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A mixed-integer linear program for supply chain management problem: case study of dairy department, the Thai Vet Army School

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Abstract

This study focuses on a mathematical model for a supply chain management problem, which consists of production planning, inventory control, and transportation problems. In this paper, milk supply chain optimization problems are formulated as a mixed-integer linear programming problem considering production lot-sizing and inventory design with a discrete time finite horizon (multi-time periods). The proposed method seeks to maximize system-wide profits while satisfying demand for milk from multiple consumers. The milk supply chain problem of the dairy department in the Thai Vet Army School are considered as a case study.

Keywords: Supply chain, Optimization, Integer program, Lot-sizing

1. Introduction

A supply chain is a system of infrastructures and activities that move the products from suppliers to consumers. In general, supply chain planning is a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and retailers, so that goods are produced and distributed at the right quantities, to the right locations, and at the right time with the optimal system-wide costs [1]. Supply chains have attracted considerable research attention, and many literatures have been published in the past few decades. Cohen and Lee [2] present a deterministic mathematical programming to solve the supply chain problem. The paper [3] proposes the idea of stochastic modelling to solve a supply chain management problem. Berry and Naim [4] apply simulation technique to imitate supply chain. In addition, the work by [5] implements artificial intelligence and soft computing to the supply chain problem. Lastly, an excellent review of research pertaining to supply chain can be found in [6].

The dairy chain is considered as one of the complex supply chain problems. The dairy chain starts at raw milk production and ends when the processed milk is delivered to the consumers. Technically, the dairy supply chain involves production schedule, transportation, and inventory management. Number of studies have been conducted to study the dairy supply chain problem, see [7] and [8] for more details. In general, the milk producer face two challenges: 1 forecasting the demand for milk and 2. Supplying the production to meet demand with an optimal profit. These two challenges are the motivation behind this study. The main contributions in this paper are two-fold. The paper applies the mathematical model to dairy supply chain problem identifying an optimal schedule for production, purchasing, transportation, and inventory. This optimization framework is generalizable and can be applied for other studies. Second, this paper applies the model with the real-life case study.

The paper is structured as follows. Section 2 presents a discussion of Problem Statement and methodology. Section 3 provides the model formulation. Section 4 discusses the results of the case study and Section 5 presents the sensitivity analysis of the mathematical program. Lastly Section 6 summarizes the research study.

2. Problem Statement and methodology

2.1 Problem statement

A Dairy department as a part of Thai Vet School is located near the Chulachomklo Royal Military Academy (CRMA). The first priority of its mission is to provide pasteurized milk to cadets daily. The excess milk production will be delivered to Kunakhon kindergarten or sold to retail markets. However, if the dairy department cannot produce raw milk to meet the demand from cadet mess hall, additional raw milk will be purchased from Weehandang milk cooperative located 65 kilometers away in Sara Buri province. Figure.1 describes the representation of a dairy department and its relationship to other organizations.

It is important to note that the orders from a cadet mess hall are placed daily depending on the number of cadets, which stochastically changes every day due to their missions. Figure 2. shows the comparison of milk consumption of cadet mess hall and production of the dairy department. The milk production in winter (Dec-Feb) is higher than other

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seasons in the year. Figure. 2 also shows that the milk production in Jan-Apr is much more than consumption. This is due to the fact that all cadets have filed training exercises in other locations so that no orders will be placed from the cadet mess hall.

In term of supply side, the diary department has the maximum production capacity of approximately 750 kilogram (k.g.) per day. Since the demand and supply for milk are not balanced in nature, the objectives of this work are to construct a mathematical model that helps a dairy department to minimize total cost of production and inventory. Also, the model should determine the optimal daily production and inventory plan. In particular, the study should answer the following questions:

1. What are the optimal production and inventory quantities?
2. What are the optimal quantities that will be purchased and sold to the Wihandang corporative?
3. How much milk should be delivered to each consumer?

The questions listed above are overly complex to be solved simultaneously, and therefore they must be addressed with the use of the optimization programming. For the supply chain planning level, this paper wishes to determine the production schedule and inventory that will meet all demands as well as identify the optimal quantities that are purchased and sold to the market in order to minimize the sum of production, inventory, and transportation costs.

