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Parametric Stability Analysis of a Floating Caliper Disk Brake

T. Jearsiripongkul and P. Hagedorn
Institut für Mechanik, TU-Darmstadt

Abstract

Considerable effort is spent in the design and testing of disk brakes of modern passenger cars. This effort can be reduced if refined mathematical-mechanical models are used for studying the dynamics of these brakes before prototypes are available. The present paper is devoted to the modeling of a floating caliper disk brake, special regard being given to squeal. A new model of a floating caliper disk brake is introduced in order to predict the onset of squeal with frequency of noise ranging between 1 to 5 kHz. The model includes the brake rotor, housing, piston and pads. In this nonlinear model all the prominent features of squeal are reproduced, such as independence of the frequency on the speed, etc. The instability appears even with a constant coefficient of friction between the disk and the pads. Limit cycles are obtained in presence of nonlinearity of the pad stiffness.

Introduction

The noise generated in an automotive disk brake is broadly classified into two main categories depending upon their frequencies. The low frequency noise having a frequency range of 100 to 1,000 Hz is known by various names such as grind, grunt, moan, groan etc. The high frequency component is commonly called squeal and has a broad frequency range between 1 to 12 kHz. Though not hazardous for the operating condition of the brakes, it is mainly the passengers' discomfort, which makes the squeal to be considered as a major problem by automobile manufacturers.

Even after significant amount of theoretical and experimental investigations over a period of nearly fifty years, the researchers are far from being in definite agreement as to the mechanism of squeal. However, it is certain that the friction force causes instability in the disk brake system, which generate noise. The onset of instability has been explained mainly on three possible grounds, namely, change of the friction characteristics with speed of the vehicle [R. A. Ibrahim. 1994., H. Ouyang and J. E. Mottershead. 1998.], change of relative orientation of the disk and the pads and thereby modification of the friction force to lead to divergence instability [N. Millner. 1978.], and flutter type instability even with a constant friction coefficient [H. V. Chowdhary, A. K. Bajaj and C. M. Krousgrill. 2001., S.W.E Earles. 1988., W. D. Iwan and T. L. Moeller. 1976., T. Jearsiripongkul, G. Chakraborty and P. Hagedorn. 2003., M. R. North., U. von Wagner, T. Jearsiripongkul, T. Vomstein, G. Chakraborty and P. Hagedorn. 2003]. Various models have been constructed by their respective proponents to justify their claims. These models are schematic and their relevance to the physical brake system is often difficult to follow. A number of FEM models have also been developed to numerically study the squealing phenomenon and to take suitable measures to alleviate it. They are, however, quite expensive and needlessly complicated.

In this paper, a new simplified model is proposed to reproduce the salient features of squeal. For a moderately wide frequency range (1 – 5 kHz) the transverse vibration of the disk plays significant role during squeal and therefore due importance is given to the vibration of the disk, which is modeled as a flexible rotating plate. The pad stiffness and damping are modeled by distributed nonlinear springs and linear dampers, respectively. The other members of the assembly are taken to be discrete systems. The floating nature of the caliper (sometimes called 'housing') has also been taken into consideration. The model can predict large amplitude oscillations of the disk brake system even with a constant friction coefficient. The effects of various important parameters like braking pressure, angular velocity of the disk etc., are obtained to be in good qualitative agreement with those found in practice.

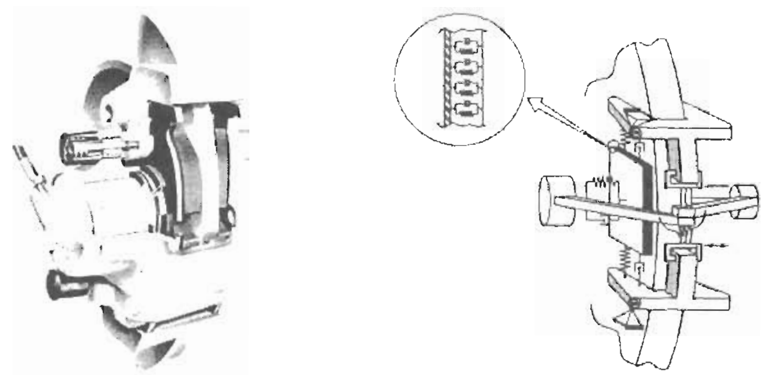
Model Description and Equations of Motion

A floating caliper disk brake, shown in figure 1(a), consists of a brake disk, housing, piston, yoke and pads. During operation oil pressure applied to the cylinder of the housing drives the piston to secure contact between the outer pad and the disk. Owing to the floating nature of the housing the whole assembly moves transverse to the disk bringing the inner pad into firm contact. A schematic model of the disk brake assembly is shown in figure 1(b).

