# Numerical Model of Flooding in the Eastern Areas of Bangkok

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## Abstract

We present basic mathematical formulations and new numerical methods to create a computer software for simulations of flooding in the Eastern areas of Bangkok. The first version of the "Flood Simulator-KMITL" makes it possible to calibrate the model by measurements and estimate an efficiency of flood protection methods in order to select the number of optimal schemes.

The global aim of the project is to construct a system of mathematical models, integrated with the GIS Arc/Info on a Convex Super Computer, to investigate hydrological cycles in flooded areas.

The real-world flood simulations demonstrate an efficiency of the proposed techniques.

# Introduction

Flooding in the Eastern areas of Bangkok is a critical problem for the agricultural, industrial and cultural development of these regions. The management and control, which involves the reduction of frequency and magnitude of floodings in specified regions, is a very difficult multipurpose problem that should be analyzed by research groups of experts in Hydrology, Agriculture, Water Systems Engineering, Computer Modelling, Numerical Analysis, Data Bases and System Analysis. In order to construct a computer software to simulate flooding in the Eastern areas of Bangkok we organized, in 1996 a research group of experts from the Royal Irrigation Department (Bangkok) and the Faculty of Engineering, KMITL.

We present the basic mathematical formulations and numerical methods to create a first version of the "Flood Simulator-KMITL" that makes it possible to estimate an efficiency of flood protection methods.

The global aim of the project is to construct a system of mathematical models, integrated with the GIS Arc/Info on a Convex Super Computer, to investigate hydrological cycles in flooded areas.

The real-world simulations of 1990 flood demonstrate an efficiency of the proposed techniques.

# Mathematical Formulations

The basic mathematical formulation derived in the frames of shallow water theory[1] is based on the Saint-Venant equations for the river (channel) flows

$$\frac{\partial w}{\partial t} + \frac{\partial Q}{\partial s} = E \qquad (1)$$

$$\frac{\partial \zeta}{\partial s} + \frac{n^2 Q|Q|}{w^2 h^{4/3}} + \frac{1}{gw}\left[\frac{\partial Q}{\partial t} + \frac{\partial}{\partial s}\left(\frac{Q^2}{w}\right)\right] = 0 \qquad (2)$$

where $t$ is the time coordinate, s the space coordinate along a river, $z \equiv z(s,t) = h + z$ the water surface level, $h \equiv h(s,t)$ the water depth, z(s) the bottom level, $w \equiv w(s,h)$ the cross-sectional area, $Q \equiv Q(s,t)$ the discharge, $n \equiv n(s)$ the Manning coefficient, $E \equiv E(s,t)$ the lateral discharge.

The surface flow is represented by the diffusion waves equation[2,3,4]

$$\frac{\partial \eta}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial \eta}{\partial x}\right) + \frac{\partial}{\partial y}\left(D\frac{\partial \eta}{\partial y}\right) + \varepsilon + R - V. \qquad (3)$$

where $\eta(x,y,t) = H(x,y,t) + Z(x,y)$ the surface flow level, $H \equiv H(x,y,t)$ the flow depth, $D \equiv D(x,y,h) = Ch^{3/2}/|grad(h)|^{1/2}$, $C \equiv C(x,y)$ the Chezy coefficient, $Z \equiv Z(x,y)$ the ground elevation level, $\varepsilon \equiv \varepsilon(x,y,t)$ the source(sink), $R \equiv R(x,y,t)$ the rainfall intensity, V- evaporation.

In order to simulate the river-surface hydraulic exchange and construct a system of simultaneous PDE's we introduce the following procedures illustrated in the Fig.1

1. " **Cut-Source procedure"** . We introduce the cuts corresponding to the river paths and impose the $\eta$-boundary conditions related to the source-function E.
2. " **Boundary-Source procedure"** . The cuts are replaced by the inner boundaries.
3. " **Source-Source procedure"** . The $\eta$-boundary conditions are substituted by the source type function defined at the transit zone of the hydraulic exchange.

Observe that the above formulations could be used to model other geophysical fluid dynamics problems, such as river-groundwater flows[5] or even pollution problems, provided that the gradient type of the hydraulic exchange is known and the momentum exchange can be neglected.

Finally, note that the key problem of the model application lies in the field of program running time and computer memory requirements. Consequently, the proposed basic mathematical formulation is oriented **for** the high performance computations strategy.

