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Multiobjective adaptive current search for engineering design optimization

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Abstract

This article proposes a multiobjective adaptive current search (MoACS) as a powerful metaheuristic optimization technique for solving multiobjective optimization problems. MoACS is a newly modified version of the current search (CS) developed from the behavior of current in an electrical network. In this study, the MoACS is developed and validated against three standard multiobjective test functions. Results obtained from the MoACS are compared with those obtained by five well-known algorithms from the literature. Then, the MoACS is applied to the design of two real-world multiobjective engineering optimization problems, i.e., welded beam design and disc brake design. It was found that MoACS provided very satisfactory solutions and had quite smooth Pareto fronts for all of the problems.

Keywords: Multiobjective adaptive current search, Metaheuristic optimization, Welded beam design, Disc brake design

1. Introduction

Engineering design can be considered as a class of an optimization divided into single-objective or multiobjective problems [1-2]. For single-objective, an optimization tends to minimize (or maximize) only one objective such as minimize loss or maximize profit. For multiobjective, it tends to minimize (or maximize) several objectives such as minimize loss and minimize cost. In fact, both loss and cost are trade-off. The less the loss, the higher the cost, and vice versa. Many real-world engineering design problems often consist of many objectives which are conflict each other [2-3]. This leads the multiobjective problems much more difficult and complex than single-objective ones. The multiobjective problem possesses multiple optimal solutions forming the so-called Pareto front [4-5]. The challenge is how to perform the smooth Pareto front containing a set of optimal solutions for all objective functions. By literatures, conventional optimization methods often face difficulties for solving multiobjective problems. One of the alternative approaches developed to solve multiobjective problems is the metaheuristics approach [1-5]. The efficient metaheuristics consecutively launched for multiobjective optimization are, for example, vector evaluated genetic algorithm (VEGA) [6], non-dominated sorting genetic algorithm II (NSGA-II) [7], differential evolution for multiobjective optimization (DEMO) [8], multiobjective cuckoo search (MOCS) [9] and multiobjective multipath adaptive tabu search (mMATS) [10].

In 2012, the current search (CS) metaheuristics was proposed to solve optimization problems [11]. The CS algorithm was developed from the behavior of electric current flown into the electrical networks. The CS was successfully applied to control engineering [12] and analog circuit design [13]. During 2013-2014, the adaptive current search (ACS) was launched [14] as a modified version of the conventional CS. The ACS was successfully applied to assembly line balancing problems [14-15] and transportation problems [16]. In this article, the multiobjective adaptive current search (MoACS) is developed as one of the most powerful metaheuristic optimization techniques for solving multiobjective optimization problems. The developed MoACS will be evaluated against three standard multiobjective test functions. Results obtained will be compared with those obtained by VEGA, NSGA-II, DEMO, MOCS and mMATS. Finally, the MoACS is then applied to design two real-world multiobjective engineering optimization problems.

2. Materials and methods

2.1 Pareto optimality

Solving multiobjective optimization problems is based on the Pareto optimality [3, 9]. Such the problem can be formulated as expressed in Equation (1), where $\boldsymbol{f}(\boldsymbol{x})$ is the multiobjective function consisting of $f_1(\boldsymbol{x}),\ldots, f_n(\boldsymbol{x})$, $n \geq 2$, $g_j(\boldsymbol{x})$, $j = 1, 2,\ldots,m$, is the inequality constraints and $h_k(\boldsymbol{x})$, $k = 1, 2,\ldots,p$, is the equality constraints. The optimal solutions, $\boldsymbol{x}^*$, are ones making $\boldsymbol{f}(\boldsymbol{x})$ minimum and both $g_j(\boldsymbol{x})$ and $h_k(\boldsymbol{x})$ satisfied.

