi = \{T_1(=4), T_2(=6)\} \), \( idle'(4) = 1 \) and \( idle'(6) = 2 \), two candidates for VS are derived: \( (1,4) \) and \( (2,6) \).

Using VS from (8) and (10), the first job of \( \tau_p \) is necessarily preempted by the other tasks and the execution is split. For example, in \textbf{Fig 10}, when \( VS = (1,4) \) is used, the first part of \( \tau_p \)'s job would be executed at the highest priority. In this case, although VS does not shorten response time, it can contribute to reducing jitters in practice if an important part of the job (e.g. polling sensors) is located in the first half of the task program code. In addition, a job does not spend its worst-case execution time in most practical situations. If the actual execution time is less than or equal to the server capacity, the response time is expected to be substantially shortened.

\textbf{Theorem 4} (Scheduleability with VS (Split of Job)). With a schedulable task set \( \Gamma \), \( \tau_p \) whose execution is split by VS derived from equations (8) and (10) meets its deadline constraint, and the new task set \( \Gamma' \) with VS is schedulable.

\textit{Proof.} After introducing VS, scheduleability for \( \Gamma_{\text{high}} = \{\tau_1, \tau_2, \ldots, \tau_{p-1}\} \), where each element has higher priority than \( \tau_p \), and that for \( \Gamma_{\text{low}} = \{\tau_{p+1}, \ldots, \tau_n\} \), which includes lower priority tasks than \( \tau_p \), are shown along with the feasibility of \( \tau_p \).

For the job of \( \tau_p \), let \( \tau_{ps} \) be the part executed by VS and \( \tau_{po} \) be the remaining part. Note that if the whole job is accommodated by VS, \( \tau_{po} \) does not exist. Obviously, early completion of a part of a job does not extend the response time. Thus, \( \tau_{po} \) meets its deadline constraint since it is included in the schedulable task set. \( \Gamma \).

Next, it is shown that \( \tau_{po} \) does not affect the scheduleability of \( \Gamma_{\text{high}} \). The existence of VS means a positive idle time \( (idle'(T_p)) \) exists at the instant \( T_p \).

By lemma 3, addition of \( \tau_{idle} \) leaves \( \Gamma_{\text{high}} \) schedulable. Since \( \tau_{idle} \) can be represented (or replaced) by VS, or \( \tau_{ps} \), \( \Gamma_{\text{high}} \) is schedulable.

For \( \Gamma_{\text{low}} \), similar to the discussion in the proof for lemma 1, exchange of the execution order among higher priority tasks than \( \Gamma_{\text{low}} \) does not change the instant when the right of execution is given to \( \Gamma_{\text{low}} \). Thus, \( \Gamma_{\text{low}} \) is schedulable.

\textbf{Example 4} (ERD method - 2) With \( \Gamma = \{(1,5), (1,6), (2,8), (4,14)\} \) and \( \tau_p = \tau_4 \), \textit{RTA} gives \( R_1 = 1, \tau_2 = 2, \tau_3 = 4, \) and \( R_4 = 14 \) (\textbf{Fig 11})
VS is given by equations (8) and (10) since \( R_1 > T_1 \). With \( \Psi = \{ T_1, T_2, T_3 \} \), \( \text{idle}'(T_i) \) is calculated for each period. Since \( \text{idle}'(T_1) = 1 \), \( VS_1 = (1, 5) \) is derived. With \( VS_1 \), \( R_1' = 14 \), which does not shorten response time (Fig 12). Similarly, \( VS_2 = (1, 6) \) is obtained from \( \text{idle}'(T_2) = 1 \). However, \( VS_2 \) still gives \( R_1' = 14 \).

