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Controllers for balancing two wheeled inverted pendulum robot with PI feedback control

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Abstract

Two-wheeled inverted pendulum (TWIP) robot has been widely investigated because the system is nonlinear and unstable. The purpose of this study is to control the balance of TWIP. The PI control is employed because the limitation of hardware. To obtain the PI gain, actual TWIP has been implemented, where K_p = 50 and K_i = 4. The results show that the balance of TWIP can achieved with an error of 0.32°.

Keywords: Two-wheeled inverted pendulum, PI controller, Lagrange equation, Manual tuning

1. Introduction

The Two-wheeled inverted pendulum (TWIP) robot is a challenge engineer problem as it is nonlinear and unstable. Therefore, many research studies conducted concerning TWIP; such as, (A) mathematical modeling of dynamic systems and (B) design of the controller.

Mathematical model is used to describe relation of parameters in TWIP. Two commonly used techniques are Newton method [1-2] and Lagrange method [3]. The Newton method employs vector concept while the Lagrange method utilizes energy consumption concept. The later has an advantage on simplicity of the computation.

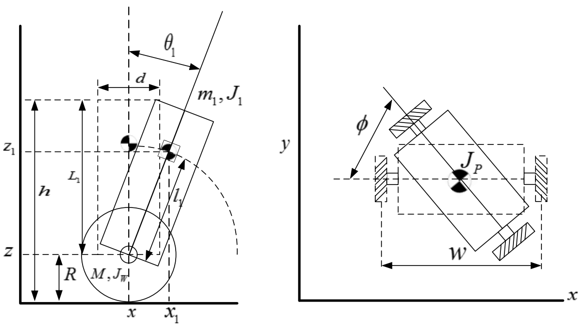
Design of the control algorithm is to obtain parameters which can balance the TWIP. Method that popularity of industrial techniques is Proportional Integral Derivative (PID). While the TWIP often uses Proportional Derivative (PD) controller [4], PID controller [5] and some research combined Linear Quadratic Controller (LQR) with PID [3].

The purpose of this research is to balance TWIP using Proportional Integral (PI) controller. This is because the onboard hardware limits its complexity of the control algorithm. The paper is structured as follows. After introduction of TWIP, TWIP hardware and mathematical model are presented, followed by the experimentation to obtain K_p and K_i. Result of balancing TWIP, and conclusions have been made, respectively.

2. TWIP hardware and mathematical model

2.1 TWIP physical hardware

TWIP consists of two main parts; two identical wheels and TWIP body (pendulum), as shown in Figure 1. The pendulum is a rack which carry controller board, battery, and sensor. All loads are distributed symmetrically.



(a) (b)

(c)

Figure 1 TWIP robot (a) side view (b) top view (c) actual robot

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2.2 TWIP electronics hardware

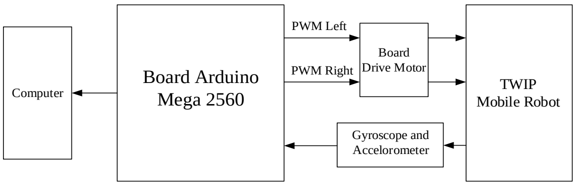

Figure 2 Electronics hardware of TWIP

Electronics hardware of TWIP (Figure 2) consists of Arduino Mega 2560 board which is a control unit and it creates signal control, pulse width modulation (PWM). Accelerometer and gyroscope module (MPU6050) is employed to measure pitch angle (θ_1) of TWIP. This angle is then fed into the control unit to be a feedback signal and used in control algorithm of TWIP. The control unit will pass information via serial communication to computer in order to record parameter's values.

2.3 Mathematical models of dynamical for TWIP

Lagrange Equation of Motion formula, Eq.1, is applied to determine the mathematical models of TWIP. Information of all parameter is represented in Table 1.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i \qquad (1)$$

Where L=T-V is Lagrange function or Lagrangian equations and it is the difference between kinetic energy (T) and potential energy (V). Q_i is generalized force which uses torque (τ) from motor DC as an input system. Let the generalized coordinate (q_i) of TWIP be $q_i = (x, \theta_1)$where x is a linear position of TWIP. θ_1 is a pitch angle of TWIP, $\dot{q}_i = (\dot{x}, \dot{\theta}_1)$ where $\dot{x}$ is a velocity of linear position and $\dot{\theta}_1$ is an angular velocity of pitch angle of TWIP. Therefore, the dynamics of Lagrange equation can be obtained as follows.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = \left(m_1 + 2M + 2\frac{J_W}{R^2}\right)\ddot{x} + m_1 l_1\left(\ddot{\theta}_1\cos\theta_1 - \dot{\theta}_1^2\sin\theta_1\right) = Q_x \qquad (2)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_1} - \frac{\partial L}{\partial \theta_1} = m_1 l_1 \ddot{x}\cos\theta_1 + \left(m_1 l_1^2 + J_1\right)\ddot{\theta}_1 - m_1 g l_1 \sin\theta_1 = Q_{\theta_1} \qquad (3)$$