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$$\left.\begin{array}{ll}\textbf{Minimize} & \boldsymbol{f}(\boldsymbol{x}) = \{f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_n(\boldsymbol{x})\} \\ \textit{subject to} & g_j(\boldsymbol{x}) \le 0, \quad j = 1, \ldots, m, \quad h_k(\boldsymbol{x}) = 0, \quad k = 1, \ldots, p\end{array}\right\}. \tag{1}$$

$$\forall i \in \{1, \ldots, n\} : u_i \le v_i \wedge \exists i \in \{1, \ldots, n\} : u_i < v_i . \tag{2}$$

$$P^* = \{\boldsymbol{x} \in F \mid \exists \boldsymbol{x}^* \in F : \boldsymbol{f}(\boldsymbol{x}^*) \prec \boldsymbol{f}(\boldsymbol{x})\} . \tag{3}$$

$$PF^* = \{\boldsymbol{s} \in S \mid \exists \boldsymbol{s}^* \in S : \boldsymbol{s}^* \prec \boldsymbol{s}\} . \tag{4}$$

A solution vector, $\boldsymbol{u} = (u_1,\ldots,u_n)^T \in S$, is said to dominate another solution vector $\boldsymbol{v} = (v_1,\ldots,v_n)^T$, denoted by $\boldsymbol{u} \prec \boldsymbol{v}$, if and only if $u_i \le v_i$ for $\forall i \in \{1,\ldots,n\}$ and for $\exists i \in \{1,\ldots,n\}$: $u_i \le v_i$. This implies that no component of $\boldsymbol{v}$ is smaller than the corresponding component of $\boldsymbol{u}$, and at least one component of $\boldsymbol{u}$ is strictly smaller stated in Equation (2). A solution $\boldsymbol{x}^* \in S$ is called a non-dominated solution if no solution can be found that dominates it. Mathematically, a solution $\boldsymbol{x}^* \in S$ is Pareto optimal if for every $\boldsymbol{x} \in S$, $\boldsymbol{f}(\boldsymbol{x}) \in F$ does not dominate $\boldsymbol{f}(\boldsymbol{x}^*) \in F$, that is $\boldsymbol{f}(\boldsymbol{x}^*) \prec \boldsymbol{f}(\boldsymbol{x})$. For a given multiobjective optimization problem, the Pareto optimal set is defined as P^* stated in Equation (3). The Pareto front PF^* of a given multiobjective optimization problem can be defined as the image of the Pareto optimal set P^* expressed in Equation (4).

2.2 MoACS algorithm

Regarding to the adaptive current search (ACS) [14-15], it possesses the memory list (ML) used to escape from local entrapment caused by any local minimum and the adaptive radius (AR) mechanism applied to speed up the search process. For the ACS algorithm, the feasible solutions called neighborhoods will be randomly generated within the limited circle area of the search space. Such the circle area is defined by the search radius R. In AR mechanism, the search radius R will be decreased once the search is close to the optimal solution. From recommendations [14-16], the initial search radius R = 20-25% of search space is suitable for most applications. The proposed multiobjective ACS called the MoACS is developed to minimize the $\boldsymbol{f}(\boldsymbol{x})$ in Equation (1). The MoACS uses the ACS as the search core. Multiobjective function $\boldsymbol{f}(\boldsymbol{x})$ in Equation (1) will be simultaneously minimized according to the equality and inequality constraints. In each iteration, the optimal solution found will be evaluated. If the optimal solution found is a non-dominated solution, it will be sorted and stored into the Pareto optimal set P^*. After the search stopped, all solutions stored in P^* will be conducted to perform the Pareto front PF^*. All solutions appeared on the PF^* are the optimal solutions of the multiobjective problem of interest. The MoACS algorithm is described as follows.