Then, from \( \text{idle}'(T_3) = 2 \), \( VS_3 = (2, 8) \) is derived and found to give \( R_1' = 10 \) (Fig 13). Finally, the results of \( VS_1 \), \( VS_2 \), and \( VS_3 \) are compared and \( VS_3 \) is chosen as VS because of the shortest response time for \( \tau_3 \).

\[
\begin{array}{c}
\text{Fig.13: Schedule by ERD with } VS_3 \text{ for } \Gamma. \\
\end{array}
\]

It is worth noting that Deadline Monotonic scheduling does not shorten the response time of \( \tau_4 \) with the above \( \Gamma \). DM is effective only when the relative deadline of \( \tau_4 \) can be made less than or equal to the deadline of a higher-priority task. However, for \( \Gamma \) in this example, if the deadline is set to be less than or equal to \( T_3 \), \( \tau_3 \)'s job misses its deadline.

The following theorem and definition bring more improvement in response time.

**Theorem 5** (Recursive Application of VS). If the response time of \( \tau_4 \) is still equal to or less than \( T_{h-1} \) after VS is applied, definition 5 can be applied again and new VS can be obtained. This is done recursively until the response time becomes longer than \( T_{h-1} \).

**Proof.** By theorem 3, \( \Gamma' = \{ \tau_1, \tau_2, ..., \tau_{h-1}, VS, \tau_h, ..., \tau_{p-1}, \tau_{p+1}, ... \} \) is a schedulable task set. Thus, definition 5 provides new VS and a new schedulable task set \( \Gamma'' \) as a result of applying VS.

**Definition 6** (Shortening Period of VS). If VS obtained by equations (7) and (9) results in \( R_p \leq T_1 \), \( T_1 \) can be \( C_p \) i.e. VS = \( \{ C_p, C_p \} \).

The ERD method can underutilize server’s capacity and fail to delegate the execution correctly when \( T_p \) and \( T_s \) are not synchronized (Section 4.4 discusses the details). This does not happen when definition 6 can be applied.

Example 5 (ERD method - 3). For \( \Gamma = \{ (2, 5), (2, 8), (2, 10) \} \) and \( \tau_3 = \tau_4 \), RTA leads to \( R_3 = 2, R_2 = 4, \) and \( R_3 = 8 \). From \( R_3 \leq T_2 (= 8) \), \( VS = \{ 2, 8 \} \) is obtained from the equations (7) and (9). Fig 14 and Fig 15 show results of scheduling by RM and ERD, respectively.

As a result of scheduling by the ERD method, \( R_3' = 2, R_2' = 8, R_3' = 4, \) and \( R_p' \leq T_1 (= 5) \). In addition to obtaining new VS = \( \{ 2, 5 \} \) by theorem 5, it is confirmed that \( R_p' \leq T_1 \), which leads to \( VS = \{ 2, 2 \} \) by definition 6. The result of scheduling with VS is shown in Fig 16.

\[
\begin{array}{c}
\text{Fig.14: Schedule by RM for } \Gamma = \{ (2, 5), (2, 8), (2, 10) \}. \\
\end{array}
\]

\[
\begin{array}{c}
\text{Fig.15: Schedule by ERD.} \\
\end{array}
\]

\[
\begin{array}{c}
\text{Fig.16: Schedule by ERD (applying theorem 5 and definition 6).} \\
\end{array}
\]

4. RTA of ERD

This section provides response time analysis (RTA) of ERD.
4.1 Interference Time

In the previous section, we reviewed several examples in which response time of a specific task \( \tau_p \) is shortened in the scheduling chart. In other words, it was no more than simulation-based confirmation. In this section, WCRT is provided from a mathematical perspective. In the model of single processor RM, WCRT of a task is equal to the response time of its first released job. On the other hand, as in multiprocessor scheduling, \( \tau_p \)'s WCRT is unknown in ERD. In some cases, \( \tau_p \)'s first job's response time becomes its WCRT like the examples in the previous section. However, Fig 17 shows the second job’s response time is longer than the first job’s one.

![Fig. 17: Second Job has longer response time.](image)

Then, it is required to build RTA of ERD. Let \( R_i^{RM} \) be a WCRT of \( \tau_i \) in RM scheduling, and \( R_i^{ERD} \) be a WCRT of \( \tau_i \) in ERD scheduling.

