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Multiple linear regression models for shear strength prediction and design of simply-supported deep beams subjected to symmetrical point loads

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Abstract

Because of nonlinear strain distributions caused either by abrupt changes in geometry or in loading in deep beam, the approach for conventional beams is not applicable. Consequently, strut-and-tie model (STM) has been applied as the most rational and simple method for strength prediction and design of reinforced concrete deep beams. A deep beam is idealized by the STM as a truss-like structure consisting of diagonal concrete struts and tension ties. There have been numerous works proposing the STMs for deep beams. However, uncertainty and complexity in shear strength computations of deep beams can be found in some STMs. Therefore, improvement of methods for predicting the shear strengths of deep beams are still needed. By means of a large experimental database of 406 deep beam test results covering a wide range of influencing parameters, several shapes and geometry of STM and six state-of-the-art formulation of the efficiency factors found in the design codes and literature, the new STMs for predicting the shear strength of simply supported reinforced concrete deep beams using multiple linear regression analysis is proposed in this paper. Furthermore, the regression diagnostics and the validation process are included in this study. Finally, two numerical examples are also provided for illustration.

Keywords : Deep beam, Efficiency factor, Multiple regression, Strut-and-tie model

1. Introduction

The method for design deep beams is considerably different from that of slender beams design because of difference in their behavior categorized by shear span-to-depth ratios, a/d. When the ratio is less than a small number, say, between 2.0 and 2.5, the mechanism of shear failure is better characterized as strut-and-tie action rather than sectional shear, and therefore the applied load is transferred directly from the load point to the support. Thus, the safe and easy-to use method namely strut-and-tie model (STM) is employed for analysis and design of deep beams available in the American codes of practice [1-2] and published literature [3-7]. The complex flow of stresses in a deep beam can be idealized by the STM as truss-like members consisting of diagonal concrete struts and steel reinforcement ties connected at each node. In general, the ultimate shear strength of a deep beam depends strongly on the capacity of the diagonal concrete strut defined in terms of the concrete cylinder strength and the efficiency factor. The concrete efficiency factor is used for representing the variation of concrete properties and the effect of compression softening of cracked concrete [8-12]. There have been a number of published articles proposed and adopted the efficiency factor used in STMs, however, some of these still give inconsistent computations reported by [5-7, 13] because they may contain lacking formulation, missing parameters, or inadequate experimental observations. As a result, the research topics in this area are still needed for developing more accurate and reliable methods.

Based on a large database of deep beam test results, several selected shapes and geometry of STM, six state-of-the-art efficiency factors, and the knowledge of relationship between shear strength and influencing parameters, the new STMs for shear strength prediction of simply supported reinforced concrete deep beams are proposed by using multiple linear regression (MLR) method. Because MLR has been considered as the most reasonable and simple method to define the relationships among the variables. Moreover, the validation process is used to confirm the accuracy and robustness of proposed STMs. Finally, some design examples are demonstrated in step by step calculations.

2. A strut-and-tie model of simply supported deep beams

An STM for a simply supported deep beam subjected to the two pointed loads is illustrated in Figure 1.

According to Hwang et al. [3] and Matamoros and Wong [4], it can reasonably be assumed that there are three failure

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mechanisms of a deep beam: 1) diagonal concrete strut mechanism, 2) vertical mechanism, and 3) horizontal mechanism.

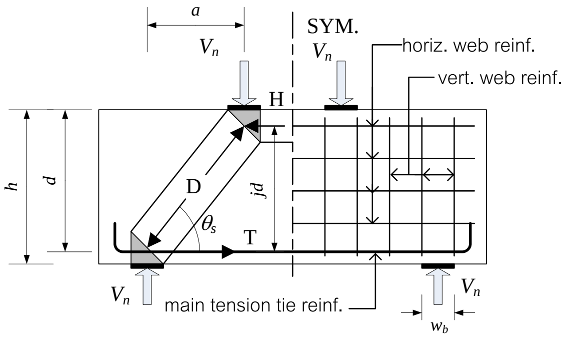

Figure 1 STM of a simply supported deep beam

Consequently, the summation of these can represent the shear strength V_n , and is written as:

$$V_n = \nu f_c' \sin\theta_s b_w w_s + A_v f_{yv} + A_h f_{yh} \tan\theta_s \quad (1)$$

