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Production lot sizing problem with sudden obsolescence and machine breakdowns

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Abstract

In this paper, an unreliable production system that produces a particular product subject to sudden obsolescence is studied. The purpose of this work is to develop an integrated product obsolescence and machine breakdowns inventory model for determining the optimal lot size that minimizes operating costs during product lifetime. Assuming that both machine failure and product obsolescence occur according to an exponential distribution, the expression for the cost function is offered. The numerical results show that both failure rate and product lifetime have significant impact in determining the optimal lot size of the integrated model.

Keywords: Production lot size, Inventory control, Maintenance, Product obsolescence

1. Introduction

Rapid changes of today's technology bring several new challenging issues to businesses. One of these issues is inventory management under the existence of product obsolescence. When a particular product becomes obsolete, the demand for that product will quickly drop and the product may not meet the needs of customers anymore. In some cases, obsolete items will be sold as scrap, causing obsolescence costs. Therefore, holding more inventory than necessary may lead to the risk of having very high obsolescence costs. One treatment used to deal with this problem is producing small lot sizes. However, producing small lot sizes can undermine the production capacity, especially when a production unit is not reliable. In general, all equipment degrades and ultimately fails from time to time due to age and usage [1]. When failures occur, the production is terminated until the production unit is completely repaired, and then the production starts over again. To compensate for the lost production capacity during breakdowns, the production lot size should be large enough. Therefore, a firm can face the dilemma whether the production lot size should be small or large when experiencing both obsolescence and reliability issues in their production. Traditionally, those two issues are treated separately in production/inventory management. In this work, a unified production/inventory model for determining the optimal lot size, where both obsolescence and reliability issues are jointly considered is proposed. The purpose of this paper is to offer the businesses a more practical model that can yield them benefits when dealing with both obsolescence and reliability issues in production/inventory management.

2. Literature review

One of the early works related to the inventory model with obsolescence is Brown et al. [2]. In that work, the authors propose an inventory model that incorporates the uncertainty of future demand resulting from obsolescence or other types of demand variation into the model. By applying a Bayesian process to adjust the future demand distributions in each period, the authors propose a Markov inventory model of the (s, S) type that can be formulated by using a dynamic programming technique. Masters [3] studied the impact of sudden obsolescence on the optimal lot size decision in the EOQ model. Assuming that the time to obsolescence is exponentially distributed and obsolete inventory is disposed of either with or without salvage value, the author concludes that the optimal lot size for the proposed model is always smaller than that of the EOQ model. Song and Zipkin [4] studied the effects of obsolescence on the inventory management decision. In that work, the authors propose an inventory model that incorporates Markovian submodel specifying factors contributing to obsolescence into the model. They conclude that obsolescence has significant effects on the inventory management decision. In addition, those effects are naturally complicated and cannot be easily treated by simple parameter adjustments. Prathumthip and Niyamosoth [5] studied the impact of sudden obsolescence on the optimal lot size decision in a manufacturing context. Using an idea similar to Masters [3], the authors offer a closed-form solution to calculate the optimal production lot size of the EPQ model subject to sudden obsolescence.

In addition to the inventory model with obsolescence, our work is also related to the integrated production/inventory and maintenance problem. One of the

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early works on this topic is Groenevelt et al. [6], in which the authors investigate the effect of machine breakdowns and subsequent corrective maintenance on the optimal lot size in the EPQ model. Assuming that the times between failures are exponentially distributed and repair times are negligible, the authors conclude that the optimal lot size for the EPQ model with machine breakdowns is always larger than that for the classical EPQ model. In addition, Groenevelt et al. [7] extend their work in Groenevelt et al. [6] by investigating the case where repair times are significant. In this case, safety stocks are incorporated into the model in order to maintain the desired service level. The authors show that the optimal lot size in this case is always larger than that for the classical EPQ model as well. Cheung and Hausman [8] propose an integrated model for determining economic production lot size when both preventive maintenance and safety stocks are jointly considered. In their work, they illustrate the trade-off between the two options and provide optimality conditions for determining which, either one or both, strategies should be implemented to minimize the cost function considering both PM times and safety stock level. Dohi et al. [9] modify the assumptions in the model proposed by Cheung and Hausman [8] and propose a new model to determine the optimal scheduled time to carry out preventive maintenance actions, and the optimal safety stock level that minimizes the expected cost per unit time. Chelbi and Ait-Kadi [10] studied a production unit subject to preventive and corrective maintenance in just-in-time operations. The authors propose a model for determining the optimal preventive maintenance period and the optimal buffer stocks the firm needs to build in order to relieve negative effects resulting from production stoppages during maintenance. Besides the papers mentioned, the integrated production/inventory and maintenance problem can be extended in many possible ways. For a comprehensive review, the reader can also consult Snyder et al. [11].