In the model as shown in figure 2, the disk is considered as an annular solid thin plate rotating with angular velocity Ω. The caliper is modeled as two masses (m and M) held together by a caliper arm (equivalent length l) and a rotational spring (stiffness k_T). The cylinder (mass M) is connected to the inner pad (mass m_p) by a spring (stiffness k_c) and damper (damping coefficient d_c). The connection between the caliper (mass m) and the outer pad (mass m_p) is assumed to be rigid. The friction material is modeled as distributed nonlinear elastic springs (k_p) and linear dampers (damping coefficient/area d_p). The applied external force (from oil pressure) P is applied in between the inner pad and the base of the cylinder. In next section, the equations of motion for this model are derived with some assumptions for further simplification.

Equations of Motion

The Hamilton's principle is used to derive the equations of motion for floating caliper disk brake model. For a given system



(a) (b)

Figure 1: A floating caliper disk brake (a) and schematic model (b)

$$\int_{t_1}^{t_2}(\delta L+\delta W)dt=0, \tag{1}$$

where $L=T-U$, T and U being the kinetic and potential energy, respectively, and W is the work done by external applied force. The expression for the kinetic energy term for the rotating brake disk, modeled as a Kirchhoff plate is

$$T_d=\frac{\rho h}{2}\iint\left(\frac{\partial w_d}{\partial t}+\Omega\frac{\partial w_d}{\partial \varphi}\right)^2 r dr d\varphi, \tag{2}$$

and the potential energy due to the flexural stiffness is given by

$$U_d=\frac{1}{2}D\iint\left\{\left(\nabla^2 w_d\right)^2+2(1-\nu)\left[\left(\frac{1}{r}\frac{\partial^2 w_d}{\partial r\partial\varphi}-\frac{1}{r^2}\frac{\partial w_d}{\partial\varphi}\right)^2 -\frac{\partial^2 w_d}{\partial r^2}\left(\frac{1}{r^2}\frac{\partial^2 w_d}{\partial\varphi^2}-\frac{1}{r}\frac{\partial w_d}{\partial\varphi}\right)\right]\right\} r dr d\varphi \tag{3}$$

where w_d is the transverse displacement of a point on the disk surface at the point located at radius r and polar angle φ at time t, D is the plate flexural rigidity and h is the equivalent thickness obtained by matching the eigenfrequency of the considered mode shape with that of the actual disk which has a complicated cross-section. Because squeal appears at low speed, the in-plane stresses associated with rotational motion are neglected. The potential energy of k_p of the outer pad is

$$T_{tot} = T_d + T_m + T_M + T_{pad}\,, \tag{9}$$

$$U_{tot} = U_d + U_{k_\varphi} + U_{k_{z\varphi}} + U_{k_t}\,. \tag{10}$$

The total virtual work can be written as,

$$\delta W = \delta W_f + \delta W_P + \delta W_d\,, \tag{11}$$

where δW_f, δW_P and δW_d are the virtual works done by the friction, applied force and damping, respectively. From the force diagram shown in figure 3, the virtual work δW_f due to the friction force can be expressed as

$$\delta W_f = \int_{-\varphi_o}^{\varphi_o}\int_{r_i}^{r_o}\left[(-f_1)(-\frac{h}{2}\frac{\partial w_d}{r\partial\varphi}) + (-f_2)(\frac{h}{2}\frac{\partial w_d}{r\partial\varphi})\right] r dr d\varphi\,, \tag{12}$$

where $f_1 = \mu F_{a1}\cos\theta$ and $f_2 = \mu F_{a2}\cos\theta$

$$F_{a1} + f_1\sin\theta = k_p^{(1)}(w_d - w_{p1}) + k_p^{(3)}(w_d - w_{p1})^3 + d_p(\dot{w}_d - \dot{w}_{p1}) \tag{13}$$

$$F_{a2} - f_2\sin\theta = k_p^{(1)}(w_{p2} - w_d) + k_p^{(3)}(w_{p2} - w_d)^3 + d_p(\dot{w}_{p2} - \dot{w}_d) \tag{14}$$