The next mathematical formulation is based on the continuity equation derived in terms of the **combined** surface and river flows [6]

$$\frac{\partial A}{\partial t} + \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = R - V \qquad (4)$$

where $A \equiv A(x,y,\eta)$ is the "apparent" cross-sectional area of the combined surface-river flow[7] defined by

$$A = \begin{cases} w_x + w_y, & \text{if} \quad \eta < Z, \\ H + w_x + w_y, & \text{otherwise} \end{cases},$$

where $w_x = w_x(x,y,h)$, $w_y = w_y(x,y,h)$ is the cross-sectional areas of the river flow at the x and y directions respectively, $h = \eta - z$, $H = \eta - Z$.

The diffusion form of the momentum equation for the surface-river flows is expressed in terms of the Chezy-Manning equations, where the inertia forces are neglected:

$$Q_{L,R} = \frac{-w_L h^{2/3}}{n\left|\frac{\partial \eta}{\partial L}\right|^{1/2}} \frac{\partial \eta}{\partial L} \qquad (5)$$

$$Q_{L,S} = \frac{-CH^{3/2}}{|\text{grad}\,\eta|^{1/2}} \frac{\partial \eta}{\partial L} \approx \nu H \frac{\partial \eta}{\partial L}$$

where L = x or L = y and $\nu$ corresponds to the linear friction.

The total discharge $Q_L$ is defined by

$$Q_L = Q_{L,R} + Q_{L,S}. \tag{6}$$

Substitution (6) to (4) and invoking the storage coefficient $\mu = \partial A/\partial \eta$ yields the Boussinesq-type equation (with the obvious porous-medium interpretation).

$$\mu \frac{\partial \eta}{\partial t} = \frac{\partial}{\partial x}\left( D_x \frac{\partial \eta}{\partial x} \right) + \frac{\partial}{\partial y}\left( D_y \frac{\partial \eta}{\partial y} \right) = R - V. \tag{7}$$

## Numerical Techniques

We apply the generalization of the two-dimensional non-negative numerical techniques proposed for the 1D/2D open flows[8,9]. The resulting scheme yields a new start-to-finish fixed boundaries algorithm based on the numerical treatment of the hyperbolic properties of the non-linear heat equations (observed first by G.I.Barenblatt and M.I. Vishik, 1956). Almost straightforward application of the techniques[9] to the equation (7) yields the numerical scheme that makes it possible to simulate small values of h as well as small H values. Moreover, the appropriate well-known numerical techniques for the diffusion waves equations could easily be adapted to the case of large h. Observe also that any kind of wet-dry-cell policies is totally superflues. Conequently he proposed procedure provides a uniform numerical treatment for the coupled surface/ river flows and satisfies the following required properties: 1. Approximation. 2. Stability. 3. H-h-non-negativity. 4. Divergence. 5. $\eta$=constant is the exact solution of the resulting non-linear finite-difference equations.

Finally the above mathematical model of the combined surface-river flows enables us to introduce the following PC-oriented

## Computer implementation

1. The combined flow level $\eta$ is interpreted as the surface flow if $\eta > Z$ and as the river flow level otherwise.
2. The equation (7) is approximated on the rectangular grid by the stable non-negative scheme.
3. The river paths are replaced by the polygon lines parallel to x or y axes.
4. The cells are marked as channelized if $w_x \neq 0$ or $w_y \neq 0$ and non-channelized otherwise.
5. The water regulators are simulated by the corresponding well-known hydraulics formulas[14], $Q_{reg} = Q_{reg}(\eta, \partial\eta/\partial L)$, where $Q_{reg}$ is the discharge of the regulator.
6. Dikes are represented by the ground level elevations. The procedures are illustrated in the Fig.2.

## Application to the Eastern areas of Bangkok

We apply the proposed technique to calibrate the model by the measurements and to simulate 1990 flooding in the Eastern areas of Bangkok. The model was calibrated by the daily measurements of the flood level at the 16 sites located at the prescribed area.The rainfall data and parameters of the major canals in the area were collected and analyzed by the officers of the Royal Irrigation Department (Bangkok).

The typical results of the model calibration and the comparison with the measurements are depicted in the Fig. 4-5. The computer plotted graphs of the flood development in the Fig.6 demonstrate the capability of the proposed technique to simulate the impact of the rainfall and the pumping stations located at the sea-side and along Chapaya-river on the flood level at the critical areas.

