



Modeling standard lines for soil compaction testing using artificial neural networks and geometric algorithms

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Abstract

Soil compaction testing is crucial for ensuring the stability and durability of infrastructure projects. Traditional methods for generating standard lines, such as averaging and polynomial fitting, often fail to capture the nonlinear relationships and variability in compaction data, leading to inaccuracies in soil property assessments. This study introduces a novel framework that leverages Artificial Neural Networks (ANNs) to dynamically model standard lines for compaction curves, addressing limitations of traditional approaches. Four activation functions—ReLU, Sigmoid, Tanh, and Swish—were evaluated, with Swish emerging as most effective for capturing complex relationships between Dry Density (DD) and Moisture Content (MC). A tolerance-based evaluation framework, incorporating tolerance levels of 2%, 5%, and 10%, was applied to analyze coverage areas. The 5% tolerance level was identified as most balanced, minimizing errors while providing reliable representations of compaction data. The study also introduced the Ray-Casting Algorithm for precise calculation of coverage areas, enabling a new performance indicator based on density of data points within the region. Results demonstrate that the ANN framework, particularly with Swish activation, outperforms traditional statistical methods in accuracy and adaptability. ReLU delivered the best performance, with the lowest prediction and percentage errors (0.1910–0.2005 and 10.66%–11.74%), while effectively explaining over 55% of the data's variability. Sigmoid showed the weakest results, with high errors and near-zero variance explanation. Tanh performed moderately, balancing accuracy and generalization with reasonable error levels and 44%–54% variance capture. Swish was consistently reliable, with stable errors and over 50% of the variance explained. This research advances compaction testing by addressing variability, operator-induced errors, and nonlinear data patterns, establishing a reliable methodology for generating standard lines. Future work could explore diverse soil types, integrate environmental factors, develop hybrid machine learning models, and improve performance indicators.

Keywords: Soil compaction testing, Artificial Neural Networks (ANNs), Standard line generation, Ray-Casting Algorithm, Tolerance-Based Evaluation, Geotechnical engineering

1. Introduction

Soil compaction testing plays a critical role in geotechnical engineering, ensuring the stability and durability of infrastructure projects. This process typically involves determining the relationship between soil density and moisture content to achieve the Maximum Dry Density (MDD) at the Optimum Moisture Content (OMC). The compaction curve generated during testing provides valuable insights into soil behavior. However, errors in compaction test results, whether due to operator variability, equipment calibration, or environmental factors, can lead to discrepancies in the compaction curves, ultimately impacting the reliability of soil assessments. Studies such as [1], [2], and [3] have highlighted how variability in soil composition, environmental conditions, and operator-induced errors can influence compaction results, emphasizing the need for more robust testing methods.

To address these inconsistencies, current methodologies often focus on generating a standard line to represent a set of compaction curves as a single unified profile. This approach is crucial for standardizing compaction results across different tests and operators. The most commonly used methods for creating a standard line include averaging compaction curves and fitting polynomial equations to represent the data, as shown in Figure 1. Averaging methods, as explored by [4], aim to create a baseline by calculating the mean compaction curve across multiple tests, while polynomial fitting methods, discussed by [5] and [6], attempt to represent the overall trend of compaction data through mathematical models. While these methods provide a simplified representation, they may not fully capture the variability and nuances of the underlying compaction data. Recent studies, including [7], [8], and [9], have explored these statistical and computational methods for standard line generation, but significant gaps remain in their ability to handle diverse datasets effectively.

Artificial Neural Networks (ANNs) have demonstrated their capability in handling complex datasets and identifying patterns in diverse applications. Their ability to model nonlinear relationships and generalize across varying conditions makes them a promising

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tool for creating a standard line of compaction curves. By training ANNs on compaction test data, it is possible to generate a representative standard line that minimizes errors and accounts for variability in the dataset. Research by [10], [11], and [12] has shown the potential of ANNs in geotechnical applications, including soil property predictions and compaction analysis, providing a strong foundation for this approach.

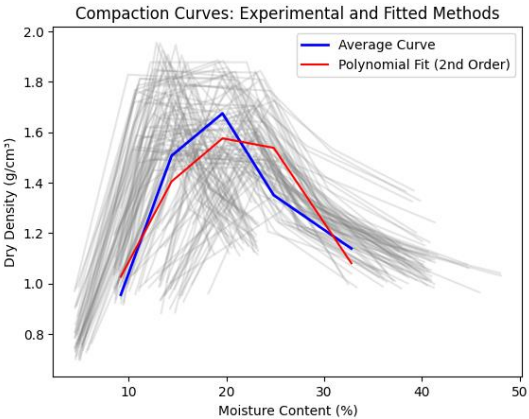


Figure 1 Standard curve by traditional approaches

The performance of a standard line can be evaluated through specific indicators, such as the coverage area created by a \pm tolerance band around the standard line, which reflects the consistency and reliability of the compaction data. However, traditional methods of evaluating coverage area often lack precision. This study introduces the Ray-Casting Algorithm as an innovative approach to calculate the coverage area and proposes a new performance indicator that incorporates the density of data points within the coverage region. Studies by [13], [14], and [15] have explored similar geometric algorithms in spatial data analysis, but their application to soil compaction testing represents a novel direction.

Table 1 Summary of recent studies about ANN and ML

| Reference / Study | Soil Compaction Prediction (ML/ANN) | Comparison of ML Models | Explainable AI / Sensitivity Analysis | Focus on Standard Line Creation | Geometric Algorithm / Ray-Casting | New Indicator / Evaluation Method |
|---------------------------|-------------------------------------|---------------------------------------|---------------------------------------|---|-----------------------------------|-----------------------------------|
| Gurtug et al. [7] | X (Statistical prediction) | | | ✓ (Standardization issues discussed) | | |
| Khatti and Grover [16] | ✓ | ✓ | | | | |
| Othman and Abdelwahab [3] | ✓ (Deep neural networks) | | | | | |
| Khatti and Grover [17] | ✓ | ✓ (LSSVM, LSTM, LSBoostRF, ANN) | | | | |
| Khatti and Grover [18] | ✓ | ✓ (multiple soft computing models) | | | | |
| Khatti and Grover [19] | ✓ | ✓ | | | | |
| Khatti and Grover [20] | ✓ | ✓ (deep learning vs traditional) | ✓ (performance analysis depth) | | | |
| Wang et al. [8] | ✓ | | | ✓ (Computational variability handling) | | |
| Khan et al. [21] | ✓ | ✓ (ANN, LR, MLR, NLR) | ✓ (Sensitivity analysis) | | | |
| Mohammed et al. [22] | ✓ | ✓ (ANN, DNN, SVR, RF, XGBoost) | ✓ (Explainable AI) | | | |
| Adamolekun et al. [23] | ✓ | ✓ (ANN, RF, SVM, XGBoost) | ✓ (Feature importance) | | | |
| This Study (2025) | ✓ | ✓ (ReLU, Sigmoid, Tanh, Swish) | ✓ Coverage Analysis | ✓ | ✓ | ✓ (Coverage points) |