2.2 Methodology

We characterize the problem (production lot-sizing, inventory, and optimal order problems) as the decision making problem in multi-time periods (daily). The first part of the problem is the production and inventory problem which is used to determine the production and inventory quantities in each time period, and the second part of the problem is the optimal order problem which determines how much milk is bought and sold to the market. This paper uses the mathematical model to formulate the problem as a deterministic mixed-integer linear program.

3. Formulation

This framework considers the time period indexed $t \in T$ and aims to optimize production and inventory quantities in given corresponding demand from cadet mess hall as well as determine the optimal purchase and sold orders in each time period t. Therefore, we discretize time on time periods (days) indexed $t \in T$ and seek to determine production and inventory quantities in each day given the corresponding demand. We begin by introducing notation which is organized into sets and indices, decision variables, and parameters.

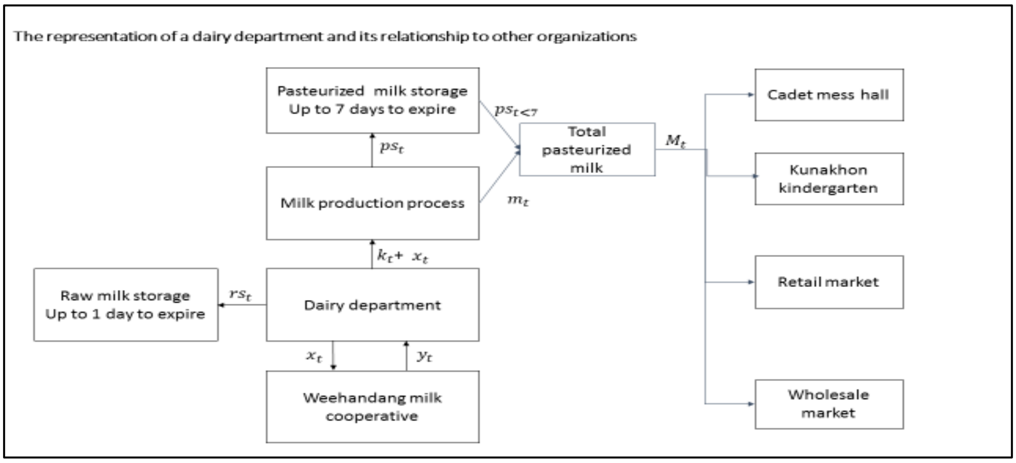

Figure 1 The representation of the milk supply chain problem

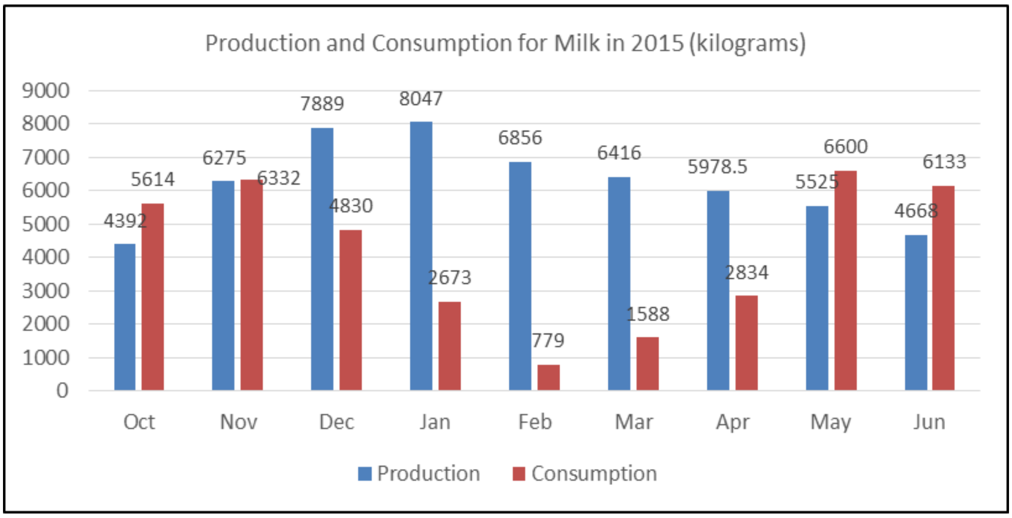

Figure 2 Milk production and consumption from cadet mess hall

Set and indexes:
j Consumers e.g., {cadet mess hall, retail market}
t Time periods (days)

