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Design of rate-compatible LDPC codes based on uniform shortening distribution

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Abstract

Information shortening is a methodology to construct rate-compatible codes. Recently, LDPC codes have been shown that uniform shortening distribution is a criterion to achieve excellent performance. The difficulty is that the information nodes must be appropriately selected. To solve this problem, this paper proposes a design of a parity-check matrix for LDPC codes in which the uniform shortening distribution is satisfied. With this technique, exceptionally high-performance rate-compatible LDPC codes are achieved.

Keywords: Insert known bit, Rate compatible, LDPC codes, Uniform shortening distribution

1. Introduction

Nowadays, error correcting codes play an important role in high speed communication systems. Reliable communication, e.g., wireless links with BER values of 10^{-6}, can be achieved by employing error correcting code [1]. Typically, the error correcting capability of the codes directly relates to a parameter called the code rate [2]. This capability is inversely proportional to the code rate, e.g., codes of rate 0.5 outperform codes of rate 0.7. Normally, the code is designed for a specific rate to combat errors introduced by noisy channels. However, code rate should be adapted according to channel conditions to apply the code to time varying channels.

Rate compatible (RC) code is one type of channel code that can support various code rates by utilizing only one encoder/decoder circuit. Therefore, RC code can be applied to many practical communication systems, such as wireless communications, over rapidly time-varying channels and hybrid automatic repeat request (HARQ) systems, among others [3-5]. HARQ systems are used to improve the efficiency of transmission by data retransmission requested from a receiver. *Shortening* is a technique commonly used to construct RC code [3-6]. The first step for constructing the RC code is to select the *mother code*. Then, shortening is achieved by inserting known bits into certain positions of information bits before encoding. The known bits affect the encoding process, but these bits will be removed before transmission through a channel. This results in a lower rate and shorter block length. At the receiver, the known bits are inserted back into the same positions that they occupied before decoding.

Due to the capacity approaching performance, this paper considers low-density parity-check (LDPC) codes as the mother codes for shortening. LDPC codes are a class of linear block codes that have been extensively employed in many communication system standards such as DVB-S2 [7], IEEE 802.16e [8], and IEEE 802.11n [9] among others. Shortening LDPC codes has been presented in the literature [4-6, 10-11] to provide lower rate codes for rate-compatible error correction schemes.

The problem that directly relates to the performance of shortening is the positions at which known bits will be inserted. It is notable that, in the literature, a shortening algorithm corresponds to a way of selecting the positions of information bits to perform shortening. Regarding LDPC codes, some bit nodes related to information bits must be devoted to shortening. A facile method is shortening the first or last part of information bits. However, this method presents inferior performance. For simplicity, this kind of shortening is called a *basic algorithm* in this paper. More sophisticated shortening algorithms have been proposed to achieve better shortening performance, e.g., the largest extrinsic sum (LES) and smallest-row variance priority (SRVP) algorithms [10-11]. These shortening algorithms provide better BER performance than the basic algorithm with the penalty of higher computational complexity. Nevertheless, the performance of shortening LDPC codes obtained from previously found shortening algorithms is poorer than that of the LDPC codes designed for a specific rate, known as dedicated LDPC codes.

To the best of our knowledge, the best known shortening LDPC codes are from the algorithm presented in [12]. This algorithm attempts to insert known bits into carefully preselected bit nodes to achieve a *uniform shortening distribution*. Interestingly, with this algorithm, the performance of the shortening LDPC codes is identical to that of dedicated LDPC codes. The shortening distribution is

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a description of the numbers of edges connected to the shortening information nodes for each check node. This means each check node possesses the same number of edges connected to the shortening information nodes. This is referred to as the uniform shortening distribution. However, there is no explicit way to obtain the group of bit nodes that yields a uniform shortening distribution. Moreover, it is almost impossible to find the group of bit nodes with a uniform shortening property [12].