3. Assumptions and notations

The model in this research is based on the famous EPQ model with machine breakdowns by Groenevelt et al. [6]. In this case, production runs subject to constant demand and constant production rate are randomly interrupted from time to time due to breakdowns. The interrupted lot is considered aborted. When all available inventories are exhausted, a new production run begins. Maintenance action is done after a breakdown or at the beginning of the production cycle for the predestined lot size, depending on whichever occurs first. However, starting a new production run after each breakdown will incur additional corrective maintenance cost. Each maintenance action restores the production unit to a ‘as good as new’ condition. For simplicity, the repair time is assumed to be very small compared to the mean time between failures (MTBF); therefore, it can be neglected. Figure 1 illustrates the progressions of production cycles for the policy on which our model is based. In each cycle, if a breakdown does not occur prior to reaching I_{max}, the maximum level of on-hand inventory, a production cycle will look like an EPQ cycle. Otherwise, a production cycle will be shorter depending on how long it runs before breakdown. In addition to the failure and repair processes mentioned, this work also assumes that a particular product is subject to sudden obsolescence that, when it occurs, makes all on-hand inventory immediately obsolete and incurs obsolescence cost. This type of obsolescence prevails in industries subject to rapid changes, such as fashion and technology industries [3]. In addition, for simplicity, the time to obsolescence is assumed to be exponentially distributed and no shortages are allowed.

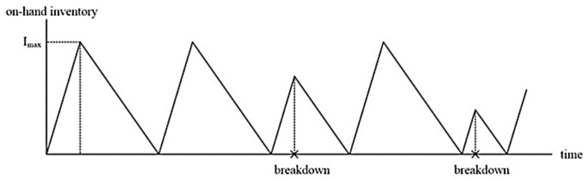

Figure 1 Progressions of production cycles

In this work, we use the following notations:

- D = demand rate
- P = production rate; $P > D$
- Y = time to obsolescence (or lifetime of an item), which is exponentially distributed with $E[Y] = L$
- K = setup cost
- C = unit cost
- i = interest rate
- h = holding cost per unit per year; $h = i.C$
- S = salvage value
- C_S = unit obsolescence cost; $C_S = C - S$
- Q = production lot size
- M = corrective maintenance cost
- T = time-to-breakdown
- $F(t)$ = cumulative density function (cdf) of T
- $f(t)$ = probability density function (pdf) of T
- $r(t)$ = failure rate function; $r(t) = f(t)/(1 - F(t))$
- λ = constant failure rate

4. Model formulation

This section shows how the mathematical model of our work is formulated. The details are as follows.

4.1 Expected cost per cycle

Costs occurred in a cycle consist of setup cost, holding cost, and corrective maintenance cost. Let CC be cost per cycle. Then,

$$CC = \begin{cases} K + M + \frac{1}{2}h(P-D)\frac{P}{D}t^2 & \text{if } t < Q/P, \\ K + \frac{1}{2}h\frac{P-D}{PD}Q^2 & \text{if } t \geq Q/P. \end{cases} \tag{1}$$

Therefore, expected cost per cycle can be defined as the following equation:

$$E[CC] = \int_0^{Q/P}\left(K + M + \frac{1}{2}h(P-D)\frac{P}{D}t^2\right)f(t)dt + \int_{Q/P}^{\infty}\left(K + \frac{1}{2}h\frac{P-D}{PD}Q^2\right)f(t)dt, \tag{2}$$

or

$$E[CC] = K + M \cdot F(Q/P) + \frac{1}{2}h(P-D)\frac{P}{D}\left((Q/P)^2(1 - F(Q/P)) + \int_0^{Q/P} t^2 f(t)dt\right). \tag{3}$$

4.2 Expected length of a cycle

Let LC be the length of a cycle. Then,

$$LC = \begin{cases} \dfrac{Pt}{D} & \text{if } t < Q/P, \\ \dfrac{Q}{D} & \text{if } t \geq Q/P. \end{cases} \tag{4}$$