Positive variables:
R_t Total amount of raw milk used for pasteurizing process at time t (k.g.)
x_t Amount of raw milk sold to the Weehandang cooperative at time t (k.g.)
y_t Amount of raw milk bought from the Weehandang cooperative at time t (k.g.)
rs_t Amount of raw milk stored in the storage at time t (k.g.)
ps_t Amount of pasteurized milk stored in the storage at time t (k.g.)
m_t Pasteurized milk produced in time t (k.g.)
M_t Total Pasteurized milk produced in time t (k.g.)
q_{jt} Amount of pasteurized milk shipped to consumer j at time t. (k.g.)

Binary variables:
$w_t \in \{0,1\}$ Impose a fixed cost for transportation to Weehandang cooperative

Parameters:
$\bar{M}$ Maximum daily milk production capacity (k.g.)
$\bar{S}$ Maximum storage capacity (k.g.)
$\overline{Sh}$ Maximum shipping capacity (k.g.)
D_{jt} Demand for milk from the consumer j (k.g.)
k_t Amount of raw milk produced by a dairy department at time t (k.g.)
Z Large positive number
$Pcost_t$ Cost of producing milk in time period t
$Bcost_t$ Cost of buying raw milk from Weehandang cooperative at time period t
$Scost_t$ Cost of storing product in time period t
$Shcost_t$ Cost of transporting milk to and from Weehandang cooperative in time period t
PC Price per k.g. of raw milk sold to Weehandang cooperative
PM_j Price per k.g. of pasteurized milk sold to Weehandang cooperative

It is important to note that we assume that raw milk, sugar, and flavour are required to produce one unit of pasteurized milk. Moreover we do not explicitly include other ingredients in the formulation. The mathematical formulation is given below, together with explanations.

3.1 Overall of a mixed-integer linear programming:

$$Maximize\ profit = \sum_t \big(PC * x_t + \sum_j PM_j * q_{jt}\big) - \sum_t (Pcost_t * m_t + Bcost_t * y_t + Scost_t * (rs_t + ps_t) + Shcost_t * w_t) \quad (1)$$

s.t.

$$R_t = k_t + y_t + rs_{t-1} - rs_t - x_t\ ,\ \forall t \quad (2)$$
$$m_t \le R_t - 2 \quad ,\forall t \quad (3)$$
$$\sum_i m_{it} \le \bar{M}\ ,\forall t \quad (4)$$
$$M_t = m_t + ps_{t-1} - ps_t\ ,\forall t \quad (5)$$
$$rs_t + ps_t \le \bar{S}\ \ ,\forall t \quad (6)$$
$$\sum_j q_{jt} \le M_t\ \ ,\forall t \quad (7)$$
$$q_{jt} \ge D_{jt},\ \forall j,t \quad (8)$$
$$x_t + y_t \le Z * w_t\ ,\forall t \quad (9)$$
$$x_t \le \bar{S}\ \ ,\forall t \quad (10)$$
$$y_t \le \bar{S}\ \ ,\forall t \quad (11)$$
$$R_t, M_t, y_t, rs_t, x_t m_t, ps_t, q_{jt} \ge 0$$
$$w_t \in \{0,1\}$$

