



Sizing of a small irrigation weir

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Abstract

This paper presents the determination of weir dimensions and its applications based on the equations derived specifically for an irrigation weir and introduced in the standard design manual entitled “KKU-2527 Weir”. The manual was used nationwide as a guideline for the construction of weirs. However, due to technical constraints that probably have a significant impact on weir design such as on-site conditions and budget constraints, among others, the main objective of this study is to determine the weir dimensions that best captures the hydraulic characteristics of flow over a weir and remedy the aforementioned technical constraints. According to the numerical solution of related equations in this study, it was found that weir dimensions are interchangeable. For example, with the significant limitations in deepening a river channel, a higher stilling basin floor elevation can be compensated by increasing the weir width. In brief, such design flexibility can be very useful for on-site weir design modifications in dealing with construction problems and dealing with the financial constraints of local administrative organizations.

Keywords: Weir design, Stilling basin elevation, Crest elevation, Weir width

1. Introduction

Water is fundamental and vital for life and nature. Its effective management is essential. Water-related problems, together with the mounting evidence of the adverse effects of increased water demand in the Northeast of Thailand, are changing the way water is handled [1]. The situation is probably even worse when the control structures (e.g., weirs) are ineffective due to inadequate maintenance and inappropriate design characteristics. As a result, water distribution systems often fail to take appropriate action or are unable to divert water to farmers' fields [2]. Therefore, a small irrigation weir is essential as it plays a significant role in managing increasing demands for river water and minimizing conflicts amongst people afterwards. In other words, it provides livelihood alternatives and enables economic activities such as raising livestock, income generation from agricultural and non-agricultural alternatives such as fisheries and tourism, among others. However, in-stream structures (e.g., weirs) that can line the entire length of a river could probably impede natural flow and disconnect the link between upstream and downstream (longitudinal connectivity), including adjacent riparian and floodplain habitats (lateral connectivity).

In the past, the government initiated and supported the construction of weirs of a type called the “KKU-2527 Weir”. This was done throughout the country by relying on voluntary local labor [3]. The aforementioned KKU-2527 Weir was originally designed and introduced by the Water

Resources and Environment Institute (WREI), Faculty of Engineering, Khon Kaen University [4]. Presently, this responsibility falls on the Sub-district Administrative Organizations (SAOs). Their experience helps to identify weir locations that provide highest benefits to local people. The SAOs are also able to construct and manage their weirs. Unfortunately, there are some constraints that are difficult to overcome including the budget, labor, and the knowledge for defining the proper weir dimensions. Moreover, it is also found that there are two primary causes of damage to existing KKU-2527 Weirs that can result in a loss of functionality of some devices/elements of the weir structure. The first is due to hydraulic pressure and the second is because of destroyed/damaged structural parts. The first cause is commonly found and it can be broken down into two main impacts. The first is erosion at the joint between the downstream side of the weir and the river channel. This results in extensive undermining of the weir foundation and could collapse it in a relatively short time. The second is erosion at the weir wall due to seepage. Therefore, the main objective of this study was to determine the optimal weir dimensions based on the consequence of hydraulic forces when the discharge over the weir is maximal and prior to the water flooding the river banks in the vicinity of the weir. Considering the erosion potential, the flow that departs the weir crest can attain a high velocity due to its static head which may cause severe erosion of river bed downstream, it is necessary to dissipate some energy of the encroaching jet entering the stilling basin. Therefore, a possible energy

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dissipating mechanism is through a “hydraulic jump” inside the stilling basin that exhibits a high resistance to erosion. This can be achieved through proper weir design.

This paper introduces the principles, methods, and techniques to be used as a guideline to determine the appropriate KKU-2527 Weir dimensions based on the principles of fluid mechanics for convenient use of SAOs.

2. Materials and methods

2.1 Theoretical background

It is well established that the design of a small irrigation weir is closely linked to rainfall, irrigation demands, and river flows, among others. Thus, from an engineering perspective, weir design must fulfil three fundamental requirements, i.e., hydraulic performance, structural stability, and environmental impact and safety aspects [5]. As such, it is clear that a proper understanding of the hydraulic characteristics of flow over a weir is important as it can certainly affect the determination of the proper weir size, shape, and its locations. Therefore, the current investigation done aimed to provide comprehensive information covering aspects of small irrigation weir behavior. The principle for defining the dimension of the proposed weir was based on the modification of standard design drawings of [3, 6-8]. To ensure the required flow is discharged over a small irrigation weir, there are a number of important dimensions (see Figure 1):

- length of the weir crest (excluding the columns on the weir crest, L);
- length of the weir crest (including the columns on the weir crest, L_M);
- width of stilling basin (B);
- upstream water depth (y_0);
- head on the weir (H);
- height of the weir crest (w);
- elevation difference between bottom elevation of the stilling basin and the river bed elevation (s).