Numerous studies have explored soil compaction prediction using machine learning techniques. Othman and Abdelwahab [3] and Khatti and Grover [16-20] applied models such as ANN, LSSVM, LSTM, and other soft computing approaches to estimate compaction parameters, demonstrating improved predictive accuracy over traditional methods. Comparative analyses of different models have been a common focus in recent works. Khan et al. [21], Mohammed et al. [22], and Khatti and Grover [17, 20] evaluated a range of algorithms, including ANN, DNN, SVR, RF, and XGBoost, to identify optimal predictive frameworks. Efforts to enhance interpretability through Explainable AI (XAI) and sensitivity analysis were highlighted in studies by Mohammed et al. [22] and Adamolekun et al. [23], who assessed feature importance to support model transparency and decision-making. Gurtug et al. [7] and Wang et al. [8] addressed standard line creation and compaction curve variability using statistical prediction and computational

strategies, marking early efforts to integrate standardization into soil compaction studies. Despite advancements in ML applications, geometric algorithms such as Ray-Casting remain largely unexplored in this context, presenting a gap for future innovation in soil behavior modeling. Few studies have proposed new indicators or evaluation metrics specific to standard compaction line accuracy or model validation, signaling a need for more comprehensive and domain-specific performance evaluation frameworks. However, no existing studies combine ML prediction, standard line generation, geometric evaluation, and the development of a new performance indicator, which this study aims to address to enhance accuracy, reliability, and standardization in soil compaction testing, as summarized in Table 1.

This paper aims to apply Artificial Neural Networks (ANN) for predicting soil compaction parameters based on data from Modified Proctor Tests, thereby improving the accuracy and efficiency of assessing compaction characteristics. It also compares the performance of the ANN model using a newly proposed indicator, the coverage points, to evaluate prediction accuracy. Additionally, the study incorporates Explainable AI (XAI) and sensitivity analysis to ensure that the model's predictions align with geotechnical principles and enhance interpretability. A key objective is the development of a standard compaction line derived from Modified Proctor Test parameters using ANN, moving beyond traditional statistical methods. Moreover, the paper introduces a geometric algorithm based on the Ray-Casting technique to construct a coverage area based on tolerance percentages, which is then used to calculate the coverage points. Finally, the study proposes this new indicator and evaluation method as a novel approach to improve the standardization and accuracy of soil compaction testing in geotechnical engineering, offering an alternative to conventional performance metrics.

2. Methodology

2.1 Notation and abbreviation

This section provides definitions of key notations and abbreviations used throughout the study for clarity and consistency as Table 2.

Table 2 The summary of Notation and Abbreviation

| Notation / Abbreviation | Definition | Notation / Abbreviation | Definition |
|----------------------------|--|----------------------------|--|
| OMC | Optimum Moisture Content | RMSE | Root Mean Square Error |
| MDD | Maximum Dry Density | MAE | Mean Absolute Error |
| MC | Moisture Content | WMAPE | Weighted Mean Absolute Percentage Error |
| DD | Dry Density | MAPE | Mean Absolute Percentage Error |
| GW | Well-Graded Gravel | PI | Performance Index |
| AASHTO | Soil classification system used in highway engineering | RSR | Ratio of Standard Deviation of Residuals to Standard Deviation of Observations |
| USCS | Unified Soil Classification System | NS | Nash-Sutcliffe Efficiency |
| ASTM | American Society for Testing and Materials | VAF | Variance Accounted For |
| ICC | Intraclass Correlation Coefficient | y_i | Actual value |
| ANN | Artificial Neural Network | \hat{y} | Predicted value |
| ReLU | Rectified Linear Unit | \bar{y} | Mean of actual values |
| Tanh | Hyperbolic Tangent | N | Number of observations |
| SL | Standard line | $var(y - \hat{y})$ | Variance of the prediction errors |
| $DD_{upper,i}$ | Dry Density value at point i on the standard line | $var(y)$ | Variance of the actual values |
| $DD_{SL,i}$ | Upper Dry Density | AVG | Average |
| $DD_{lower,i}$ | Lower Dry Density | SD | Standard Deviation |
| ε | Tolerance value | | |

2.2 Soil compaction data and experiment

The soil used in this study is classified as GW under the USCS and as A-2-5 (Clayey Gravel) under the AASHTO classification system. Physically, it is well-graded gravel with a minor silt content. The classifications from both systems align consistently, indicating its suitability as a subbase material for highway construction in accordance with government standards. In terms of basic soil properties, the liquid limit is 49.16, the plasticity index is 3.89, and the specific gravity ranges from 2.65 to 2.70. Figure 2 illustrates the grain size distribution curve for the soil studied.

In the experimental procedure, the soil was sieved through a No. 4 (4.75 mm) sieve, and the moisture content was adjusted incrementally before compaction. The compaction process used a cylindrical mold, with the soil compacted in three layers by a 4.54 kg hammer dropped from a height of 457 mm, as specified by ASTM D1557. This systematic approach ensured consistent and reliable results across all test sets.

The soil compaction experiments in this study followed the Modified Proctor Compaction Test (ASTM D1557) to determine the MDD and OMC under high-energy compaction conditions, which use higher compactive effort compared to the Standard Proctor Test (ASTM D698) [24]. The experiments involved a total of 120 test sets (Figure 1), with each test repeated five times using varying moisture contents of 2%, 4%, 6%, 8%, and 10%. Three operator groups conducted the experiments, with each group performing 30 test sets. The raw data is presented in the appendix, ensuring a comprehensive analysis. Moreover, based on the uncontrollable or variable factors for soil specimens, including inconsistent compaction, uneven hammer strikes, uneven moisture distribution, weighing errors, mould wear, operator skill variation, temperature fluctuations, drying time differences, moisture evaporation, organic material interference, and weighing equipment accuracy, the data were tested for reliability analysis, as shown in Table 3. The results indicate that both DD (0.7799) and MC (0.8604) exhibit good consistency based on the Intraclass Correlation Coefficient (ICC) values, which fall within the range of 0.75 to 0.9. The ICC values reflect the level of agreement or reliability in the measurements, with values above 0.75 indicating a strong degree of consistency. In addition, both the dry density and moisture content datasets were plotted using Bland–Altman plots with confidence intervals to visually show the agreement between repeated measurements, as shown in Figure 3.

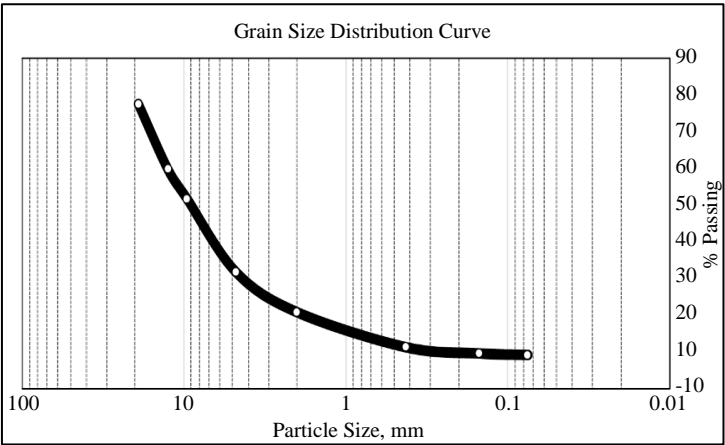


Figure 2 Grain Size Distribution Curve

Table 3 The results of reliable analysis using ICC (One-Way Random Model: ICC(1))

| Dataset | ICC(1) | Consistency |
|---------|--------|-------------|
| DD | 0.7799 | Good |
| MC | 0.8604 | Good |