To solve the aforementioned shortening problem, this paper proposes a novel design using a parity-check matrix of LDPC codes suitable for shortening. These LDPC codes are designed based on a uniform shortening distribution. With this design, the uniform shortening distribution that guarantees excellent performance is satisfied by the basic algorithm. As a result, the shortening LDPC codes constructed from the proposed parity-check matrix exhibit identical performance comparable with dedicated LDPC codes for a variety of code rates and lengths.

This paper is structured as follows. Section 2 describes LDPC codes and shortening. In section 3 the design of RC LDPC codes based on uniform shortening distribution is presented. The simulation results and discussion are given in Section 4. Finally, the conclusions are presented in Section 5.

2. Shortening low density parity check code

2.1 LDPC codes

Low-density parity-check (LDPC) block codes were initially introduced by Gallager [13]. They were capable of approaching capacity on a variety of channels. LDPC codes are defined by a sparse parity check matrix with z size of $M \times N$ that denoted is by (N, K) LDPC code. The number of "$1's$" in each row and column are called row weight (w_r) and column weight (w_c) , respectively. Commonly, there are two types of LDPC codes. The first one is the regular LDPC code, which contain constant row and column weights. The other type is the irregular LDPC code which do not have constant row and column weights.

The vector $\mathbf{m} = (m_1, m_2, \ldots, m_K)$ denotes the binary data source that is encoded into the codeword $\mathbf{c} = (c_1, c_2, \ldots, c_N)$ by an LDPC encoder of code rate K/N.

2.2 Shortening LDPC codes

A shortening technique is used to achieve lower code rates by insert known bits into certain information bits. The known bits can be zeros or ones before encoding with a fixed rate encoder using mother codes. Then, the known bits are removed before transmitting. The receiver must insert known bits before encoding, and remove them to keep the actual information.

Figure 1 shows a block diagram of shortened LDPC codes with a rate of K_s/N_s and length N_s from mother LDPC codes with a rate of N_m/K_m and length N_s. The shortened LDPC can be easily explained by the following example.

Example 1 Consider the parity check matrix and Tanner graph of a $(16{,}8)$ LDPC code with the rate of $1/2$ as shown in Figure 2. If this code is shortened into a $(12{,}4)$ LDPC with a rate of $1/3$, the Tanner graph is shown in Figure 3.

2.3. Position selection of shortening

The bit error rate (BER) performance of shortened LDPC codes depends on the position of the shortening. The easiest way to shorten is by inserting known bits into the first or last bits of information, as shown in Figure 4. This shortening pattern referred to as the *basic pattern* henceforth. Nevertheless, the performance is not good for every construction. Many selection algorithms were introduced to increase the performance. Selection algorithms with a uniform shortening distribution [12] have gained interest since they can achieve good performance for various rates of shortening. The shortening distribution denotes the distribution of many edges connected to the shortening information nodes for each check node. For this, we need to define shortening with uniform and non-uniform distributions.

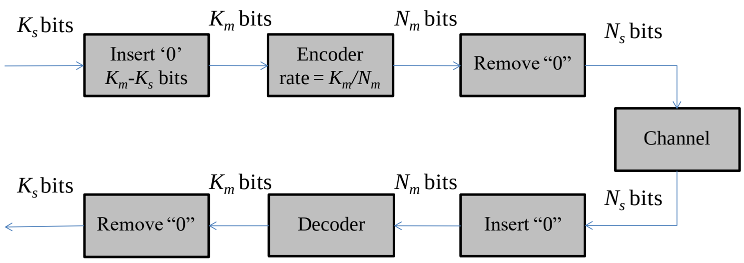

Figure 1 Block diagram of shortening rate K_m/N_m into K_s/N_s.