## References


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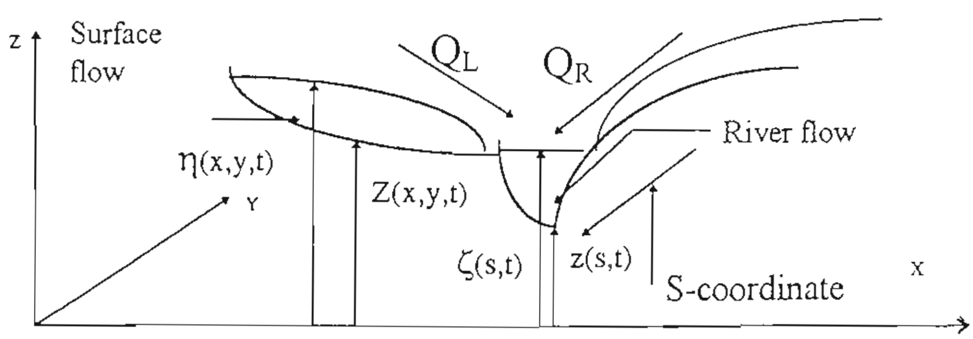



**Fig. 1.1 Surface-river flow hydraulic exchange. Basic model.**

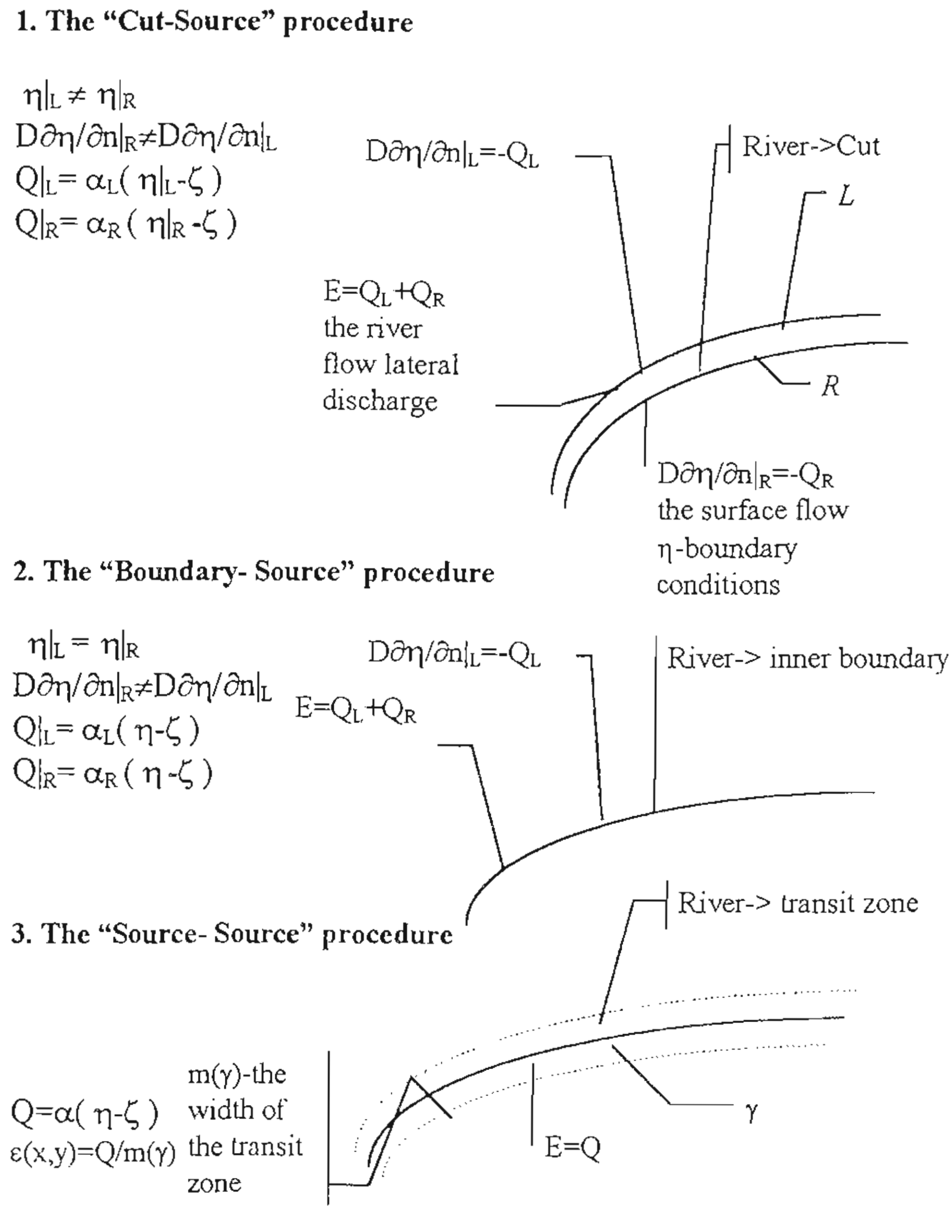



Fig. 1.2 The coupling procedures.