When $ICC < 0.5$ (Poor consistency), $0.5 \leq ICC < 0.75$ (Moderate consistency), $0.75 \leq ICC < 0.9$ (Good consistency) and $ICC \geq 0.9$ (Excellent consistency)

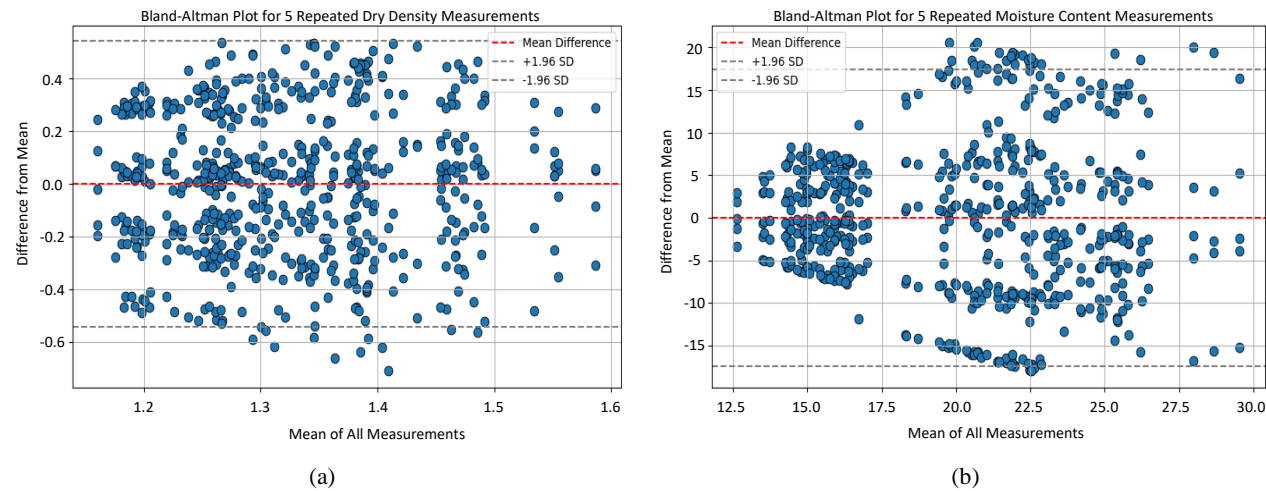


Figure 3 ICC plots using Bland–Altman plot with confidence intervals, (a) Dry Density, (b) Moisture Content

2.3 Artificial Neural Network (ANN) architecture and activation functions

ANNs have emerged as a robust computational tool for modeling nonlinear relationships in complex datasets. The architecture of an ANN typically consists of an input layer, one or more hidden layers, and an output layer. Each layer is composed of neurons, and the activation function in each neuron determines the output by introducing nonlinearity into the model [25, 26]. Choosing appropriate activation functions is critical for enhancing the learning ability and performance of the ANN [27, 28].

This study incorporates four activation functions—ReLU, Sigmoid, Tanh, and Swish—due to their unique characteristics and suitability for specific applications in modeling soil compaction data. These functions were chosen because they address key challenges such as nonlinearity, vanishing gradients, and model interpretability in the context of predicting standard lines for compaction testing [10, 12, 27].

2.4 Configuration analysis

The sensitivity analysis in this study evaluated the impact of key configurations on the performance of the ANN model, focusing on the number of hidden layers (3, 4, and 5) and the number of dense units (32, 64, and 128). These parameters were systematically varied to assess their influence on the model’s ability to predict the optimal standard curve line for soil compaction. To ensure the analysis focused solely on these variables, other configurations were held constant, including a test size of 0.2, a random state of 45 for reproducibility, 1000 epochs, a batch size of 10, and a validation split of 0.2. This approach provided a controlled environment to isolate the effects of the number of hidden layers and dense units, offering valuable insights into optimizing the ANN model for reliable and accurate soil compaction predictions. The summary is shown in Table 4.

Table 4 Summary of Configuration

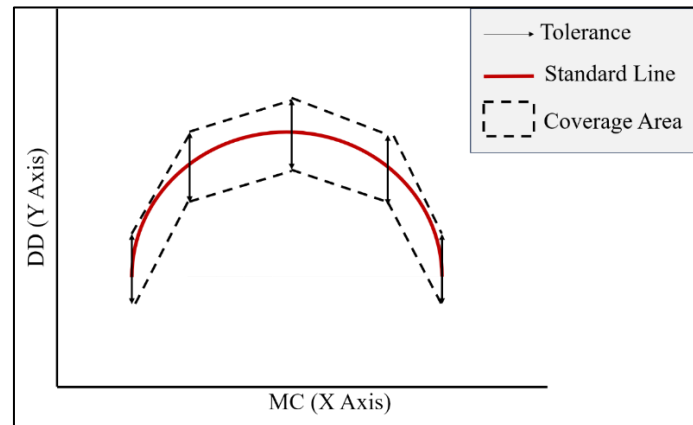
| Configuration | Test size | Random state | Epochs | Batch size | Validation split | Hidden layers | Dense units |
|---------------|-----------|--------------|--------|------------|------------------|---------------|-------------|
| 1 | 0.2 | 45 | 1000 | 10 | 0.2 | 3 | 32 |
| 2 | | | | | | 3 | 64 |
| 3 | | | | | | 3 | 128 |
| 4 | | | | | | 4 | 32 |
| 5 | | | | | | 4 | 64 |
| 6 | | | | | | 4 | 128 |
| 7 | | | | | | 5 | 32 |
| 8 | | | | | | 5 | 64 |
| 9 | | | | | | 5 | 128 |

Moreover, the model is configured with the Adam optimizer and a mean squared error (MSE) loss function. Adam is a popular optimization algorithm known for its efficiency and adaptive learning rate, making it well-suited for training deep learning models. The choice of MSE as the loss function is commonly used for regression tasks, as it penalizes large deviations between predicted and actual values, helping the model learn to minimize the error in predictions effectively.

2.5 Coverage area analysis and ray-casting algorithm

The coverage area analysis was conducted to evaluate how well the standard line (SL) represents the compaction data by determining the number of data points that fall within a defined tolerance region around it, as shown in Figure 4. The tolerance region was defined by two boundaries, represented by two equations. The upper boundary for each data point i was calculated as $DD_{upper,i} = DD_{SL,i} + \varepsilon$ where $DD_{SL,i}$ is the Dry Density value at point i on the standard line, and ε is the tolerance value. The lower boundary for each data point i was calculated as $DD_{lower,i} = DD_{SL,i} - \varepsilon$.

Moreover, to determine whether each data point falls within this coverage area as the coverage points, the Ray-Casting Algorithm was employed. This algorithm works by casting a horizontal ray from the data point and counting the number of intersections it makes with the polygon's edges. If the number of intersections is odd, the point is classified as being inside the polygon; if even, it is classified as outside. The steps of the algorithm involve iterating through all edges of the polygon, checking for ray-edge intersections, and toggling the classification based on the number of intersections. The area between these boundaries forms a polygonal region around the standard line that represents the allowable deviation for DD across the specified range of data, as shown in Figure 5.

