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Determination of inventory replenishment policy with the open vehicle routing concept in a multi-depot and multi-retailer distribution system

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Abstract

The Inventory Open Vehicle Routing Problem (IOVRP) involves decisions in inventory replenishment and vehicle routing of a system having multiple depots and multiple retailers. The research objective of this work is to develop practical replenishment decisions by applying meta-heuristics of an Ant Colony Algorithm. The routing solutions applying IOVRP and an Inventory Routing Problem (IRP) that are solved and compared to measure the methods' performance. The result shows that the IOVRP gives 24.66% better solutions in term of total costs than the IRP. Additionally, sensitivity analysis of related factors, i.e., inventory holding costs, ordering cost and vehicle capacity, was performed on the percentage deviation of total costs. Based on the analysis of variance, there is an advantage of IOVRP over IRP when the problem involves small vehicle capacity, low ordering costs, and high holding costs.

Keywords: Inventory open vehicle routing problem, Multi-Retailer, Multi-Depot distribution system, Ant colony optimization

1. Introduction

The Inventory Routing Problem (IRP) involves the determination of an inventory replenishment policy and vehicle routes in a system composed of a number of depots and retailers. Each depot supplies items to those retailers located within a certain area. For the problem with multiple depots, the approach is first to form several clusters. Each cluster is composed of a single depot and a number of retailers assigned to it. The problem, then, is reduced to a number of single depot with multi-retailer problems. The next step is to determine the proper inventory replenishment policy for each cluster, along with the number of vehicles to be used, and the distribution path of each vehicle with starting and ending points at the depot. A tutorial paper involving IRP has been presented [1]. To solve the problem, [2] developed a two-phase approach based on decomposing a set of decisions, i.e., a delivery schedule and delivery routes. The first phase utilized integer programming, whereas the second phase employed routing and scheduling heuristics. A multi-period IRP with constant demand was discussed by [3]. The problem was formulated as linear mixed-integer programming and solved using an approach based on a Lagrangian relaxation method and an assignment problem. The approach using an Ant Colony System with a Hybrid Local Search to solve the vehicle routing problem with a single depot with known customer requirements is discussed by [4]. Most of the works associated with the IRP assume closed loop vehicle routing in which each vehicle has its own starting and ending points at the same location, which is the depot. The Open Vehicle Routing Problem (OVRP) is a special form of vehicle routing problem in which a vehicle does not have to return to its starting point at the end of the route. First introduced to solve the air cargo routing problem of the FedEx Company in 1983, the OVRP concept has gained more attention today, especially for commodity distribution activities of supermarket businesses having multiple depots and retailers. The work of [5] presents a heuristic method based on the minimum spanning tree with penalties assessed to solve the OVRP problem. A Tabu Search heuristic was proposed by [6] to determine the number of enabled vehicles along with their routes, to serve all customers, in a system having single depot and a set of customers. The metaheuristic search based on Ant Colony Optimization (ACO) hybridized with a Tabu Search to solve the OVRP was presented by [7]. The study of [8] discussed a method based for modification of ACO solving the OVRP and claiming that the method is successful in finding the solutions within 1% of known optimal solutions. The OVRP with driver ending nodes and time window deadlines was studied by [9]. They proposed a Tabu Search method to determine the proper solution to the problem. The work of [10] presented an attribute-based hill climber heuristic, which is a parameter-free variant of the Tabu Search principle, to solve the OVRP.

Unlike most of the traditional IRPs that consider the case of closed loop vehicle routing, this research studied the integration of an opened loop vehicle routing concept to the

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IRP. In the research, the system is composed of two depots with a number of retailers. Each retailer has its own product demand, holding costs, and order cost rates. The objective is to determine the proper inventory replenishment policy, number of vehicles needed, and the routing of each vehicle. Here, each vehicle is allowed to start its route at one depot but end the route at a different depot.

2. Problem characteristics

In the current study, the distribution system consists of two depots supplying a product to a group of retailers. Each retailer has its own annual demand rate, item holding costs, and order costs. Product delivery is done using a number of trucks having the same capacity limitations. The delivery route must start and end at a depot, either the same or a different one. The objective is to determine the proper inventory replenishment policy along with delivery routes to minimize the annual system costs that are comprised of the annual variable costs incurred from the replenishment policy and the annual transportation costs incurred from annual delivery. The related assumptions are:

1. We consider a single product with a deterministic and known demand.
2. The delivery lead time is known and delivery time window is enough to complete each delivery route.
3. There is no allocation between retailers.
4. The vehicles are identical and capacitated.
5. Associate costs are inventory setup costs or ordering costs, inventory holding costs, and transportation costs.
6. Vehicles are available at all depots.
7. There are no inventory costs occurring at depots.
8. Product shortage is not allowed.

