Loosening of Nut by Vibration

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Abstract

The objective of this paper is to study, theoretically, the effects of various parameters on the amount of nut loosening per dynamic loading cycle applied in the direction of bolt axis. The results show that the capability in loosening resistance of bolt-nut connections can be improved by decreasing the helix angle and/or increasing thread friction, base friction, bolt length, nut wall thickness, and nut height.

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การคลายตัวของแป้นเกลียวเนื่องจากการสั่นสะเทือน

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บทคัดย่อ

จุดมุ่งหมายของบทความนี้ก็เพื่อศึกษาถึงอิทธิพลของตัวพารามิเตอร์ต่างๆที่มีต่อปริมาณการคลายตัวของแป้นเกลียวต่อแรงพลวัตที่กระทำในทิศทางของแกนสลักเกลียว 1 ไซเกิ้ลในทางทฤษฎี จากผลของการศึกษาทำให้ทราบว่า เราสามารถปรับปรุงความต้านทานต่อการคลายตัวของแป้นเกลียวได้โดยการลดมุมลาดของเกลียว และ/หรือ การเพิ่มความเสียดทานที่ฟันเกลียวและที่ฐานของแป้นเกลียว การเพิ่มความยาวของตัวสลัก ความหนาของผนังแป้นเกลียว และความสูงของตัวแป้นเกลียว

1. Introduction

In the past, several investigators have attempted to study how bolt-nut connections loosen. Most of their experimental results showing the effects of some parameters on the amount of nut loosening are quite in an agreement. However, only Goodier and Sweeney proposed a theory of nut loosening by vibrations. In their article, they derived the equations of external torques required to loosen a nut in both an increasing bolt load process and a decreasing bolt load process. The nut, according to their

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theory, will loosen if these external torques become negative or at least zero.

The objective of this article is to develop a more complete theory of nut loosening by vibrations and to study the effects of various parameters on the amount of nut loosening.

The following assumptions are made:

1. Stresses in a bolt and nut are below the elastic limit,
2. pressure on nut bearing surface is linearly distributed with outside pressure equal to m times average pressure,
3. the clamped part are rigid,
4. a cornerless nut is used.

2. Development of the Theory

When a bolt-nut connection is loaded, the bolt diameter contracts while the nut expands radially. There are two sliding motions observed in the connection. One is the sliding of the nut threads on the bolt threads; the other is the sliding of the nut on the clamped parts. The sliding motion between the threads deviates from the radial direction because the tangential thread load distributed along the entire contacting length will produce a torque to twist the bolt shank by a small angle. The sliding motion of the nut on the clamped parts would be in radial direction if there were no rotary motion of the nut. However, a loosening torque produced by the tangential thread load on nut threads tries to rotate the nut in the loosening sense as the bolt-nut connection is loaded This causes a "loosening rotation" of the nut when all contacting points on the nut bearing surface start to slide. It should be noted here that all contacting points on the nut bearing surface do not start sliding at the same time when a preload in the connection starts to change. The contacting points lying on the innermost circle start first; then those lying on the outer circles start; and the ones lying on the outermost circle start last. The connection is said to be losing the capability of loosening resistance when all of these contacting points are sliding.

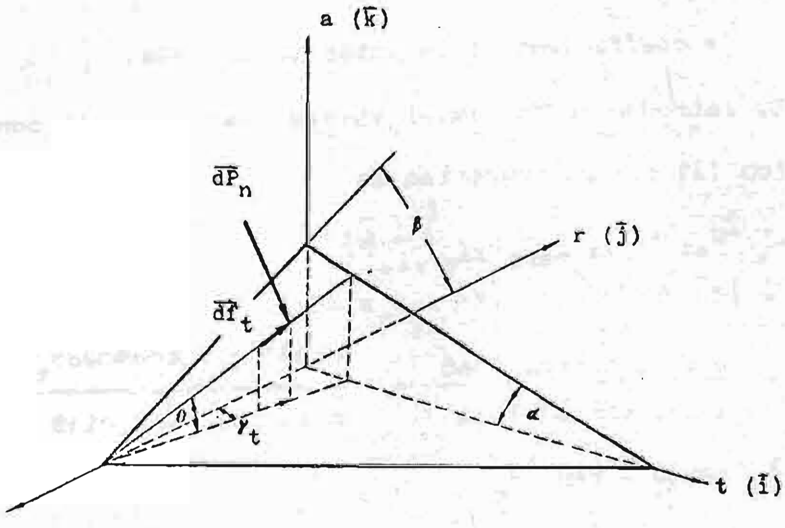

Fig.1 An inclined plane representing a nut thread contacting surface

Consideration of the load on an arbitrary contacting point on the nut thread surface as shown in Fig.1, the normal and friction forces can be expressed vectorially by

