Load and Deflection Analysis of Finned Cooling Panels

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Abstract

The objective of this paper is to study and analyse the load and deflection of finned cooling panels. By comparing theoretical results to experimental results, we found that the error is depend on fin length. The reason is that the major part of strain energy is stored in the beam body while the fins store very little. However, strain energy is, theoretically, uniformly stored throughout the finned cooling panels.

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การวิเคราะห์ภาระและการโก่งงอของแผ่นระบายความร้อน

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บทคัดย่อ

จุดมุ่งหมายของบทความนี้ก็เพื่อศึกษาและวิเคราะห์ถึงภาระและการโก่งงอของแผ่น
ระบายความร้อน จากการเปรียบเทียบผลลัพธ์ในทางทฤษฎีกับผลจากการทดลองพบว่า ค่าผิด
พลาดขึ้นอยู่กับความยาวของครีบระบายความร้อน ทั้งนี้เนื่องจากพลังงานความเครียดส่วนใหญ่จะ
สะสมอยู่ในตัวคาน บริเวณที่เป็นครีบจะมีสะสมอยู่น้อยมาก แต่ในทางทฤษฎีพลังงานความเครียด
จะสะสมอยู่อย่างสม่ำเสมอตลอดทั้งแผ่นระบายความร้อน

1. Introduction

The objective of this work is to develop the expression
showing the relationship between load and deflection for finned
cooling panels by theoretical method and to compare theoretical
results to the experimental results.

The following development treats the finned cooling panel
as a beam having ratio of the thickness to the width greater than
one-fourth. Any expression developed later is therefore based on
beam theory.

2. Theoretical Analysis

Our work is concerned with a centrally-loaded, simply-

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supported, non-uniform beam as shown in Fig.1. The first step of our analysis is to calculate total strain energy stored in the beam. We may neglect strain energy due to transverse shear since it is very small when comparing to strain energy due to bending.
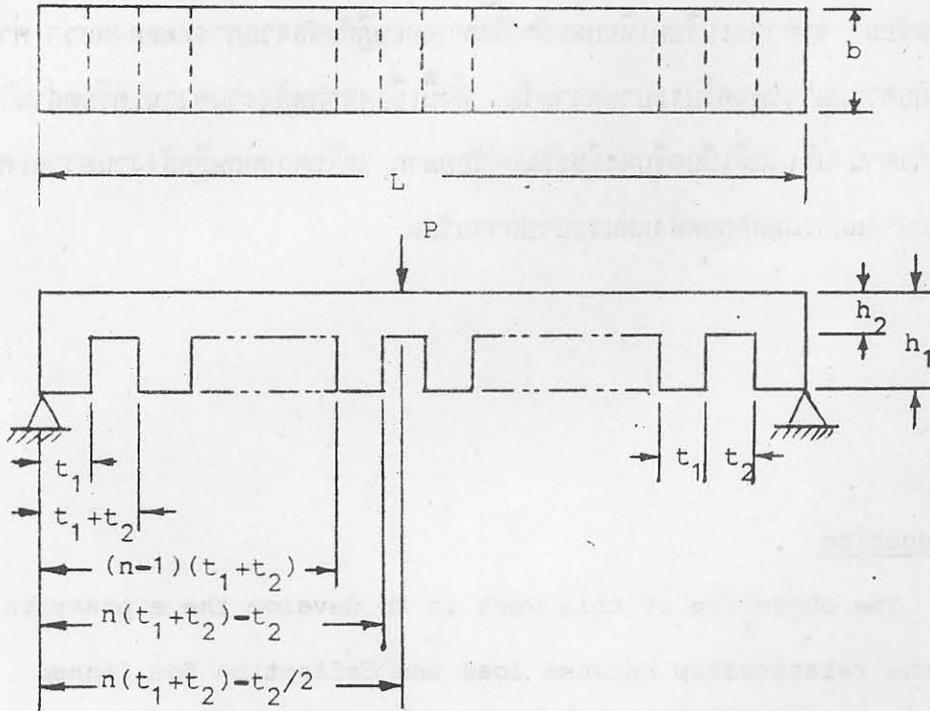

Fig.1 Simply-supported beam subjected to point load at mid-span

The total strain energy can be expressed as

$$U_T = 2[\sum_{i=1}^{n} \frac{P^2}{8E} \int_{i-1}^{i(t_1+t_2)-t_2} \frac{x^2 dx}{I_1} + \sum_{i=1}^{n-1} \frac{P^2}{8E} \int_{i(t_1+t_2)-t_2}^{i(t_1+t_2)} \frac{x^2 dx}{I_2}$$

$$+ \frac{P^2}{8E} \int_{n(t_1+t_2)-t_2}^{n(t_1+t_2)-t_2/2} \frac{x^2 dx}{I_2} \tag{1}$$

where $I_1 = bh_1^3/12$ and $I_2 = bh_2^3/12$.