**Definition 7** (\( \tau_{vs} \) and \( \tau_h \)): \( \tau_{vs} \) is a task whose period is the same as VS and whose priority is treated as being lower than VS. \( \tau_h \) is a task whose priority is higher than \( \tau_p \) but excludes \( \tau_{vs} \).

\( \tau_2 \) in Fig 9, \( \tau_1 \) in Fig 12, and \( \tau_3 \) in Fig 13 are \( \tau_{vs} \).

**Lemma 4** (Upper bound of WCRT in ERD). For \( \tau_p \),

\[
R_p^{ERD} \leq R_p^{RM}.
\]

**Proof.** In the ERD method, \( \tau_p \) can be executed if VS whose priority is higher than \( \tau_p \) has execution right. As a result, \( \tau_p \)'s execution is advanced. Otherwise, \( \tau_p \) is scheduled according to the same rule as the RM method. Therefore, the response time can be no longer than that in the RM method. \( \Box \)

**Theorem 6** (WCRT of High Priority Task). A task subset \( \Gamma_{hp} \) consists of tasks whose priority is higher than \( \tau_p \). For any \( \tau_i \in \Gamma_{hp} \) \((i < p)\), a response time of \( \tau_i \)'s first job exhibits \( R_i^{ERD} \).

**Proof.** For \( \tau_p \), VS can be regarded as a task with execution time \( C_i \) and period \( T_i \). Let VS be appended as a periodic task to \( \Gamma_{hp} \), and make a task set \( \Gamma_{hp'} \). After scheduling \( \Gamma_{hp'} \) with RM rule, WCRT of \( \tau_i \) can be retrieved from Definition 2.3 \( \Box \)

**Definition 8** (Interference Time). Interference time \( I_{p}^{ERD} \) is the maximum time during which some higher-priority task \( \tau_i \) affects response time of \( \tau_p \) under ERD scheduling. \( I_p^{RM} \) is the case under RM scheduling and calculated as \( I_p^{RM} = \left\lceil \frac{R_p^{RM}}{T_i} \right\rceil C_i \).

**Lemma 5** (Upper bound of Interference Time of \( \tau_h \)). For \( \tau_h \in \Gamma_{hp} \) \((h < p, h \neq vs)\), \( I_{p}^{ERD} \leq I_{p}^{RM} \)

**Proof.** From a contribution of VS, \( I_{h}^{ERD} \leq \frac{R_{hp}^{ERD}}{T_h} C_h \). From Lemma 4, \( \frac{R_{hp}^{ERD}}{T_h} C_h \leq \left\lceil \frac{R_{hp}^{RM}}{T_h} \right\rceil C_h \). Thus, \( I_{h}^{ERD} \leq I_{p}^{RM} \)

Now that we have obtained an upper bound of interference time of \( \tau_h \), with regards to the estimate of interference time of \( \tau_{vs} \), we start a method based on Bertogna’s way [13]. We focus on the interference time while a task \( \tau_{vs} \) with higher priority than \( \tau_p \) operates in an interval \( L \). \( L \) is divided into 3 sub intervals (Fig 18):

- \( L_{body} \) is the interval consisting of \( \tau_{vs} \)'s successive periods which are totally included in \( L \).
- \( L_{carry-in} \) or \( L_{carry-out} \) is the interval between the beginning of \( L \) and the beginning of \( L_{body} \). \( L_{carry-out} \) or \( L_{carry-out} \) is the interval between the end of \( L_{body} \) and the end of \( L \). Similar to the single processor RTA in Definition 1, ERD RTA repeats the calculations so that the right and left sides get equal. Let the initial value of \( L \) be the response time of \( \tau_p \) in the RM method.

\[
L = R_p^{RM}
\]

Now we consider values for \( L_{cin} \), \( L_{body} \), and \( L_{co} \) to maximize an interference time of \( \tau_{vs} \). There are two cases in which the interference time is maximized. \( L \) starts at the time when the job of \( \tau_{vs} \) starts executing, and the job finishes at the end of \( L \). The same discussion can be made for both cases. We choose the former case here.