$$\vec{dP}_n = |\vec{dP}_n|(-\sin\alpha\cos\beta'\ \vec{i} + \sin\beta'\ \vec{j} - \cos\alpha\cos\beta'\ \vec{k}) \qquad (1)$$

and

$$\vec{df}_t = \mu_t|\vec{dP}_n|(\cos\theta\sin\gamma_t\ \vec{i} + \cos\theta\cos\gamma_t\ \vec{j} + \sin\theta\ \vec{k}). \qquad (2)$$

The resultant force of these two forces is

$$\begin{aligned}\vec{dR}_i = |\vec{dP}_n|[&(-\sin\alpha\cos\beta' + \mu_t\cos\theta\sin\gamma_t)\ \vec{i} \\ &+(\sin\beta' + \mu_t\cos\theta\cos\gamma_t)\ \vec{j} \\ &+(-\cos\alpha\cos\beta' + \mu_t\sin\theta)\ \vec{k}\ ]. \end{aligned} \qquad (3)$$

Here: $|\vec{dP}_n|$ = normal force magnitude

$\tan\beta' = \tan\beta\cos\alpha$

$\tan\Theta = \tan\beta\cos\gamma_t - \tan\alpha\sin\gamma_t$

γ_t = angle between the projection of frictional force on tr-plane and the r-axis

β = half thread angle

α = helix angle

μ_t = coefficient of friction in threads.

By introducing the axial thread load per unit contacting length, q_{ai}, equation (3) can be rewritten as

$$d\vec{R}_i = -q_{ai}\,ds(C_{ti}\,\vec{i} - C_{ri}\,\vec{j} + \vec{k}) \qquad (4)$$

where $q_{ai} = |d\vec{P}_n/ds|(\cos\alpha\cos\beta' - \mu_t\sin\Theta)$

$$C_{ri} = \frac{\text{radial thread load}}{\text{axial thread load}} = \frac{\sin\beta' + \mu_t\cos\Theta\cos\gamma_t}{\cos\alpha\cos\beta' - \mu_t\sin\Theta}$$

$$\approx \tan(\beta - \tan^{-1}\mu_t)$$

$$C_{ti} = \frac{\text{tangential thread load}}{\text{axial thread load}} = \frac{\sin\alpha\cos\beta' - \mu_t\cos\Theta\sin\gamma_t}{\cos\alpha\cos\beta' - \mu_t\sin\Theta}$$

$$\approx \frac{\tan\alpha + \mu_t\tan\gamma_t}{1 + \mu_t\tan\beta}$$

s = distance along the helix.

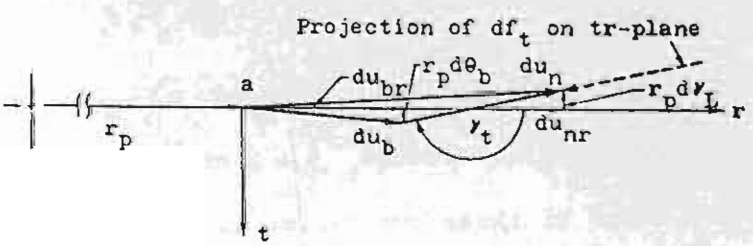

Fig.2 Displacement diagram and direction of the projection of thread friction on tr-plane

The direction of thread friction as shown in Fig.2 can be obtained from the relations

$$\tan\gamma_t = -\frac{r_p(d\theta_b - |d\gamma_L|)}{du_{nr} - du_{br}} \tag{5}$$

where θ_b = twisting angle of the bolt body

γ_L = loosening angle of the nut

u_{nr} = radial displacement of nut threads

u_{br} = radial displacement of bolt threads

The normal and frictional forces at an arbitrary point on nut bearing surface can be expressed as

$$\vec{dN} = pdA\,\vec{k} \tag{6}$$

and $$\vec{df} = \mu_b pdA(-\sin\gamma_r\,\vec{i} - \cos\gamma_r\,\vec{j}) \tag{7}$$

where p = pressure on nut bearing surface at radius r

dA = a differential area on nut bearing surface at radius r

μ_b = coefficient of friction between nut and clamped parts

γ_r = an angle between a differential base friction at radius r and the r-axis.