The friction coefficient is assumed to be constant. The virtual work due to the applied external force P is given by

$$\delta W_P = -P\delta w_{c2} + P\delta w_{p2}\,. \tag{15}$$

Similarly to the virtual work δW_d due to damping, it also can be obtained with the same approach. Finally from the Hamilton's principle the non-linear equations of motion are obtained. Subsequently, these equations of motion are linearized around the equilibrium points with a constant braking pressure. The equilibrium deflection of the disk is too small to be taken into consideration. In addition because of small value of the angular velocity while braking, the centrifugal effect is neglected. The linearized equations of motion are discretized using Rayleigh-Ritz method with the help of the following shape function

$$w_d(r,\varphi,t) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} R_{m,n}(r)\left[\cos(m\varphi)A_{m,n}(t) + \sin(m\varphi)B_{m,n}(t)\right] \tag{16}$$

where $R_{m,n}(r)$ represents a mode shape of the stationary disk in terms of number of nodal diameters (m) and nodal circles (n) and is determined by applying boundary conditions given by

$$w_d(r=a,\varphi,t) = 0,\quad \frac{\partial w_d}{\partial r}(r=a) = 0 \quad \text{(clamped inside)} \tag{17}$$

$$M_r(r=b,\varphi,t) = 0,\quad V_r(r=b,\varphi,t) = 0 \quad \text{(free inside)} \tag{18}$$

where

$$M_r = -D\left[\frac{\partial^2 w_d}{\partial r^2} + \nu\left(\frac{1}{r}\frac{\partial w_d}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w_d}{\partial \varphi^2}\right)\right] \quad (19)$$

and

$$V_r = -D\left[\frac{\partial}{\partial r}\left(\nabla^2 w_d\right) + (1-\nu)\frac{\partial}{r\partial\varphi}\left(\frac{1}{r}\frac{\partial^2 w_d}{\partial r\partial\varphi} - \frac{1}{r^2}\frac{\partial w_d}{\partial \varphi}\right)\right] \quad (20)$$

are the bending moment and the shear force, respectively. The equations of motion can be written in the matrix form as,

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{G}+\mathbf{D})\dot{\mathbf{q}} + (\mathbf{C}+\mathbf{N})\mathbf{q} = \mathbf{0}\,, \quad (21)$$

where $\mathrm{q} = \left\{A_{3,1}, B_{3,1}, w_{c1}, w_{p2}, w_{c2}\right\}^T$, $\mathrm{M}^T = \mathrm{M}, \mathrm{D}^T = \mathrm{D}, \mathrm{G}^T = -\mathrm{G}, \mathrm{C}^T = \mathrm{C}$ and $\mathbf{N}$ is asymmetric due to the friction force. The equation of motion (21) represents a dynamic system with linear constant coefficients whose complex eigenvalues can be obtained easily. The system becomes unstable when one of the complex eigenvalues has positive real part. The exponentially increasing amplitudes of the free vibration are limited by the nonlinear terms.