- Step 1 Initialize the multiobjective function $\boldsymbol{f}(\boldsymbol{x}) = \{f_1(\boldsymbol{x}), f_2(\boldsymbol{x}),\ldots,f_n(\boldsymbol{x})\}$, $\boldsymbol{x} = (x_1,\ldots,x_d)^T$, the constraints $g_j(\boldsymbol{x})$ and $h_k(\boldsymbol{x})$, search spaces and search radius R.
- Step 2 Initialize $ACS_1,\ldots,ACS_k$ and Pareto optimal set P^*, $j_{\max}$ and $k = j = 1$.
- Step 3 Uniformly random initial solution $\boldsymbol{X}_i$, $i = 1,\ldots,N$ within the search space.
- Step 4 Evaluate the multiobjective function $f(\boldsymbol{X}_i)$ for $\forall \boldsymbol{X}$ according to the constraints $g_j(\boldsymbol{X}_i)$ and $h_k(\boldsymbol{X}_i)$, then rank $\boldsymbol{X}_i$, where $X_1 < X_2 < \cdots < X_N$.
- Step 5 Let $\boldsymbol{x}_0 = \boldsymbol{X}_k$ as selected initial solution.
- Step 6 Uniformly random neighborhood $\boldsymbol{x}_i$, $i = 1,\ldots,n$ around $\boldsymbol{x}_0$ within radius R.
- Step 7 Evaluate the multiobjective function $f(\boldsymbol{x}_i)$ for $\forall \boldsymbol{x}$ according to the constraints $g_j(\boldsymbol{x}_i)$ and $h_k(\boldsymbol{x}_i)$. A solution giving the minimum objective function is set as $\boldsymbol{x}^*$.
- Step 8 Check if it is Pareto optimal (non-dominated solution), sort and store it into the Pareto optimal set P^*.
- Step 9 If $f(\boldsymbol{x}^*) < f(\boldsymbol{x}_0)$, set $\boldsymbol{x}_0 = \boldsymbol{x}^*$, set $j = 1$ and return to Step 6. Otherwise update $j = j+1$.
- Step 10 Activate the AR by adjusting $R = \rho R$, $0 < \rho < 1$.
- Step 11 If $j < j_{\max}$, return to Step 6. Otherwise set $j = 1$ and update $k = k+1$.
- Step 12 If $k > N$, terminate the search process, and sort the current Pareto optimal solutions. Pareto front PF^* is then performed by the sorted Pareto optimal solutions. Otherwise return to Step 5.

In this work, the proposed MoACS algorithm was coded by MATLAB run on Intel Core2 Duo 2.0 GHz 3 Gbytes DDR-RAM computer. The search parameters of the MoACS consist of number of initial solutions N, number of neighborhood member n, number of solution cycling $j_{\max}$, initial search radius R and AR mechanisms. These parameters are priori set from preliminary parametric study [14-16] as follows: N = 60, n = 40, $j_{\max}$ = 10, R = 20% of search space and AR = 2 states {(i) at 750th iteration, adjusting $R = 0.1R$ and (ii) at 1,500th iteration, adjusting $R = 0.01R$}. *MaxIteration* = 2,000 is set as the termination criteria (TC). This parameter set will be used for all problems in this article. Each problem will be conducted with 100 trial runs under the same condition in order to obtain the best solution found.

2.3 Standard test functions

The proposed MoACS is evaluated against three standard multiobjective test functions [9-10]. They are the convex front ZDT1 stated in Equation (5) where d is the number of dimensions, the concave front ZDT2 stated in Equation (6) and the discontinuous front ZDT3 expressed in Equation (7), where g and x_i in Equation (6) and Equation (7) are the same as in Equation (5). In comparison, the error E_f between the estimated Pareto front PF_e and its correspondingly true Pareto front PF_t is evaluated via the formulation stated in Equation (8), where N is the number of sorted solutions.

$$\left.\begin{array}{l}\text{ZDT1:} \quad f_1(x) = x_1, \quad f_2(x) = g(1-\sqrt{f_1/g}), \\ \quad g = 1 + \dfrac{9\sum_{i=2}^{d} x_i}{d-1}, \quad x_i \in [0,1], \quad i = 1,\ldots,30\end{array}\right\}. \tag{5}$$

$$\text{ZDT2:} \quad f_1(x) = x_1, \quad f_2(x) = g\left(1-\frac{f_1}{g}\right)^2 . \tag{6}$$

$$\text{ZDT3:} \quad f_1(x) = x_1, \quad f_2(x) = g\left(1-\sqrt{\frac{f_1}{g}}-\frac{f_1}{g}\sin(10\pi f_1)\right). \tag{7}$$

$$E_f = \|PF_e - PF_t\| = \sum_{j=1}^{N} (PF_e^j - PF_t)^2 \,. \qquad (8)$$

2.4 Real-world engineering applications

The MoACS is also applied to design two real-world multiobjective engineering optimization problems. They are welded beam design and disc brake design. Their details are described as follows.

2.4.1 Welded beam design

As shown in Figure 1, the schematic structure of a welded beam design shows that it has four design variables [9, 17], i.e. the width w and length L of the welded area, the depth d and thickness h of the main beam. The objective of this problem is to minimize both the fabrication cost and the end deflection on the beam δ. Therefore, the problem can be formulated according to the multiobjective optimization problems as stated in Equation (9) – (11), where $f_1(\boldsymbol{x})$ is the fabrication cost, $f_2(\boldsymbol{x})$ is the end deflection of the beam, τ is shear stress, σ is bending stress in the beam and P is bucking load on the bar.