where ν is the efficiency factor which can be obtained from one of the five selected formulations listed in Table1. The term f_c', f_{yv}, and f_{yh} are the concrete compressive strength, and the yield strengths of vertical and horizontal reinforcement respectively; The other terms are defined as follows: b_w is the width of beam section; A_v and A_h are the areas of vertical and horizontal web reinforcement, respectively; and θ_s represents the angle between horizontal direction and the diagonal concrete strut. Here, $\theta_s = \tan^{-1}(jd/a)$, where j is a well-know parameter derived from the elastic theory for a beam section with singly reinforcement: $j = 1 - k/3$,where $k = \sqrt{(n\rho)^2 + 2(n\rho)} - (n\rho)$, n is the modular ratio and ρ is the main steel ratio excluding the amount of horizontal web reinforcement. It should be noted that there is a difference between the main tension reinforcement and the horizontal web reinforcement. The former is provided for resisting the tension force in the bottom portion of beam, while the latter for crack control and serviceability purposes. The term of w_s is the width of strut which can be assumed to be prismatic strut having a uniform cross section over their length as shown in Figure 1. Several approaches to determine the width of strut defined by the codes of practice and literature are considered as follows:

$$w_s = \sqrt{(kd)^2 + w_b^2} \quad (2a)$$

$$w_s = kd \quad (2b)$$

$$w_s = \min(w_t \cos\theta_s + w_b \sin\theta_s, kd\cos\theta_s + w_b \sin\theta_s) \quad (2c)$$

Eq. (2a), (2b), and (2c) are provided by Hwang et al. [3], Hwang et al. [14], and ACI [1], respectively. Where w_b and $w_t = 2(h-d)$ are the width of bearing plate and the effective width of tie, respectively. Lastly, the principal tensile strain of concrete ε_1 is needed to determine the efficiency factors as exhibited in Table 1, and can be defined as:

$$\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002)/\tan^2\theta_s \quad (3a)$$

where ε_s is the tensile strain of main steel. This formula has been used extensively in the AASHTO code [2]. However, for simplicity, ε_1 can be approximated as:

$$\varepsilon_1 = 0.005 \quad (3b)$$

This was proposed by Hwang and Lee [15].

3. Methodology

To develop the mathematical models for predicting the shear strengths of reinforced concrete deep beams, MLR is adopted in conjunction with the STMs and the experimental observations. This section consists of four stages: 1) collecting data; 2) constructing models; 3) regression diagnostics and 4) validating the developed models.

3.1. Collecting the experimental data

A database of 406 reinforced concrete deep beams test results collected by Chetchotisak et al. [16] is used in this study. This data set is assembled from the published literature, where the shear strengths and details of the simply supported deep beams subjected to either one or two point loads acting symmetrically with respect to the centerline of the beam span is considered. In addition, the distribution of influencing parameters of tested beams is shown in Figure 2. In order to avoid overfitting problem (i.e., a developed model has a good accuracy on the data for building but a poor performance in predicting the new unseen data), the experimental data have been divided into two data sets. Of the total 406 samples in this database, 70%, i.e., 284 samples are randomly selected to use as a data set for development (training set), while the remaining 122 samples are treated as a validation set, also called unseen data set.

3.2. Constructing the regression models

From Eq. (1), the shear strength of deep beam V_n can be rewritten in the form of MLR as:

$$V_n = \beta_0 + \beta_1 V_D + \beta_2 V_v + \beta_3 V_h + e \quad (4)$$

where β_0, β_1, β_2, and β_3 are the regression coefficients and e is the error of model. V_D, V_v and V_h are the independent variables representing the load-carrying capacity of the STM consisting of diagonal concrete strut, vertical and horizontal truss, respectively, and can be described as:

$$V_D = \nu f_c' \sin\theta_s b_w w_s \quad (5)$$

$$V_v = A_v f_{yv} \quad (6)$$

$$V_h = A_h f_{yh} \tan\theta_s \quad (7)$$

Table 1 Summary of the existing concrete efficiency factors

Model	Efficiency Factor Models
Vecchio and Collins [8]	$\nu = \dfrac{1}{0.8 + 170\ \varepsilon_1} \le 0.85$
Warwick and Foster [9]	$\nu = 1.25 - \dfrac{f_c'}{500} - 0.72\, a/d + 0.18(a/d)^2 \le 1$ for $a/d < 2$ and $\nu = 0.53 - f_c'/500$ for $a/d \ge 2$
Zhang and Hsu [10]	$\nu = \dfrac{5.8}{\sqrt{f_c'}}\dfrac{1}{\sqrt{1+400\varepsilon_1}} \le \dfrac{0.9}{\sqrt{1+400\varepsilon_1}}$
Kaufmann and Marti [11]	$\nu = \dfrac{1}{(0.4+30\varepsilon_1) f_c'^{1/3}}$
Zwicky and Vogel [12]	$\nu = (1.8 - 38\varepsilon_1)\cdot(f_c')^{-1/3}$ and $0.85\cdot(f_c')^{-1/3} \le \nu \le 1.6\cdot(f_c')^{-1/3}$

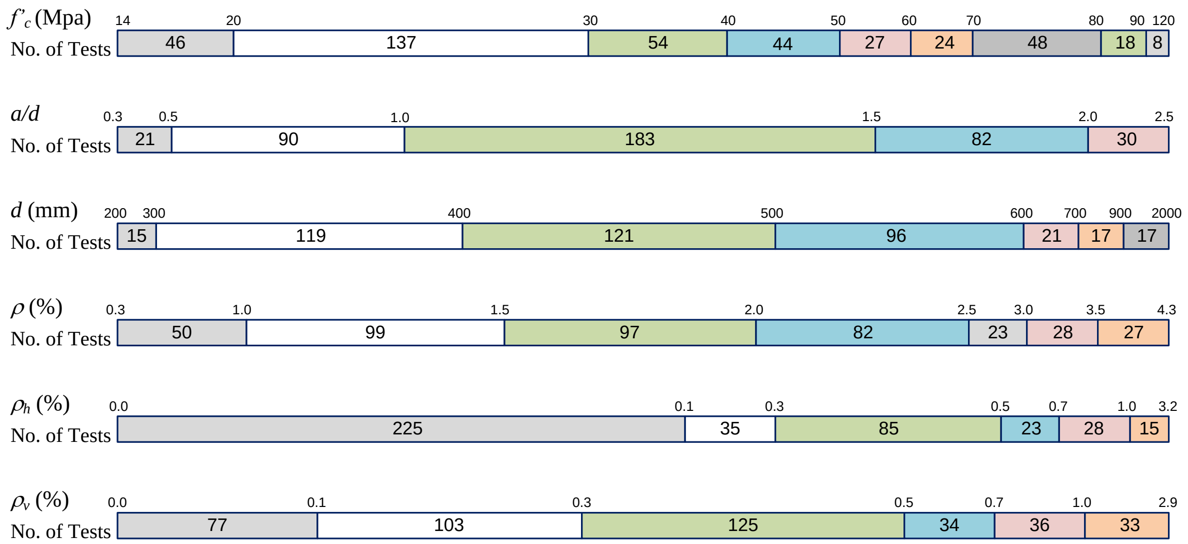

Figure 2 Distributions of parameter values of R.C. deep beams in shear database (406 test results)

In this study, all of w_s and ε_1 in Eq. (2) and Eq. (3) are evaluated in conjunction with each of selected ν. Consequently, it is found that the expressions in Eq. (2a) and Eq. (3b) result in the best accuracy indicated by the satisfactions of several statistical tests described in section 3.4.

3.3. Regression models and diagnostics

The developed models and statistical analysis performed using Microsoft Excel spreadsheet are shown in Table 2. These statistical parameters are used to apply regression diagnostics including: 1) *p*-value of *F*-test, 2) *R*-square for the model, and 3) *p*-value of the *t*-test for the regression coefficients. To confirm linear relationship among the response variable (V_n) and the independent variables, a hypothesis testing is achieved to accept or reject the null hypothesis (H_0) assuming that $\beta_0 = \beta_1 = \beta_2 = \beta_3 = 0$. When the null hypothesis is rejected, the alternative hypothesis (H_1) assuming that at least one of them is nonzero is accepted. As observe from the sixth column of Table 2, *p*-values for all models are less than 0.01. Such results imply that there are linear relation between V_n and at least one of the independent variables. In addition, *R*-square known as the coefficient of multiple determination is used to evaluate the proportional of the total variation in response variable accounted for a group of independent variables. The *R*-square of all model is greater than 0.85 as shown in the seventh column of Table 2, this argues that the developed model yields a good fit for the data.