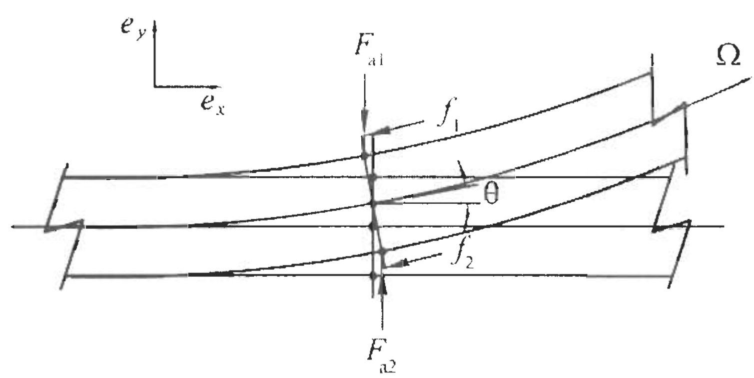


Figure 3: Force diagram on the disk

Numerical Results and Discussions

A radially slotted disk with radius 0.153 m and thickness 0.0264 m is considered whose frequency associated with (3,1) mode is experimentally found out to be 1763.42 Hz during squeal. The disk is modeled as a Kirchhoff's plate of uniform cross-section with equivalent thickness 0.0154 m. The values of the other system parameters are either taken from literature and experiment or realistically assumed. The complex natural frequencies of the linearized system have been calculated with constant friction coefficient. Beyond a limiting value of the friction coefficient, hereafter named as the critical coefficient and denoted by μ_{cr}, the disk motion becomes unstable through a flutter-type instability (see figure 4). The figure shows the eigenvalues of the system with a variation of friction coefficient and the friction coefficient 0.43 is found to be μ_{cr}.

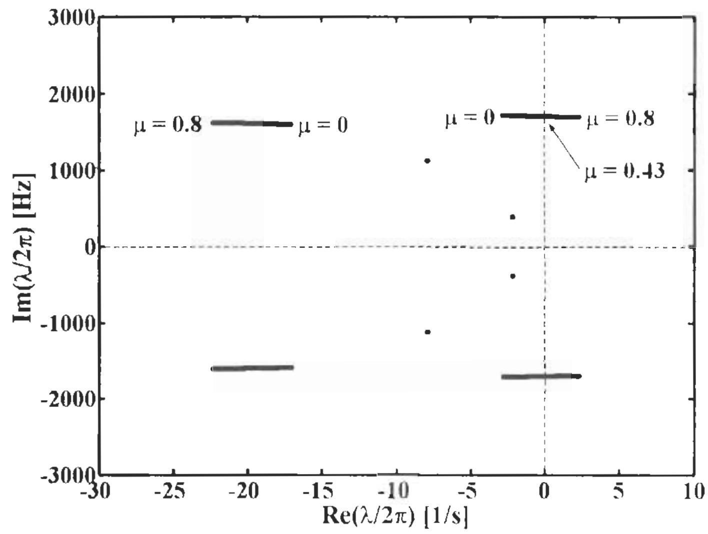


Figure 4: Flutter instability (μ -variation)

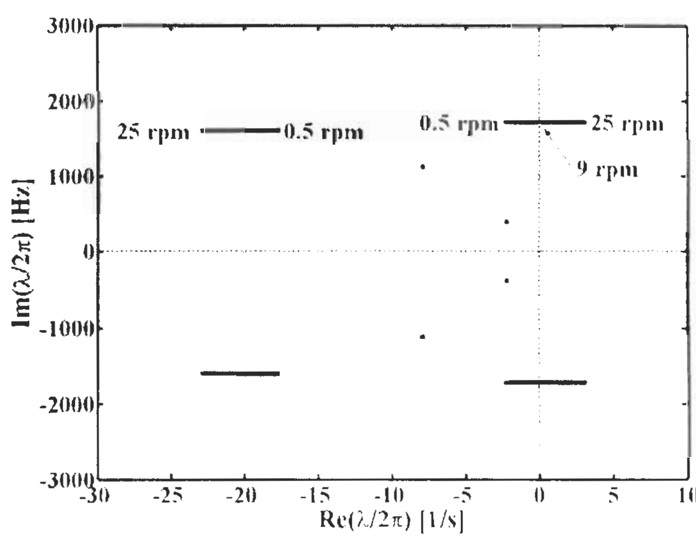


Figure 5: Flutter instability (Ω -variation)

Similarly, the disk motion becomes unstable when the angular velocity of the disk goes beyond the limiting value 9 rpm as shown in figure 5. The value of μ_{cr} decreases with increasing braking pressure (figure 6). The time responses of $A_{3,1}$ is plotted in figure 7, where the instability and the limit cycles can be seen. The frequency of the response is near to the natural frequency of the disk vibration and does not depend significantly on either the coefficient of friction or the angular velocity of the disk (figure 8).