The following notation is used throughout the paper.

TSC = total system costs ($/year)
TVC = total variable costs ($/year)
TTC = total transportation costs ($/year)
D_i = annual demand of retailer *I* (unit/year)
Q_i = replenishment quantity of retailer *i* (unit/order)
T_i = replenishment interval of retailer *i* (year)
d_{ij} = distance from *i* to *j* (km)
C = per unit cost of the product ($/unit)
S_i = inventory setup cost of retailer *i* ($/order)
f_i = holding cost fraction of retailer *i*
H_i = inventory holding cost of retailer *i* ($/unit/year) = f_i*C
n = number of retailers
X = vehicle capacity (unit)
α, β = constant parameters used to control the amount of pheromone and the distance, respectively
P_i = probability of travelling from location *i* to location *j*
η_{ij}^{α} = amount of the deposited pheromone when travelling from location *i* to location *j*
τ_{ij}^{β} = distance influence of travelling from location *i* to location *j*
Att_{ij} = attractiveness of travelling from location *i* to location *j*
$CumuProb_{ij}$ = cumulative probability of travelling from location *i* to location *j*
L_i = total travel distance of the best solution of all ants for each iteration (km)
ρ = updated pheromone value
m = number of ants
r_v = number of vehicles or routes in period *v*
a = fixed transportation costs ($/route)
b = variable transportation costs ($/km)

The total system cost can be calculated using equations 1, 2, and 3:

$$TSC = TVC + TTC \tag{1}$$

$$TVC = \sum_{i=1}^{n}\left(\frac{S_i}{T_i} + \frac{H_i D_i T_i}{2}\right) \tag{2}$$

$$TTC = \sum_{v=1}^{50}\left[ar_v + b\sum_{r=1}^{k}\left\{\min(d_{D_j,i}) + \sum_{n\in l} d_{i,n} + d_{n,D_j}\right\}\right] \tag{3}$$

where ar_v is the fixed transportation costs and,

$$b\sum_{r=1}^{k}\left\{\min(d_{D_j,i}) + \sum_{n\in l} d_{i,n} + d_{n,D_j}\right\}$$

is the variable transportation costs. The characteristics of the problem can be seen in Example 1.

Example 1: Given that a distribution system composed of two depots is supplying a product to eight retailers. The delivery truck has a capacity limitation of 200 units. The cost of the product is $10 per unit. The information associated with the product demand at each retailer and all distances related to the distribution system are given in the Tables 1 and 2, respectively.

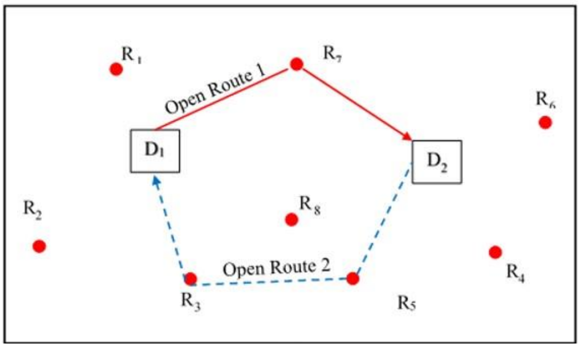
Table 1 Information about product demand at each retailer

Retailer	Annual demand (units)	Holding Cost Fraction	Order Cost ($)
1	1500	0.20	40
2	1000	0.30	40
3	3000	0.25	50
4	2500	0.20	50
5	3000	0.25	50
6	1200	0.30	80
7	4500	0.20	30
8	800	0.25	80

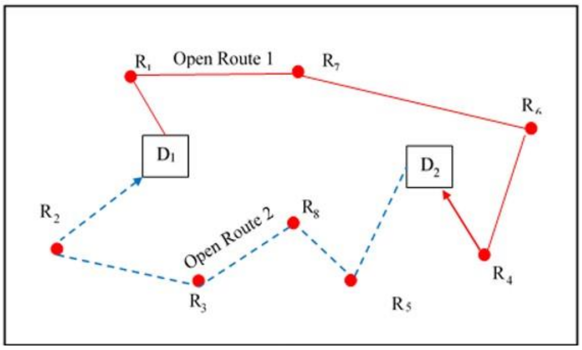
Table 2 Distances between any two locations in the distribution system