Next, to test that whether each of the independent variables has relationship with V_n, the *t*-tests can be deployed for every developed model. The hypothesis *t*-test of null hypothesis assumes that $\beta_i = 0$ while the alternative hypothesis assumes that $\beta_i \neq 0$. Where $i = 0$ is used for the intercept while $i = 1,2,3$ for the coefficients of the independent variables, respectively. The results of these *t*-tests are given in the eighth to the eleventh column of Tables 2 in the form of a *p*-value for each test. All models give large *p*-values (i.e., $p > 0.05$) for the intercept therefore indicate that $\beta_0 = 0$ for all models. Furthermore, for four-fifths of all models [8, 10-12], alternative hypothesis is accepted at 1% levels of significance, indicates that $\beta_1 \neq 0$, $\beta_2 \neq 0$ and $\beta_3 \neq 0$ at 99% confidence. This may implies that the amount of vertical and horizontal web reinforcement can assist to enhance the shear capacity of reinforced concrete deep beams. However, the remainder developed from Warwick and Foster [9] gives high *p*-values for V_h, this suggests that V_h is not needed in this model. This has been reduced to a two-term formulation and the value of β_1 and β_2 have changed slightly as seen in Table 3. Moreover, the residual analysis has been checked using visual inspection of the residual plots and scatter plots, respectively. From all diagnoses, it can be concluded that the principal assumptions based on the MLR have not been violated.

3.4. Validation of regression models

To assure the accuracy of the developed models, the predicted shear strength computed from the regression equations are compared to actual shear strength from validation data set. Several statistics such as the average (AVG), standard deviation (SD), and coefficient of variation (COV) for the strength ratio (experimental strength/calculated strength), including the mean absolute percentage error (MAPE) are used to evaluate the precision of regression models. Here, AVG can be roughly treated as an implication of safe or unsafe of the developed models, while the SD and COV can indicate model consistency. Finally, the MAPE is used to measure the model accuracy as well.

The statistics from Table 3 reveal that most of developed models have approximately the same safety and accuracy, however, the three models developed from efficiency factor proposed by Vecchio & Collins [8], Kaufmann and Marti [11] and Zwicky and Vogel [12] give the best performance in prediction indicated by the lowest SD, COV and MAPE. This seems to agree with Foster and Malik [13] who concluded that the multi-parameter models and models based on the modified compression field theory give the best correlation with the test results. In addition, other STMs from available literature and U. S. codes of practice are selected and computed for comparison. The details of these are summarized in Table 4.

It is found from Figure 3 and Table 5 that both American codes of practice provide relatively conservative and inconsistent prediction, while Russo et al. [5] and Arabzadeh et al. [7] result in reasonable accuracy. However, the proposed STM using Zwicky & Vogel [12] shows the best performance.

4. Illustrated example

4.1 An RC deep beam with web reinforcement

Here, refer to Figure 1, the beam specimen No. ACI-I tested by Aguilar et al. [17] is used to illustrate the worked example, details of the specimen are as follows: Geometry: a = 915 mm, d = 791 mm, h = 915 mm, b_w = 305 mm, w_b = 305 mm; concrete strength: f'_c = 32 MPa; longitudinal reinforcement: ρ = 1.27%, f_y = 420 MPa; vertical web reinforcement: ρ_v = 0.31, f_{yv} = 450 MPa; and horizontal web reinforcement: ρ_h = 0.35, f_{yh} = 450 MPa.

Step 1: Determine geometric properties

From f'_c = 32 MPa, the classical bending parameters can be calculated here: n = 7.52, k = 0.352, j = 0.883,
$\theta_s = \tan^{-1}(jd/a) = \tan^{-1}(0.883 \times 791/915) = 0.652$ -rad. = 37.35 deg.
From Eq. (2a)

$$w_s = \sqrt{(kd)^2 + w_b^2} = \sqrt{(0.352 \times 791)^2 + 305^2}$$
$$= 412.92 \text{ mm.}$$
$$A_v = \rho_v\, b_w\, a = 0.31 \times 305 \times 915 = 865.13 \text{ sq. mm.}$$
$$A_h = \rho_h\, b_w\, d = 0.35 \times 305 \times 791 = 844.39 \text{ sq. mm.}$$