**Figure 4** Coverage area From Standard \pm Tolerance

```
def is_point_in_polygon(point, polygon):
    x, y = point
    n = len(polygon)
    inside = False
    px, py = polygon[-1]
    for i in range(n):
        sx, sy = polygon[i]
        if ((sy >= y) != (py >= y)) and (x < (px - sx) * (y - sy) / (py - sy) + sx):
            inside = not inside
        px, py = sx, sy
    return inside
```

Figure 5 Ray-Casting Algorithm in Python

Moreover, while the study accounts for variations in DD by applying a tolerance band, the MC is treated as constant and represented by its average value across the dataset. This approach simplifies the analysis, focusing on the variability in DD while maintaining a consistent reference for MC. By combining these methods, the study evaluates the coverage area and quantifies the number of data points included within the tolerance-defined region using the Ray-Casting Algorithm.

2.6 Tolerance scenario

To evaluate the performance of the standard line in representing soil compaction data, three tolerance scenarios—2%, 5%, and 10%—were applied. The 5% tolerance level was selected based on recommendations from the Civil Engineering and Development Department (CEDD) of Hong Kong, which uses this threshold as a standard in soil compaction assessments. The 2% and 10% scenarios were included to analyze the effects of selecting tolerances that are either too low or too high. A 2% tolerance represents a strict scenario, capturing minimal deviations from the standard line, which may exclude valid data points and lead to overly restrictive evaluations. Conversely, a 10% tolerance represents a lenient scenario, allowing for broader deviations, which may compromise the reliability of the standard line by overestimating the acceptable range. These varying tolerance levels directly impact the number of data points covered within the defined coverage area, with lower tolerances reducing the number of covered points and higher tolerances increasing it. By comparing these three scenarios, this study aims to understand how tolerance levels influence the number of points within the coverage area and, consequently, the accuracy and applicability of the standard line in compaction testing.

2.7 Performance evaluation

The regular performance metrics of an ANN model were evaluated using four activation functions—ReLU, Sigmoid, Tanh, and Swish—across nine configurations (1 to 9), based on multiple indicators. RMSE, as shown in Equation 1, and MAE, as shown in Equation 2, measure the magnitude of prediction errors, with lower values indicating better performance [29, 30]. WMAPE, as shown in Equation 3, and MAPE, as shown in Equation 4, express prediction errors in relation to actual values, where lower values are preferable [31, 32]. PI, as shown in Equation 5, assesses model effectiveness, with higher values indicating better performance [33]. RSR, as shown in Equation 6, should be minimized for improved accuracy [34]. NS, as shown in Equation 7, ranges from negative values to 1 and is optimal when closer to 1, representing higher predictive power [35]. VAF, as shown in Equation 8, measures model fit, with higher values suggesting a better match between predicted and actual outcomes [36].

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (1)$$

$$\text{MAE} = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i| \quad (2)$$

$$\text{WMAPE} = \frac{\sum_{i=1}^n \sum_{l=1}^n |y_i - \hat{y}_l|}{\sum_{i=1}^n y_i} \quad (3)$$

$$\text{MAPE} = \frac{100\%}{n} \sum_{i=1}^n \left| \frac{y_i - \hat{y}_i}{y_i} \right| \quad (4)$$

$$\text{PI} = \frac{1}{n} \sum_{i=1}^n \left(1 - \frac{|y_i - \hat{y}_i|}{y_i} \right) \times 100 \quad (5)$$

$$\text{RSR} = \frac{\text{RMSE}}{\sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \bar{y})^2}} \quad (6)$$

$$\text{NS} = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \quad (7)$$

$$\text{VAF} = \left[1 - \frac{\text{var}(y - \hat{y})}{\text{var}(y)} \right] \times 100\% \quad (8)$$

3. Results

3.1 Comparison of compaction curves using different activation functions

The performance of the ANN was evaluated by examining the compaction curves generated using different activation functions: ReLU, Sigmoid, Tanh, and Swish. The compaction curves, represented by DD versus MC, were analyzed across nine configurations to assess the behavior of each activation function in modeling the standard line.

The compaction curves produced using the ReLU activation function (Figure 6a) exhibit a well-defined peak corresponding to the MD at the OMC. The results indicate that ReLU effectively captures the nonlinear relationship between DD and MC, maintaining consistent patterns across all configurations. The sharp transitions in the curves highlight the capability of ReLU to handle abrupt changes in data, making it suitable for modeling compaction characteristics.

In contrast, the Sigmoid activation function (Figure 6b) displays flatter and less dynamic compaction curves. The results reveal that Sigmoid struggles to capture the peak MDD and OMC accurately, particularly in higher configurations. This limitation can be

attributed to the vanishing gradient problem associated with the Sigmoid function, which leads to reduced sensitivity to changes in the input data. Consequently, the compaction curves show minimal variation across configurations, indicating suboptimal performance in representing the standard line.

The Tanh activation function (Figure 6c) produces compaction curves with clear peaks, similar to ReLU. The results demonstrate that Tanh effectively captures the nonlinear relationship between DD and MC, with smoother transitions than ReLU. However, some configurations exhibit steeper gradients, indicating a more aggressive response to changes in MC. This makes Tanh a robust choice for modeling compaction data while maintaining sensitivity to input variability.

The Swish activation function (Figure 6d) generates compaction curves that combine the strengths of ReLU and Tanh. The curves display well-defined peaks and smooth transitions, highlighting the effectiveness of Swish in modeling the standard line. The results suggest that Swish maintains a balanced response to input changes, avoiding the vanishing gradient issue of Sigmoid while capturing the nonlinear behavior effectively. Its performance is consistent across all configurations, making it a promising activation function for this application.

The comparative analysis of the four activation functions indicates that ReLU, Tanh, and Swish perform well in modeling compaction data, with Swish providing the most balanced and consistent results. Sigmoid, however, underperforms due to its inherent limitations in handling complex nonlinear relationships. This evaluation provides valuable insights into selecting the most suitable activation function for predicting compaction characteristics using ANN.

