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# Frequency domain identification of volterra model for separating ultraharmonic using the technique of half-frequency of the input signal

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**Abstract**

This paper presents a method for the identification of ultraharmonic component with Volterra kernels from a simulated ultrasound echo signal. Due to the fact that using a Volterra series cannot respond to the ultraharmonic component, the purpose of the paper is to develop the identification method for the ultraharmonic using the excitation technique of half-frequency of the input signal. In addition, the model can still be held at subharmonic component. Finally the identifications of Volterra system were studied in frequency domain. Application of this method is to separate only the ultraharmonic component or ultra-subharmonic components for improving contrast-to-tissue ratio (CTR) of ultrasound imaging.

**Keywords:** Volterra filter, Nonlinear ultrasound signal, Ultraharmonic imaging

## 1. Introduction

Studies in the issue of ultrasound contrast agents (UCAs) have reported of several investigators on the clinical applications as described in [1-2]. Ultrasound echoes from UCAs include multiple frequency components at fundamental ($f_0$), subharmonic ($f_0/2$), second harmonic ($2f_0$), and ultraharmonic ($3f_0/2$) [3]. Advancements in ultrasound imaging based on contrast agents have been increased in the specificity and sensitivity of diagnostic. Improving image quality in ultrasound is using harmonic components to create an image. However, image contrast enhancement must have good performance of the tools used for separating the harmonic, which is measured by contrast-to-tissue ratio (CTR) [4].

Second harmonic imaging (HI) has been used for many years. However from generating the second harmonic in tissue surrounding, it suffers reduced CTR. Subharmonic imaging (SHI) has been investigated for use owing to the lack of generation in tissue. SHI should have a better CTR due to its higher exciting frequency [5]. Recently, these have been interest in the potential of using ultraharmonic for imaging. In [6-7], they have been reported, including the generating frequency of contrast, use, advantages and a transducer design for ultraharmonic imaging (UHI). UHI possesses over HI and SHI, such as greater Doppler and image resolution. However, it has been hampered by lower signal-to-noise ratio issues.

To create harmonic images using signal processing tools, from the literature review, they consist of three strategies: bandpass filter (BPF), pulse inversion (PI) and Volterra filter (VF). In [8], it has been demonstrated by HI that the VF is better than PI and BPF in terms of CTR. In this paper, we are interested in the capability of the VF for use with ultraharmonic. However for in this case, the VF cannot be modeled. In our previous study of modeled by [9-10], multiple input signal output (MISO) Volterra series was introduced to overcome this problem. This method is success that can be predicted, but cannot be used for filtering frequency components.

Nevertheless identification techniques based on single input signal output (SISO) VF need to identify for the solution from this problem. In order to improve the CTR using the ultraharmonic and VF, the purpose of this paper is to develop an estimation method of the VF identification using the excitation technique of half-frequency of the input signal. The Volterra system resulted in [10] was extended in this study. The results obtained the identification VF in terms of first-, second-, and third-order are analyzed in frequency domain on the issue of separating the ultraharmonic.

## 2. Materials and methods

### *2.1 Ultrasound echo signal*

In this section, the ultrasound echo signal (UES) from nonlinear bubble oscillations is described. UCAs consist of small gas bubbles that are injected into the blood flow to enhance the ultrasound signals from vessels. There are several models on presenting bubble behavior [11]. However for generating of ultraharmonic, the UES can be generated by Church model [12]. Previous paper has reported on the UES simulated from the Church equation, as detailed in [10]. In this paper, we have used by the Church equation. A UES

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of nonlinear bubble system obtained from simulation as displayed in Figure 1.

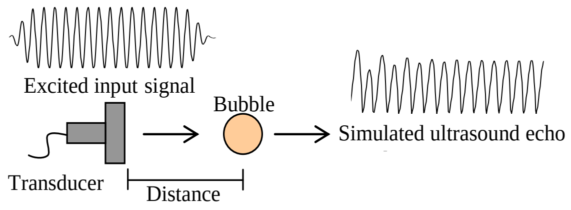


**Figure 1** A nonlinear ultrasound echo system using Church model

*2.2 SISO volterra system*

A single input-single output (SISO) Volterra series is representation for a nonlinear system. The equation of relationship between input and output a discrete-time SISO Volterra series can be written in the form [13]

$$y(t) = h_0 + \int_{-\infty}^{\infty} h_1(\tau_1)x(t-\tau_1)d\tau_1 + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_2(\tau_1,\tau_2)x(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2 + ... + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}...\int_{-\infty}^{\infty} h_k(\tau_1,\tau_2,...,\tau_k)x(t-\tau_1)x(t-\tau_2)...x(t-\tau_k)d\tau_1 d\tau_2 ..d\tau_k \quad (1)$$

where $x(t)$ represents the dynamic of input signal and $y(t)$ represents the system response or output signal. The sum of the convolution integrals contains a kernel or identification, $h_k(\tau_1, \tau_2, \ldots, \tau_k)$, which represents the impulse response of the $k$th-order systems.