Step 2: Use the efficiency factor proposed by Vecchio & Collins [8]

$$\nu = \frac{1}{0.8 + 170\varepsilon_1} = \frac{1}{0.8 + 170 \times 0.005} = 0.606$$

Step 3: Determine the shear strength V_n from Eqs. (4)-(7)

$$V_D = \nu\, f'_c b_w w_s \sin\theta_s / 1000$$
$$= 0.606 \times 32 \times 305 \times 412.92 \times \sin 37.35^o / 1000$$
$$= 1482 \text{ kN}$$
$$V_v = A_v f_{yv} / 1000 = 865.13 \times 450 / 1000 = 389 \text{ kN}$$
and
$$V_h = A_h f_{yh} \tan\theta_s / 1000$$
$$= 844.39 \times 450 \times \tan 37.35^o / 1000 = 290 \text{ kN}$$
$$V_n = 0.971\, V_D + 0.161\, V_v + 0.265\, V_h$$
$$= 0.971 \times 1482 + 0.161 \times 389 + 0.265 \times 290$$
$$= 1578 \text{ kN}$$

This is quite close to the experimental strength of 1,357 kN, thus, the absolute percentage error is 16%.

4.2 An RC deep beam without web reinforcement

The beam specimen No. UH10-100 (see Fig.1) from Yang et al. [18] is used in this example, details of the specimen are as follows: Geometry: a = 1000 mm, d = 935 mm, h = 1000 mm, b_w = 160 mm, w_b = 100 mm; concrete strength: f'_c = 78.5 MPa; longitudinal reinforcement: ρ = 0.90%.

Step 1: Determine geometric properties

From f'_c = 78.5 MPa, n = 4.80, k = 0.254, j = 0.915,
$\theta_s = 0.708$ rad. = 40.56 deg.
From Eq. (2a), $w_s = 257.65$ mm.

Table 2 Results of regression analysis

Efficiency Factor Model	Regression coefficients β_0	β_1	β_2	β_3	P (F)	R^2	P (t) Intercept	P (t) V_D	P (t) V_v	P (t) V_h
Vecchio & Collins [8]	4.46	0.971	0.161	0.265	< 0.01	0.94	0.668	< 0.01	< 0.01	< 0.01
Warwick & Foster [9]	11.59	0.679	0.368	-0.07	< 0.01	0.87	0.456	< 0.01	< 0.01	0.336
Zhang & Hsu [10]	-1.17	0.968	0.204	0.242	< 0.01	0.93	0.688	< 0.01	< 0.01	< 0.01
Kaufmann & Marti [11]	5.00	0.972	0.159	0.263	< 0.01	0.94	0.630	< 0.01	< 0.01	< 0.01
Zwicky & Vogel [12]	5.36	0.972	0.158	0.262	< 0.01	0.94	0.967	< 0.01	< 0.01	< 0.01

Table 3 Validation of Regression Models

Efficiency Factor Model	Regression Model	AVG	SD	% COV	MAPE
Vecchio & Collins [8]	$V_n = 0.971V_D + 0.161V_v + 0.265V_h$	1.00	0.16	16%	12%
Warwick & Foster [9]	$V_n = 0.676\,V_D + 0.371\,V_v$	1.07	0.22	20%	16%
Zhang & Hsu [10]	$V_n = 0.968\,V_D + 0.204\,V_v + 0.242\,V_h$	1.00	0.18	18%	14%
Kaufmann & Marti [11]	$V_n = 0.972\,V_D + 0.159\,V_v + 0.263V_h$	1.00	0.16	16%	12%
Zwicky & Vogel [12]	$V_n = 0.972\,V_D + 0.158\,V_v + 0.262\,V_h$	1.00	0.16	16%	12%

Table 4 Selected existing shear strength models of reinforced concrete deep beams