	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8
D_1	10	15	18	25	30	22	16	14
D_2	20	25	30	15	8	12	15	15
R_1	-	20	15	28	28	27	18	13
R_2	20	-	7	26	29	20	17	13
R_3	15	7	-	21	25	29	20	14
R_4	28	26	21	-	6	13	19	23
R_5	28	29	25	6	-	5	12	18
R_6	27	20	29	13	5	-	4	14
R_7	18	17	20	19	12	4	-	10
R_8	13	13	14	23	18	14	10	-

One possible inventory replenishment policy is to refill the product at retailers 3, 5, and 7 every week (T = 1/50 = 0.02 years) and refill the product at all remaining retailers every two weeks (T = 2/50 = 0.04 years). Alternatively, for odd-weeks (weeks 1, 3, 5…), the product will be replenished at the retailers 3, 5, and 7, while for even-weeks (weeks 2, 4, 6…), the product will be replenished at all retailers. Note that this solution may not be optimal.



(a)



(b)

Figure 1 (a) Delivery route for odd-weeks. (b) Delivery route for even-weeks

Assuming a single delivery route as shown in Figure 1, the total system cost is comprised of the total inventory, variable and total transportation costs, which can be calculated using equations 1, 2, and 3 as follows:

$$TVC=\left(\frac{40}{0.04}+\frac{40}{0.04}+\frac{50}{0.02}+\frac{50}{0.04}+\frac{50}{0.04}+\frac{80}{0.04}+\frac{30}{0.02}+\frac{80}{0.04}\right)$$
$$+\left(\begin{array}{l}\frac{20*1500*0.04}{2}+\frac{30*1000*0.04}{2}+\frac{25*3000*0.02}{2}+\frac{20*2500*0.04}{2}\\+\frac{25*3000*0.02}{2}+\frac{30*1200*0.04}{2}+\frac{20*4500*0.02}{2}+\frac{25*800*0.04}{2}\end{array}\right)$$
$$=\$19{,}470$$

TTC = (100*2)*50+(31+51)*25+(60+62)*2

TSC = 19,470+15,100 = \$34,510

3. Methodology

The developed algorithm is comprised of two calculations. The first phase generates an initial solution by applying classical inventory theory with a power-of-two policy. Then, all retailers are clustered according to their replenishment period. The second phase is routing for retailers in each cluster by applying ACO.

Phase I Solving the inventory problem to find a practical replenishment schedule.

Step 1: Solve for the finite replenishment planning horizon using classical EOQ, Equation (4), with a power-of-two policy. The replenishment interval, T_i, calculated from equation (5) is in yearly units. However, the replenishment period used in this research is in weekly units, so the result of calculating T_i must multiplied by 50 (assuming that there are 50 weeks in a year). If we let T_i be the optimal power of two-reorder interval, the optimal k in Equation (9) is the smallest integer k satisfying Equations (6)-(8):

$$Q_i=\sqrt{\frac{2SiD_i}{H_i}} \quad (4)$$

$$T_i=\frac{50D_i}{Q_i} \quad (5)$$

$$2^k\le T_i\le 2^{k+1} \quad (6)$$

$$f(T_i)=TVC_i(T_i)=\frac{S_i}{T_i}+\frac{H_iD_iT_i}{2} \quad (7)$$

$$f(T_B2^k)\le f(T_B2^{k+1}) \quad (8)$$

The replenishment interval (in weeks) is revised and becomes:

$$T_i=T_B2^k \quad (9)$$

Then, the replenishment quantity is calculated as:

$$Q_i=\frac{D_iT_B2^k}{50} \quad (10)$$

The results from Equations (4-10) are the replenishment schedule of each retailer of the resulting replenishment quantities and periods at every 1, 2, 4, 8…, 2^k weeks.

Step 2: Calculate the distance between depots and retailers according to Equation (11):

$$d_{ij}=\sqrt{(x_i-x_j)^2+(y_i-y_j)^2} \quad (11)$$

Step 3: Clustering retailers according to their replenishment period.

The retailers with the same replenishment period will be clustered into the same delivery schedule. For example, retailers in the first cluster will be supplied every week if T_i is equal to 2^0, retailers in the second cluster will be supplied every two weeks if T_i equal to 2^1, and so on. The developed algorithm is applied to solve a problem with 20 retailers. The depot and retailer locations (x, y) are randomly generated in the range of 0 and 400 km. The retailers' demands are randomly generated so that they are between 2,000 and 20,000 units with a unit price of \$100. The inventory parameters are inventory holding cost rate (randomly generated from 10% to 30%) and the inventory setup cost rate (randomly generated from 100 to 300 \$/order). The vehicles used are identical and vehicle capacity is equal to 4,000 units. The computational results of the case study are displayed in Table 3.

Table 3 Computational results of the case study applying the first phase (D_1 and D_2 are the first and the second depots, respectively).