$$H = \left(\begin{array}{cccccccc|cccccccc}
\multicolumn{8}{c|}{\text{parity column}} & \multicolumn{8}{c}{\text{information column}} \\
1&0&0&0&1&0&1&0&1&0&0&0&0&0&0&0\\
0&1&0&0&0&1&0&1&0&0&0&0&1&0&0&0\\
0&0&1&0&0&0&0&0&0&1&1&1&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&1&1&1\\
0&0&0&0&1&0&0&0&0&1&1&1&0&0&0&0\\
1&1&0&0&0&0&0&1&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&1&1&1\\
0&0&1&1&0&1&0&0&1&0&0&0&0&0&0&0
\end{array}\right)$$

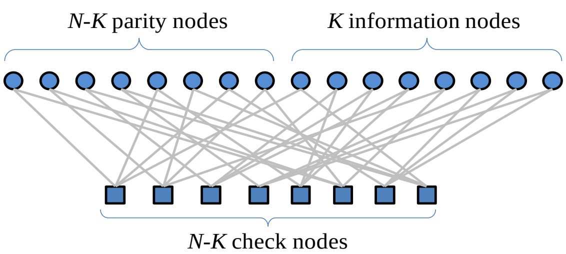

Figure 2 The parity check matrix (left) and corresponding Tanner graph of a $(16{,}8)$ LDPC code (right).

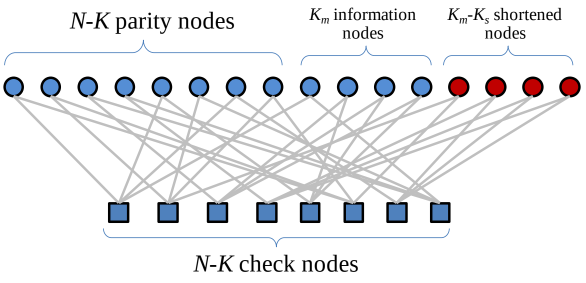

Figure 3 Tanner graph of shortening (16,8)LDPC code into (12,4)LDPC code.

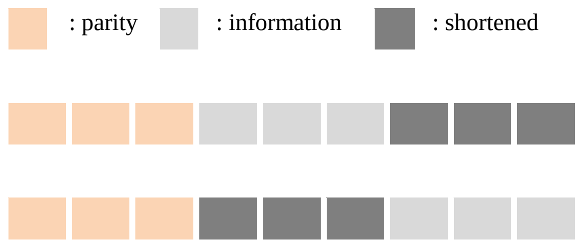

Figure 4 The basic pattern of shortening

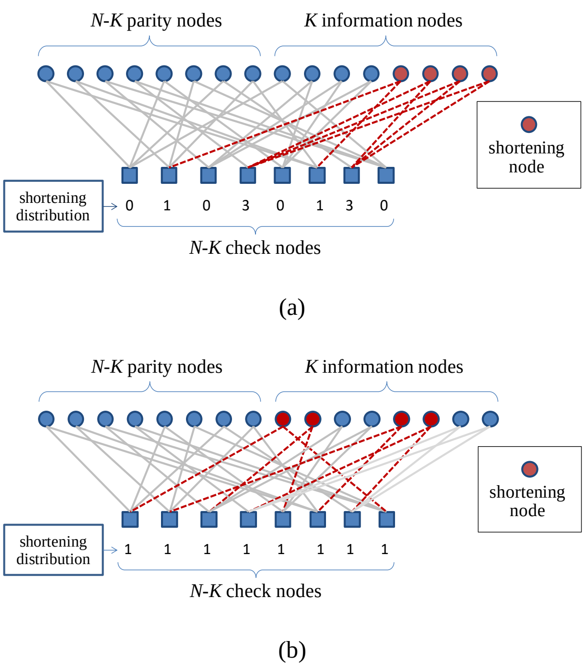

Figure 5 Tanner graph of shortening in Example 1 with (a) non-uniform and (b) uniform shortening distributions.

Figure 5 shows a Tanner graph of shortening in *Example 1* with various shortened positions. Shortening with the last part of information bits is shown in Figure 5(a) and its non-uniform shortening distribution is shown in Figure 6(a). Alternatively, Figure 5(b) shows the shortened nodes with uniform distribution as shown in Figure 6(b).