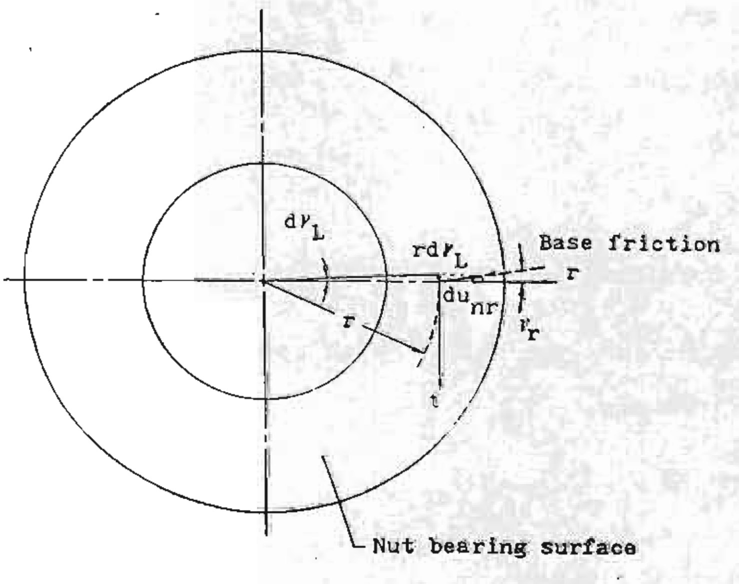

Fig.3 Displacement diagram and direction of base friction on nut bearing surface

The direction of a differential base friction as shown in Fig.3 can be determined from the relations

$$\tan\gamma_r = \frac{r|d\gamma_L|}{du_{nr}} \tag{8}$$

or, for a very small γ_r,

$$\sin\gamma_r = \frac{r|d\gamma_L|}{du_{nr}} \tag{9}$$

where r = radius

u_{nr} = radial displacement of a contacting point on nut bearing surface at radius r.

A loosening torque, T_{Li}, and a resisting torque, T_{Ri}, on the nut can be given by

$$T_{Li} = \int_0^{H/\sin\alpha} r_p C_{ti} q_{ai} ds \tag{10}$$

and $$T_{Ri} = \int_{D_b/2}^{D_n/2} \mu_b p dA \sin\gamma_r r. \tag{11}$$

A twisting torque, T_{Ti}, and an elastic reaction torque, T_{Ei}, on the bolt are

$$T_{Ti} = \int_0^{H/\sin\alpha} r_p C_{ti} q_{ai} ds \tag{12}$$

and $$T_{Ei} = k_b \theta_b \tag{13}$$

where k_b = torsional stiffness of bolt body.

By letting $q_{am}=F_b\sin\alpha/H$, $q_{ri}=q_{ai}/q_{am}$, and $\xi=s\sin\alpha/H$, equations (10) and (12) become

$$T_{Li} = T_{Ti} = F_b r_p \int_0^1 C_{ti} q_{ri} d\xi. \tag{14}$$

Substitution of C_{ti} gives

$$T_{Li} = T_{Ti} = \frac{F_b r_p \tan\alpha}{1+\mu_t\tan\beta} + \frac{\mu_t F_b r_p}{1+\mu_t\tan\beta} \int_0^1 q_{ri}\tan\gamma_t d\xi . \tag{15}$$

Substitution of $\tan\gamma_t$ yields

$$T_{Li} = T_{Ti} = [I_{1i} - I_{2i}(\frac{d\theta_b}{dF_b} + \left|\frac{d\gamma_L}{dF_b}\right|)]\, F_b \qquad (16)$$

where $I_{1i} = \dfrac{r_p \tan\alpha}{1+\mu_t \tan\beta}$

$$I_{2i} = \frac{\mu_t r_p}{1+\mu_t \tan\beta} \int_0^1 \frac{q_{ri}}{\frac{1}{r_p} \frac{d}{dF_b}(u_{nr} - u_{br})} d\xi$$

F_b = bolt load

and q_{ri} = the ratio of load distribution on threads derived by Harnchoowong[1].

By substituting $\sin\gamma_r$ into equation (11) and rearranging gives

$$T_{Ri} = I_{3i} F_b \left|\frac{d\gamma_L}{dF_b}\right| \qquad (17)$$

where $I_{3i} = \dfrac{2\pi\mu_b R^3}{F_b} \displaystyle\int_{1-t/2R}^{1+t/2R} \frac{p(r/R)^3}{\frac{1}{r} \frac{d}{dF_b}(u_{nr})} d(r/R)$

$R = \frac{1}{2}(D_n + D_b)$

$t = \frac{1}{2}(D_n - D_b)$

D_n = nut outside diameter

D_b = bolt diameter.