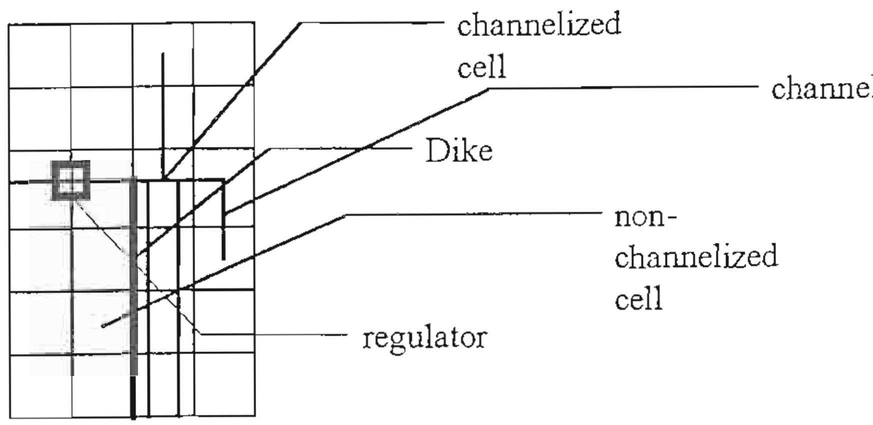



**Fig. 2** Finite-difference grid for the combined surface-river flows.

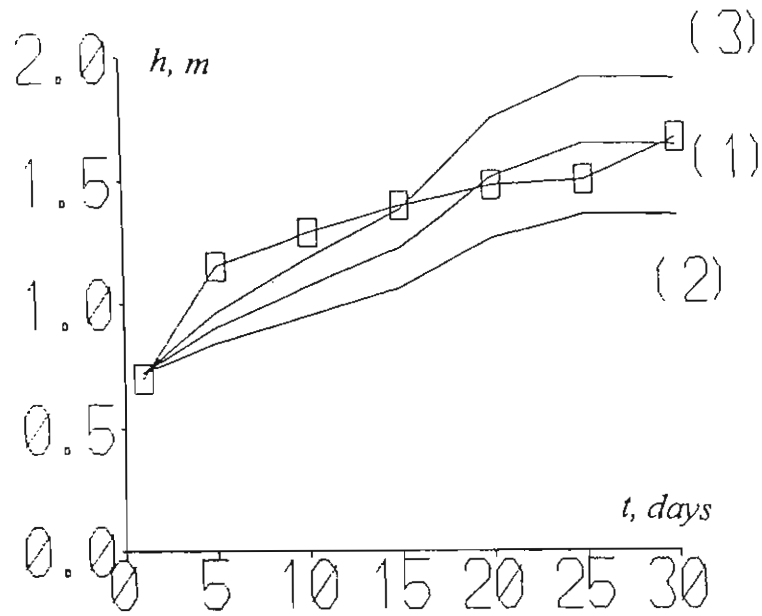



**Fig. 3** The measured ( □ ) and the simulated (---) flood level,

September, 1990, (1) $\nu$ = 0.0003, (2) $\nu$ = 0.0008, (3) $\nu$ = 0.0001