Therefore, expected length of a cycle can be defined as the following equation:

$$E[LC] = \int_0^{Q/P} \frac{Pt}{D} f(t)dt + \int_{Q/P}^{\infty} (Q/D) f(t)dt, \tag{5}$$

or

$$E[LC] = (P/D)\int_0^{Q/P} tf(t)dt + (Q/D)(1 - F(Q/P)). \tag{6}$$

4.3 Expected number of cycles during the lifetime of an item

Let *N(Y)* be the number of cycles during the lifetime of an item, *Y*. Given that *Y* is exponentially distributed with *E[Y] = L*, then the expected number of cycles during the lifetime of an item can be calculated by the following equation:

$$E[N(Y)] = \frac{E[Y]}{E[LC]} = \frac{L}{E[LC]}. \tag{7}$$

4.4 Expected lifetime obsolescence cost

According to Masters [3], the expected lifetime obsolescence cost can be defined as the unit obsolescence cost, C_S, times the long-run average inventory level, *AI*. Note that the unit obsolescence cost is defined as the unit cost less the salvage cost of an item. Therefore,

$$C_S = C - S. \tag{8}$$

By applying the Renewal Reward Theorem [12], the long-run average inventory level, *AI*, can be defined as in the following equation:

$$\text{long-run average inventory level} = \frac{E[\text{on-hand quantities held in a cycle}]}{E[\text{length of a cycle}]} \tag{9}$$

Let *OH* be on-hand quantities held in a cycle. Thus,

$$OH = \begin{cases} \dfrac{1}{2}(P-D)\dfrac{P}{D}t^2 & \text{if } t < Q/P, \\ \dfrac{Q^2}{2}\left(\dfrac{P-D}{PD}\right) & \text{if } t \geq Q/P. \end{cases} \tag{10}$$

Therefore, the long-run average inventory level, *AI*, can be calculated by the following equation.

$$AI = \frac{E[OH]}{E[LC]} = \frac{\int_0^{Q/P} \frac{1}{2}(P-D)\frac{P}{D}t^2 f(t)dt + \int_{Q/P}^{\infty} \frac{Q^2}{2}\left(\frac{P-D}{PD}\right) f(t)dt}{E[LC]}. \tag{11}$$

Let *OC* be the lifetime obsolescence cost. Thus, the expected lifetime obsolescence cost can be calculated by the following equation:

$$OC = C_S.AI. \tag{12}$$

4.5 Expected total costs

As mentioned before, the objective of the model is to determine the optimal lot size Q^* that minimizes the expected total costs during the lifetime of a particular product. Let *TC* (*Q*) be the lifetime total costs of an item. Then, the expected total costs during the lifetime of an item can be calculated by the following equation:

$$E[TC(Q)] = \frac{L}{E[LC]} \cdot E[CC] + OC = \frac{L \cdot E[CC] + C_S.E[OH]}{E[LC]}. \tag{13}$$

Substituting all model parameters for the right-hand size of (13), we obtain an expression of the objective function in our model as a function of *Q* as the following equation:

$$E[TC(Q)] = \frac{L \cdot \left(K + M \cdot F(Q/P) + \frac{1}{2}h(P-D)\frac{P}{D}\left((Q/P)^2(1-F(Q/P)) + \int_0^{Q/P} t^2 f(t)dt \right) \right) + C_S \cdot \left(\int_0^{Q/P} \frac{1}{2}(P-D)\frac{P}{D}t^2 f(t)dt + \frac{Q^2}{2}\left(\frac{P-D}{PD}\right)(1-F(Q/P)) \right)}{(P/D)\int_0^{Q/P} tf(t)dt + (Q/D)(1-F(Q/P))}. \tag{14}$$