Referring to Figure 1, the parameters, i.e., w , s , and y_0 , are measured at the bed level of the river at the proposed weir site (note: the parameter “ s ” is positive when the bottom elevation of stilling basin is lower than the river bed level at the proposed site). The main criteria for the design of the proposed weir can be classified in two ways:

- 1) The proposed weir should be placed in the river;
- 2) The proposed weir should be combined with the construction of berms to confine the flow of the river and allow/direct the excess flow to take a “short cut” from upstream to downstream during the peak period.

Based on the above criteria, the design of a weir can be made by considering only the maximum discharge of the river before it will overflow its lower bank. In this study, the flow between upstream and downstream sections (at locations 0 to 3, respectively, as depicted in Figure 1) is assumed to be one dimensional, steady, incompressible, and fully developed turbulent flow. The specified energy equation can be presented as Equation 1 in which the energy loss is assumed to be negligible.

$$y_0 + \frac{v_0^2}{2g} + s = y_1 + \frac{v_1^2}{2g} \quad (1)$$

where y is the depth of flow, v is the average flow velocity, g is the acceleration due to gravity, and s is the elevation difference between bottom elevation of stilling basin and river bed elevation. Subscripts 0 and 1 are upstream and downstream sections, respectively.

In Figure 1, between Sections 1 and 2 (located inside the proposed weir), a hydraulic jump is produced as a consequence of the impact of free-flowing water from upstream of the weir into downstream stilling basin. Consequently, the energy of water can be dissipated and will also prevent scouring on the downstream side of the weir. The momentum equation for a simple hydraulic jump can be written as:

$$\frac{y_2}{y_1} = \frac{1}{2} \times \left[\left(\sqrt{1 + 8Fr_1^2} \right) - 1 \right] \quad (2)$$

where Fr_1 is the Froude number in Section 1, which can be calculated using Equation 3:

$$Fr_1^2 = \frac{v_1^2}{gy_1} \quad (3)$$

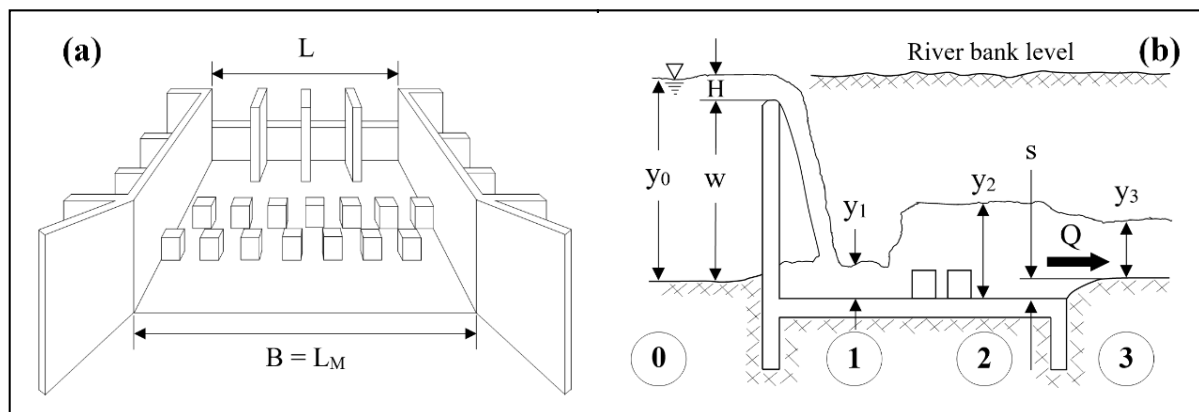


Figure 1 The KKU-2527 Weir: a) typical weir cross section, and b) typical centerline side view (modified from [8])