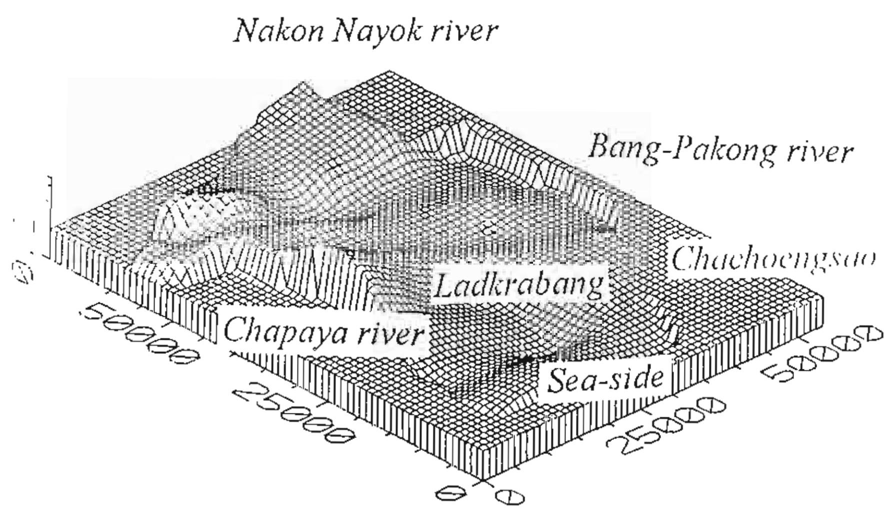
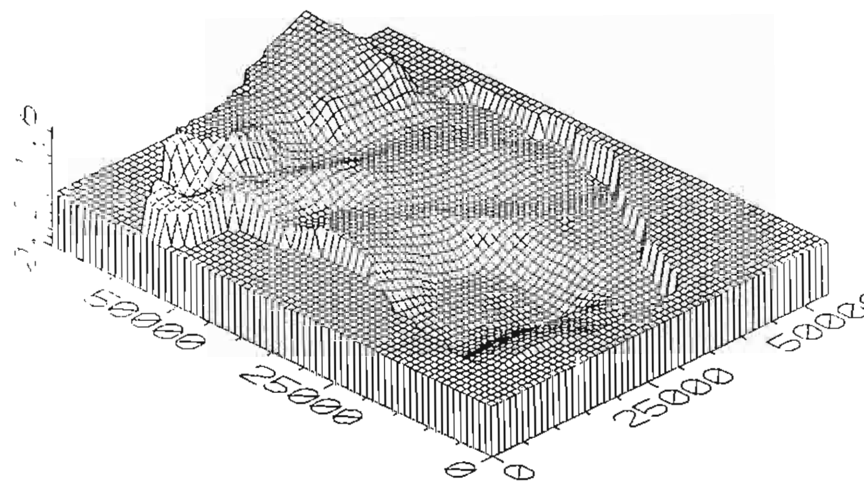



**Fig. 4.1 Flood level, South Rangsit, September, 4, 1990.**




Flood level, South Rangsit, September, 9, 1990 (measured)

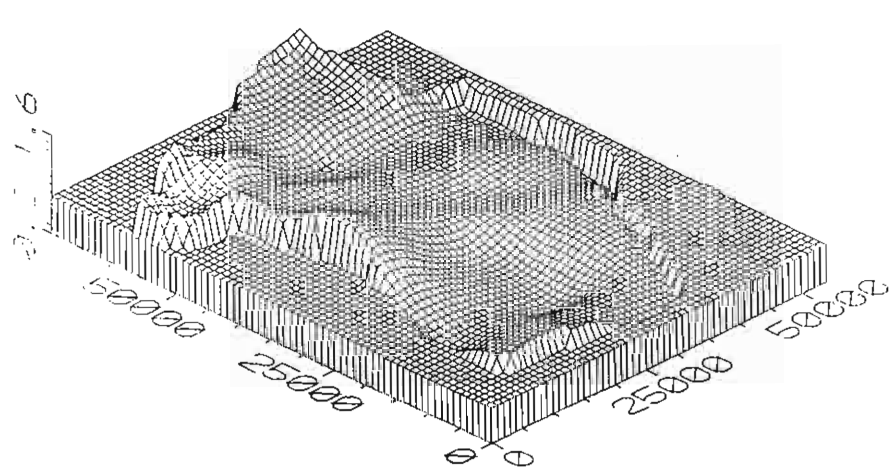
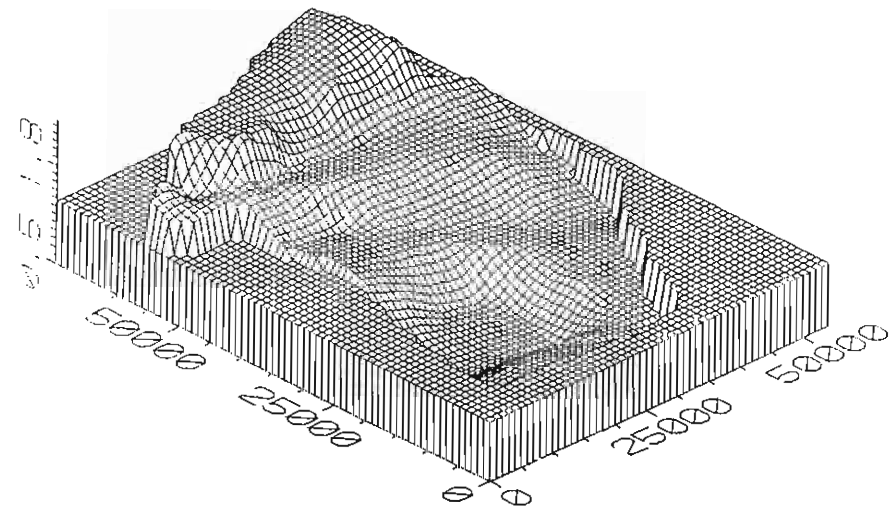