![Fig.18: Body, Carried-in and Carried-out jobs.](image)

In the interval \( L_{cin} \), in order to maximize interference time of \( \tau_{vs} \), the whole job is required to be executed. On the other hand, in order to maximize the length of \( L_{body} \) plus \( L_{co} \), \( L_{cin} \) must be minimized.
Such an $L_{cin}$ is calculated as:

$$L_{cin} = T_{vs} - R_{vs}^{ERD} + C_{vs}, \quad (12)$$

where $R_{vs}^{ERD}$ is derived from Theorem 6. With $L_{cin}$, $L_{body}$ and $L_{co}$ are:

$$
\begin{align*}
nbody & = \left\lfloor \frac{L - L_{cin}}{T_{vs}} \right\rfloor \\
L_{body} & = nbody \times T_{vs} \\
L_{co} & = L - (L_{cin} + L_{body}).
\end{align*}
$$

Thus, the execution times, $cin$ and $body$, of $\tau_{vs}$’s jobs in the intervals $L_{cin}$ and $L_{body}$, respectively, are calculated as follows:

$$
\begin{align*}
cin & = C_{vs} \\
body & = nbody \times C_{vs}
\end{align*}
$$

**Lemma 6** (Carried-out job of $\tau_{vs}$) Upper bound of the execution time, $co$, of $\tau_{vs}$’s job in the interval $L_{co}$ is calculated as follows:

$$co = \min(\max(L_{co} - C_{vs}, 0), C_{vs})$$

*Proof.* From Definition 7, $\tau_{vs}$’s priority is evidently lower than VS and its period is the same as VS. That is, at the release instant of $\tau_{vs}$’s job, VS is ready to execute. If there is no higher priority task’s job than VS, $\tau_{p}$’s job is executed. In the case that a higher priority task’s job is released, that job is executed. In any case, $\tau_{vs}$’s job must wait for $C_{vs}$, since $\tau_{p}$’s job does not finish before the end of $L$. \hfill \Box

**Lemma 7** (Interference Time of ERD). Interference Time $I_{i}^{ERD}$ of $\tau_i$ during an interval $L$ is calculated as follows.

$$I_{i}^{ERD} = \begin{cases} 
\text{cin} + \text{body} + co & \text{if } \tau_i = \tau_{vs} \\
I_{i}^{RM} & \text{otherwise}
\end{cases} \quad (13)$$

$$I_{i}^{ERD} = \text{cin} - \text{empty}(L'') \quad (14)$$

*Proof.* By Lemmas 5, 6 and the discussion in this section. \hfill \Box

### 4.2 Reducing Carry-in

The discussion in the previous subsection supposes that, in an interval $L$, the interference time to $\tau_p$ is the maximum one. Thus, the response time of $\tau_p$ is to be pessimistic. In regard to $\tau_{vs}$’s carry-in job, we consider whether it is possible to be shorten it or not. First, focus on the interval $L'$ which is $\tau_p$’s period immediately before $L$ (Fig 19).

![Fig 19: Body, Carried-in and Carried-out length in $L'$](image)

Each sub interval of $L'$ is calculated as follows:

$$
\begin{align*}
L' & = T_p \\
L'_{co} & = T_{vs} - L_{cin} \\
nbody' & = \left\lfloor \frac{L' - L'_{co}}{T_{vs}} \right\rfloor \\
L'_{body} & = nbody' \times T_{vs} \\
L'_{cin} & = L' - (L_{co} + L_{body})
\end{align*}
$$

Thus, $co'$, $body'$, and $cin'$ are as follows:

$$
\begin{align*}
co' & = 0 \\
body' & = nbody' \times C_{vs} \\
cin' & = \min(\max(L'_{cin} - T_{vs} + R_{vs}^{ERD}, 0), C_{vs})
\end{align*}
$$