Retailer NO. (*i*)	Coordinates		D_i	Q_i	T_i	2^k	2^{k+1}	TVC (2^k)	TVC (2^{k+1})	T_i	Q_i
	x	*y*									
1	300.42	181.73	12788	642	2.33	2	4	11084.6	13004.8	2	550
2	106.85	153.11	9686	419	3.57	2	4	9850.4	4775.8	4	470
3	388.07	32.26	12848	659	2.27	2	4	11687	13306	2	580
4	386.57	377.75	14428	481	3.11	2	4	10595.2	6265.4	4	619
5	380.46	376.72	2472	385	3.89	2	4	9480.4	4035.8	4	396
6	176.95	158.58	9519	738	2.03	2	4	14633.4	14779.2	2	727
7	138.80	217.40	11161	289	5.19	4	8	12255.8	5978.8	4	222
8	72.91	315.10	2450	675	2.22	2	4	6071.1	13590.4	2	609
9	274.16	88.00	3265	336	4.46	4	8	12025.4	6766.8	4	301
10	169.93	81.71	6447	472	3.17	2	4	10476.8	6028.6	4	595
11	287.13	327.93	9596	456	3.29	2	4	10276	5627	4	555
12	31.86	223.56	3441	669	2.24	2	4	12297.4	13475.2	2	597
13	305.27	82.55	12389	434	3.45	2	4	10016.4	5107.8	4	503
14	208.24	231.49	7996	677	2.22	2	4	12445.3	13611.2	2	611
15	243.53	105.82	16078	681	2.20	2	4	12467.8	13696.4	2	619
16	260.64	98.34	17726	624	2.40	2	4	10471	12698	2	519
17	33.95	89.59	14551	545	2.75	2	4	7999.8	11462.4	2	396
18	95.88	229.95	15791	299	5.01	4	8	4811.9	6137.2	4	238
19	390.32	120.81	14382	385	3.90	2	4	9476.4	4027.8	4	395
20	175.97	110.51	9365	637	2.35	2	4	10895.8	12910.4	2	541
D1	189.84	116.27									
D2	359.27	44.18									

Step 4: Go to routing phase (Phase II).

Phase II Routing retailers in each cluster. The nearest depot is selected as the departure depot and the route is formed according to the developed algorithm based on ACO. If the vehicle is full, it returns to the nearest depot. Thus, the route may be a closed or open route. Ant colony optimization algorithm (ACO), which is a probabilistic technique for solving computational problems for finding good paths through graphs, is applied to find the best path through the distribution system. At each iteration of the algorithm, each ant moves from a state *i* (retailer *i*) to state *j* (retailer *j*), corresponding to a more complete intermediate solution. Thus, each ant *k* computes a set of feasible expansions to its current state in each iteration, and moves to one of these probabilistically. For ant *k*, the probability of moving from retailer *i* to retailer *j* depends on a combination of two values, i.e., attractiveness and the pheromone level of the move. The ant chooses to move from one location to another location according to some rules:

1. It must visit each city exactly once.
2. A distant city has less chance of being chosen (it is less visible).
3. The more intense the pheromone trail laid out on an edge between two cities, the greater the probability that edge will be chosen.
4. Having completed its journey, the ant deposits more pheromones on edges that it travelled, if the journey is short.
5. After each iteration, trails of pheromones evaporate.

In the routing phase, the calculation steps are shown in Figure 2. The related initial parameters values are calculated as follows:

Step 1: Initialization- calculate the value of the pheromone level and attractiveness according to Equations (12) and (13):

$$\tau_{ij}^{\beta} = \left(\frac{1}{distance_{ij}} \right)^{\beta} \quad (12)$$

$$Att_{ij} = \eta_{ij}^{\alpha} \times \tau_{ij}^{\beta} \quad (13)$$

Att_{ij} is the attractiveness which is a function of the amount of pheromone deposited for transition from state *i* to *j* and $\alpha \geq 0$ and $\beta \geq 0$ are parameters to control the influence of the pheromone and the distance in consecutive steps. The value of η_{ij}^{α} is the pheromone parameter weighting by parameter α, while τ_{ij}^{β} is a constant in the TSP problem, and is usually set to the reciprocal of the distance between *i* and *j*, as shown in Equation (13). The parameters, α and β, are normally set to a value of 1-5. In the current study, the values of α and β were set to 5. The initial value of the pheromone parameter is 1 for travelling from retailer *i* to retailer *j* and to 0 for retailer *i* to *i*, or *j* to *j*. The number of iterations was 5 and for each iteration the number of ants was set to be 3 (*m*=3). Therefore, the initial values of attractiveness (*i*, *j*) were calculated using Equation (13).