The objective function, Equation (1), is to maximize the total profit over time horizon. The term, $\sum_t \big(PC * x_t + PM_j * q_{jt}\big)$, represents sales while the term, $\sum_t (Pcost_t * m_t + Bcost_t * y_t + Scost_t * (rs_t + ps_t) + Shcost_t * w_t)$, presents the total cost, including the cost of producing and storing raw and pasteurized milk, and shipping cost. Equation (2) is a set of raw milk balance constraints that identify the amount of raw milk, R_t, which results from raw milk bought from the Weehandang cooperative, y_t, plus raw milk produced at the dairy department, k_t, and the milk stored in the storage at a day1 before day t, rs_{t-1}, and less the raw milk sold to the Weehandang cooperative at time t, x_t, and raw milk stored in the storage at time t, rs_t. Equation (3) is a set of resource constraints that limit the total of pasteurized milk production at time period t less than the total raw milk at time t less 2 k.g. of raw milk. The milk production process takes about 2 k.g. of raw milk each time of production for start-up process. Equation (4) is a set of production capacity constraints that restrict the total of pasteurized milk at time t less than the maximum capacity of milk production. Equation (5) is a set of pasteurized milk inventory balance constraints indicate the total milk in the t equals the pasteurized milk produced at time period t and the milk stored in the storage one time period before the current time period less the pasteurized milk stored in the storage at time period t, ps_t. Note that the processed milk can be stored up to seven days. Equation (6) is a set of storage capacity constraints that provide the upper bound for storing raw and pasteurized milk. Equation (7) is a set of availability of pasteurized milk constraints that limit the total pasteurized milk transported to the consumers less than the total amount of pasteurized milk in time period. Equation (8) is a set of demand requirement constraints which ensure that demand for pasteurized milk for each consumer is met. Equation (9) impose a fixed shipping cost with binary variables if the transportation to Weehandang corporative is required. Equations (10-11) restrict the shipping capacity each trip from and to Weehandang cooperative.

Finally, mathematical program (1)-(11) can be solved using any commercial optimization package including the General Algebraic Modeling System (GAMS) (Rosenthal, 2015). The output of the model will provide answers to the first three questions listed in the problem statement.

4. Results

This section presents a numerical example to illustrate the proposed model. The model uses the actual historical data in October, 2014 from the dairy department. Moreover, the model has been ran in GAMS up 15 time periods (days). This example has 181 continuous variables, 15 discrete variables, and 196 constraints.The production schedule, inventory plan, and distribution are presented in Table 1, 2, and 3. The model finds the optimal profit of 72,694 baths. Moreover, as shown in Table 1, the results indicates that the dairy department purchases a lot of raw milk from Weehandang corporative, produce pasteurized milk , and store some of pasteurized milk before delivery to the consumers. Moreover, Table 2. shows that all demand for milk from multiple consumers is satisfied.

[1] It is important to note that the raw milk can be stored only 1 day in the refrigerator.

Table 1 Raw milk in litters purchase, production, and storage planning

Day	Purchase	Production	Storage
1	204	102	0
2	192	104	0
3	300	109	0
4	0	110	0
5	0	113	0
6	291	115	0
7	193	120	0
8	196	117	0
9	169	120	0
10	195	117	29
11	0	120	0
12	0	112	0
13	0	119	0
14	313	113	0
15	180	126	0

Table 2 Pasteurized milk production and storage planning

Day	Production	Storage	Total Sales
1	304	0	304
2	304	0	304
3	407	83	324
4	108	30	161
5	101	0	131
6	404	0	404
7	314	0	314
8	314	0	314
9	284	0	284
10	284	0	284
11	139	109	30
12	118	187	40
13	117	0	304
14	424	0	424
15	304	0	304

5. Sensitivity Analysis

The sensitivity analysis on the variation of the objective function coefficient has been conducted. Various objective function coefficients (milk prices and different costs) were chosen and ran in the model. Changing coefficients of milk prices has a smaller range as compared to the change in coefficient of production, shipping, inventory costs before affecting the optimal solution.

6. Conclusions

This paper presents a technique for optimizing a milk supply chain management problem for the dairy department, the Thai Army Vet School. The comprehensive deterministic mixed-integer linear model has been proposed to maximize the total profit of supply chain system.

Table 3 Distribution of milk to each consumer

Day	Mess hall	Retails	Kunakhon	Thai Vet
1	254	30	0	20
2	254	30	0	20
3	254	50	0	20
4	91	70	0	0
5	91	40	0	0
6	254	60	90	0
7	254	60	0	0
8	254	30	0	0
9	254	30	0	0
10	254	30	0	0
11	0	40	0	0
12	0	50	0	0
13	254	80	90	0
14	254	50	0	0
15	254	40	0	0

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