Let $\text{empty}(l)$ be an idle/empty time in the interval $l$. Intuitively, $l$ minus the sum of $C_p$ and the interference time of all tasks of higher priority than $\tau_p$ is idle/empty time. If there is idle/empty time, it means there is room to advance the carry-in job of $\tau_{vs}$ in $L_{cin}$. Then, we calculate empty time in the interval $L'$ as follows:

$$\text{empty}(L') = \max(L' - (cin' + body' + co') - C_p - \sum_{j=1,j\neq vs}^{p-1} I_{i}^{ERD}, 0)$$

$empty(L')$ is the amount of time by which the carry-in job is advanced. That is, a part of the carry-in job in $L_{cin}$ is moved to $L_{co}$. Then, we update the execution time of the carry-in job as $ncin$.

$$ncin = cin - \text{empty}(L')$$

Compared to the formula (12), $L_{cin}$ of interval $L$ can be reduced as follows:

$$L_{cin} = T_{vs} - R_{vs}^{ERD} + ncin$$

We can replace $cin$ and $L_{cin}$ in the formulas in the previous subsection by $ncin$ and the reduced $L_{cin}$ respectively, and therefore can alleviate the pessimism of the interference time.
4.3 Updating $I$

With the updated $I_i^{ERD}$, the initial value of $R_p^{ERD}$ is calculated as follows:

$$R_p^{ERD} = \sum_{i<p} I_i^{ERD} + C_p$$

Recalling (11), $L$ can be updated by $R_p^{ERD}$. In addition, the formula (14) in Lemma 7 can be updated since the response time of $\tau_p$ is now shortened compared with RM (see Definition 8). Consequently, the formula of Lemma 7 is updated as follows:

$$I_i^{ERD} = \begin{cases} 
  cin + body + co & \text{if } \tau_i = \tau_{vs} \\
  \left[ \frac{R_p^{ERD}}{T_i} \right] C_i & \text{otherwise}
\end{cases}$$

Theorem 7 (RTA of ERD). In ERD, $\tau_p$’s WCR$T$ is calculated as:

$$R_p^{ERD} = \sum_{i<p} I_i^{ERD} + C_p,$$

where the initial value of $R_p^{ERD}$ is $R_p^{RM}$. Calculation is repeated until $R_p^{ERD}$ is unchanged.

Proof. By the discussion in this section. \qed

5. EVALUATION

5.1 Task Sets and Simulation Environment

In this section, we evaluate the proposed method by showing how much the analyzed WCRT is shortened compared with RM. For comparison, we show the results of ERD RTA and DM. In addition, the longest response times in the task scheduling simulation with ERD (ERD simulation) are shown for reference\textsuperscript{11}. We also provide simulation results for AFJ\textsuperscript{12}. For evaluation, 30,000 task sets are synthetically generated. Each task set is schedulable (with no deadline misses) under RM and has processor utilization of 72% to 96%. When generating task sets, tasks’ periods are decided by random numbers based on the uniform distribution between 10 and 100. Their execution times are decided by random numbers based on the exponential distribution where the upper bound, $U_{mt}$, is 25% or 10% of the corresponding periods. On average, each task set has 5.06 tasks and 9.38 tasks in case of $U_{mt} = 25\%$ and $U_{mt} = 10\%$, respectively.

For the evaluation, the target task $\tau_p$ whose response time should be shortened is a task with the lowest priority (longest period). To raise the priority of $\tau_p$, we applied the following policy. For DM, $\tau_p$ was given the shortest deadline possible while the schedulability was guaranteed. For ERD, after several candidates of VS were derived, the best server for the results of $\tau_p$ was selected.

The simulation period is 10,000 ticks. The same task sets are used for ERD RTA and DM as well as ERD simulation. We use Python 3.6.5 for the simulation. For task set generation, we use schedcat [22] modified for our experiment environment. For simulation, the source code including the ERD scheduling algorithm is available in the author’s repository [23].