Step 2: Calculate the probability of travelling from location *i* to *j* using Equation (14):

$$P_{ij} = \frac{Att_{ij}}{\sum_{j=1}^{S} Att_{ij}} \quad (14)$$

Step 3: Apply a Russian roulette concept for randomly selecting the next retailer to be visited by first calculating the cumulative probability of travelling from *i* to *j* using Equation (15):

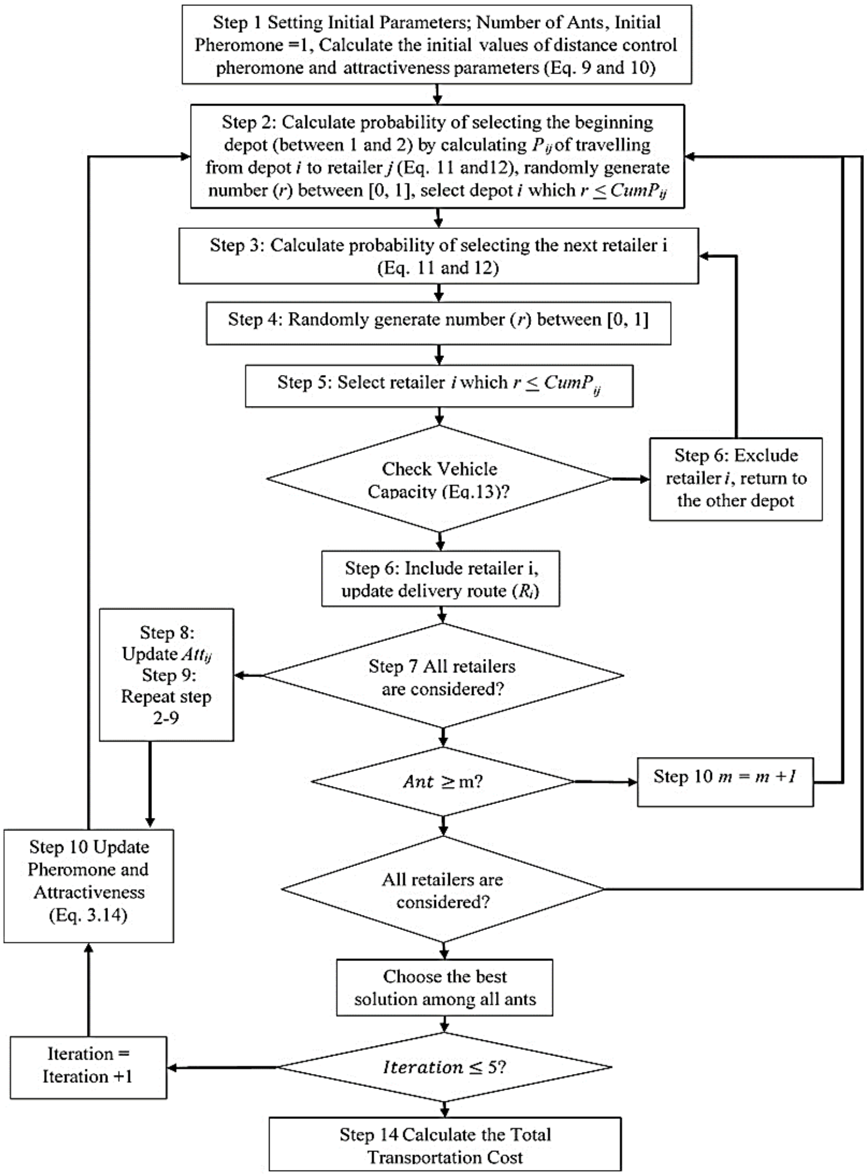


Figure 2 Flow chart of the routing phase (second phase)

$$CumP_{ij} = \sum_{k=1}^{j} P_{ik} \tag{15}$$

Step 4: Randomly generate a number (r) between [0, 1]

Step 5: If the number falls within the cumulative probability of a retailer ($r \leq CumP_{ij}$), that retailer is selected to be the next visited location.

Step 6: If the last retailer violates the vehicle capacity constraint according to Equation (16), exclude the retailer and go to Step 7.

$$\sum_{i=1}^{n} Q_i \leq X \tag{16}$$

Step 7: Repeat Steps 2-6 by recalculating the probability and cumulative probability for the remaining retailers.

Step 8: Recalculate Att_{ij} according to Equation (13).

Step 9: Repeat Steps 2-9 until all retailers are routed. The solution is the set of delivery routes of each ant.