Fig. 4.2 Flood level, South Rangsit, September, 9, 1990 (simulated)




Flood level, South Rangsit, October, 4, 1990 (measured)

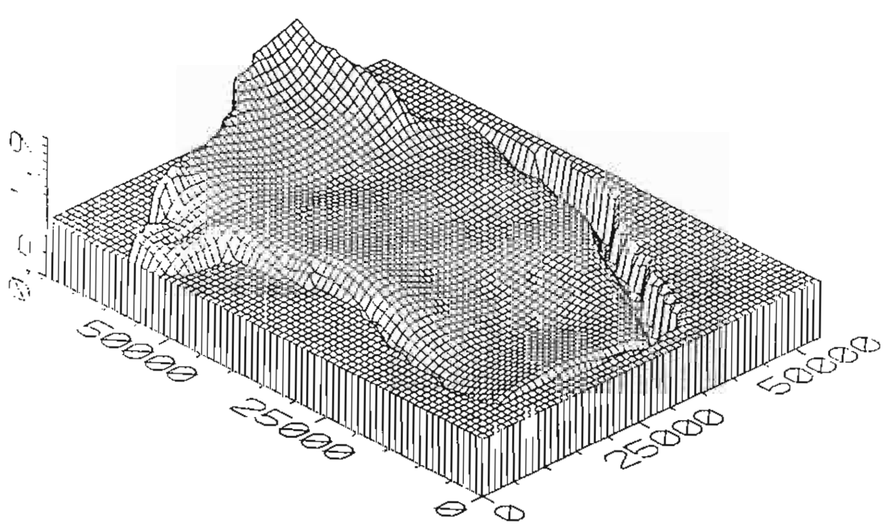



Fig. 4.3 Flood level, South Rangsit, October, 4, 1990 (simulated)