The objective function $E[TC(Q)]$ in (14) is rather complicated, especially when $F(t)$ is a general distribution where the failure rate function, $r(t)$, derived from the distribution is also a function of *t*. However, the complexity of $E[TC(Q)]$ can be reduced by assuming that the failure rate is constant and time-independent, that is, $r(t)=\lambda$. In this case, time to failure, *T*, becomes an exponentially distributed random variable with cdf and pdf defined as follows [13]:

$$F(t) = 1 - e^{-\lambda t}, t \geq 0, \tag{15}$$

and

$$f(t) = \lambda e^{-\lambda t}. \tag{16}$$

Substituting (15) and (16) for $F(t)$ and $f(t)$ respectively in (14), we obtain an expression for the objective function in our model as a function of *Q* as in the following equation:

$$E[TC(Q)] = -\frac{C_S(D-P)\left(\left(-1+e^{\frac{Q\lambda}{P}}\right)P - Q\lambda\right) + L\left(-hP\left(\left(-1+e^{\frac{Q\lambda}{P}}\right)P - Q\lambda\right) + D\left(-\left(-M+e^{\frac{Q\lambda}{P}}(K+M)\right)\lambda^2 + h\left(\left(-1+e^{\frac{Q\lambda}{P}}\right)P - Q\lambda\right)\right)\right)}{\left(-1+e^{\frac{Q\lambda}{P}}\right)P\lambda}. \quad (17)$$

The objective function in (17) is a continuous function. There are several software packages that can solve for the optimal value of Q that minimizes $E[TC(Q)]$. Unfortunately, because it is difficult to prove whether or not $E[TC(Q)]$ is convex, the optimal solution to our model may be not the global optimum. Note that when $\lambda \to 0$, our proposed model will reduce to Prathumthip and Niyamosoth's model [5]. In this case, the optimal value of Q can be derived in a closed-form solution as the following equation:

$$Q^* = \sqrt{\frac{2KLD}{(hL+C_S)(1-D/P)}} = \sqrt{\frac{2KD}{h(1-D/P)}} \cdot \sqrt{\frac{L}{L+(C_S/h)}} = EPQ\sqrt{\frac{L}{L+(C_S/h)}}. \quad (18)$$

Because the factor $\sqrt{\frac{L}{L+(C_S/h)}}$ is less than one, we conclude that the optimal production lot size for the classical EPQ model is always larger than that for the EPQ model with sudden obsolescence. In addition, when $L \to \infty$, Q^* in (18) will reduce itself to EPQ.

5. Numerical experiments

In the following sections, the numerical results and the sensitivity analyses are provided. The test parameters are taken from a real case in the mechanical parts industry. The software package used in solving the model is MATLAB toolbox.

5.1 Test parameters

For the numerical experiments, we use the following test parameters:

L = 5 (years), K = 2,175 (THB), M = 5,000 (THB), i = 0.25, C = 294.45 (THB per unit), S = 24.75 (THB per unit), C_S = C – S = 269.70 (THB per unit), h = $i \times C$ = 73.6125 (THB per unit per year), D = 1,885 (units per year), P = 11,457 (units per year), λ = 12 (times per year).

5.2 Numerical results and sensitivity analyses

In this section, the numerical tests and the sensitivity analyses are performed. The results are shown in the following tables:

Table 1 The effect of L on the optimal solutions

λ=0			λ=12		
L	Q^*	$E[TC(Q^*)]$	L	Q^*	$E[TC(Q^*)]$
0.5	126.5138	32,401.9288	0.5	129.3895	38,069.5637
1.0	169.0790	48,496.5676	1.0	174.2202	59,842.9138
1.5	196.7979	62,498.7699	1.5	203.7974	79,529.2181
2.0	216.9804	75,580.5696	2.0	225.5199	98,298.5137
2.5	232.5440	88,152.6849	2.5	242.3798	116,560.4598
3.0	244.9960	100,406.7542	3.0	255.9376	134,506.0541
3.5	255.2240	112,446.8044	3.5	267.1201	152,238.9046
4.0	263.7953	124,335.0250	4.0	276.5234	169,820.9129
4.5	271.0936	136,111.2051	4.5	284.5532	187,291.6627
5.0	277.3894	147,802.1277	5.0	291.4974	204,677.7855

From Table 1, it can be seen that the expected lifetime of an item, L, has a significant impact on the optimal solutions of the problem. According to the numerical test, we can conclude that, for a fixed value of λ, when the expected lifetime of an item, L, increases, the optimal production lot size, Q^*, and the expected total costs during the lifetime of an item, $E[TC(Q^*)]$, also increase. In other words, the firm tends to produce a smaller lot size when the lifetime of an item is short and vice versa. In addition, we also conclude that, for a fixed value of L, the optimal values of Q^* and $E[TC(Q^*)]$ for the model with machine breakdowns (λ = 12) are always larger than those for the model without machine breakdowns (λ = 0). The results in Table 1 can be illustrated in Figure 2 and Figure 3.