Eq. 2 and Eq. 3 are the nonlinear equation of TWIP. To linearize these equations, Taylor Series is applied where a normal operating point or equilibrium point is set as $\theta_1 \approx 0$, resulting $\cos\theta_1 \approx 1$, $\sin\theta_1 \approx \theta_1$, $\dot{\theta}_1^2 \approx 0$. Cramer's rule is then used to find acceleration of linear position ($\ddot{x}$) and angular acceleration of pitch angle ($\ddot{\theta}_1$). These can be calculated as in Eq. 4 and Eq. 5

$$\ddot{x} = 194.62\tau - 14.24\theta_1 \qquad (4)$$

$$\ddot{\theta}_1 = -3394.51\tau + 419.48\theta_1 \qquad (5)$$

Let's X = $[x\ \dot{x}\ \theta_1\ \dot{\theta}_1]^T$ where X is a state vector.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & -14.24 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 419.48 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 194.62\\ 0\\ -3394.51\end{bmatrix}\tau \qquad (6)$$

$$Y = \begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}$$

Converting state space equation, Eq.6, to determine system transfer function; $\frac{Y(s)}{U(s)} = C[sI - A]^{-1}B + D$. Therefore, the system has a right half plane pole, thus, TWIP is unstable, as shows in Eq.7. Symbols and value of parameters are represent in Table 1.

$$\frac{\theta_1}{\tau} = \frac{-3394.51s^2}{s^4 - 419.48s^2} = \frac{-3394.51}{s^2 - 419.48} = \frac{-3394.51}{(s+20.48)(s-20.48)}$$
(7)

Table 1 Parameters of TWIP

Order	symbol and value	Unit	Parameter
1	$g = 9.81$	$m/\sec^2$	Gravity acceleration
2	$m_1 = 0.585$	Kg	Mass of the pendulum
3	$M = 0.052$	Kg	Mass of one wheel
4	$R = 0.034$	m	Radius of the wheel
5	$L_1 = 0.129$	m	Distance of the pendulum $L_1 = (h - R)$
6	$l_1 = 0.043$	m	Distance from the point z to the center of gravity
7	$w = 0.130$	m	Distance between the left and right wheel
8	$d = 0.070$	m	Distance width of TWIP
9	$h = 0.163$	m	Distance height of TWIP
10	$J_1 = 0.00036$	Kgm^2	Moment of inertial of the platform about the pitch-axis
11	$J_P = 0.00106$	Kgm^2	Moment of inertial of the platform about the yaw-axis
12	$J_W = 0.00003$	Kgm^2	Moment of inertial of the wheel

3. Designed gain K_p and K_i

A control law can be written as $u = K_p E(s) + \frac{K_i}{s}E(s)$. In order to find the gain of PI controller using manual tuning [6], firstly, determine the K_p by set $K_i = 0$ and then vary K_p. Observe pitch angle (output) of TWIP until it starts to oscillate, set K_p to be critical gain (K_{cr}). The designed gain K_p is then calculated $K_p = K_{cr}/2$. Figure 3 shows the time response of gain K_p where K_{cr} is 100. Thus, suitable K_p is 50.

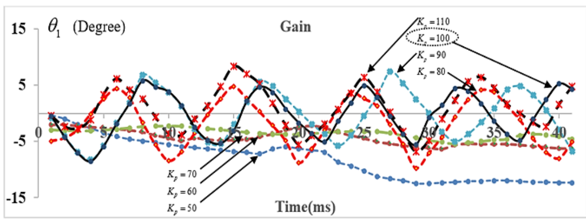

Figure 3 Time response to find gain K_{cr} from experiment

Secondly find the gain K_i where K_p is set to 50. Increased value K_i from 0 until system is stable. Figure 4 shows the time response related to gain K_i, where selected K_i

is 4. Therefore, the designed gain of TWIP is $K_p = 50$ and $K_i = 4$.

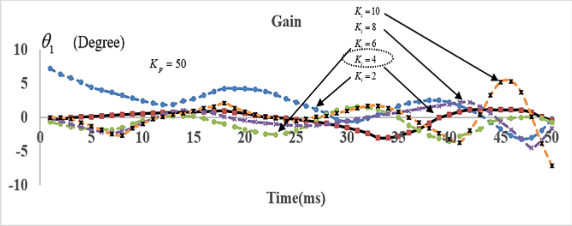

Figure 4 Time response to find gain K_i from experiment where $K_p = 50$.

4. Results

TWIP is then applied the designed gain, $K_p = 50$ and K_i=4, and it gives a satisfied result, as shown Figure 5.

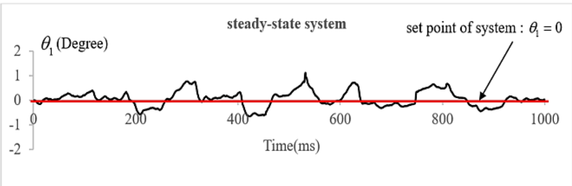

Figure 5 Time response TWIP

Figure 5 shows time response of TWIP that the pitch angle has a small error of 0.32° calculated by root mean square (RMS) of actual $\theta_1, \theta_{1RMS} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\theta_i^2}$.

5. Conclusions

This research proposes a mathematical model of TWIP using Lagrange method. Equation of TWIP is nonlinear. To linearize these equations, Taylor Series is applied. PI control is employed because of hardware limitation. The gain K_p and K_i is designed from experiment where $K_p = 50$ and K_i= 4. The TWIP shows that it can balance well with a small error of 0.32°.

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