Since $T_{Li} = T_{Ri}$ and $T_{Ti} = T_{Ei}$, thus

$$I_{1i} - I_{2i}(\frac{d\theta_b}{dF_b} + \left|\frac{d\gamma_L}{dF_b}\right|) = I_{3i} \left|\frac{d\gamma_L}{dF_b}\right| \qquad (18)$$

and $$[I_{1i} - I_{2i}(\frac{d\theta_b}{dF_b} + \left|\frac{d\gamma_L}{dF_b}\right|)]\, F_b = k_b \theta_b. \qquad (19)$$

Differentiation of equation (19) with respect to F_b yields

$$I_{1i} - I_{2i}\left(\frac{d\theta_b}{dF_b} + \left|\frac{d\gamma_L}{dF_b}\right|\right) = k_b \frac{d\theta_b}{dF_b} \qquad (20)$$

By solving equations (18) and (20) simultaneously gives

$$\frac{d\theta_b}{dF_b} = \frac{I_{1i} I_{3i}}{k_b(I_{2i}+I_{3i})+I_{2i} I_{3i}} \qquad (21)$$

and $$\left|\frac{d\gamma_L}{dF_b}\right| = \frac{I_{1i}}{I_{2i}+I_{3i}+I_{2i} I_{3i}/k_b} . \qquad (22)$$

The solutions $d\theta_b/dF_b$ and $|d\gamma_L/dF_b|$ for a decreasing bolt load process can also be obtained by the same procedures except the directions of thread friction on nut threads and base friction on nut bearing surface are outward. Thus,

$$\frac{d\theta_b}{dF_b} = \frac{I_{1d} I_{3d}}{k_b(I_{2d}+I_{3d})+I_{2d} I_{3d}} \qquad (23)$$

and $$\left|\frac{d\gamma_L}{dF_b}\right| = \frac{I_{1d}}{I_{2d}+I_{3d}+I_{2d} I_{3d}/k_b} \qquad (24)$$

where $$I_{1d} = \frac{r_p \tan\alpha}{1-\mu_t \tan\beta}$$

$$I_{2d} = \frac{\mu_t r_p}{1-\mu_t \tan\beta} \int_0^1 \frac{q_{rd}}{\frac{1}{r_p}\frac{d}{dF_b}(u_{nr}-u_{br})} d\xi$$

and $$I_{3d} = \frac{2\pi\mu_b R}{F_b} \int_{1-t/2R}^{1+t/2R} \frac{p(r/R)^3}{\frac{1}{r}\frac{d}{dF_b}(u_{nr})} d(r/R) .$$

As mentioned earlier, the connection losses the capability of loosening resistance as the contacting points lying on the outermost circle on the nut bearing surface are sliding. The relations between bolt-load and the radial displacements of those contacting points can be given by:

a) for an increasing bolt load process,

$$F_b = k_i u_{ro} \tag{25}$$

b) for a decreasing bolt load process,

$$F_b = k_d u_{ro} \tag{26}$$

Here: k_i = stiffness of the outermost contacting points on nut bearing surface in an increasing bolt load process

k_d = stiffness of the outermost contacting points on nut bearing surface in a decreasing bolt load process

and u_{ro} = radial displacements of the outermost contacting points on nut bearing surface.

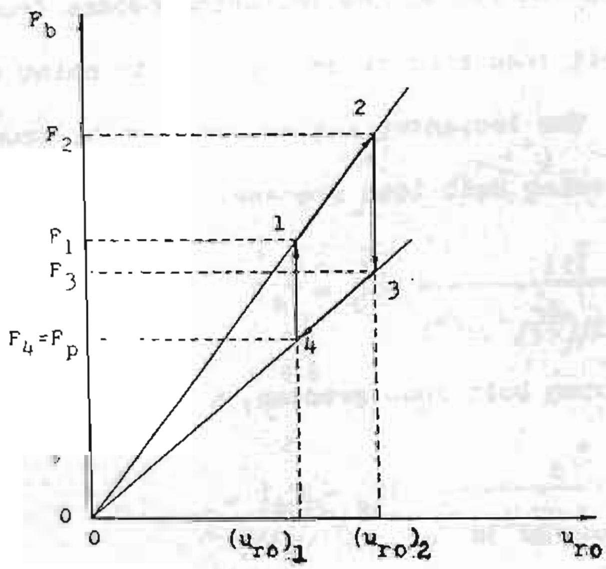

Fig.4 The relationship between bolt load and the radial displacement of the contacting points lying on the outermost circle on the nut bearing surface during a loading cycle

Fig.4 shows the relationship between bolt load and the radial displacements of the outermost contacting points on nut bearing surface during a complete cycle of dynamic loading. Bolt loads at points 1, 2, 3, and 4 can be given by

$$F_1 = \frac{k_i}{k_d} F_p$$

$$F_2 = (1 + DSR)\, F_p \qquad (27)$$

$$F_3 = \frac{k_d}{k_i} (1 + DSR)\, F_p$$

and $F_4 = F_p$

where $DSR = F_d/F_p$

F_p = bolt preload

and F_d = dynamic load.