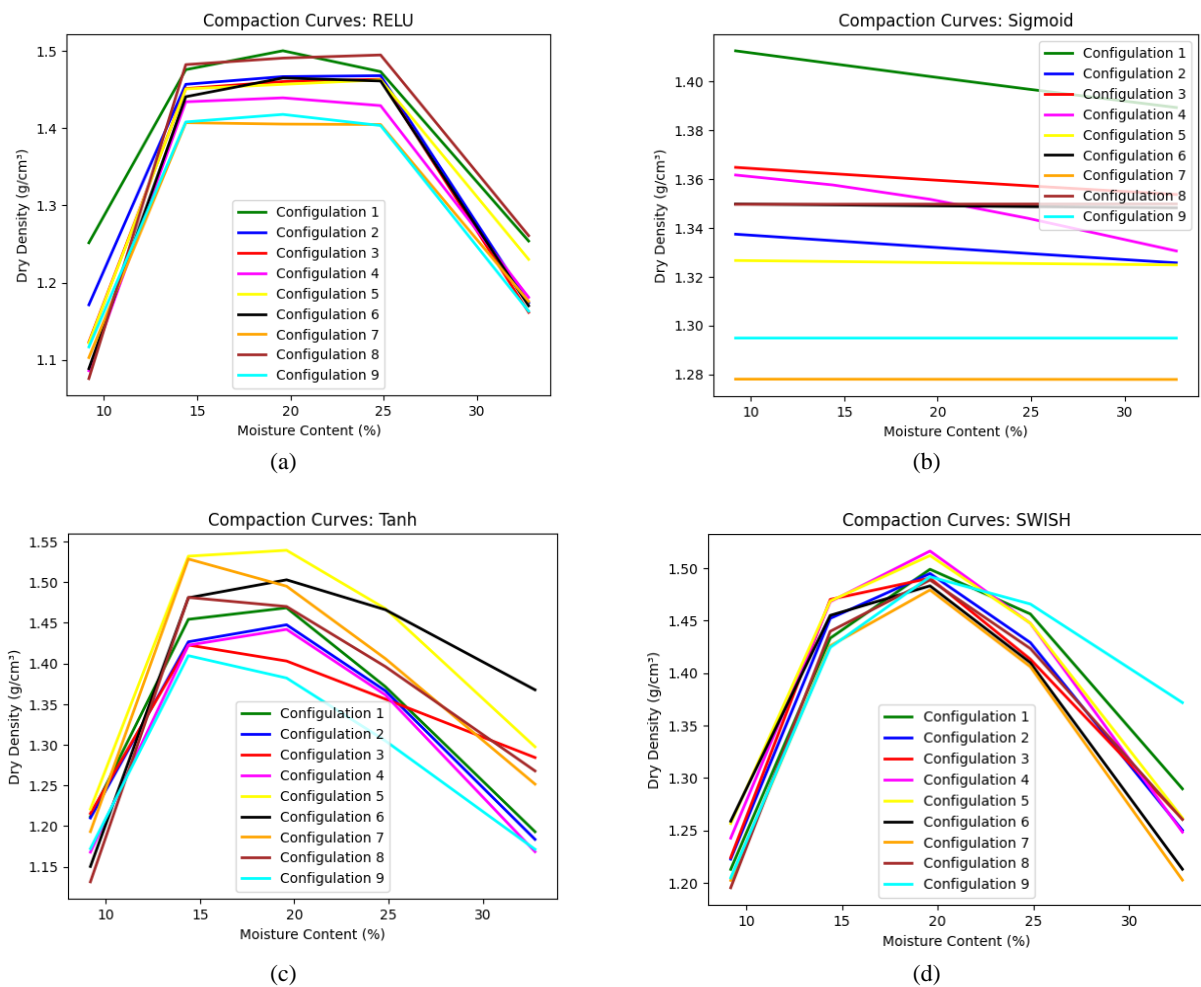


Figure 6 Compaction curve prediction, (a) RELU, (b) Sigmoid, (c) Tanh, (d) Swish,

3.2 Coverage analysis of compaction data across tolerance levels

The evaluation of compaction curves was extended to analyze the coverage area using three tolerance levels: 2%, 5%, and 10%. The number of DD spots covered by the standard line under each tolerance scenario was calculated, with higher coverage indicating the method's effectiveness in capturing the variability within the compaction data. Below is a detailed analysis of each tolerance level, supported by bar charts and boundary coverage figures.

The 2% tolerance level resulted in the lowest number of DD spots across all configurations and activation functions (Figure 7). The strict tolerance tightly constrained the boundaries, leading to overfitting and multiple overlapping peaks in the coverage areas. The average method (Figure 8a) highlighted the limitations of a fixed approach, producing narrow boundaries and failing to generalize effectively. Swish performed relatively well, particularly Swish1 and Swish4 (Figures 8b and 8c), compared to ReLU and Tanh. However, Sigmoid consistently underperformed, capturing fewer DD spots. ReLU1 and ReLU2 (Figures 8d and 8e) showed slightly better performance, while Tanh5 and Tanh7 (Figures 8f and 8g) captured some variability but still exhibited multiple peaks. Overall, the 2% tolerance was too restrictive, resulting in poor generalization and reduced model accuracy.

