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# Hybrid of scattering matrix method and wave iterative algorithm for waveguide cascaded irises

Pinit Nuangpirom\*1) and Somsak Akatimagool<sup>2)</sup>

<sup>1)</sup>Faculty of Engineering, Rajamangala University of Technology Lanna, Chiang Mai, Thailand.

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#### Abstract

In this paper, the application of the wave scattering matrix method of two domains using wave iterative algorithm is presented in order to obtain the incident wave, reflected wave and transmitted wave parameters of the waveguide iris. Traditionally, the conversion of the wave scattering matrix to the frequency response and electromagnetics characterization has been used in order to perform the cascaded connection in waveguide filters. The numerical results of waveguide bandpass filter using the wave scattering parameters integrated with wave iterative procedures, were reported as efficient examples.

Keywords: Scattering matric method, Wave iterative algorithm, Waveguide cascaded irises

### 1. Introduction

Modern analysis of microwave circuits with the help of powerful simulation tool makes to precision analysis. In the design and analysis of microwave components is being used by numerical method, such as the finite Differential Time Domain (FDTD) [1], Method of Moments (MOM) [2]. Most high frequency two port networks are characterized in terms of scattering parameters, which have been arranged in matrix forms and related transmitted and reflected waves in the input and output ports [3]. Likewise, The modeling of microwave systems formed by subsystems usually characterized by mono modal scattering matrix. On the other hand, the characterization of microwave systems or components formed by networks connected in usable cascaded connection of two-port scattering matrix in order to obtain the overall scattering parameters. Especially, one of the classic problems in waveguide iris filter is a design and analysis of an optimum equalizer to match an arbitrary multilayer load (cascaded irises) to generators with complex internal impedance of two port network [4].

In this paper, we present a novel numerical technique for the efficient cascaded connection of N multimodal two-port scattering matrix. The proposed methods based on wave iterative algorithm apply to analyze waveguide filter.

### 2. The modal scattering matrix of length of waveguide

Considering the 2-port network as shown in Figure 1, where Vn is the amplitude of the voltage wave on port n, and In is the amplitude of the current wave on port n. The modal Scattering matrix, or S matrix, is defined in relation to these incidents (an) and reflected waves (bn) as [5].

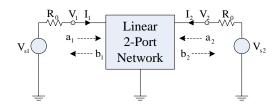


Figure 1 Two port network with scattering waves

$$\begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{bmatrix} \tag{1}$$

A specific element of the S matrix can be determined as

$$S_{ij} = \frac{b_i}{a_j} \tag{2}$$

In equation (2) says that Sij is the transmission coefficient from port j to port i, Sii is the reflection coefficient seen looking into port I when all other ports are terminated in matched loads. Therefore, the form of the scattering matrix of a length of waveguide for a propagation mode is often taken as [6].

$$\begin{bmatrix} 0 & e^{-\gamma l} \\ e^{-\gamma l} & 0 \end{bmatrix}$$
 (3)

Where  $\gamma$  is the propagation constant of the mode above cutoff frequency.

<sup>&</sup>lt;sup>2)</sup>Faculty of Technical Education, King Mongkut's University of Technology North Bangkok, Thailand.

### 3. The scattering matrix of waveguide iris

A centered symmetrical obstacle of zero thickness with its edges to the electric field (TE<sub>10</sub>-mode in rectangular guide) [7], as shown in Figure 2.