By considering Fig.4, the loosening occurs from point 1 to point 2 in an increasing bolt load process and point 3 to point 4 in a decreasing bolt load process. The loosening angles can then be found:

a) in an increasing bolt load process,

$$\gamma_L = \frac{I_{1i}}{I_{2i}+I_{3i}+I_{2i}I_{3i}/k_b} (F_2 - F_1) \qquad (28)$$

b) in a decreasing bolt load process,

$$\gamma_L = \frac{I_{1d}}{I_{2d}+I_{3d}+I_{2d}I_{3d}/k_b} (F_3 - F_4)\ . \qquad (29)$$

The total loosening angle per cycle is

$$\gamma_{L,total} = F_p [(1+DSR- \frac{k_i}{k_d}) \frac{I_{1i}}{I_{2i}+I_{3i}+I_{2i}I_{3i}/k_b}$$

$$+ (\frac{k_d}{k_i}(1+DSR) - 1) \frac{I_{1d}}{I_{2d}+I_{3d}+I_{2d}I_{3d}/k_b}]\ . \qquad (30)$$

3. Numerical Results

A standard one inch nominal size bolt and a cornerless nut with pitch diameter of 0.9188 inch are selected for the numerical test. The material is steel with $E=30\times10^6$ psi and $\nu=0.27$.

The parameters to be studied are coefficients of thread and base frictions, thread pitch, nut wall thickness, nut height, bolt length, dynamic-static load ratio (DSR), and bolt preload.

4. Conclusions

From the numerical test results, the following conclusions can be drawn.

1. The amount of loosening decreases as the thread friction increases.
2. The amount of loosening decreases as the base friction increases.
3. Fine threads yield less amount of loosening than the coarse threads do.
4. The amount of loosening decreases as the nut wall thickness increases.
5. The amount of loosening decreases as the nut height increases.
6. Increasing bolt length will reduce the amount of loosening.
7. The amount of loosening is linearly dependent of the dynamic-static load ratio (DSR).
8. The amount of loosening is linearly dependent of the bolt preload while fixing the DSR.