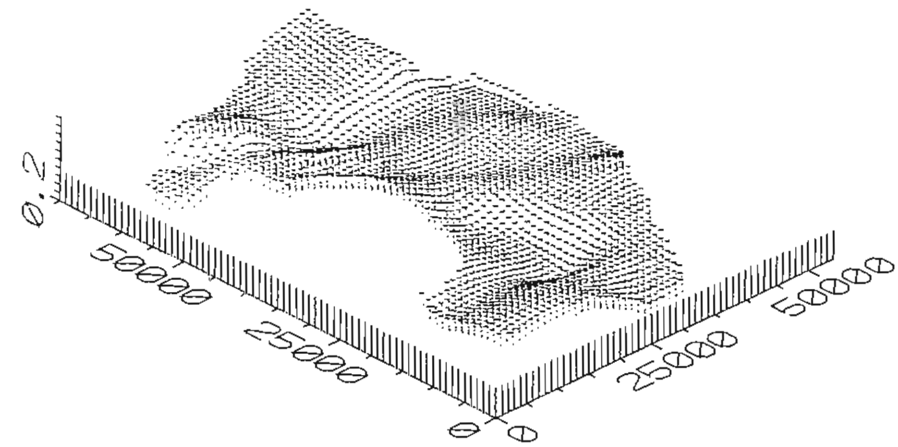



**Fig. 5.1 Flood developmentl, South Rangsit, September, 4, 1990 (-) non-flooded areas**

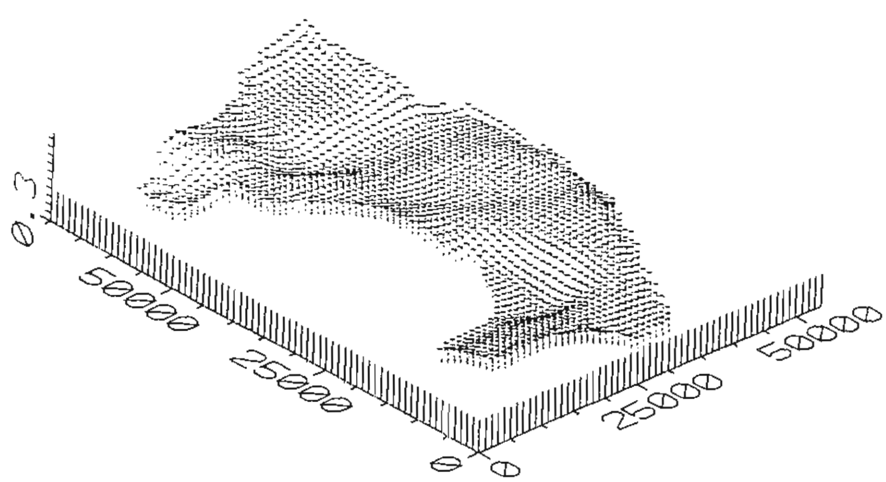



**Fig. 5.2 Flood developmentl, South Rangsit, September, 9, 1990 (-) non-flooded areas**

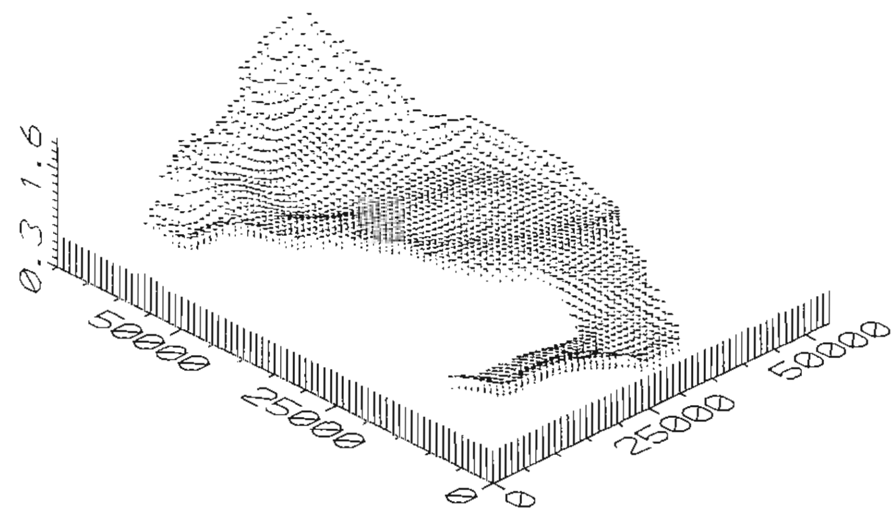



Fig. 5.3 Flood developmentl, South Rangsit, September, 12, 1990 (-) non-flooded areas

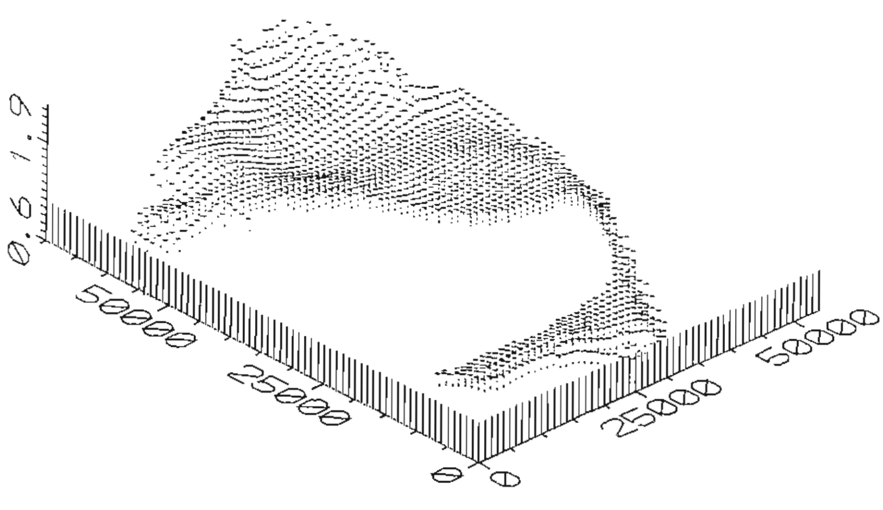



Fig. 5.4 Flood developmentl, South Rangsit, September, 16, 1990 (-) non-flooded areas