Regarding the flow from Sections 2 to 3 (original river bed), the energy loss is assumed to be relatively small and is thus neglected. The specific energy equation can be expressed as:

$$y_2 + \frac{v_2^2}{2g} = y_3 + \frac{v_3^2}{2g} + s \quad (4)$$

Moreover, at the end of Section 2 (Figure 1), the elevation of stilling basin can be raised and it is the so-called “sill” to avoid sediment deposition inside the stilling basin. It is noteworthy that the river bed downstream of the weir or sill can be raised as long as the energy loss is not too large. It can be ignored when the flow is passing the sill. Based on the continuity equation, at any section, flow entering the weir must be equal to the flow leaving the weir. This can be written as:

$$Q = v_0 A_0 \quad (5)$$

$$Q = v_1 y_1 B \quad (6)$$

$$Q = v_2 y_2 B \quad (7)$$

$$Q = v_3 A_3 \quad (8)$$

where Q is flow over the weir crest, A is the cross sectional area of flow determined from flow depth and field cross sectional survey using Equations 9 and 10:

$$A_0 = f(y_0) \quad (9)$$

$$A_3 = f(y_3) \quad (10)$$

In Section 3, the hydraulic radius (R_3) can be calculated using:

$$R_3 = \frac{A_3}{P_3} \quad (11)$$

where P_3 is the wetted perimeter of the cross-sectional area of flow and can be calculated as:

$$P_3 = g(y_3) \quad (12)$$

The flow over the weir crest is a function of the head on the weir. It can be calculated using the discharge equation for a sharp-crested weir [9]. Generally, it can also be used to calculate discharge coefficients for various flow conditions as:

$$Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2} \quad (13)$$

where Q is the flow over the weir crest, C_d is the dimensionless discharge coefficient derived by [10]. This depends on weir geometry and the measured head above the weir. It is about 0.60. L is the length of the weir crest, H is the head on the weir and can be described by:

$$H = y_0 - w \quad (14)$$

Focusing on the downstream Section 3 (original river bed), the flow regime is assumed to be uniform which allows the use of Manning's equation to predict the velocity of a uniform flow based on the velocity, slope, and channel conditions. It can be expressed as:

$$v_3 = \frac{1}{n} R_3^{2/3} S_0^{1/2} \quad (15)$$

where n is Manning's roughness coefficient that relates to roughness of the river channel, S_0 = slope of the river bed (dimensionless). It is obtained from a detailed survey or estimates from a topographic map.

An indirect benefit of a weir is that it can be used as a bridge over the river in which its span is above the piers. Accounting for the entire width of all piers, the length of the weir crest (L) where the water overflows will be decreased. Therefore, the modified weir crest length would need to be extended. In this paper, the width of all piers is defined as $0.2L$ in which the modified weir crest length (L_M) will be equal to $1.2L$ (see Figure 1). Additionally, due to the ease of construction and low cost efficiency, the width of stilling basin (B) is defined as equal to L_M and can be written as in Equation 16:

$$B = L_M = 1.2L \quad (16)$$

2.2 Equation solution

2.2.1 Grouping constant physical and hydraulic properties

All of the above equations are necessary conditions for determining weir dimensions. Each equation needs to be solved. Consequently, it was found that some parameters can be substituted for one other, such as between B , w , and s (see Figure 1). This can be considered a flexibility of weir design in the case of a space constraint.

With respect to the solution process, some parameters will first be grouped into three categories, i.e., a group of constant values/area-specific values (five parameters), a group of flow properties (fourteen parameters), and group of important weir dimensions (four parameters), as described below:

- **Group of constant values/area-specific values**, g , C_d , y_0 , S_0 , and n can be obtained from a field survey or estimated from the literature. This is considered to be preliminary data obtained prior to the determination of weir dimensions.
- **Group of flow properties**, y_1 , y_2 , y_3 , v_0 , v_1 , v_2 , v_3 , Q , F_{r1} , H , A_0 , A_3 , P_3 , and R_3 are unknown parameters and need to be calculated using the aforementioned equations.
- **Group of important weir dimensions**, L , w , B , and s are also unknown parameters and need to be calculated using the aforementioned equations.