A general class by SISO system, the frequency responses are giving higher multiple harmonic ($2f_0$, $3f_0$, …, etc.), when the input excitation is single tone frequency ($f_0$). However, assumption-based modeling in the case of ultraharmonic, the SISO system is by applying the excitation of half-frequency of the input signal ($f_0/2$). Thus, the system responses are adaptation at sub-frequency ($f_0/2$, $3f_0/2$, …, etc). In order to obtain the ultraharmonic at $3f_0/2$, using equation (1) for this work is the truncated third-order Volterra filter (TVF). The relation of the input $x(n)$-output $y(n)$ of system when the Volterra identifications have a finite memory length $N$, which is represented by

$$y(n) = h_0 + \sum_{k_1=0}^{N-1} h_1(k_1)x(n-k_1) + \sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1} h_2(k_1,k_2)x(n-k_1)x(n-k_2) + \sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1}\sum_{k_3=0}^{N-1} h_3(k_1,k_2,k_3)x(n-k_1)x(n-k_2)x(n-k_3) \quad (2)$$

where $h_0, h_1(k_1), h_2(k_1k_2)$ and $h_3(k_1, k_2, k_3)$ are the bias, linear, quadratic and cubic discrete Volterra identifications, respectively. The frequency domain response of the TVF system can be calculated by discrete Fourier transform (DFT) of equation (2).

$$\mathbf{Y}(m) = \mathbf{H}_0 + \mathbf{H}_1(m)\mathbf{X}(m) + \mathbf{H}_2(m_1,m_2)\mathbf{X}(m_1)\mathbf{X}(m_2) + \mathbf{H}_3(m_1,m_2,m_3)\mathbf{X}(m_1)\mathbf{X}(m_2)\mathbf{X}(m_2) \quad (3)$$

where $\mathbf{X}(m)$ and $\mathbf{Y}(m)$ represent the number of $N$ point DFTs of $x(n)$ and $y(n)$, respectively; likewise $\mathbf{H}_0$, $\mathbf{H}_1(m)$, $\mathbf{H}_2(m)$ and $\mathbf{H}_3(m)$ represent $h_0, h_1(k_1), h_2(k_1k_2)$ and $h_3(k_1, k_2, k_3)$, respectively. Here, those of $\mathbf{H}_1(m)$, $\mathbf{H}_2(m)$ and $\mathbf{H}_3(m)$ are called Volterra frequency responses (VFR) of linear, quadratic and cubic terms, respectively.

*2.3 Identification algorithm*

In this section, we describe a method on TVF identification of the sub-frequency by exciting of half-frequency of the input signal. The diagram of system identification is as shown in Figure 2. In order to estimate identification of the TVF model, the equation (2) can be written in the form of linear algebra equation. Let $y(n)$ denote output vector $y$, $x(n\text{-}k_i)$ denote input matrix $\mathbf{X}$, and let $h_i(k_1, k_2, \ldots, k_i)$ denote $\mathbf{h}$, therefore the linear algebra equation is written as

$$\mathbf{y} = \mathbf{X}\mathbf{h} \quad (4)$$

where

$$\mathbf{y} = \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ \vdots \\ y(n) \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} 1 & x(0) & x(-1) & \cdots & x^i(n-M+1) \\ 1 & x(1) & x(0) & \cdots & x^i(n-M+2) \\ 1 & x(2) & x(1) & \cdots & x^i(n-M+3) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x(n) & x(n-1) & & x^i(n-M+L) \end{bmatrix}$$

$$\mathbf{h} = \begin{bmatrix} h_0 \\ h_1(0) \\ h_1(1) \\ \vdots \\ h_i(M-1,...,M-1) \end{bmatrix}$$

and $L$ is input sequence length, $M$ is memory length, and $i$ is *ith*-order Volterra identifications which the order is 3.