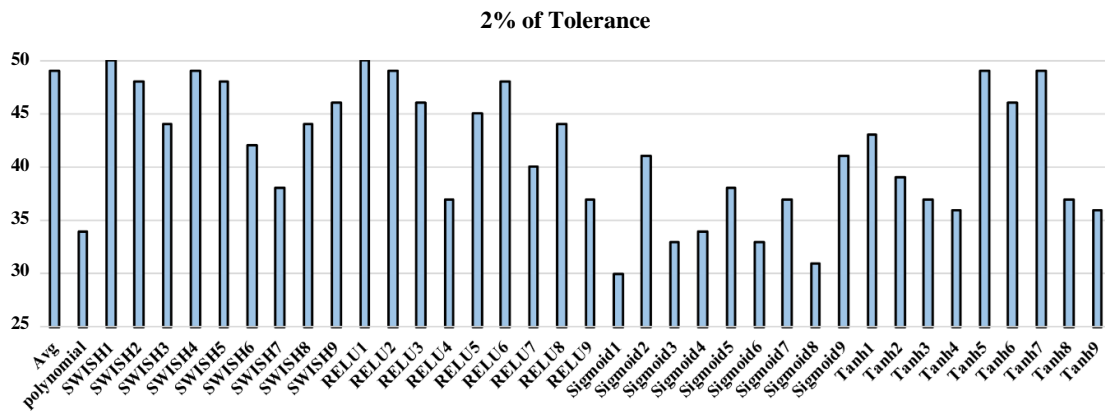


Figure 7 Coverage points at 2% of tolerance

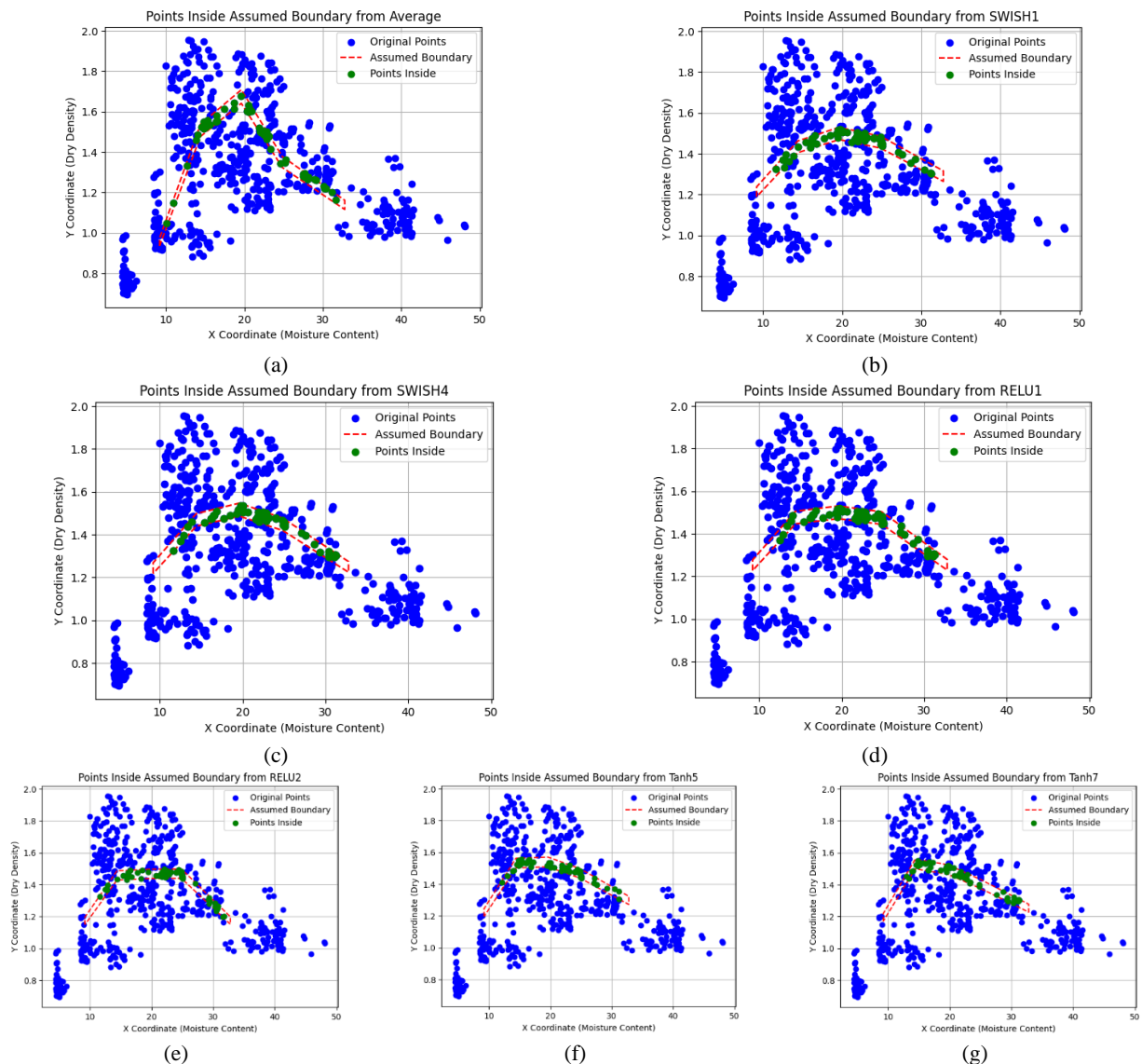


Figure 8 Comparison the peak from 2% tolerance result among, (a) Average, (b) SWISH1, (c) SWISH4, (d) RELU1, (e) RELU2, (f) Tanh5, (g) Tanh7

The 5% tolerance level, as shown in the bar chart (Figure 9), exhibited a balanced increase in DD spots covered across all configurations. This level is considered optimal, balancing strictness and coverage. Swish6 achieved the highest coverage (Figure 10a), capturing the single peak corresponding to the MDD at the OMC. The boundary effectively modeled the compaction curve, maintaining high sensitivity to the data's nonlinear behavior. In contrast, Sigmoid1 showed the lowest coverage (Figure 10b), with flatter boundaries that failed to represent the MDD accurately. The comparative figures demonstrate Swish6's ability to model the curve effectively, while Sigmoid1 struggled to capture critical features.

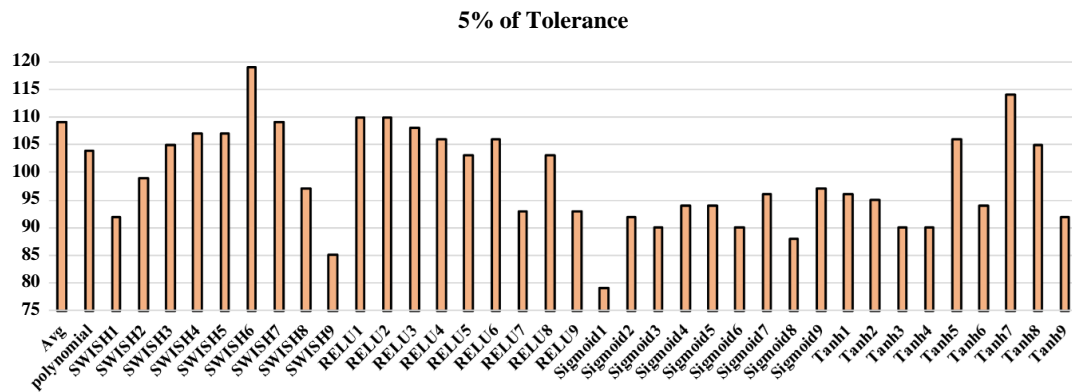


Figure 9 Coverage points at 5% of tolerance

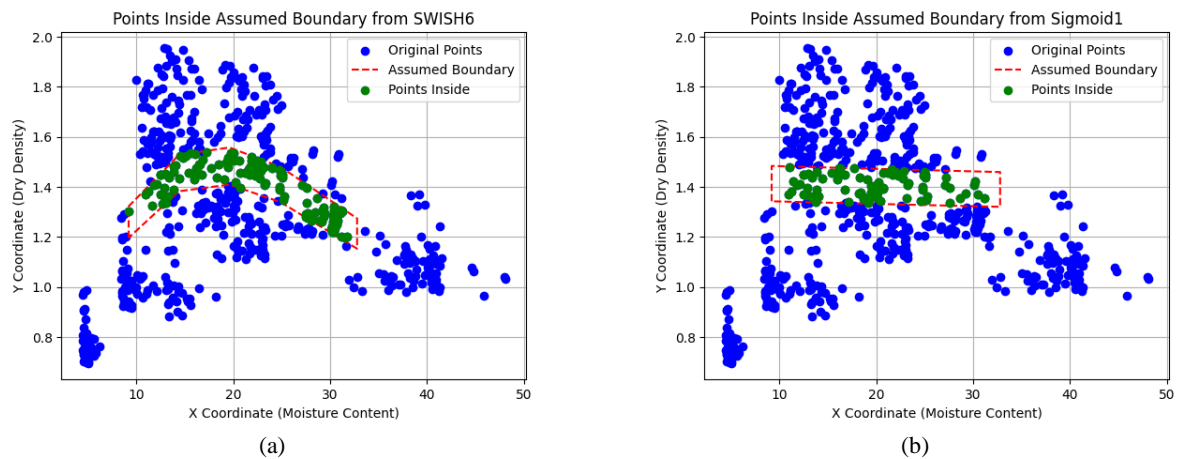


Figure 10 Comparison the highest and the lowest from 5% tolerance result between, (a) SWISH6, (b) Sigmoid1

The 10% tolerance level produced the highest number of DD spots covered, as illustrated in Figure 11. This lenient tolerance allowed for broader deviations, capturing a wider range of compaction data. However, it also introduced errors by overgeneralizing the data representation. Tanh3 (Figure 12a) maintained the expected curve shape, accurately capturing the MDD, while Sigmoid1 (Figure 12b) displayed a nearly rectangular boundary that failed to reflect the nonlinear compaction behavior. The figures highlight the trade-off at this level: while higher tolerances increase coverage, they compromise accuracy by overgeneralizing the coverage area, particularly for models like Sigmoid.