2.2.2 Solution method

Since the 18 unknown parameters from the latter groups are more than the number of equations, two parameters in the third group must be specified. For practical reasons, the parameters, w and B , are usually specified. Basically, the solution consists of 2 steps. The first is to use the parameter y_0 obtained from field measurements to determine A_0 using Equation 9. Assuming y_3 , the parameters A_3 , P_3 , R_3 , v_3 , Q , v_0 , H , and L can be determined from Equations 10, 12, 11, 15, 8, 5, 14, and 16, respectively. Then, the particular value of y_3 that makes both sides of Equation 13 equal is found. Next, a value of the parameter y_1 is assumed and the values of v_1 , s , Fr_1 , y_2 , and v_2 are determined from Equations 6, 1, 3, 2, and 7, respectively, so that both sides of Equation 4 are equal. Regardless of the solution technique, all 16 equations must be used to determine these 16 parameters, i.e., A_0 , A_3 , P_3 , R_3 , v_3 , Q , v_0 , H , L , y_3 , v_1 , s , Fr_1 , y_2 , v_2 , and y_1 . It is possible to obtain more than one set of correct answers.

The relationship between the number and size of the piers in Equation 16 can be presented in Equations 17 and 18. The piers are useful for (i) stream crossing by connecting piers, and (ii) inserting pieces of wood between piers to temporarily raise the weir crest to store more water upstream when flooding recedes. Both are usually required and can be done by the community.

$$B = (M \times c) + L \quad (17)$$

$$L = (M + 1) \times p \quad (18)$$

where M is the number of piers, c is the thickness of each pier along the direction of weir crest length, p is the distance between two sides of adjacent piers.

In Equations 17 and 18, the parameters B and L are known whereas M , c , and p are unknowns. If one of the unknowns is specified, the other two can be determined.

2.3 Field work

The field work, in which the weir dimensions are determined, can be summarized as the following. First, the benefits of the designated area, where the proposed weir will be constructed (including alternative sites), should be assessed with water use as the first priority. The river channel should be straight, with a uniform cross section, require no major changes, and allow water to flow into the floodplain on one or both river banks. Next, the river cross section must be determined at selected locations by measuring its depth and width at each elevation. Finally, the lowest point of the cross section and the distance to the downstream cross section (not less than 1 km) is found to determine the slope of river bed (S_0).

After the field work, analysis was then done. First, the values of n and S_0 were calculated. Then, the cross sections were plotted to determine their cross sectional areas and wetted perimeters. Finally, y_0 was determined as indicated in Figure 1.

3. Results

3.1 Flexibility in KCU-2527 Weir design

Using the equations discussed in Section 2.1, the data was processed in Microsoft Excel to solve the appropriate

equations and enhance flexibility in designing a KCU-2527 Weir. There are two types of relationships that govern the design flexibility that can be illustrated as follows.

3.1.1 Relationship between discharge and w/y_0

The design of a KCU-2527 Weir needs to be tailored to address specific project objectives regarding how water discharge can be matched with the proper design of a weir. Weir design can be done in two ways. First, in defining the discharge over a weir (Q), hydrologic analysis and flow measurements should be done at the proposed weir site. The water depth upstream of the weir (y_0) and the required ratio of crest height (w) over upstream water depth (y_0) was found and w/y_0 determined. For example, if the discharge over a weir is set to $50 \text{ m}^3/\text{s}$, the water depth upstream of the weir is 4 m, and w/y_0 is 0.6. In Figure 2e, when y_0 is equal to 4 m and C_d is 0.6, the crest length (L) is 14 m, as illustrated in the figure. Several crest lengths based on different discharges over weir and different ratios of w/y_0 are given in Figure 2. The second method of weir design is in case the crest length of 14 m is greater than the river width at the proposed weir site. In this context, a modification can be made by setting a new crest length (for instance 12 m) where the discharge over weir is still $50 \text{ m}^3/\text{s}$. Thereafter, a new ratio of w/y_0 can be determined as 0.56 by drawing a horizontal line from the specified discharge over weir ($50 \text{ m}^3/\text{s}$) to the right until it intersects the graph at the new crest length ($L = 12 \text{ m}$). Then, the height of crest (w) and the width of stilling basin (B) can be determined from Equations 14 and 16, respectively.

3.1.2 Relationship between B/y_0 and s/y_0

The simplicity and flexibility of the KCU-2527 Weir design makes it easier to locate/design/build effective weirs at suitable sites based on the principles of fluid mechanics. The relationships of some the significant parameters, i.e., width of the stilling basin (B), the upstream water depth (y_0), and elevation difference between the bottom elevation of stilling basin and river bed elevation (s) possibly affect the discharge over the crest of the weir. Therefore, the aforementioned parameters need to be carefully identified and will be discussed in this section.