The shortening distribution is an important factor that affects the BER performance [12]. Shortening with a uniform distribution can reach better performance than for a non-uniform distribution, although its complexity is higher than shortening with only the first/last part of the information.

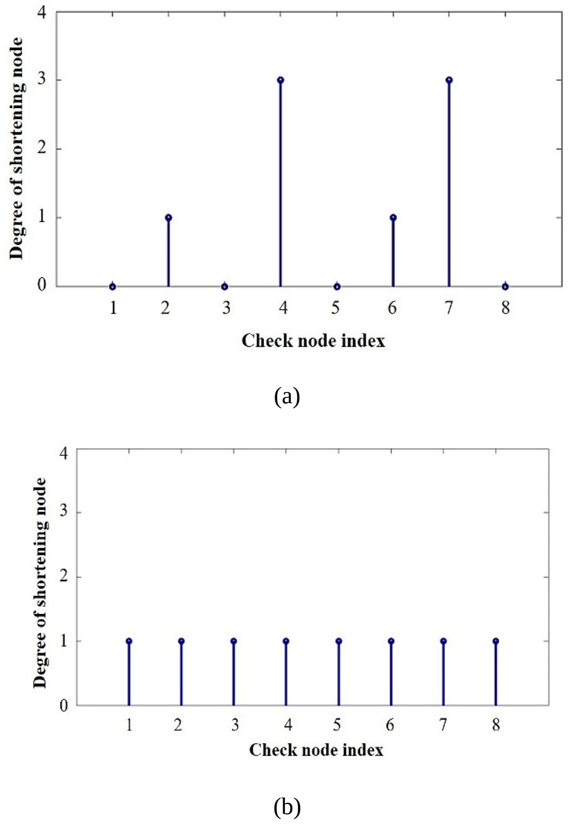

Figure 6 The shortening distributions of (a) non-uniform and (b) uniform shortening distributions.

3. Design of shortening low density parity check codes based on uniform shortening distribution

In this section, the construction of LDPC codes based on a uniform shortening distribution is proposed. This constructs only regular LDPC codes because it is much simpler to design using a uniform shortening distribution. This paper proposes an algorithm designed to construct the LDPC code with $w_c = 3$ since the performance of LDPC code using this algorithm with $w_c = 3$ was better than with $w_c > 3$. Moreover, to construct LDPC code with $w_c > 3$, the complexity of computation is higher, since it requires more parameters. The parity check matrix of this proposed construction is as follows:

$$H = \begin{bmatrix} I^{a_{1,1}} & I^{a_{1,2}} & \cdots & I^{a_{1,n}} \\ I^{a_{2,1}} & I^{a_{2,2}} & \cdots & I^{a_{2,n}} \\ \vdots & \vdots & \ddots & \vdots \\ I^{a_{m,1}} & I^{a_{m,2}} & \cdots & I^{a_{m,n}} \end{bmatrix}_{mL \times nL} \quad (1),$$

where I = identity matrix of size of $L \times L$ and, $a_{i,j}$= the right shift number of identity matrix by $a_{i,j} \in \{0, 1, 2, ..., L-1\}$.

This proposed construction utilizes the identity matrix's shifting to construct a parity check matrix because it can make a uniform shortening. In fact, the most important thing for the design of LDPC code is the girth. This paper proposes a method to design parity check matrix for LDPC code without a girth 4 and 6 by designing the best shift number of the identity matrix. The right shift number of the identity matrix is denoted by the matrix A as shown in (2).

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{bmatrix}_{m\times n} \quad (2),$$

where $a_{i,j} \in \{0,1,2,\ldots,L-1\}$.