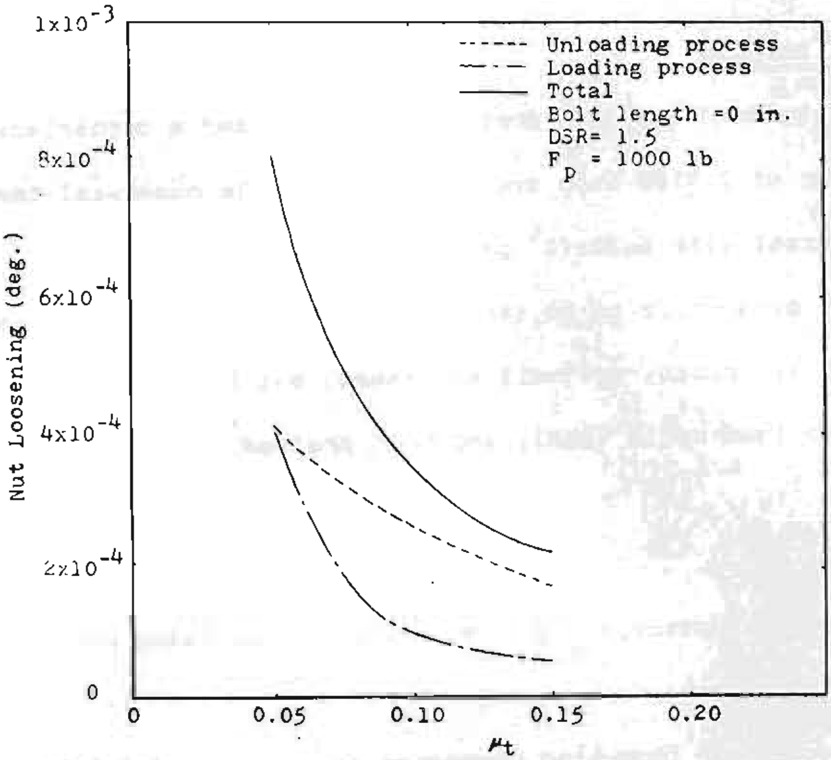

Fig.5 Effect of thread friction on nut loosening

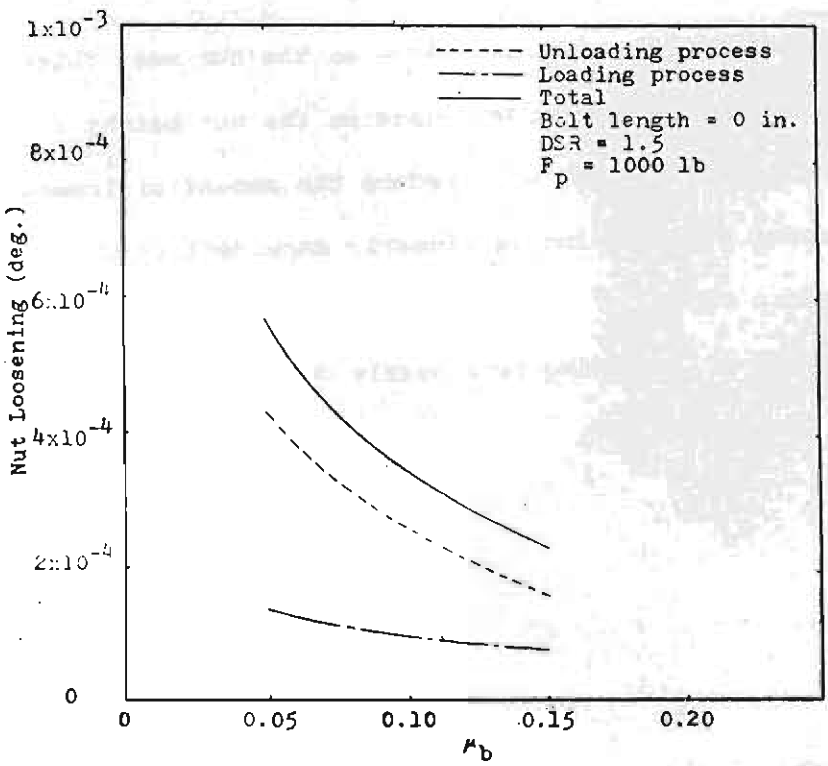

Fig.6 Effect of base friction on nut loosening

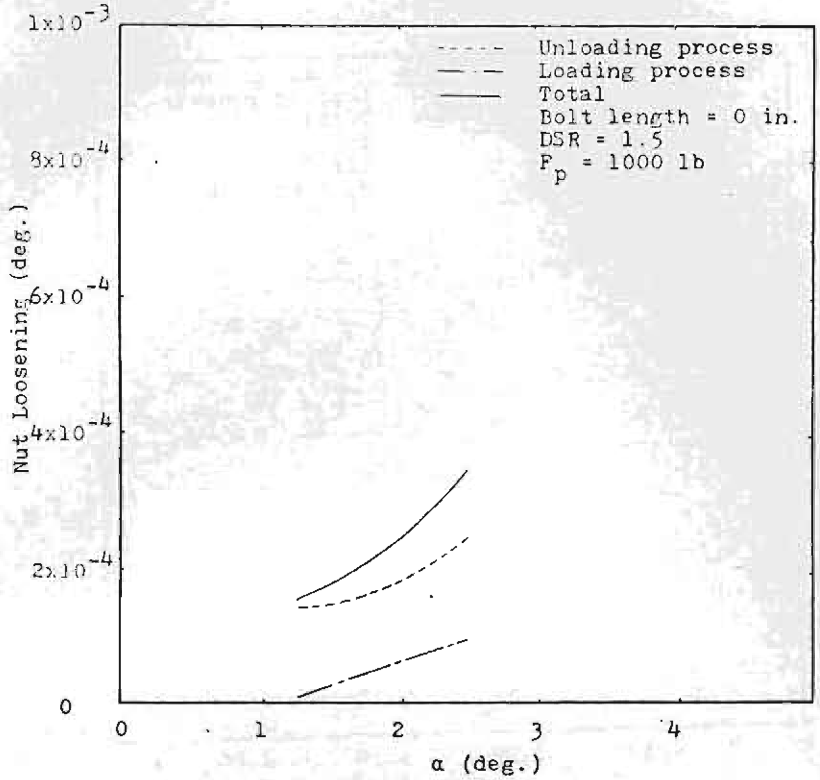