Based on the representative river cross section depicted in Figure 3 and the input values from Table 1, the solutions to the 16 equations and M , c , and p were determined and are shown in Table 2. Fr_1 values ranging from 4.5 to 9 are recommended by [9] for a stable and well-balanced hydraulic jump that is not overly sensitive to tailwater fluctuations. Once completed, verification can be done by substituting solution values indicated in Table 2 into the equations to see if they all are satisfied.

In the case where w and B are treated as unknowns, the 16 equations will have 18 unknowns and hence will be unsolvable. Analytically, 15 of the 16 equations can be used to eliminate 15 unknowns. This results in one equation with three unknowns. If B , s , and Fr_1 are chosen as unknowns, an equation can be written symbolically as $f(B, s, Fr_1) = 0$ where f indicates a function and B , s , Fr_1 are its parameters. Assigning a value to Fr_1 , the plot of B against s shown in Figure 4 was made using the inputs indicated in Table 1. The division of B and s by y_0 was performed to establish a dimensionless coordinate system for easier plotting.

The aforementioned graph, which illustrates the flexibility in weir design based on actual conditions and SAO planning, can be used in conjunction with the following process.

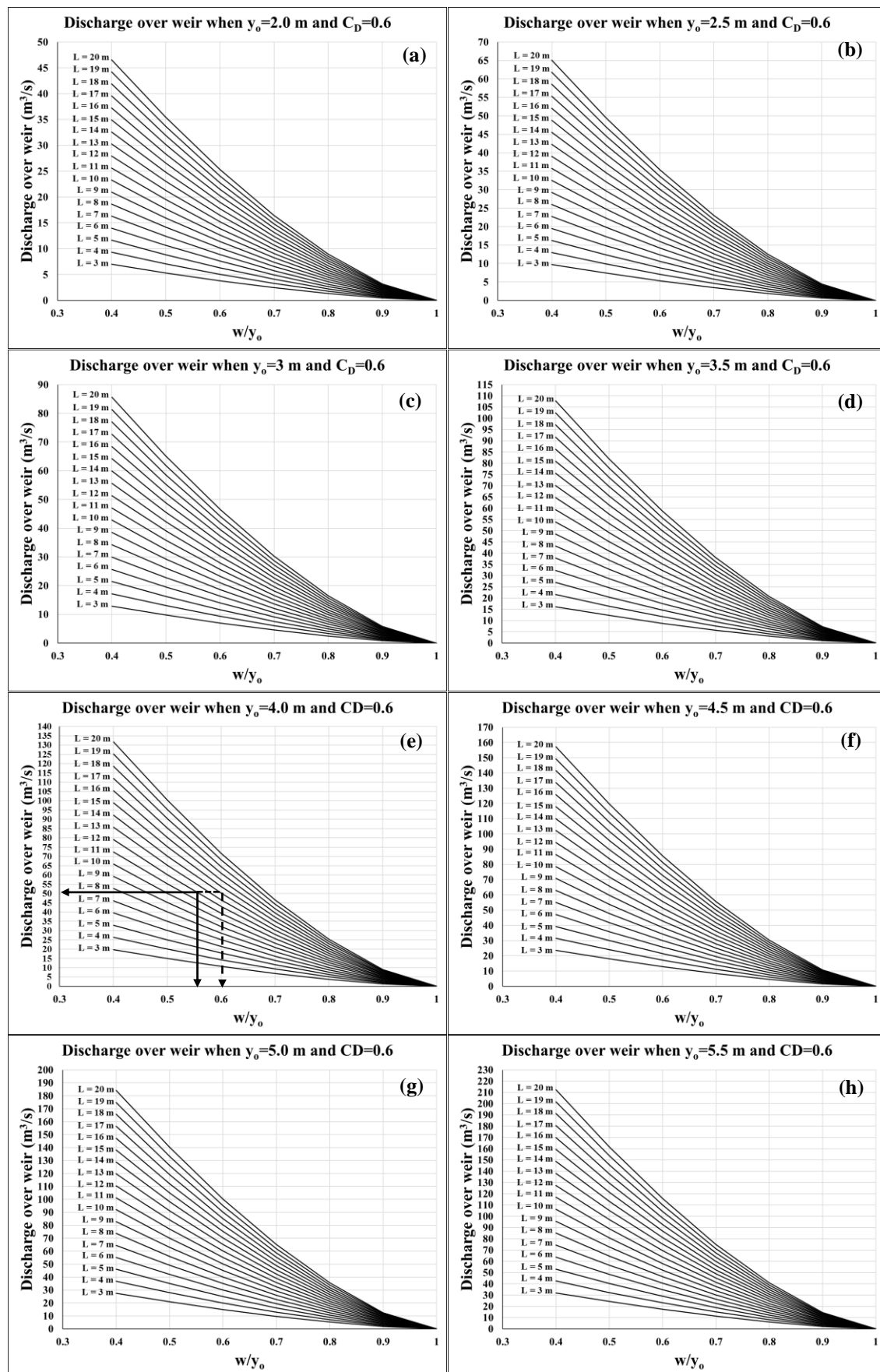


Figure 2 The relationship between the discharge over weir (Q) and the w/y_0 ratio for different upstream water depths (y_0) and various crest lengths

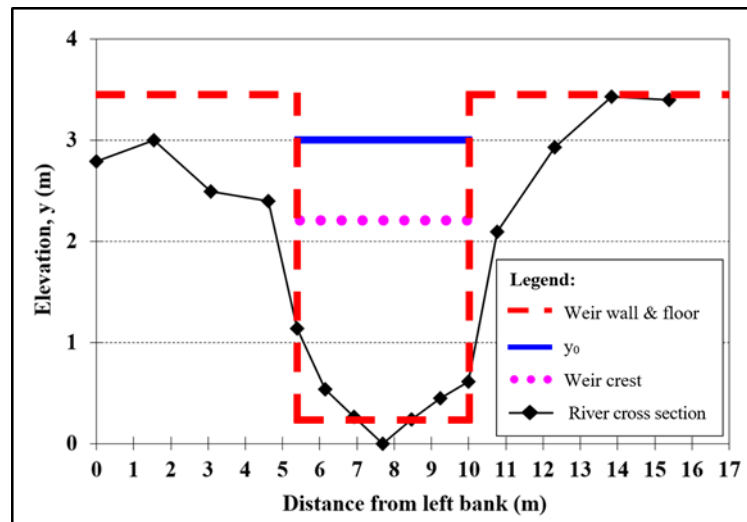


Figure 3 A representative river cross section selected for this study

Table 1 A set of cross sectional, constant, and area specific parameters

y (m)	A (m ²)	P (m)
0.0	0.00	0.00
0.5	0.77	3.75
1.0	2.67	6.00
1.5	5.08	10.00
2.0	7.79	12.60
2.5	11.23	16.00
3.0	15.83	18.25

Note: $g = 9.8 \text{ m/s}^2$, $C_d = 0.6$, $y_0 = 3 \text{ m}$, $S_0 = 0.0001$, $n = 0.013 \text{ s/m}^{1/3}$, $w = 2.2 \text{ m}$, and $B = 4.46 \text{ m}$