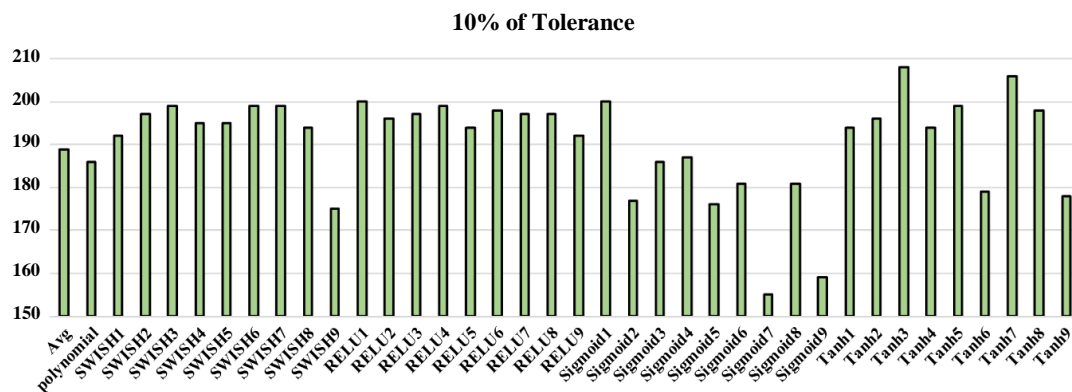


Figure 11 Coverage points at 10% of tolerance

The comparative analysis across tolerance levels revealed that Swish consistently outperformed other activation functions in terms of coverage, regardless of the tolerance. ReLU and Tanh also delivered strong results, especially under 5% and 10% tolerances, while Sigmoid underperformed due to its inherent limitations. The findings suggest that the 5% tolerance level offers the best balance between strictness and coverage, aligning with practical recommendations for compaction analysis. A 2% tolerance was too restrictive, resulting in multiple overlapping peaks and reduced accuracy, while a 10% tolerance was too lenient, introducing overgeneralization errors. The 5% tolerance emerged as the most reasonable option, providing a balanced representation of the compaction data and allowing for accurate modeling of the MDD and OMC. Swish, ReLU, and Tanh demonstrated strong performance, with Swish achieving the best overall results.

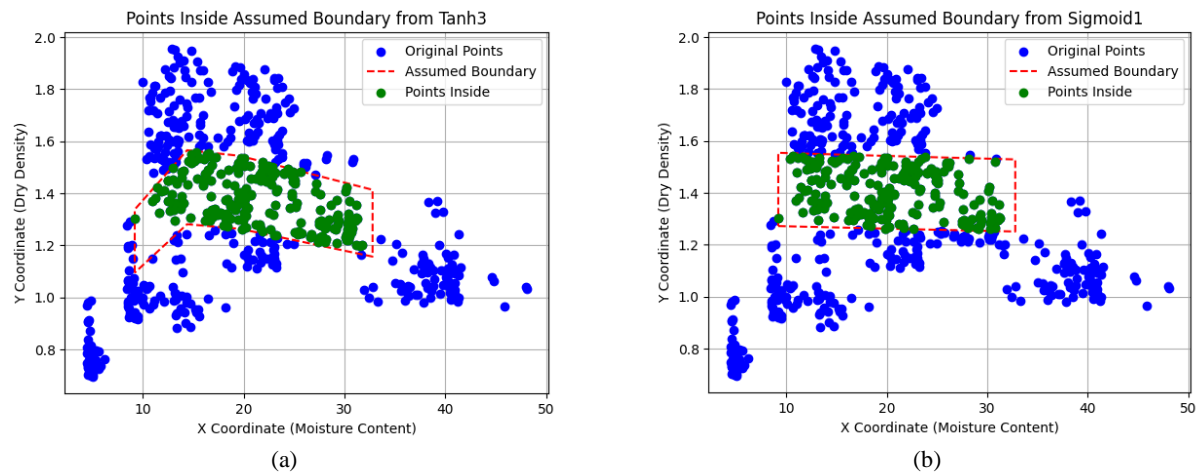


Figure 12 Comparison the peak from 10% tolerance result between, (a) Tanh3, (b) Sigmoid1

In addition, Swish emerged as the most effective model for generating a standard line for compaction testing. Its superior ability to capture the nonlinear relationship between DD and MC ensures that the compaction curve is accurately represented while maintaining high coverage under varying tolerances. These qualities make Swish the ideal activation function for establishing reliable and practical standards for compaction testing, offering precision, adaptability, and robustness across diverse scenarios.

Table 5 Results of regular performance measurements

| Activation Functions | Config. | Regular Performance Measurements | | | | | | | |
|--------------------------|---------|----------------------------------|--------------|----------------|---------------|--------------|--------------|--------------|---------------|
| | | RMSE (Lowest) | MAE (Lowest) | WMAPE (Lowest) | MAPE (Lowest) | PI (Highest) | RSR (Lowest) | NS (Highest) | VAF (Highest) |
| Ideal Values | | 0 | 0 | 0 | 0 | 100 | 0 | 1 | 100 |
| ReLU | 1 | 0.1934 | 0.1452 | 11.2299 | 11.3964 | 60.8333 | 0.6449 | 0.5806 | 58.2058 |
| | 2 | 0.1915 | 0.1422 | 10.9949 | 11.0462 | 62.5000 | 0.6387 | 0.5887 | 58.8662 |
| | 3 | 0.1961 | 0.1423 | 11.0051 | 10.6556 | 59.1667 | 0.6540 | 0.5687 | 58.6992 |
| | 4 | 0.1925 | 0.1434 | 11.0843 | 11.2353 | 61.6667 | 0.6418 | 0.5846 | 58.5389 |
| | 5 | 0.1946 | 0.1428 | 11.0413 | 11.0634 | 62.5000 | 0.6488 | 0.5755 | 57.6434 |
| | 6 | 0.1910 | 0.1407 | 10.8800 | 10.9784 | 64.1667 | 0.6370 | 0.5909 | 59.1227 |
| | 7 | 0.1942 | 0.1441 | 11.1432 | 11.1470 | 56.6667 | 0.6478 | 0.5768 | 57.8088 |
| | 8 | 0.2005 | 0.1511 | 11.6865 | 11.7390 | 51.6667 | 0.6687 | 0.5490 | 55.3482 |
| | 9 | 0.1940 | 0.1437 | 11.1138 | 11.2214 | 65.0000 | 0.6469 | 0.5780 | 57.8206 |
| Sigmoid | 1 | 0.2989 | 0.2444 | 18.8998 | 21.3936 | 34.1667 | 0.9968 | -0.0020 | 1.0456 |
| | 2 | 0.2982 | 0.2449 | 18.9359 | 21.2040 | 33.3333 | 0.9946 | 0.0025 | 0.6734 |
| | 3 | 0.3015 | 0.2452 | 18.9614 | 21.7158 | 34.1667 | 1.0056 | -0.0197 | 0.5981 |
| | 4 | 0.2995 | 0.2471 | 19.1055 | 21.4010 | 32.5000 | 0.9987 | -0.0057 | 0.0580 |
| | 5 | 0.2957 | 0.2412 | 18.6492 | 20.5824 | 33.3333 | 0.9863 | 0.0191 | 1.9414 |
| | 6 | 0.3003 | 0.2463 | 19.0429 | 21.5484 | 34.1667 | 1.0015 | -0.0114 | 0.2077 |
| | 7 | 0.3007 | 0.2478 | 19.1632 | 21.6294 | 31.6667 | 1.0027 | -0.0139 | -0.0027 |
| | 8 | 0.3073 | 0.2512 | 19.4213 | 22.4933 | 30.8333 | 1.0248 | -0.0591 | 0.0150 |
| | 9 | 0.2988 | 0.2481 | 19.1817 | 20.8916 | 32.5000 | 0.9965 | -0.0014 | 0.0064 |
| Tanh | 1 | 0.2171 | 0.1737 | 13.4294 | 13.7667 | 44.1667 | 0.7240 | 0.4714 | 47.1652 |
| | 2 | 0.2118 | 0.1711 | 13.2295 | 13.7324 | 44.1667 | 0.7063 | 0.4970 | 50.2719 |
| | 3 | 0.2225 | 0.1819 | 14.0617 | 14.5970 | 40.0000 | 0.7421 | 0.4446 | 46.9425 |
| | 4 | 0.2099 | 0.1639 | 12.6739 | 13.2412 | 47.5000 | 0.6999 | 0.5061 | 50.6213 |
| | 5 | 0.2014 | 0.1528 | 11.8120 | 11.9442 | 50.8333 | 0.6718 | 0.5449 | 54.7848 |
| | 6 | 0.2171 | 0.1689 | 13.0623 | 12.7692 | 49.1667 | 0.7240 | 0.4714 | 54.6921 |
| | 7 | 0.2121 | 0.1646 | 12.7271 | 13.8099 | 50.0000 | 0.7074 | 0.4954 | 50.2030 |
| | 8 | 0.2019 | 0.1592 | 12.3088 | 12.9965 | 48.3333 | 0.6735 | 0.5426 | 56.5586 |
| | 9 | 0.2093 | 0.1687 | 13.0436 | 13.5280 | 47.5000 | 0.6980 | 0.5087 | 52.9026 |
| Swish | 1 | 0.2072 | 0.1620 | 12.5220 | 12.7061 | 49.1667 | 0.6908 | 0.5187 | 51.8785 |
| | 2 | 0.2035 | 0.1574 | 12.1708 | 12.7326 | 52.5000 | 0.6786 | 0.5356 | 54.0790 |
| | 3 | 0.2067 | 0.1655 | 12.7973 | 13.2418 | 45.8333 | 0.6892 | 0.5210 | 53.4562 |
| | 4 | 0.2062 | 0.1614 | 12.4802 | 12.8740 | 50.0000 | 0.6878 | 0.5230 | 52.3308 |
| | 5 | 0.2123 | 0.1687 | 13.0454 | 14.0815 | 50.8333 | 0.7079 | 0.4947 | 51.0623 |
| | 6 | 0.2042 | 0.1561 | 12.0712 | 12.5091 | 50.8333 | 0.6810 | 0.5323 | 53.2408 |
| | 7 | 0.2100 | 0.1667 | 12.8911 | 13.5991 | 51.6667 | 0.7004 | 0.5053 | 50.8801 |
| | 8 | 0.2035 | 0.1556 | 12.0278 | 11.9762 | 50.8333 | 0.6786 | 0.5356 | 53.7964 |
| | 9 | 0.2053 | 0.1582 | 12.2352 | 12.6837 | 51.6667 | 0.6847 | 0.5273 | 52.7333 |
| Best Value | | 0.1910 | 0.1407 | 10.8800 | 10.6556 | 65.0000 | 0.6370 | 0.5909 | 59.1227 |
| Best Activation Function | | ReLU | ReLU | ReLU | ReLU | ReLU | ReLU | ReLU | ReLU |