Step 10: Update the pheromone by decreasing it to 10%, ($\rho = 0.1$). If i to j are not the selected arc, then the new pheromone decreases. Alternatively, the new pheromone of the selected arc i to j is decreased by 10% and increased by the reciprocal of the total travel distance of the best solution from the first iteration according to Equation (17):

$$\eta_{ij}^{n} = \begin{cases} (1-\rho)(\eta_{ij}^{n-1}) + \tau^{l}, for(i,j), \\ \text{if arc } (i,j) \text{ is included in the best solution} \\ (1-\rho)(\eta_{ij}^{n-1}), \text{Otherwise} \end{cases} \tag{17}$$

ρ is the parameter that controls the evaporation of the pheromone and it is set to 0.1. τ^{l} is the best travel distance of the first through third ants. The pheromone trail starts to evaporate, thus reducing its attractive strength. The more time it takes for an ant to travel down the path, the more the pheromones evaporate. A short path, by comparison, gets marched over more frequently, and thus the pheromone density becomes higher on shorter paths. Pheromone evaporation also has the advantage of avoiding convergence to a locally optimal solution. If there was no evaporation at all, the paths chosen by the first ants would tend to be excessively attractive to subsequent ones. In this case, exploration of the solution space would be constrained.

Step 11: Repeat Steps 2-10 for all three ants (m=3) and choose the best answer among the three.

Step 12: Repeat Step 10 by updating the pheromone of the best answer.

Step 13: Repeat Steps 2-10 for 5 iterations and choose the best answer.

Step 14: Calculate the total transportation costs using Equations (18) and (19).

$$TTC = \sum_{v=1}^{50} (a \times NoRoutes + b \times TotalTravelDistance) \quad (18)$$

where a and b are the fixed transportation costs per route and variable transportation cost per kilometer, respectively. In this research, the values of a and b were set to \$100 and \$1, respectively. The objective function minimized the total transportation costs consisting of fixed and variable transportation costs. The fixed transportation costs depend on the number of travel routes (v) and the variable transportation costs depends on the total travel distance. The best travel route is a set of retailers that are sequenced as $\{D_j, i, i+1 \ldots k, D_j\}$. Thus, the objective function from Equation (18) is transformed to Equation (19).

$$TTC = \sum_{v=1}^{50} ar_v + b\sum_{k=1}^{m}\left\{ \min\left[d_{D_{j,i}}\right] + \sum_{n\in l} d_{i,n} + d_{D_j} \right\} \quad (19)$$

Computational examples of 20 retailers with 10 replicates by applying the IRP and IOVRP of three ants, five iterations, and the values of pheromone control (α and β) set to five are shown in Table 4. The travel distance of each replenishment period, every odd week, every other week, and every four weeks, were calculated for both depots. The computational results in Table 4 show travel distances by applying IRP using Nearest Neighborhood and IOVRP in ACO. For IOVRP, the route begins at either Depot 1 or Depot 2 and ends at the nearest depot.

To compare the performance of IOVRP and IRP, the difference in total transportation costs for IOVRP and IRP were compared by calculating the percentage of deviation (Equation 20). The total transportation costs are composed of fixed and variable costs. The fixed transportation costs depend on the number of delivery routes, whereas the variable transportation costs depend on the travel distances.

$$\%Deviation = \frac{TotalCost_{IRP} - TotalCost_{IOVRP}}{TotalCost_{IRP}} \times 100 \quad (20)$$

The average value of the percentage deviation of 10 experiments was about 24.66% as the results show in Table 5. Based on the computational examples, the IOVRP gave a better solution than IRP.

4. The sensitivity analysis

The example of a 20 retailer distribution system was generated by varying several factors to analyze the sensitivity of related factors to the solutions. Each retailer's location was generated from number between 0 and 400. Retailer demands were generated as values between 2,000 and 20,000. The vehicle capacity levels were 4000, 8000, and 10,000 units. The setup cost levels were 100, 200 and 300. The holding cost levels were 10%, 20% and 30%. The response was expressed as percent deviation and the experimental objective as to perform sensitivity analysis on factors related factors to percent deviation. These results are shown in Table 6.