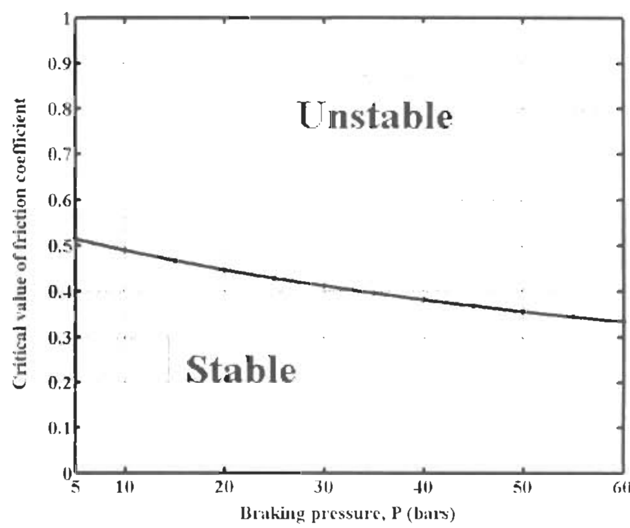


Figure 6: Effect of braking pressure on squeal

Conclusions

The paper presents a new model of the floating caliper disk brake to study its stability. The model incorporates the main components of the disk brake assembly, namely, disk, pads and housing. The disk is modeled as a flexible Kirchhoff's plate with constant cross section, the pad as a mass with distributed dampers and nonlinear springs. The other parts are modeled as discrete linear systems. The friction coefficient between the pads and the disk is assumed to be constant. Flutter-type instability is observed beyond a critical value of the friction coefficient and the angular velocity. The variations of the critical friction coefficient with various operating conditions and system parameters justify reliability of the model. Experiments to be carried out at a special test rig will be used to further validate the model.

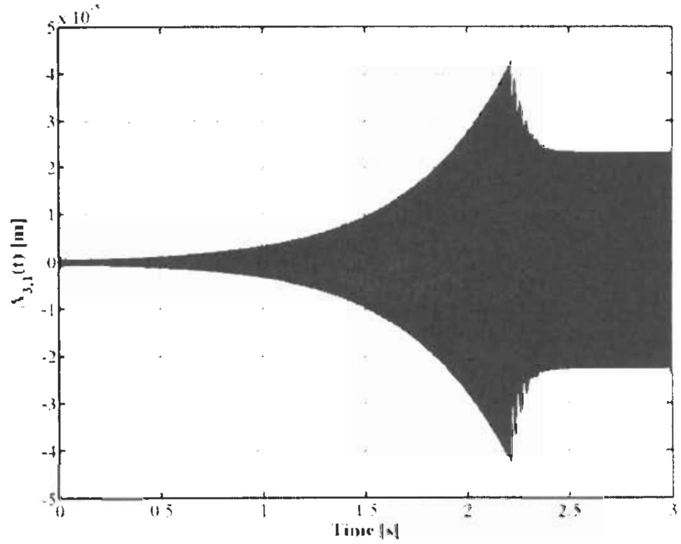


Figure 7: Time response

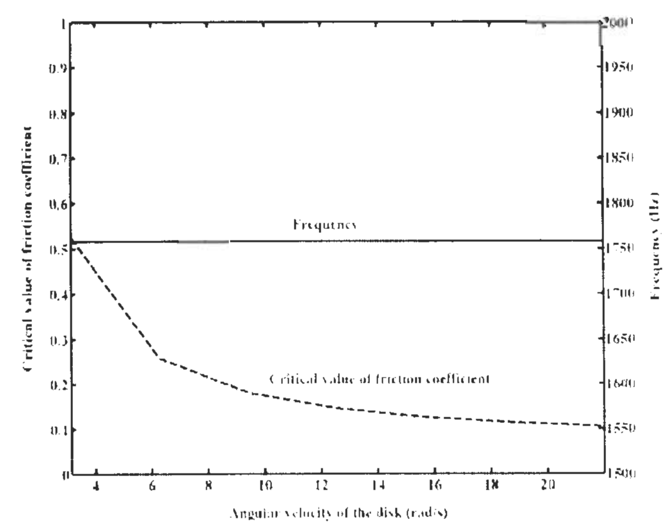


Figure 8: Effect of angular velocity with the frequency of squeal

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