Model	Efficiency Factor Models
ACI 318-11 [1]	$V_n = f_{ce}\sin\theta_s b_w w_s$ where $f_{ce} = 0.85\beta_s\, f'_c$, $w_s = \min[(w_t\cos\theta_s + w_b\sin\theta_s),(h_c\cos\theta_s + w_b\sin\theta_s)]$, $\beta_s = 0.75$ for bottle-shaped struts with reinforcement satisfying Section A.3.3 of ACI 318-11 $\beta_s = 0.60$ for bottle-shaped struts without reinforcement satisfying Section A.3.3 of ACI 318-11
AASHTO-LRFD 2008 [2]	$V_n = f_{cu}\sin\theta_s b_w w_s$ where $f_{cu} = \dfrac{f'_c}{0.8+170\varepsilon_1} \le 0.85 f'_c$, $\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002)/\tan^2\theta_s$, $w_s = w_t\cos\theta_s + w_b\sin\theta_s$
Russo et al. [5]	$V_n = 0.545\left(k\chi\, f'_c\cos\alpha_s + 0.25\rho_h f_{yh}\cot\alpha_s + 0.35\dfrac{a}{d}\rho_v f_{yv}\right)b_w d$ where $\chi = 0.74\left(\dfrac{f'_c}{105}\right)^3 - 1.28\left(\dfrac{f'_c}{105}\right)^2 + 0.22\left(\dfrac{f'_c}{105}\right) + 0.87$, and $\tan\alpha_s = \dfrac{a}{0.9d}$
Arabzadeh et al. [7]	$V_n = \dfrac{f'^{\,0.70}_c}{0.5+0.1\left(\dfrac{a}{d}\right)^2} A_{str}\sin\theta_s + 0.09\rho_p^{-0.35}A_{wp}\cos\theta_s$, $\rho_p = \rho_v\cos^2\theta_s + \rho_h\sin^2\theta_s$ $A_{wp} = \rho_p A_{str(t)}$, $A_{str(t)}$ = the area of tangential-section of strut

Step 2: Use the efficiency factor proposed by Zwicky & Vogel [12]

$$\nu = (1.8 - 38\varepsilon_1)\cdot(f'_c)^{-1/3} = (1.8 - 38\times 0.005)\cdot(78.5)^{-1/3}$$
$$= 0.376$$

$$\nu \le 1.6\cdot(f'_c)^{-1/3} = 1.6\times(78.5)^{-1/3} = 0.374$$

Step 3: Determine the shear strength V_n from Eqs. (4)-(7)

$$V_D = \nu\, f'_c b_w w_s \sin\theta_s$$
$$= 0.374\times 78.5\times 160\times 257.65\times\sin 40.56^o/1000$$
$$= 786.3 \text{ kN}$$

Because the specimen used in this example does not contain web reinforcement, $V_v = V_h = 0$.

$$V_n = 0.972\times 786.3 = 764 \text{ kN}$$

This is very close to the tested strength of 769 kN.

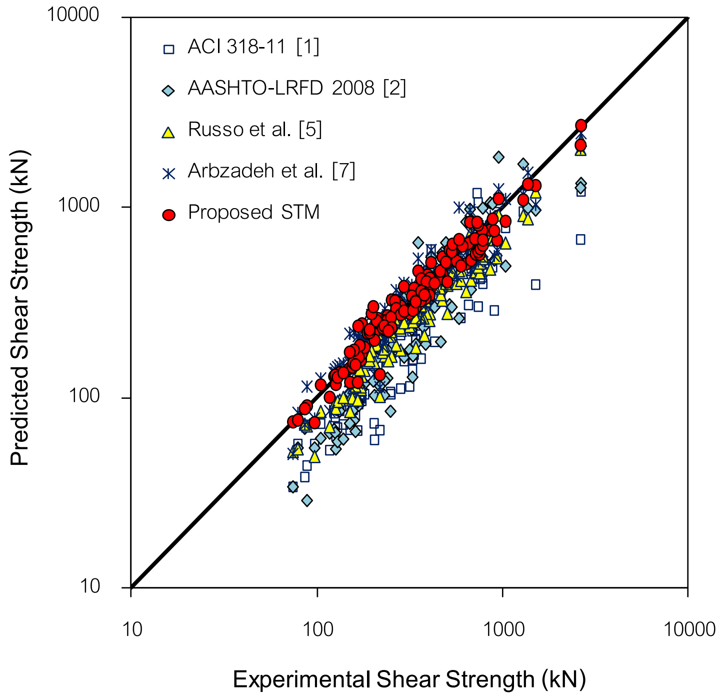

Figure 3 Comparison of the predicted shear strengths of deep beams obtained by five different STMs to the experimental shear strengths