The experiment was a full factorial design or FAT 3X3X3. The replenishment quantities depended on the setup and holding cost rates. Low setup costs with high holding costs gave small replenishment quantities while high setup costs and low holding costs required larger replenishment

Table 4 Comparing total transportation costs by applying IRP and IOVRP of two depots and twenty retailers in the distribution system

Experiment No.	Travel Distance								
	IRP (Neighborhood) (Depot 1)			IRP (Neighborhood) (Depot 2)			IOVRP (Between Depot 1 and Depot 2)		
	Every Odd Week	Every Other Week	Every Four Week	Every Odd Week	Every Other Week	Every Four Week	Every Odd Week	Every Other Week	Every Four Week
1	0	1131.53	1528.08	0	702.65	1312.64	0	1286.59	3353.06
2	0	1423.78	1500.7	0	922.46	1468.10	0	1585.81	3107.89
3	0	759.96	1446.1	0	874.01	1315.9	0	1387.69	3695.80
4	0	1036.01	1446.1	0	968.46	1312.64	0	2146.33	3567.36
5	0	1252.34	1500.74	0	730.35	1312.62	0	2350.11	3370.49
6	79.75	1052.47	1446.14	625.3	1393.15	1397.09	275.99	2505.03	3440.17
7	0	531.83	1539.34	0	857.82	1421.97	0	1186.97	3286.18
8	0	980.49	1446.14	0	940.713	1421.97	0	1188.25	3286.18
9	226.56	1492.7	1446.14	280.73	280.734	280.73	183.19	2180.81	3394.30
10	0	1036.00	1036.005	0	968.46	968.46	0	2146.33	3567.36

Table 5 Comparison of transportation costs between IRP and IOVRP by calculating the percentage of deviation

No.	Total Transportation Cost		*%Deviation*
	IRP	IOVRP	
1	24674.92	14639.67	40.67
2	25315.07	19693.71	22.21
3	24396.12	20083.50	17.68
4	24763.25	20713.70	16.35
5	24796.09	20720.61	16.44
6	35993.96	31221.20	13.26
7	24350.98	14473.16	40.56
8	24789.33	19474.44	21.44
9	35575.90	20758.32	41.65
10	24763.25	20713.70	16.35
Average	26941.89	20249.20	24.66

Table 6 Computation of experimental results at various combinations of vehicle capacity, ordering cost rate, and inventory holding cost rate

Vehicle Capacity	Ordering Cost	Inventory Holding Cost	Total Annual Cost (*TSC*)		
			IRP	OVRP	%Difference
4000	100	10%	56,553.29	37,298.75	34.05
		20%	56,022.53	35,223.99	37.13
		30%	53,250.47	35,685.03	32.99
	200	10%	55,888.60	48,232.80	13.70
		20%	53,224.88	44,330.12	16.71
		30%	50,290.53	42,297.79	15.89
	300	10%	53,255.59	48,936.73	8.11
		20%	50,004.97	44,677.58	10.65
		30%	49,801.88	42,502.02	14.66
8000	100	10%	33,299.03	23,400.50	29.73
		20%	31,244.53	19,299.72	38.23
		30%	30,504.86	23,599.33	22.64
	200	10%	33,255.73	28,922.55	13.03
		20%	27,068.86	23,481.51	13.25
		30%	26,248.20	21,843.03	16.78
	300	10%	37,392.04	31,658.28	15.33
		20%	27,155.66	21,894.73	19.37
		30%	27,769.95	22,008.57	20.75
10000	100	10%	32,294.03	22,465.50	30.43
		20%	30,625.53	18,294.72	40.26
		30%	29,566.86	22,779.33	22.96
	200	10%	32,348.73	27,154.55	16.06
		20%	26,118.86	21,288.51	18.49
		30%	25,538.20	20,402.03	20.11
	300	10%	36,754.04	30,846.28	16.07
		20%	26,375.66	21,175.73	19.71
		30%	27,140.95	21,317.57	21.46

quantities. Since one constraint of this problem was the vehicle capacity, the computational method utilized vehicle capacity, as well. Vehicles with high capacities for delivery small replenishment quantities may be advantageous in decreasing transportation costs, especially when the fixed transportation costs are high. Thus, analysis of variance, ANOVA, was applied and the results are shown in Table 7.