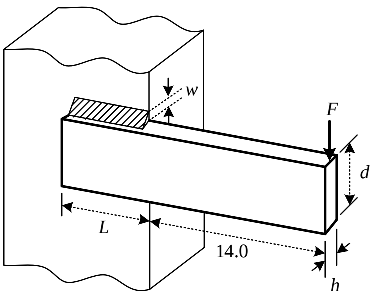

Figure 1 Welded beam design problem

$$\left.\begin{aligned} &\textbf{Minimize } \boldsymbol{f}(\boldsymbol{x}) = \{f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x})\},\\ &f_1(\boldsymbol{x}) = 1.10471 w^2 L + 0.04811 dh(14.0 + L),\\ &f_2(\boldsymbol{x}) = \delta \end{aligned}\right\} . \qquad (9)$$

$$\left.\begin{aligned} &\text{Subject to :}\\ &g_1(\boldsymbol{x}) = w - h \le 0,\\ &g_2(\boldsymbol{x}) = \delta(\boldsymbol{x}) - 0.25 \le 0,\\ &g_3(\boldsymbol{x}) = \tau(\boldsymbol{x}) - 13{,}600 \le 0,\\ &g_4(\boldsymbol{x}) = \sigma(\boldsymbol{x}) - 30{,}000 \le 0,\\ &g_5(\boldsymbol{x}) = 0.10471 w^2 + 0.04811 dh(14.0 + L) - 5.0 \le 0,\\ &g_6(\boldsymbol{x}) = 0.125 - w \le 0,\\ &g_7(\boldsymbol{x}) = 6{,}000 - P(\boldsymbol{x}) \le 0,\\ &0.1 \le L, d \le 10.0,\\ &0.125 \le w, h \le 2.0 \end{aligned}\right\} . \qquad (10)$$

$$\left.\begin{aligned} &\text{where :}\\ &\sigma(\boldsymbol{x}) = \frac{504{,}000}{hd^2},\quad Q = 6{,}000\left(14.0 + \frac{L}{2}\right),\quad D = \frac{1}{2}\sqrt{L^2 + (w+d)^2},\\ &J = \sqrt{2} wL\left[\frac{L^2}{6} + \frac{(w+d)^2}{2}\right],\quad \delta = \frac{65{,}856}{30{,}000 hd^3},\quad \beta = \frac{QD}{J},\quad \alpha = \frac{6{,}000}{\sqrt{2} wL},\\ &\tau(\boldsymbol{x}) = \sqrt{\alpha^2 + \frac{\alpha\beta L}{D} + \beta^2},\quad P = 0.61423\times 10^6 \frac{dh^3}{6}\left(1 - \frac{d\sqrt{30/48}}{28}\right) \end{aligned}\right\} . \qquad (11)$$

2.4.2 Disc break design

The structure of a multiple disc break design is shown in Figure 2. This design problem consists of four design variables [9, 18], i.e. the inner radius r and outer radius R of the discs, the engaging force F and the number of the friction surface S. The objective of this problem is to minimize the braking time and the overall mass. By this, the design problem can be performed as expressed in Equation (12) – (13), where $f_1(\boldsymbol{x})$ is the braking time, $f_2(\boldsymbol{x})$ is the overall mass.

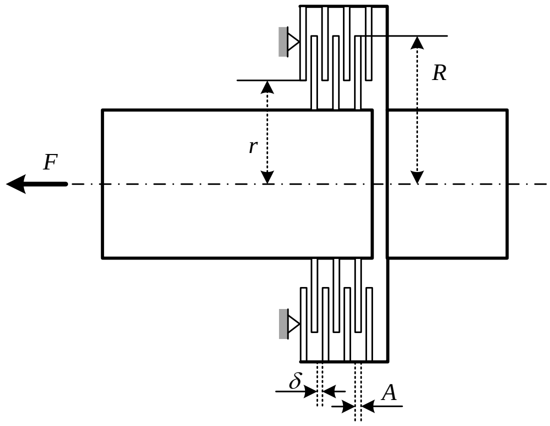

Figure 2 Disc break design problem

$$\left.\begin{aligned} &\textbf{Minimize } \boldsymbol{f}(\boldsymbol{x}) = \{f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x})\},\\ &f_1(\boldsymbol{x}) = 4.9\times 10^{-5}(R^2 - r^2)(S - 1),\\ &f_2(\boldsymbol{x}) = \frac{9.82\times 10^6 (R^2 - r^2)}{FS(R^3 - r^3)} \end{aligned}\right\} \qquad (12)$$