Table 2 Summary of calculated parameters based on the equations solved for designing a weir*

Parameter	Value	Unit
c	0.25	m
H	0.80	m
L	3.72	m
M	3	piers
p	0.93	m
Q	4.69	m ³ /s
s	-0.93**	m
A ₀	15.83	m ²
v ₀	0.30	m/s
Fr ₁	4.70	-
y ₁	0.17	m
v ₁	6.11	m/s
y ₂	1.06	m
v ₂	0.99	m/s
A ₃	8.28	m ²
y ₃	2.07	m
v ₃	0.57	m/s
P ₃	13.08	m
R ₃	0.63	m

Note: * w and B were set to 2.2 m and 4.46 m, respectively

** A negative “s” value indicates a higher bottom elevation of stilling basin compared to the river bed level at the proposed site (see Figure 1b).

First, the selection of weir site must be undertaken to ensure that it meets the community’s needs. A crest height (w) can then be defined in which the value of y_0 can be set based on the difference in elevation between the bottom of the river bed and the river bank, focusing on the lower side (Tables 1 and 2). Next, the parameters B and s need to be defined in the way that the formation of a hydraulic jump with a suitable Froude number (Fr_1) can take place in a stilling basin downstream of a weir. Under these defined conditions, a set of parameters B and s can be varied based upon the aforementioned equations and can be plotted in the form of Figure 4. For instance, if B and Fr_1 are set to 4.46 m and 4.70, respectively, s is equal to -0.93 m and other parameters indicated in Table 2 can be obtained from the solution. Table 2 lists the calculated values of relevant parameters determined by varying the target parameters, i.e., w or B. The ratios of $s/y_0 = -0.310$ and $B/y_0 = 1.487$ can be plotted in the form of Figure 4 where y_0 is constant and equal to 3 m. In the case of economic constraints or technical difficulties, the value of B can be reduced to 4 m and B/y_0 will decrease to 1.333. As a result, the value of parameter s is -0.918 m with this updated B/y_0 of 1.333. The ratio of s/y_0 is -0.306 from Figure 4.