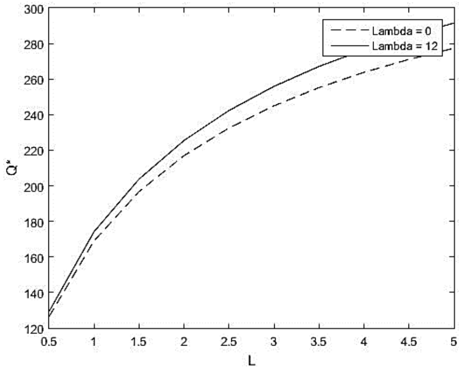

Figure 2 The effect of L on Q^*

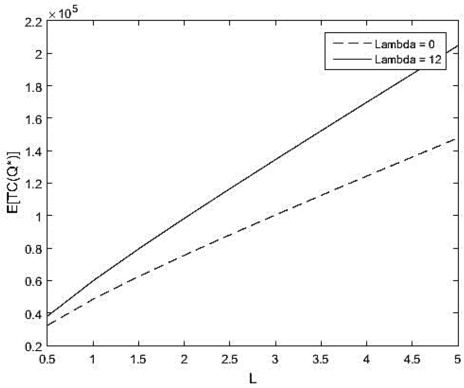

Figure 3 The effect of L on $E[TC(Q^*)]$

In this section, the effect of failure rate, λ, on the optimal solutions of the model is investigated. The results are shown in Table 2.

Table 2 The effect of λ on the optimal solutions

λ(L=2)	Q^*	$E[TC(Q^*)]$
0	216.9804	75,580.5696
6	221.1687	86,911.1817
12	225.5199	98,298.5137
18	230.0142	109,745.1198
24	234.7401	121,253.5969
30	239.6243	132,826.5653
36	244.7010	144,466.6453
42	249.9779	156,176.4290
48	255.4622	167,958.4473

From Table 2, it can be seen that the failure rate also has a significant impact on the optimal solutions of the problem. The results show that, for fixed value of L, the optimal values of Q^* and $E[TC(Q)]$ for the model with machine breakdowns ($\lambda > 0$) are always larger than those of the model without machine breakdowns ($\lambda = 0$). In addition, according to the numerical test, we also conclude that, for a fixed value of L, when the failure rate increases, the optimal production lot size, Q^*, and the expected total costs during the lifetime of an item, $E[TC(Q^*)]$, also increase. The results in Table 2 can be illustrated in Figure 4 and Figure 5.

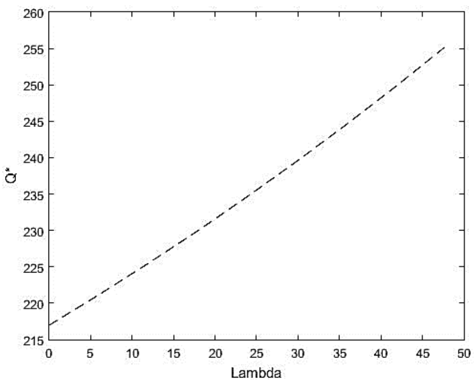

Figure 4 The effect of λ on Q^*

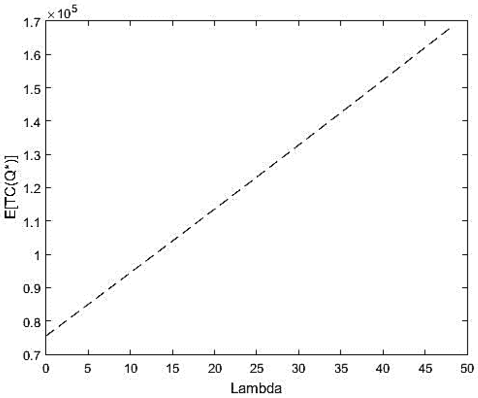

Figure 5 The effect of λ on $E[TC(Q*)]$

6. Conclusions

In this paper, we study the production lot sizing problem considering both product obsolescence and machine breakdowns. The experimental results show that both failure rate and product lifetime have significant impact in determining the optimal lot size of the integrated model. This model can help the firm determine the production lot size more appropriately, especially in a high-tech industry where the model of a product changes rapidly and the production unit is prone to failures due to its complexity.

7. Acknowledgements

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8. References

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