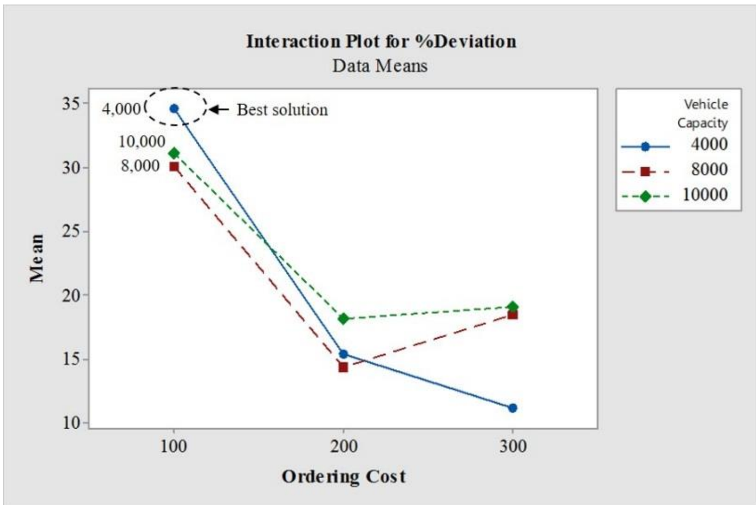
Form Table 7, there is significant difference in the percent deviation of ordering and holding costs since their *P-values* are lower than 0.05. However, there was not a significant *percent deviation* based on vehicle capacity. There was a significant interaction effect for ordering and holding costs and an interaction effect of vehicle capacity and ordering costs. Since two-way interactions exist, graphical analysis is needed. According to Figure 3(a), a vehicle capacity of 4,000 with a setup cost of 100 gave the highest percentage deviation. In accordance with Figure 3(b), a setup cost of 100 and a holding cost of 20% resulted in the highest percentage deviation. Based on the evidence, use of IOVRP is advantageous over IRP when with small vehicle capacities for supplying products with low ordering costs and high holding costs. This results in a smaller replenishment quantities.

5. Conclusions and recommendations

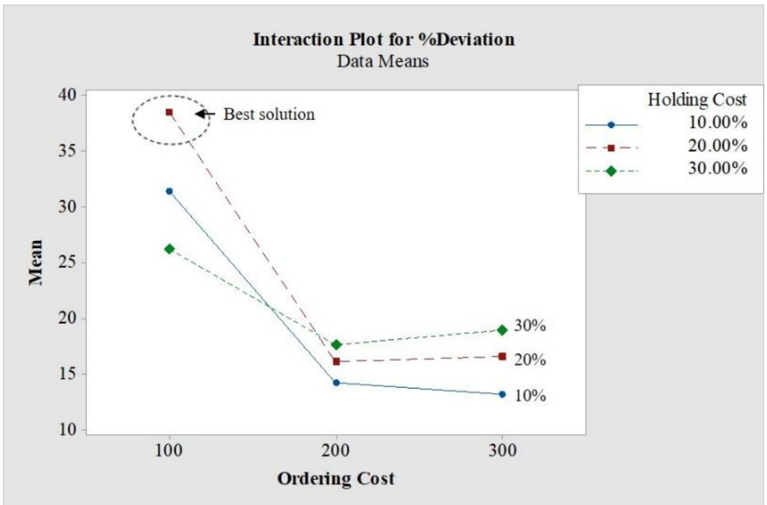
This research applies the economics of order quantities with a power-of-two policy and Ant Colony Optimization to solve for practical replenishment schedules in a multiple depot and multiple retailer distribution system. The delivery routes were created using the IOVRP concept developed through application of an Ant Colony Algorithm. The solutions of a closed route (IRP) and opened route (IOVRP) were compared by calculating their percentage deviation.

Table 7 Analysis of variance of FAT 3X3X3 percent deviation

Source	DF	Adj. SS	Adj. MS	F-value	P-value
Model	14	1995.40	142.528	29.19	0.000
Linear	6	1631.70	271.951	55.69	0.000
Vehicle Capacity	2	28.40	14.198	2.91	0.093
Ordering Costs	2	1522.51	761.257	155.88	0.000
Holding Costs	2	80.79	40.397	8.27	0.006
2-way Interactions	8	363.69	45.462	9.31	0.000
Vehicle Capacity*Ordering Costs	4	146.60	36.651	7.50	0.003
Ordering costs*Holding Costs	4	217.09	54.273	11.11	0.001
Error	12	58.60	4.884		
Total	26	2054.00			



(a)



(b)

Figure 3 Interaction plot of (a) vehicle capacity and ordering costs, and (b) holding costs and ordering costs

From the experimental results, IOVRP gave lower total transportation costs than IRP, since the average percent deviation was about 24.66%. Additionally, the IOVRP gave lower fixed-transportation costs because the number of routes was less than for the IRP. According to the sensitivity analysis, a small vehicle capacity supplying products with low setup costs and high holding costs using the open route concept of IOVRP gave lower transportation costs than a closed route IRP. However, this research is constrained by various parameters of the case study. These are the inventory parameters, related ACO parameters such as the weighted control of pheromone and attractiveness, the number of ants, and the assigned vehicle capacity. Even though some of the parameters were studied through application of sensitivity analysis, the solutions may be valid only for the given study parameters' interval. Extended research is needed to increase the number of retailers and compare the computational times of other algorithms in the literature.

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