Interpretation of Figure 4 can be done by scientific explanation. Referring to Figure 3, due to the hard bedrock below river bed at the weir sites, river bed excavation would need to be done to the designed width and depth of the proposed channel to obtain the required value of s. However, excavation can be costly and time-consuming. The excavation and quantity of rock removed should be minimized by increasing the stilling basin width (B). In this context, to avoid river bottom excavation, the graph suggests that the stilling basin width (B) can be extended by raising the bottom of stilling basin. Above all, weir dimensions can be adjusted to meet the constraints of local conditions, depending on the stilling basin width (B), crest height (w), and the specified parameter values in Table 1.

4. Discussion

Based on the solution presented in the current study, there can be flexibility in designing/sizing of a KKU-2527 Weir that can ensure that its size will be adequate for the required flow. The graphical relationship between the s/y_0

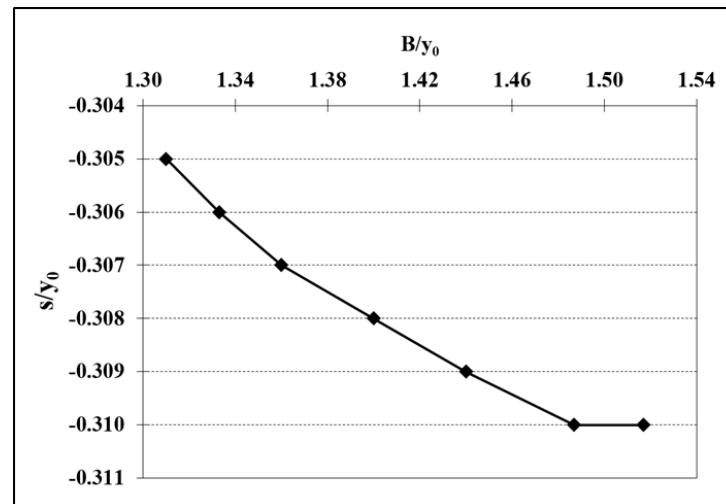


Figure 4 The relationship between B/y_0 and s/y_0 using $Fr_1 = 4.7$ and the values in Table 1

and B/y_0 ratios shows how cross sectional area, some constant values, and area-specific parameters shown in Table 2 can be controlled. The results of the current study demonstrate that the crest height (w) can be increased when the stilling basin elevation is raised, i.e., the differences in the elevations of the stilling basin and the river bed can be decreased. The graph presented in Figure 4, which has a negative slope, illustrates weir design flexibility. It indicates that decreasing the value of s by raising the stilling basin level can be compensated for the increased B value by widening the stilling basin.

In short, the flexibility in the KKKU-2527 Weir design proposed herein will definitely help with technical adjustments required by on-site conditions and budget constraints. However, the proposed weir construction should address other related problems such as seepage, strength/robustness of the weir, ease of weir construction, quantity of material estimated for weir construction, and estimated cost of weir construction, which were not considered in this study.

Seepage occurs underneath and through the sides of weirs. It can be reduced through some standard construction techniques. Side seepage can be reduced by back filling and compacting around the flank of walls and sides of weir foundations with excavated earth. Weir walls should be vertical since inclined walls do not permit visual observation of the unavoidable earth subsidence until the walls collapse which is costly to remedy. Subsidence of vertical walls is easily seen and filled with earth.

In future work, the design procedure will be simplified to avoid or reduce complex computation. Suitable dimensionless numbers will be prepared specifically for Thailand's conditions.

5. Conclusion

In view of hydraulics, the determination of suitable weir dimensions can be accomplished using a group of related equations explained in the previous section and in the standard design manual entitled "KKU-2527 Weir". Additionally, these equations provide the basis for adjusting some parameters to deal with on-site problems and budget shortfalls. By exploiting this set of equations, weir dimensions are found to be related, e.g., increasing the weir crest will result in a higher bed level and wider stilling basin.

However, there are other aspects of a highly functional weir that need to be considered such as construction site selection and development, and cost, as well as ease of operation and maintenance.

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