\textbf{Fig.20: WCR$T$ Evaluation : Ratio of task sets with shortened WCR$T$ ($U_{mt} = 25\%$).}

5.2 Results

The ERD method shortens the response time by using a high-priority virtual server, while the DM method can shorten the response time when the priority of $\tau_p$ can be raised by regulating its relative deadline. \textbf{Fig 20} shows the ratio of task sets which can reduce WCR$T$ when $U_{mt}$ is 25\%. The horizontal axis indicates the processor utilization of the task sets, and the vertical axis indicates the ratio of the task sets that can have WCR$T$ reduced. In the ERD simulation, the longest response times can be reduced for almost all task sets whose processor utilization is up to 84\%. Even when the utilization is 96\%, 95\% of the task sets succeeded to reduce $\tau_p$’s response time.

In the DM method, it is confirmed that fewer task sets than in ERD RTA can have shortened WCR$T$ for $\tau_p$.

\textsuperscript{11} In ERD simulation, the obtained longest response times might not be WCR$T$s, since the simulation period is limited.

\textsuperscript{12} We exclude RFJ since there was not big difference between AFJ and RFJ.
Fig. 21: WCRT Evaluation: Average WCRT normalized to RM \((U_{mt} = 25\%\)).

Fig. 21 shows the average WCRT normalized to RM RTA. While ERD RTA exhibits shorter WCRT than DM, it is longer than ERD simulation due to its pessimism, which implies there is room for further improving the analysis. When the utilization is 86\%, it is confirmed that ERD can reduce the average WCRT by 13.45\% at maximum compared to the DM.

Fig. 22 shows the results when \(U_{mt} = 10\%\). Similar trends to Fig. 20 and Fig. 21 can be seen. Fig. 22 exhibits better results compared to Fig. 20. This shows that if \(U_{mt}\) is lower, VS could easily obtain high priority in ERD case and \(\tau_p\) could increase its priority in DM case even when it has the lowest priority.

Fig. 24: Jitter Evaluation: comparison of average AFJ \((U_{mt} = 25\%\)).

Fig. 24 shows simulation results of the average AFJ when \(U_{mt} = 25\%\). The vertical axis indicates absolute value of the average AFJ. In RM, AFJ is around 30 to 35 when the utilization is in the 72\% to 78\% range. Over the 80\% range, AFJ is around 40. The AFJ of ERD and DM is less than 5 when utilization is below the 80\% range and up to 10 or 11 when utilization is more than 80\%. Fig. 25 shows the results when \(U_{mt} = 10\%\). In this case, the AFJ of RM is higher than that for \(U_{mt}\) which is 25\%. The AFJ of ERD and DM is almost the same as \(U_{mt}\) is 10\%.

For jitter, it can be said that ERD does not have a big advantage over DM when the utilization is between 72\% and 78\%. These methods show good results compared with RM.
The algorithm we proposed in this paper handles only one target (important) task, \( \tau_p \). Theoretically, the capacity of VS can be used for any task whose priority is higher than \( \tau_p \). However, this approach is not sufficient to shorten multiple target tasks’ WCRT due to the limited capacity of VS on a single processor. Since multi processor systems have more capacity than a single processor system, building a multi processor version of ERD which serves multiple target tasks is a goal of future work.

6. CONCLUSIONS

Execution Right Delegation (ERD) is a real-time scheduling technique that improves real-time performance of tasks which have long periods but are of high importance, while RM necessarily gives low priority to tasks with long periods. In this paper, after reviewing ERD, we showed a response time analysis (RTA) method of ERD and evaluated it in comparison with DM and the simulated behavior of ERD. The results showed that ERD RTA can give shorter worst-case response times (WCR Ts) than DM but is still a pessimistic analysis compared with the simulation results. There is room to improve the analysis for further reduction of WCRT.

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