Fig.7 Effect of helix angle on nut loosening

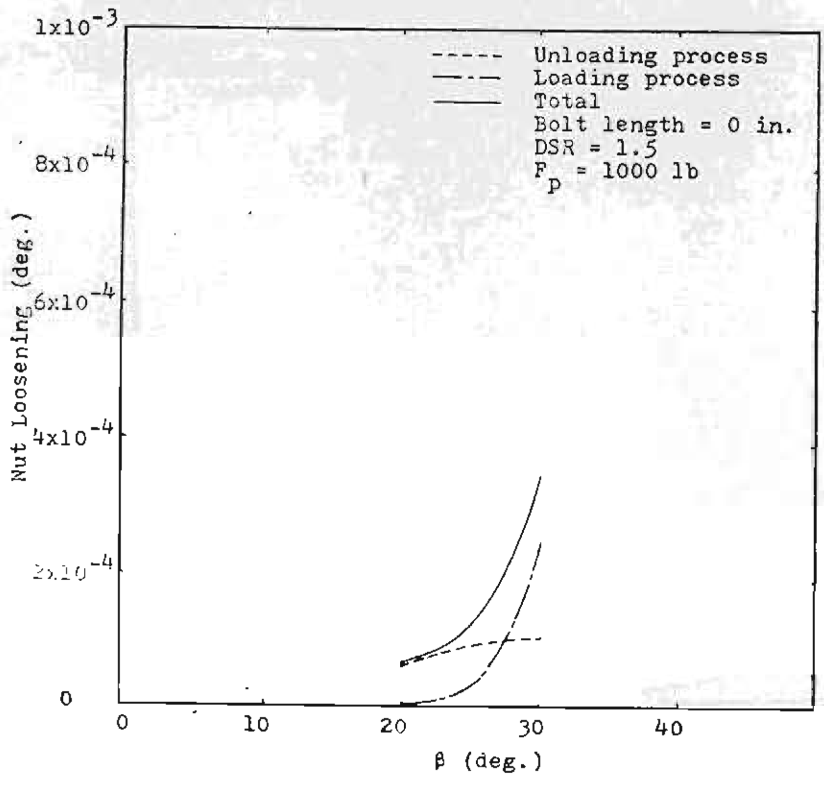

Fig.8 Effect of thread angle on nut loosening

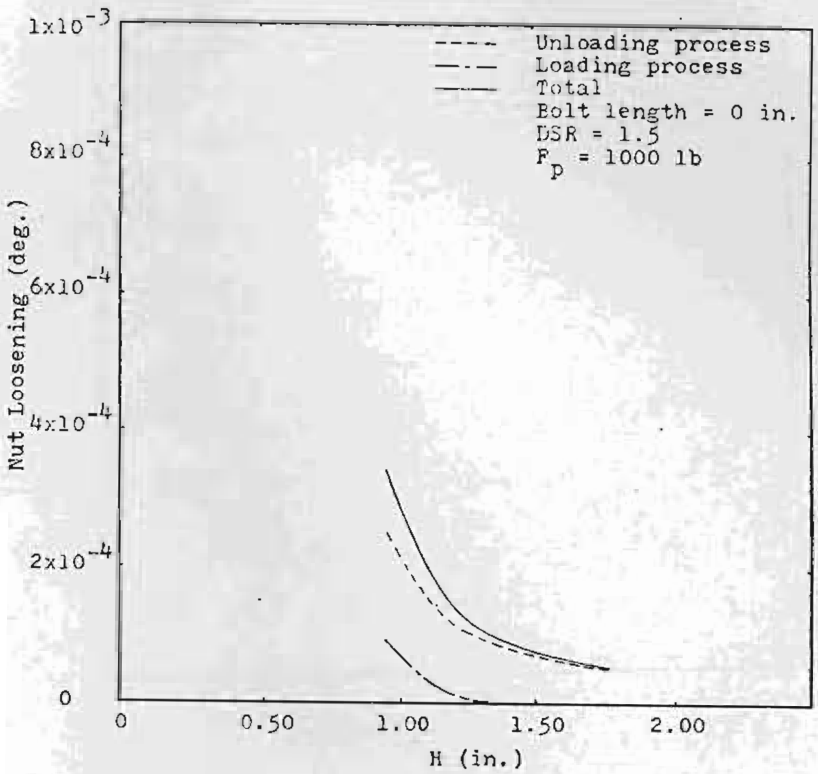

Fig.9 Effect of nut height on nut loosening