An algorithm to construct matrix A was used to construct a parity check matrix of LDPC codes of size $mL \times nL$ and a rate of $((nL-mL))/nL$ as follows:

1. Define $L, m = w_c$ and $n = w_r$

2. Random $a_1 = [\,a_{1,1}\, a_{1,2} \ldots a_{1,n}]$
   where $a_{1,i} \neq a_{1,j}$
   $i,j \in \{1,2,\ldots,n\}$
   $a_{1,i}, a_{i,2} \in \{0,1,2,\ldots,L-1\}$

3. Random $a_2 = [a_{2,1}\, a_{2,2} \ldots a_{2,n}]$
   where $a_{2,i} \neq a_{2,j}$
   $b_i = (a_{2,i} - a_{1,i})\%L$
   $b_i \neq b_j$
   $i,j \in \{1,2,\ldots,n\}$
   $a_{2,i}, a_{2,j}, b_i, b_j \in \{0,1,2,\ldots,L-1\}$

4. Random $a_3 = [a_{3,1}\, a_{3,2} \ldots a_{3,n}]$
   where $a_{3,i} \neq a_{3,j}$
   $c_i = (a_{3,i} - a_{2,i})\%L$
   $c_i \neq c_j$
   $d_i = (a_{3,i} - a_{1,i})\%L$
   $d_i \neq d_j$
   $E = [e_{i,j}]_{n\times n}$
   $e_{i,j} = (b_i + c_j)\%L$
   $e_{i,j}(\forall i = j) \neq e_{i,j}(\forall i \neq j)$
   $i,j \in \{1,2,\ldots,n\}$
   $a_{3,i}, a_{3,j}, c_i, c_j, d_i, d_j \in \{0,1,2,\ldots,L-1\}$

5. $A = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$

For example, if we need to construct a $(300,150)$ LDPC code with $w_c = 3, w_r = 6$, we can use a proposed algorithm as follows:

Step 1 Initialize three parameters of $m = w_c = 3, n = w_r = 6$ and $L = \frac{N}{w_r} = \frac{300}{6} = 50$.

Step 2 Calculate the set a_1 of the first row in matrix A by random n values from $\{0,1,2,\ldots,49\}$

$a_1 = [\;2 \;\; 15 \;\; 4 \;\; 7 \;\; 42 \;\; 36\,]$

Step 3 Calculate the set a_2 of the second row in matrix A with random n values from $\{0,1,2,\ldots,49\}$ with the condition of $b_i \neq b_j$ where $(b_1, b_2, \ldots, b_n) = (a_2 - a_1)\%L$. This condition is defined to avoid the 4-cycles in matrix H.

$a_2 = [\,16 \;\; 48 \;\; 2 \;\; 23 \;\; 20 \;\; 39\,]$

$b = (\,36, 17, 2, 34, 22, 47\,)$

Step 4 Calculate the set a_3 of the third row in matrix A by random n values from $\{0,1,2,\ldots,49\}$ with three conditions of $c_i \neq c_j$, $d_i \neq d_j$ and $e_{i,j}(\forall i = j) \neq e_{i,j}(\forall i \neq j)$ where $(c_1, c_2, \ldots, c_n) = (a_3 - a_2)\%L$, $(d_1, d_2, \ldots, d_n) = (a_3 - a_1)\%L$ and $e_{i,j} = (b_i + c_j)\%L$. The first and second conditions are defined to avoid the 4-cycles in matrix H, and the third condition is used to avoid 6-cycles in matrix H.

$a_3 = [\,40 \;\; 10 \;\; 25 \;\; 36 \;\; 34 \;\; 30]$

$c = (\,26, 38, 27, 37, 36, 9\,)$

$d = (\,12, 5, 29, 21, 8, 6)$

$$E = \begin{bmatrix} 12 & 24 & 13 & 23 & 22 & 45 \\ 43 & 5 & 44 & 4 & 3 & 26 \\ 28 & 40 & 29 & 39 & 38 & 11 \\ 10 & 22 & 11 & 21 & 20 & 43 \\ 48 & 10 & 49 & 9 & 8 & 31 \\ 23 & 35 & 24 & 34 & 33 & 6 \end{bmatrix}$$

Step 5 We obtain matrix A that can be used to construct matrix H.