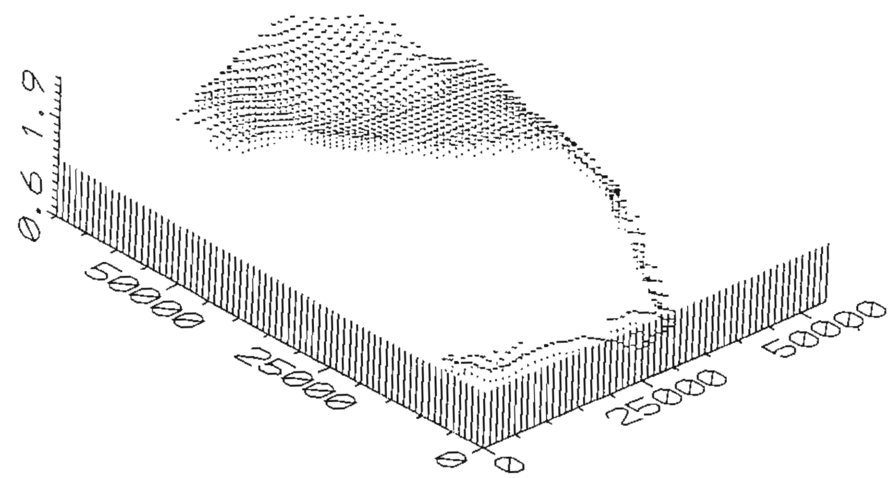



Fig. 5.5 Flood developmentl, South Rangsit, September, 20, 1990 (-) non-flooded areas

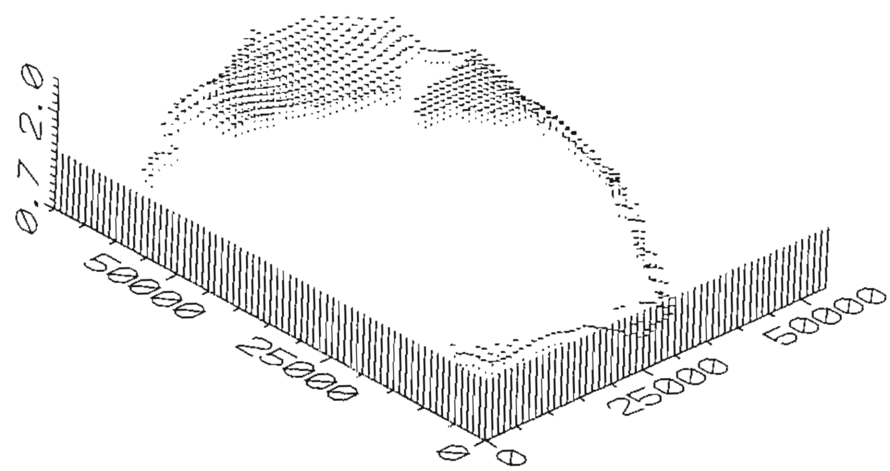



Fig. 5.6 Flood developmentl, South Rangsit, September, 24, 1990 (-) non-flooded areas

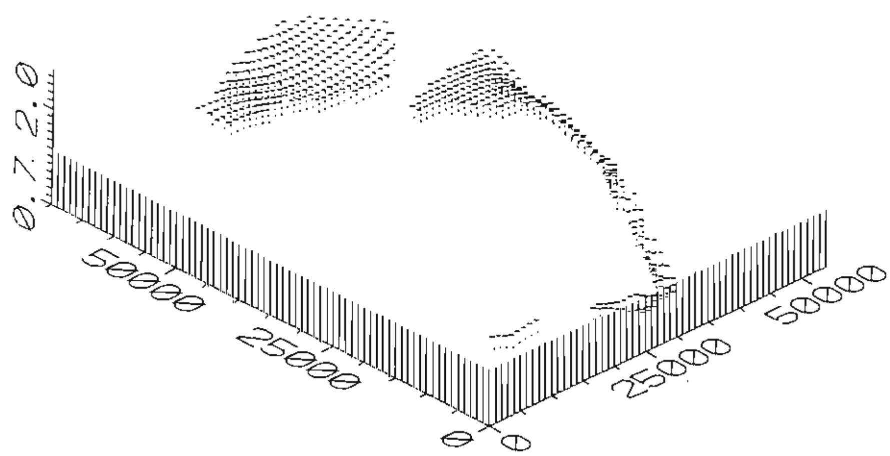



Fig. 5.7 Flood developmentl, South Rangsit, September, 28, 1990 (-) non-flooded areas