$$\left.\begin{aligned} &\text{Subject to :}\\ &g_1(\boldsymbol{x}) = 20 - (R - r) \le 0,\\ &g_2(\boldsymbol{x}) = 2.5(S + 1) - 30 \le 0,\\ &g_3(\boldsymbol{x}) = \frac{F}{3.14(R^2 - r^2)} - 0.4 \le 0,\\ &g_4(\boldsymbol{x}) = \frac{2.22\times 10^{-3} F(R^3 - r^3)}{(R^2 - r^2)^2} - 1 \le 0,\\ &g_5(\boldsymbol{x}) = 900 - \frac{0.0266 FS(R^3 - r^3)}{(R^2 - r^2)} \le 0,\\ &55.0 \le r \le 80.0,\quad 75.0 \le R \le 110.0,\\ &1{,}000 \le F \le 3{,}000,\quad 2.0 \le S \le 20.0 \end{aligned}\right\} . \qquad (13)$$

3. Results

3.1 Results of standard test functions

For comparison to five selected well-known algorithms from literatures, results of the functions ZDT1 – ZDT3 obtained by the VEGA, NSGA-II, DEMO and MOCS from [9] and those by the MATS from [10] are summarized in Table 1. With the defined TC and 100 trial runs, the best results of the functions ZDT1 – ZDT3 obtained by the proposed MoACS are also summarized in Table 1. The Pareto fronts obtained by the MoACS and the true fronts of functions ZDT1 – ZDT3 are depicted in Figure 3 – 5, respectively.

Table 1 Evaluation results of standard multiobjective test functions

Algorithms		Error E_f		
		ZDT1	**ZDT2**	**ZDT3**
VEGA	[9]	3.79e-02	2.37e-03	3.29e-01
NSGA-II	[9]	3.33e-02	7.24e-02	1.14e-01
DEMO	[9]	1.08e-03	7.55e-04	1.18e-03
MOCS	[9]	1.17e-04	2.23e-05	2.88e-05
mMATS	[10]	1.14e-04	2.15e-05	1.07e-05
MoACS		**1.02e-04**	**2.01e-05**	**1.01e-05**