Reformulating equation (4) into finding Volterra identification to solve that the equation (4) can be expressed as

$$\mathbf{h} = \mathbf{X}^{\dagger}\mathbf{y} \quad (5)$$

where $\mathbf{X}^{\dagger}$ is generalized inverse which is defined by performing of the singular value decomposition (SVD) method. Expanding equation (5) gives

$$\mathbf{X}^{\dagger} = (\mathbf{U}\mathbf{S}\mathbf{V}^T) = \sum_{i=1}^{r} \sigma_i \mathbf{u}_i \mathbf{v}_i^T . \quad (6)$$

Then, the identification gives

$$\mathbf{h} = (\mathbf{U}\mathbf{S}\mathbf{V}^T)^{\dagger}\mathbf{y} = \sum_{i=1}^{r} \mathbf{v}_i \frac{1}{\sigma_i} \mathbf{u}_i^T \mathbf{y} \quad (7)$$

where **S** is a $M \times N$ diagonal matrix with singular values sorted in descending order $\sigma_1 \geq \sigma_2 \geq \sigma_3 \ldots \geq \sigma_r \geq 0$ ($r$ = min{$M$, $N$}). $\mathbf{U}(M{\times}M)$ and $\mathbf{V}(N{\times}N)$ are unitary matrices, which $\mathbf{U} = (\mathbf{u}_1, \ldots, \mathbf{u}_M)$ and $\mathbf{V} = (\mathbf{v}_1, \ldots, \mathbf{v}_N)$ are orthonormal columns.

The VFR of identifications $h_i(k_1, k_2, \ldots, k_i)$ can be obtained by the DFT, which can be written as

$$\mathbf{H}_i(f_1,\ldots,f_i)=\sum_{k_1=0}^{M-1}\cdots\sum_{k_i=0}^{M-1}h_i(k_1,k_2,\ldots,k_i)\times e^{-(j\frac{2\pi}{M}k_1p+\ldots+j\frac{2\pi}{M}k_ip)} \qquad (8)$$

where $p$ = 0, 1,…, $N$-1.

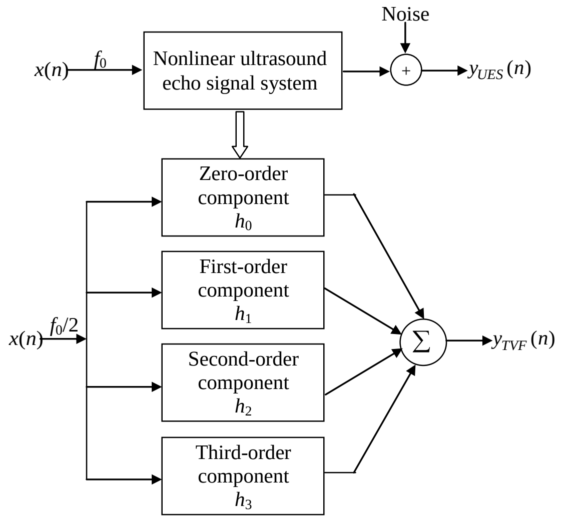


**Figure 2** Identification of the UES using the TVF model

## 3. Results

For in this study, the excitation input signal of the UES simulation is sinusoidal wave at a frequency ($f_0$) of 5 MHz with 1 MPa sound pressure, containing 16 cycles. The UES is added with Gaussian white noise, which signal-to-noise ratio (SNR) of the use is 50 dB. The reception response of transducer is Gaussian - modulated sinusoidal pulse with fraction bandwidth 60% of center frequency 5 MHz. The frequency sampling is 100 MHz. Figure 3 shows the system response of simulating and modeling of the UES and the TVF, respectively. The church model is excited with $f_0$ = 5 MHz obtaining that the UES consists of fundamental ($f_0$ = 5 MHz), subharmonic ($f_0/2$ = 2.5 MHz), ultraharmonic ($3f_0/2$ = 7.5 MHz) and second-harmonic ($2f_0$ = 10 MHz) components.

For the modeling of the UES with the TVF model, using the excitation technique of half-frequency of the input signal can successfully to model sub-frequency. The filter length $N$ is 22. As shown in Figure 3, the predicted result of the TVF in frequency domain is fundamental ($f_0$ = 5 MHz), subharmonic ($f_0/2$ = 2.5 MHz), ultraharmonic ($3f_0/2$ = 7.5 MHz). This is advantage of this technique, in which the generality of Volterra model cannot able to model.
The estimation of filter identification can be calculated by equation (7). Utilization of identification is investigating the frequency response property for filtering frequency. The DFTs of linear, quadratic and cubic terms can be calculated by the equation (8). The frequency response characteristics of these identifications are plotted in Figures 4-6. For investigating in the performance for frequency filtering will be a discussion in section 4.