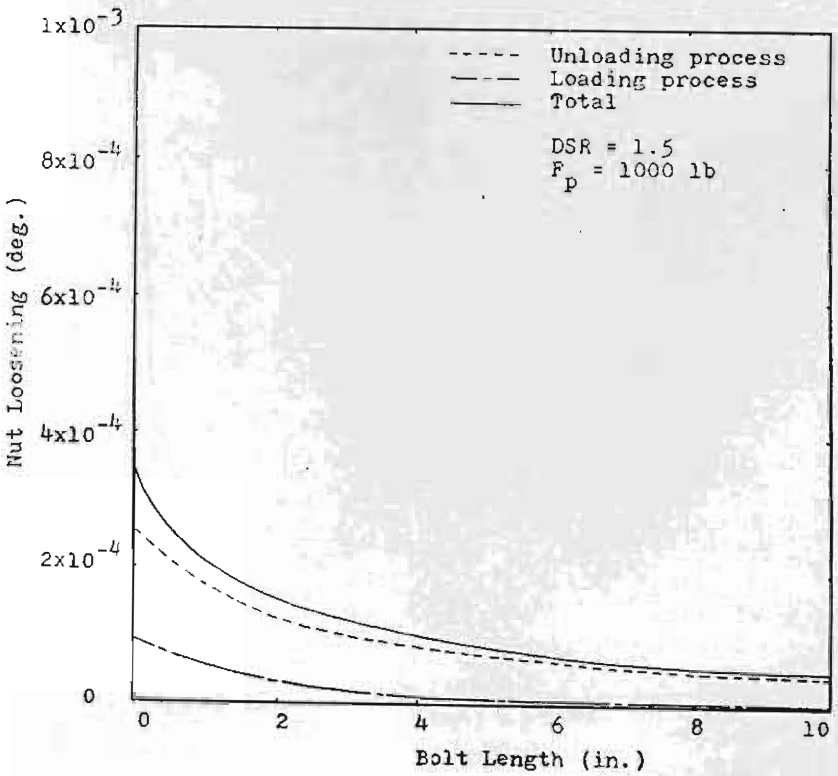

Fig.10 Effect of bolt length on nut loosening

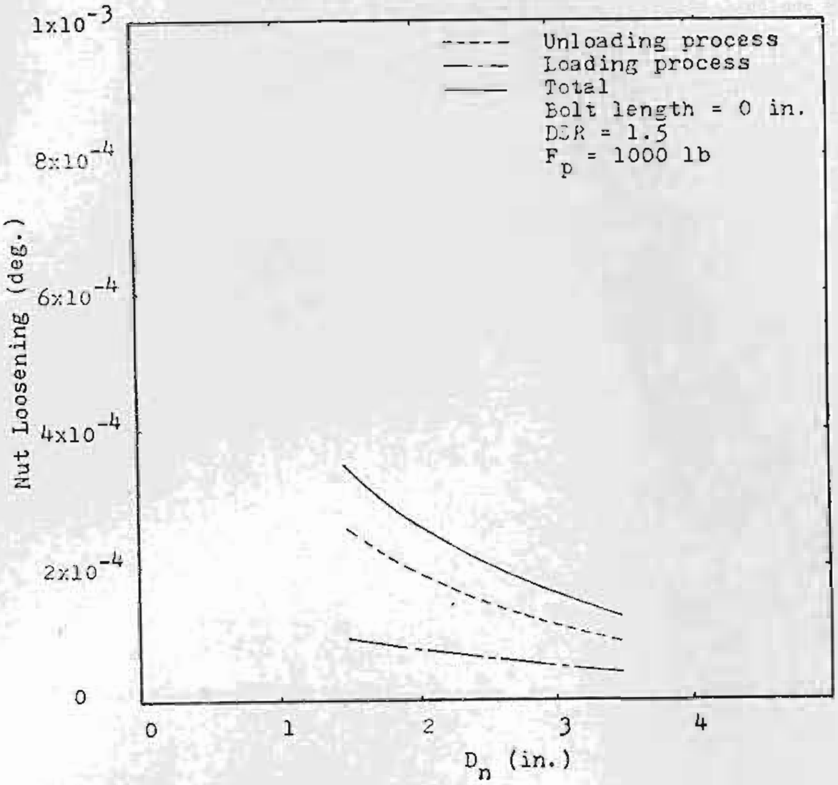

Fig.11 Effect of nut wall thickness on nut loosening

5. References

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