Since n is much greater than 1, the upper limit of the last integral in Eq.(1) is approximate to $n(t_1+t_2)-t_2$. Eq.(1) can then be

approximate to

$$U_T = \frac{P^2}{4E} \sum_{i=1}^{n} [\int_{(i-1)(t_1+t_2)}^{i(t_1+t_2)-t_2} \frac{x^2}{I_1}dx + \int_{i(t_1+t_2)-t_2}^{i(t_1+t_2)} \frac{x^2}{I_2}dx] \qquad (2)$$
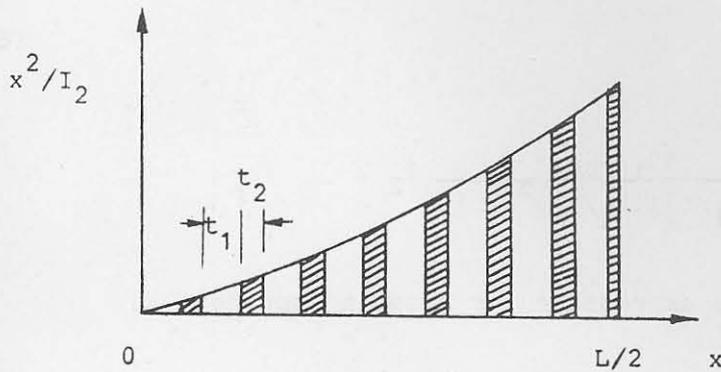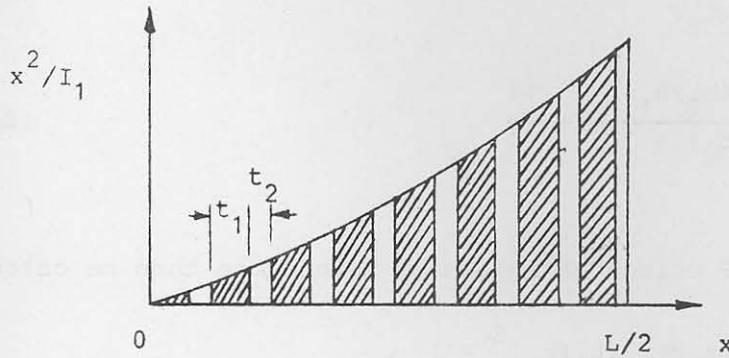

Fig.2 Graphical evaluation of the integrals in Eq.(2)

It can be seen from Fig.2 that the first integral is equal to the shaded area (A_{s1}) and the second one is equal to the shaded area (A_{s2}). The values of these shaded areas are approximate to:

$$A_{s1} = \frac{t_1}{t_1+t_2} \int_0^{L/2} \frac{x^2}{I_1}dx = \frac{t_1}{t_1+t_2} \frac{L^3}{24I_1} \qquad (3)$$

and

$$A_{s2} = \frac{t_2}{t_1+t_2} \int_0^{L/2} \frac{x^2}{I_2}dx = \frac{t_2}{t_1+t_2} \frac{L^3}{24I_2} . \qquad (4)$$

By substituting the values of A_{s1} and A_{s2} into Eq.(2) gives

$$U_T = \frac{P^2}{4E} \left[\frac{t_1}{t_1+t_2} \frac{L^3}{24I_1} + \frac{t_2}{t_1+t_2} \frac{L^3}{24I_2} \right]$$

$$= \frac{P^2 L^3}{96EI_2} \frac{[t_1(I_2/I_1) + t_2]}{(t_1+t_2)} \tag{5}$$

or $\quad U_T = \frac{P^2 L^3}{96EI_2} \frac{[(t_1/t_2)(h_2/h_1)^3 + 1]}{[(t_1/t_2) + 1]}$ $\tag{6}$

Deflection under load P using Castigliano's method can then be calculated:

$$\delta = \frac{\partial U_T}{\partial P} = \frac{PL^3}{48EI_2} \frac{[(t_1/t_2)(h_2/h_1)^3 + 1]}{[(t_1/t_2) + 1]} \tag{7}$$

or $\quad \delta = \frac{PL^3}{48EI_{eq}}$ $\tag{8}$

where $\quad I_{eq} = \frac{(t_1/t_2) + 1}{(t_1/t_2)(h_2/h_1)^3 + 1} I_2 \,.$

We can also develop the expression for a spring constant k,

$$k = \frac{P}{\delta} = \frac{48EI_{eq}}{L^3} \tag{9}$$

3. Experimental Analysis

We have 9 specimens dimensioning in Table 1 for beam deflection testing. The instrumentation are set as shown in Fig.3. Experimental data comparing with theoretical one are shown in Fig.4 and 5.
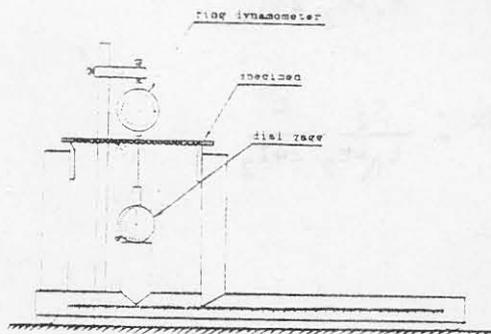

Fig.3 Beam deflection testing instrumentation.