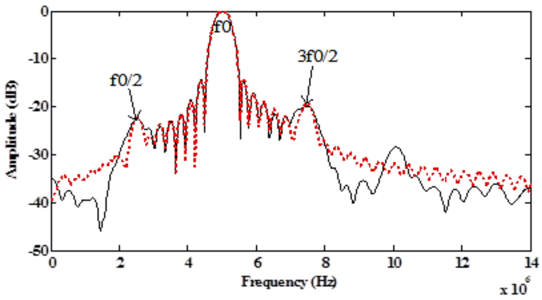


**Figure 3** Results in frequency domain of simulating the UES (solid) and modeling the TVF (dotted). The SNR is 50 dB.

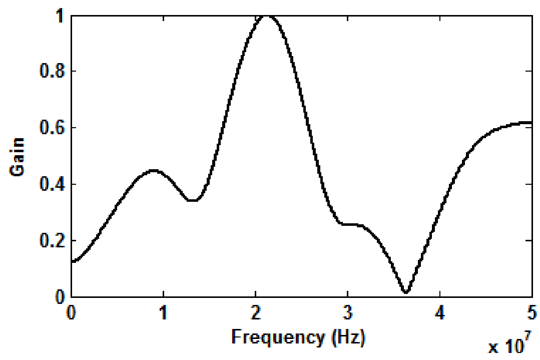


**Figure 4** The VFR of linear filter identifications

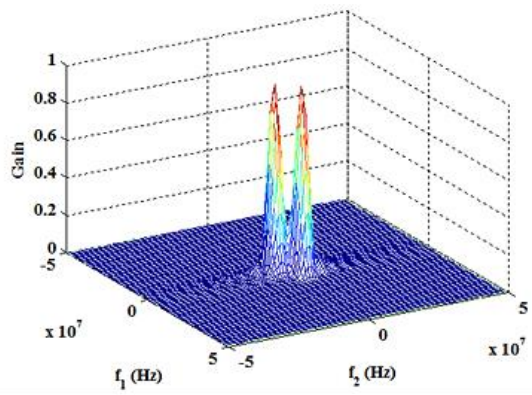


**Figure 5** The VFR of quadratic filter identifications

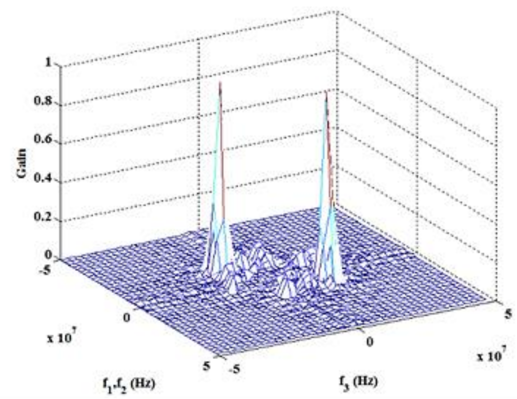


**Figure 6** The VFR of cubic filter identifications

## 4. Discussion

Figure 4 shows the frequency response of linear filter $\mathbf{H}_1$. It can be seen that the peck gain is about 22 MHz. However, it has brandish in some frequency. This result is from frequency translation of the input signal. Only consider of the frequency response $\mathbf{H}_1$ may be sufficient to produce, for this it should be demonstrated in the linear output of frequency response function (FRF). The FRF is relation of cross-identification and input transforms, which is investigation in response to input signals.

Figure 5 shows the response output of quadratic filter $\mathbf{H}_2$. From Figure 5, it can be seen that gain is clearly strong on frequency of passband centered around (5, 5) and (-5, -5) MHz. This is capable of modeling in capture of the fundamental frequency (tissue echo) band. This $\mathbf{H}_2$ ensure a complete for removing the fundamental frequency from tissue echo, which helps improving the CTR.

The third-order model is in the associated frequency response for ultraharmonic, and results in Figure 6. The gain of cubic filter $\mathbf{H}_3$ response is the apparent frequency at around 7.5 MHz (ultraharmonic), and it is little frequency response at around 2.5 MHz (subharmonic). With these frequency responses, the capability of cubic filter can be used for separating the sub- and ultraharmonic components. However, advantage of summing both frequencies is higher CTR value. Therefore, the use of only ultraharmonic for imaging must add filter to remove subharmonic.

## 5. Conclusions

This paper presents the method based on SISO Volterra model of identification nonlinear USE for separating ultraharmonic frequency component. To identify the component of ultraharmonic, we use the method based on third-order Volterra model with the excitation of half-frequency of the input signal. The method has been successfully modeled for the identification of ultraharmonic component. In this work we have been investigated the frequency response of the *k*th-order Volterra identifications for ultraharmonic filtering applications. The results of investigation that cubic term can give the response frequency to ultraharmonic component. This method is significant improvement over conventional linear filtering such as bandpass filter. The practical application of the method can improve CTR. However, in order to demonstrate its performance, we will more progress consider on cross-kernels with input signal, which this issue will be discussed in the future.

## 6. Acknowledgements

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