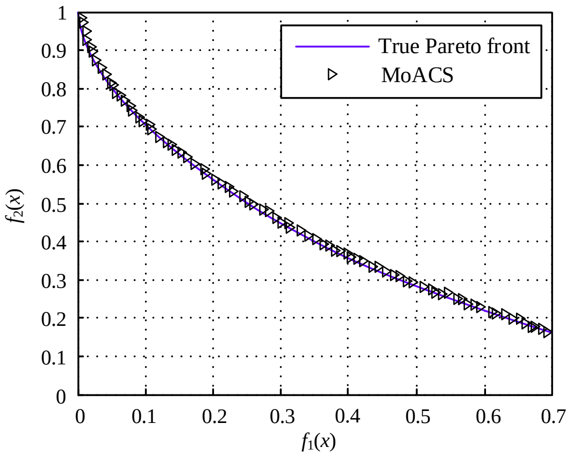

Figure 3 Pareto front of ZDT1 **(**convex front**)**

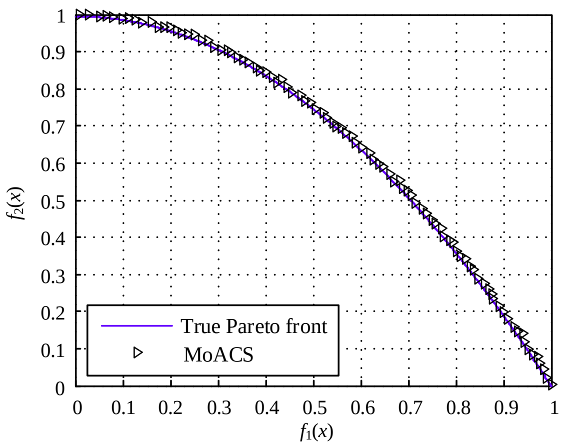

Figure 4 Pareto front of ZDT2 **(**concave front**)**

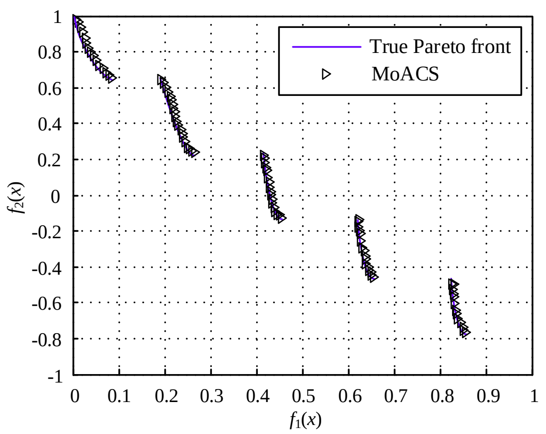

Figure 5 Pareto front of ZDT3 **(**discontinuous front**)**

3.2 Results of real-world engineering applications

By applying the MoACS to two selected real-world engineering applications with the same parameter setting and the preset TC, a set of optimal (non-dominated) solutions obtained performs the well-smooth Pareto fronts as depicted in Figure 6 – 7.

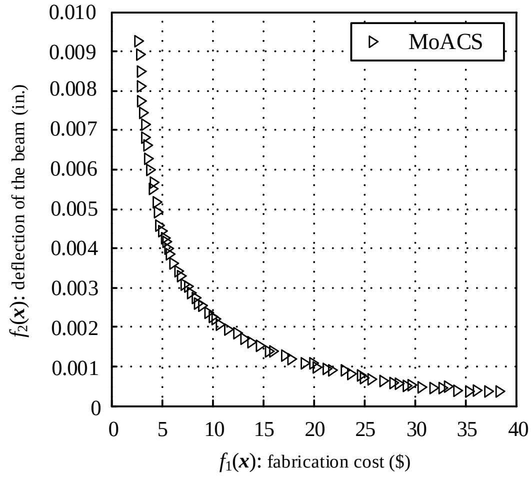

Figure 6 Pareto front of welded beam design problem

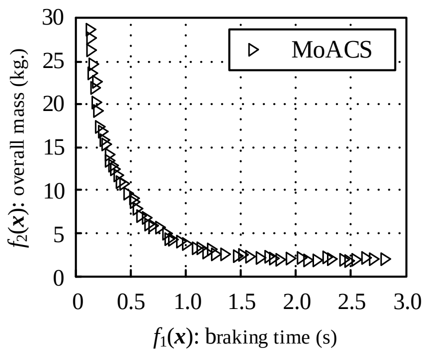

Figure 7 Pareto front of disc break design problem

4. Discussion

For results of standard test functions, referring to Table 1, the proposed MoACS shows superior results to other algorithms with less error E_f. Moreover, Figure 3 – 5 reveal that the MoACS can provide the smooth Pareto fronts very coincide with the true fronts of each standard multiobjective test function with good distribution and good spread. For results of real-world engineering applications, referring to the well-smooth Pareto fronts depicted Figure 6 – 7, it was found that both welded beam design and disc break design problems can be satisfactory solved by the proposed MoACS.

5. Conclusions

The multiobjective adaptive current search (MoACS) has been developed and proposed in this article for solving multiobjective optimization problems. Based on the conventional current search (CS), developed algorithm of the MoACS has been elaborated. The proposed MoACS has been evaluated against three standard multiobjective test functions in order to perform its effectiveness. As results, it was found that the MoACS could provide superior solutions to VEGA, NSGA-II, DEMO, MOCS and mMATS over three selected test functions with less error. In this article, the MoACS has been also applied to design two real-world multiobjective engineering optimization problems, i.e. welded beam design and disc brake design. As simulation results, it was found that a set of non-dominated solutions of each application obtained by the MoACS could perform the well-smooth (good distribution and spread) Pareto fronts. It can be concluded that the proposed MoACS is one of the most efficient multiobjective optimizers. For the future trends, the popular performance metrics such as the Generational Distance (GD), Generalized Spread (GS) and Hypervolume (HV) will be emphasized for performance comparison of the proposed algorithm with other well-known methods for multiobjective optimization problems. In addition, hybridization of the proposed algorithm with other selected algorithms will be developed and proposed.

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