$$A = \begin{bmatrix} 2 & 15 & 4 & 7 & 42 & 36 \\ 16 & 48 & 2 & 23 & 20 & 39 \\ 40 & 10 & 25 & 36 & 34 & 30 \end{bmatrix}$$

$$H = \begin{bmatrix} I^{2} & I^{15} & I^{4} & I^{7} & I^{42} & I^{36} \\ I^{16} & I^{48} & I^{2} & I^{23} & I^{20} & I^{39} \\ I^{40} & I^{10} & I^{25} & I^{36} & I^{34} & I^{30} \end{bmatrix}$$

The proposed algorithm was used to construct regular LDPC codes that can achieve high performance for shortening by selecting the first or last part of information bits. Figure 7 shows the positions of shortening mother LDPC codes with the rate K_m/N_m into rate $K_{s1}/N_{s1}, K_{s2}/N_{s2}$ and K_{s3}/N_{s3} by shortening the last part of the information bits, where $K_{s1}/N_{s1} > K_{s2}/N_{s2} > K_{s3}/N_{s3}$.

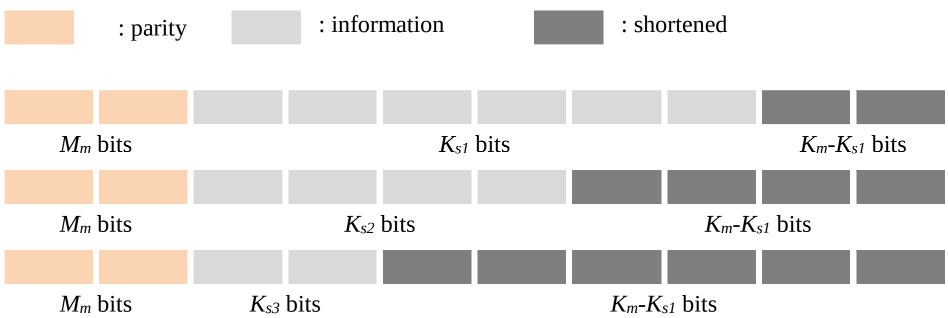

Figure 7 The shortening node for proposed LDPC codes.

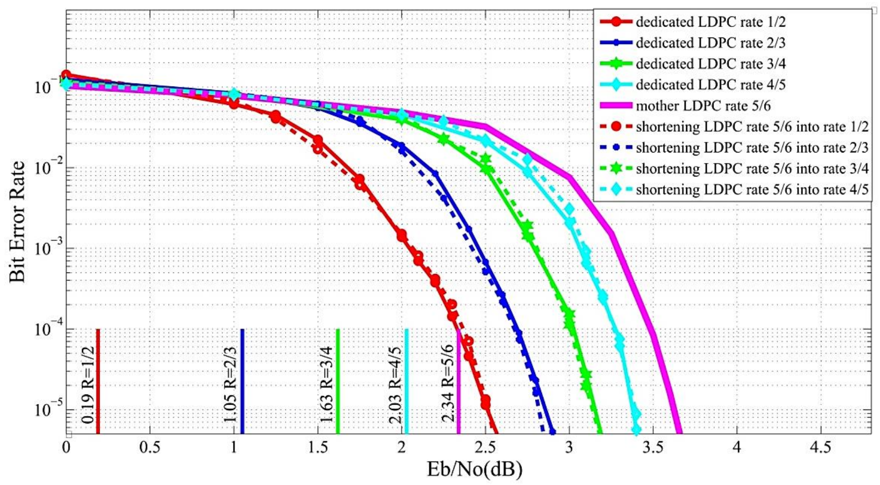

Figure 8 The BER performance of shortening LDPC rates, 1/2, 2/3, 3/4 and 4/5 comparing with dedicated LDPC with same rate and length.