At the iris surfaces of the discontinuity, the boundary conditions of tangential fields are expressed in terms of waves which consist of two conditions as

Case 1, on the metal regions (*M*) (Figure 3a), we have the condition;  $E_1 = E_2 = 0$  thus the wave relation in the region 1 and 2 can be represented as follows:

$$B_1 = -A_1 \tag{4}$$

$$B_2 = -A_2 \tag{5}$$

Matrix defined as

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \tag{6}$$

Case 2, on the dielectric regions (D) (Figure 3b), we have the conditions;  $E_1=E_2$  and  $J_1+J_2=0$  the wave relation can be represented as follows:

$$B_1 = A_2 \tag{7}$$

$$B_2 = A_1 \tag{8}$$

The matrix can written as,

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \tag{9}$$

Considering S matrix, the summation of (6) and (9) as follow:

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} -S_M & S_D \\ S_D & -S_M \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$$
 (10)

Where  $S_M$  and  $S_D$  are the scattering matrix on metal and dielectric regions, respectively.

## 4. The hybrid of scattering matrix method and wave iterative algorithm

The efficient Wave Iterative Algorithm (WIA) has been developed by improving the wave concept iterative procedure (WCIP) [8]. In this paper we illustrate an optimized WIA approach to electromagnetic analysis applied to waveguide filter, as shown in Figure 4.

Starting from iterative procedure, the exciting wave  $(A_{\theta})$  is converted to the reflected wave  $B_{i}^{(n)}$  of i waveguide side by the S matrix. Upon application of equation (10), we obtain

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}^{(n)} = \begin{bmatrix} -S_M & S_D \\ S_D & -S_M \end{bmatrix}^{(n-1)} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} + \begin{bmatrix} -S_M A_0 \\ 0 \end{bmatrix}$$
(11)

We have related all wave  $A_n$  and  $B_n$  by S matrix and determine the scattering parameters of the two-port network. Using equation (11) is as follow:

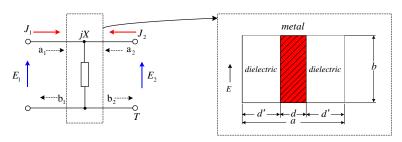
$$B_1^{(n)} = -S_M A_1^{(n-1)} + S_D A_2^{(n-1)} - S_M A_0$$
 (12)

$$B_2^{(n)} = S_D A_1^{(n-1)} - S_M A_2^{(n-1)}$$
(13)

The same procedure can be followed in order to relate wave  $\underline{A}_n$  and wave  $B_n$  by using modal reflection coefficient and S matrix of a length of waveguide, as follows:

$$A_{1} = \left[\Gamma\right] B_{1} \quad , \quad A_{2} = \left[e^{-\gamma l_{1}}\right] B_{3} \quad , \quad A_{3} = \left[e^{-\gamma l_{1}}\right] B_{2} \quad ,$$

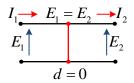
$$A_{4} = \left[e^{-\gamma l_{2}}\right] B_{5} \quad , \quad A_{5} = \left[e^{-\gamma l_{2}}\right] B_{4} \quad , \quad A_{1} = \left[\Gamma\right] B_{6} \quad (14)$$



(a) Scattering wave of waveguide iris

(b) Inductive Iris Structure

Figure 2 inductive obstacle structures



(a) Metal region

$$I_1 \longrightarrow E_1 = E_2 \longrightarrow I_2$$

$$E_1 \uparrow \qquad \uparrow E_2$$

$$J_1 + J_2 = 0$$

(b) Dielectric region

Figure 3 Equivalent circuit of iris discontinuity

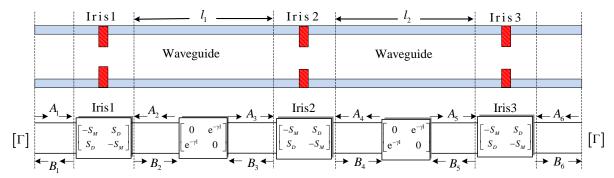


Figure 4 Wave Propagation in waveguide irises

The expression of modal reflection coefficient at the input and output side of waveguide in the spectrum domain is given by

$$\Gamma_{i}^{TE/TM} = \frac{1 - Z_{0i} Y_{m,n}^{TE/TM}}{1 + Z_{0i} Y_{m,n}^{TE/TM}}$$
(15)

where the  $TE_{m,n}$ ,  $TM_{m,n}$  mode admittances in the metallic box are  $Y_{m,n}^{TE} = \frac{\gamma}{j\omega\mu_0\mu_r}$ ,  $Y_{m,n}^{TM} = \frac{j\omega\varepsilon_0\varepsilon_r}{\gamma}$  respectively,  $\gamma = \sqrt{(m\pi/a)^2 + (n\pi/b)^2 - k_0^2\varepsilon_r}$ , and  $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$ .