Table 1 Dimension and property of specimens

specimen no.	t_1(in)	t_2(in)	h_1(in)	h_2(in)	b(in)	L(in)	E(psi)
1	1/16	1/8	1/4	7/32	1/2	8.125	10×10^6
2	1/16	1/8	1/4	7/32	1/2	8.125	10×10^6
3	1/16	1/8	1/4	7/32	1/2	8.125	10×10^6
4	1/8	1/8	1/4	7/32	1/2	8.125	10×10^6
5	1/8	1/8	1/4	7/32	1/2	8.125	10×10^6
6	1/8	1/8	1/4	7/32	1/2	8.125	10×10^6
7	3/16	1/8	1/4	7/32	1/2	8.125	10×10^6
8	3/16	1/8	1/4	7/32	1/2	8.125	10×10^6
9	3/16	1/8	1/4	7/32	1/2	8.125	10×10^6
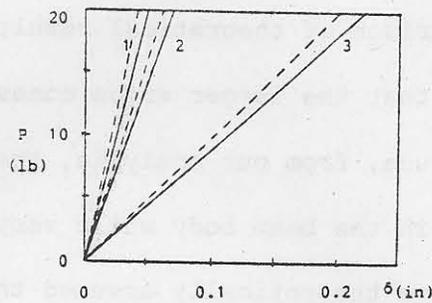
(a) specimen no. 1, 2, and 3.
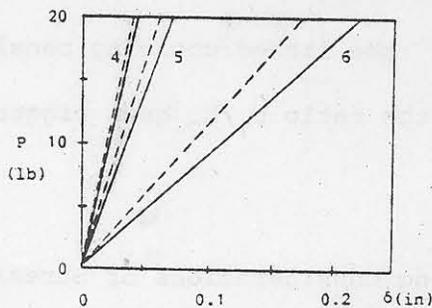
(b) specimen no. 4, 5, and 6.
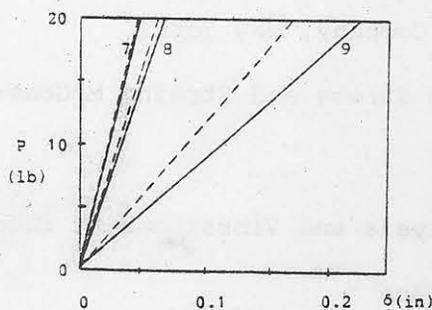
(c) specimen no. 7, 8, and 9.

- - - Theoretical results
—— Experimental results

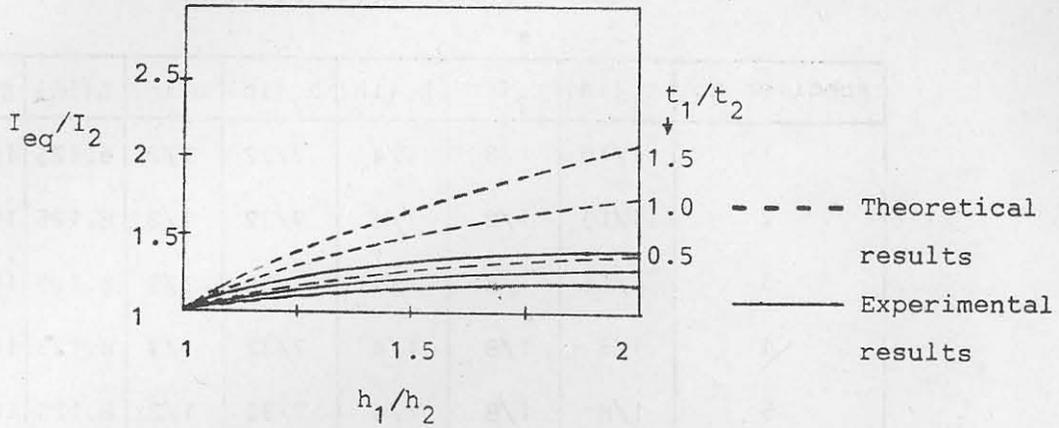Fig.4 Relationship between load and deflection.

Fig.5 Curves I_{eq}/I_2 plotted as a function of t_1/t_2 and h_1/h_2.

4. Conclusion and Discussion

In this paper, the development of equivalent moment of inertia of area equation is the main purpose. Bending stress can be determined by using bending stress equation ($\sigma=Mc/I$) without any modification.

Fig.4 and 5 show the comparison of theoretical results with experimental results. It is obvious that the larger erroe comes from the greater ratio of h_1/h_2. We can conclude, from our analysis, that there is a great deal of strain energy stored in the beam body while very small amount stored in the fins. However, we theoretically assumed that strain energy is uniformly stored throughout the finned cooloing panel. This certainly causes the larger error as the ratio h_1/h_2 gets bigger.

5. References

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