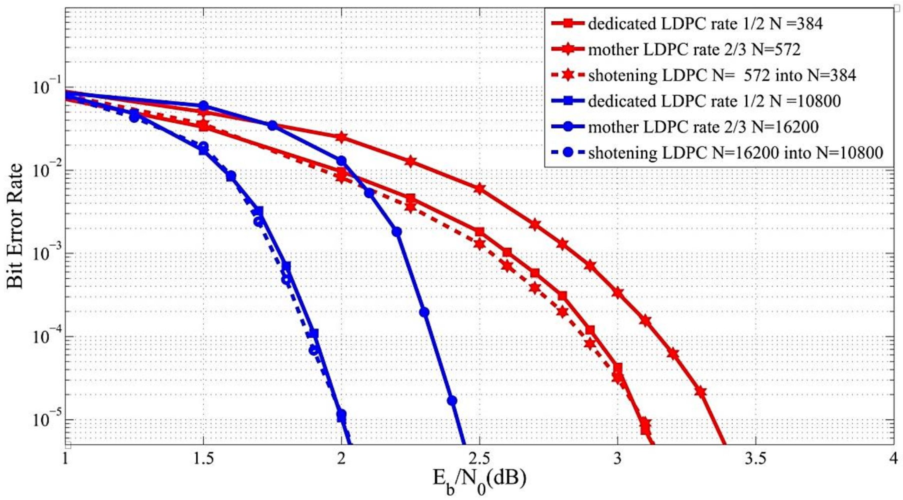

Figure 9 The BER performance of shortening LDPC rate $1/2$ with length 572 and 10800 compared with dedicated LDPC with the same rate and length.

This shortening position can obtain uniform distribution for all lengths $of\ iL$ where $i = 1, 2..., n-1$ with the number of edges connected to shortening information nodes for each check node as:

$$d_s = \frac{w_c \times (K_m - K_s)}{M_m} \quad (3)$$

4. Simulation results

In this section, we show the BER performance of the proposed LDPC codes over an additive white Gaussian noise (AWGN) channel by computer simulation. As a result, the minimum sum product (min-sum) algorithm with the maximum iterations of 15 is employed because the mother codes were constructed from the proposed algorithm with a w_c of 3, in addition to its simplicity.

Firstly, the BER performance of the shortened mother LDPC code with a rate of $5/6$, length of 3888, $w_r = 18$, $w_c = 3$ and $L = 216$ into the lower rate was as shown in Table 1 by inserting known bits into last part of the information bits. This was compared with dedicated LDPC with the same rate and length.

Table 1 Parameters of shortening LDPC codes

Rate	Length (K_s)	Number of shortened bits
1/2	1296	2592
2/3	1944	1944
3/4	2592	1296
4/5	3240	648

From the results in Figure 8, the performance of shortened LDPC codes was similar to dedicated LDPC codes for all the rates with a gap between LDPC codes and the capacity of low rates was wider than high rates because the length of low rates was shorter than high rate.

The next result shows the BER performance of shortened mother LDPC codes with other lengths. Since the previous results showed only the performance shortened mother LDPC codes with a medium length, thus this result shows the BER performance of shortened mother LDPC codes with short and long block lengths. Figure 9 shows the BER performance of mother LDPC codes with a rate of $2/3$ into a rate of $1/2$ by shortening from length 572 and 16200 into 384