Table 5 Performance in computation for the different STMs

STM	AVG	% COV	MAPE
ACI 318-11 [1]	1.52	42%	32%
AASHTO-LRFD [2]	1.44	33%	32%
Russo et al. [5]	1.31	17%	23%
Arabzadeh et al. [7]	1.01	21%	16%
The proposed STM	1.00	16%	12%

5. Conclusions

Based on a large database of test results and six state-of-the art efficiency factor models as well as the selected shape and geometry of the STM, the STMs developed using MLR for predicting the shear strength of single-span deep beams subjected to symmetrical point loads are presented in this paper. The following conclusions can be drawn for the present study:

• In general, there is strong linear relationship between the deep beam shear strength and the vertical load capacities of STM consisting of diagonal strut, vertical and horizontal reinforcement.

• It can be concluded that the calibration of the STMs using MLR analysis in this study shows good agreement with test results.

• The three developed models using efficiency factor obtained from Vecchio & Collins [8], Kaufmann and Marti [11] and Zwicky and Vogel [12] provide the same level of accuracy.

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7. References

[1] ACI Committee 318. Building Code Requirements for Structural Concrete (ACI 318-11) and Commentary. Detroit: American Concrete Institute; 2011.
[2] AASHTO. AASHTO LRFD Bridge Design Specifications. 4th ed. Washington: AASHTO; 2008.
[3] Hwang S, Lu W, Lee H. Shear Strength Prediction for Deep Beams. ACI Str J 2000;97(3):367-376.
[4] Matamoros AB, Wong KH. Design of Simply Supported Deep Beams Using Strut-and-Tie Models. ACI Str J 2003;100(6):704-712.
[5] Russo G, Venir R, Pauletta M. Reinforced Concrete Deep Beams—Shear Strength Model and Design Formula. ACI Str J 2005;102(3):429-437.
[6] Park JW, Kuchma D. Strut-and-Tie Model Analysis for Strength Prediction of Deep Beams. ACI Str J 2007;104(6):657-666.
[7] Arabzadeh A, Rahaie AR, Aghayari R. A Simple Strut-and-Tie Model for Prediction of Ultimate Shear Strength of RC Deep Beams. Int J of Civil Eng 2009;7(3):141-153.
[8] Vecchio FJ, Collins MP. The Modified Compression Field Theory for Reinforced Concrete Elements Subjected to Shear. ACI J 1986;83(2):219-231.
[9] Warwick WB, Foster SJ. Investigation into the Efficiency Factor Used in non-Flexural Reinforced Concrete Member Design. Kensington: University of New South Wales; 1993. UNICIV Report No. R-320.
[10] Zhang LX, Hsu TTC. Behavior and Analysis of 100 MPa Concrete Membrane Elements. J of Str Eng ASCE 1998;124(1):24-34.

[11] Kaufmann W, Marti P. Structural Concrete: Cracked Membrane Model. J of Str Eng ASCE 1998;124(12):1467-1475.
[12] Zwicky D, Vogel T. Critical Inclination of Compression Struts in Concrete Beams. J of Str Eng ASCE 2006;132(5):686-693.
[13] Foster SJ, Malik AR. Evaluation of Efficiency Factor Models used in Strut-and-Tie Modeling of Nonflexural Members. J of Str Eng ASCE 2002;128(5):569-577.
[14] Hwang SJ, Lu WY, Lee HJ. Shear Strength Prediction for Reinforced Concrete Corbels. ACI Str J 2000;97(4):543-552.
[15] Hwang SJ, Lee HJ. Strength Prediction for Discontinuity Regions by Softened Strut-and-Tie Model. J of Str Eng ASCE 2002;128(12):1519-1526.
[16] Chetchotisak P, Teerawong J, Yindeesuk S, Song J. New Strut-and-Tie-Models for Shear Strength Prediction and Design of RC Deep Beams. Computers and Concrete 2014;13(7):19-40.
[17] Aguilar G, Matamoros AB, Parra-Montesinos GJ, Ramirez JA, Wight JK. Experimental Evaluation of Design Procedures for Shear Strength of Deep Reinforced Concrete Beams. ACI Str J 2002;99(4):539-548.
[18] Yang KH, Chung HS, Lee ET, Eun HC. Shear Characteristics of High-Strength Concrete Deep Beams without Shear Reinforcements. Eng Struct 2003;25:1343-1352.