To simplify the propagated wave calculation of two domains, the Modal - FFT function permits movement the transverse filed components from the real domain to the spectrum domain, we can write the  $TE_{m,n}$  mode wave equation as follows:

$$\begin{bmatrix} B_{(m,n)}^{TE} \\ B_{(m,n)}^{TM} \end{bmatrix} = Q_{m,n} \begin{bmatrix} n/b & -m/a \\ m/a & n/b \end{bmatrix} FFT \begin{bmatrix} B_x \\ B_y \end{bmatrix}$$
(16)

On the other hand, the Modal - FFT function permits movement the modal filed components from the spectrum domain comeback to the real domain, we can write the wave equation in x, y direction as follows:

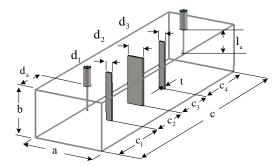
$$\begin{bmatrix} A_{x} \\ A_{y} \end{bmatrix} = FFT^{-1} \left\{ \frac{1}{Q_{m,n}} \begin{bmatrix} n/b & -m/a \\ m/a & n/b \end{bmatrix}^{-1} \begin{bmatrix} A_{(m,n)}^{TE} \\ A_{(m,n)}^{TM} \end{bmatrix} \right\}$$
(17)

where: 
$$Q_{m,n} = \sqrt{\frac{ab}{2\Phi_{m,n}}} \frac{1}{\sqrt{(m/a)^2 + (n/b)^2}}, \Phi_{m,n} = \begin{cases} 2 \text{ if } m, n \neq 0 \\ 1 \text{ if } m, n \neq 0 \end{cases}$$

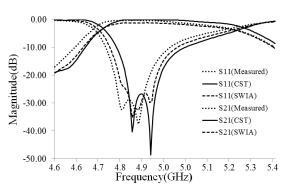
m,n refers the modes number, a,b refers the waveguide dimension.

### 5. Numerical results

To illustrate the validity of this approach, the discontinuity of waveguide cascaded irises was investigated. The propose filter circuit was implement on an aluminum material. The dimensions of the considered waveguide bandpass filter were a =48 mm, b =32 mm, c= 168 mm, c1=c2= 50.6mm, c2=c3=33.4mm, da =30 mm, d1=d3=4mm, d2=11mm, la =18.75 mm, and t= 1 mm, as shown in Figure 5.



(a) The Structure of waveguide iris filter



(b) Frequency response of BPF filter

Figure 5 Waveguide BPF filter

Figure 5 presents the comparison of dB(S11) and dB(S21) of waveguide BPF filter using three inductive irises among measurement, CST simulation and the proposed SWIA (Scattering Wave Iterative Algorithm) method at the operating frequency of 4.6-5.4 GHz (C band). The proposed BPF filter provides the center frequency at 4.96 GHz and bandwidth equal to 510 MHz. The good agreement both three methods are presented.

### 6. Conclusions

The novel mathematical analysis model of the scattering matrix based on the Wave Iterative Algorithm (SWIA) has been presented. The wave scattering matrix method that characterizes the cascaded irises of waveguide BPF filter, was implemented. We have also seen that the simple model functions may not emulate the behavior of the unknown propagated waves well. Thus, the proposed scattering matrix functions of two domains are easy and conveniently to electromagnetic problems solving, which engineers can use

to be a new ideal of particular numerical analysis for microwave filters or related.

### 7. Acknowledgements

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