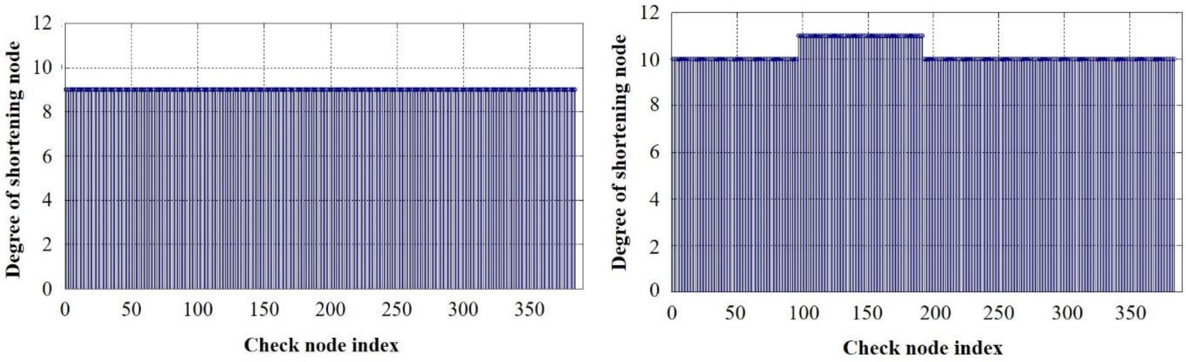

Figure 10 The shortening distribution of proposed LDPC codes (left) and shortened LDPC codes in IEEE802.16e (WiMAX) from a rate of $5/6, N = 2304$ into a rate of $2/3, N = 1152$.

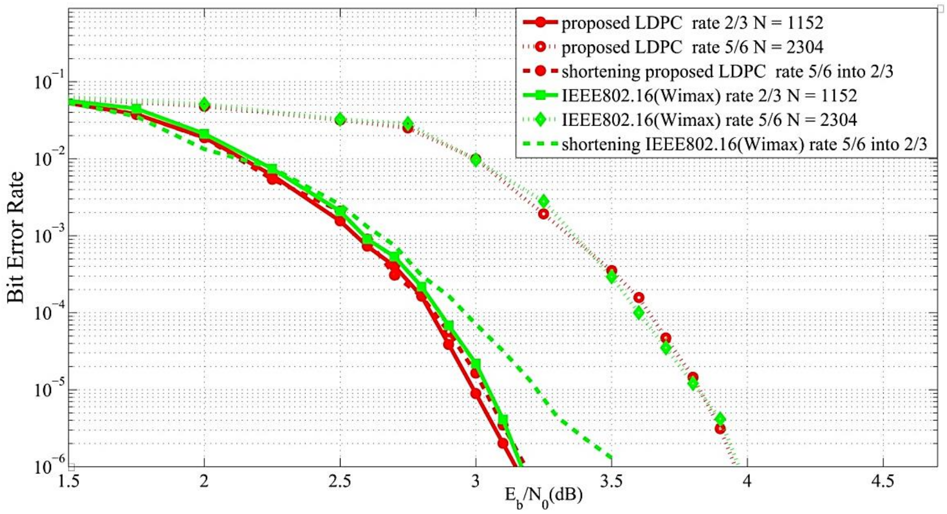

Figure 11 The BER performance of shortening proposed LDPC codes and shortening LDPC codes in IEEE802.16e (WiMAX) from a rate of $5/6, N = 2304$ into a rate of $2/3, N = 1152$.

and 10800, respectively. The performance of shortened LDPC codes was compared to the performance of dedicated LDPC codes in both short and long block lengths.

Finally, a comparison of proposed LDPC codes and LDPC codes based on the IEEE802.16e (WiMAX) standard [14] with shortening is presented. Figure 10 shows the shortening distribution of proposed LDPC and WiMAX LDPC codes with a shortening from rate of $5/6$ and a length of 2304 into a rate of $2/3$ and length of 1152 by shortening the last part of the information bits. Unquestionably, the shortening distribution of the proposed LDPC codes was uniform and the shortening distribution of WiMAX LDPC codes was non-uniform. The BER performance of this comparison is shown in Figure 11. The BER performance of proposed LDPC codes was better than for the WiMAX LDPC codes by approximately 0.4 dB at 10^{-6}.

5. Conclusions

This paper presents a design of rate-compatible LDPC codes based on a uniform shortening distribution that can achieve higher BER performance compared to that of dedicated LDPC codes with the same rate and length. These LDPC codes can provide a uniform shortening distribution by shortening them with a basic algorithm that is simple. The rate-compatible LDPC codes are appropriate for use in modern communication systems that require a flexibility of rate using only one encoder/decoder.

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