3.3 Performance measurement results

As shown in Table 5, the evaluation of activation functions in the ANN model highlights distinct performance trends. ReLU performed well, with Config 6 (RMSE: 0.1910, MAE: 0.1407) and Config 2 (RMSE: 0.1915, MAE: 0.1422) showing the lowest errors, along with strong WMAPE and MAPE values. The highest Performance Index (PI) of 65.0000 appeared in Config 9, while VAF ranged from 55.3482 to 59.1227, indicating reasonable variance explanation. Sigmoid performed the worst, with significantly higher RMSE and MAE across all configurations, and the best NS value of only 0.0191 (Config 5), indicating poor predictive power, while VAF values remained near zero. Tanh performed better than Sigmoid but slightly worse than ReLU and Swish, with Config 5 (RMSE: 0.2014, MAE: 0.1528, VAF: 54.7848) and Config 8 (RMSE: 0.2019, MAE: 0.1592, VAF: 56.5586) showing the best results. PI values

for Tanh ranged from 40.0000 to 50.8333, reflecting moderate performance. Swish emerged as one of the best activation functions overall, with Config 2 (RMSE: 0.2035, MAE: 0.1574, VAF: 54.0790) and Config 8 (RMSE: 0.2035, MAE: 0.1556, VAF: 53.7964) demonstrating optimal performance. The best RSR value of 0.6786 (Configs 2 and 8) signaled superior model stability, while NS values were consistently higher than those of Tanh and significantly better than Sigmoid.

However, the best-performing activation function across all regular performance metrics was ReLU, demonstrating the lowest errors and highest predictive accuracy. Config 6 achieved the best results for RMSE (0.1910), MAE (0.1407), WMAPE (10.8800), RSR (0.6370), NS (0.5909), and VAF (59.1227), making it the most effective configuration overall. Config 3 provided the lowest MAPE (10.6556), while Config 9 achieved the highest PI of 65.0000, further confirming ReLU's superiority in this ANN model evaluation. This demonstrates that the proposed method, Coverage Analysis of Compaction Data Across Tolerance Levels, yields different performance measurements compared to those that produce the same results as ReLU.

4. Discussion

This study significantly advances the field of soil compaction testing by addressing key gaps identified in prior research and providing novel contributions to the modeling of standard lines for compaction curves. Traditional studies, such as those by [6] and [7], focused on statistical and mean-based methods for generating standard lines. These approaches, while foundational, lacked the capacity to adapt to nonlinear data and failed to capture variability effectively, particularly in complex soil compositions, as highlighted by [3]. This study bridges this gap by leveraging Artificial Neural Networks (ANNs) to model compaction data dynamically, enabling the generation of standard lines that account for nonlinear relationships. Moreover, while previous works like [8] and [9] emphasized the challenges posed by environmental conditions and operator-induced errors in compaction testing, this study incorporates a robust tolerance-based evaluation framework to mitigate these issues. By applying tolerance levels (2%, 5%, and 10%) to analyze coverage areas, this research introduces a performance evaluation metric grounded in geometric algorithms, building on prior geometric methods explored by [1] and [6]. The use of the Ray-Casting Algorithm to determine the inclusion of data points within tolerance boundaries represents a novel approach in soil mechanics, as suggested by [13] and [23].

This study also diverges from earlier research, such as [6] and [7], which primarily focused on unifying compaction curves through computational techniques but did not provide a clear evaluation metric for standard line performance. By integrating Swish, ReLU, Tanh, and Sigmoid activation functions, this research aligns with recent advancements in machine learning, including studies by [24] and [28]. The comparative analysis across activation functions revealed that Swish consistently outperformed the others, providing the most accurate and adaptable standard line for compaction testing, further validating findings by [26] and [27] on the effectiveness of Swish in capturing nonlinear patterns.

This study shows that the Swish activation-based ANN model outperforms the ReLU, Sigmoid, and Tanh-based ANN models in the context of compaction data characteristics. Swish is a smooth and non-monotonic activation function, which enables the model to learn more complex and subtle nonlinear relationships between input variables, such as Dry Density (DD) and Moisture Content (MC) [24, 27]. This is essential for modeling compaction curves, which do not follow simple linear or monotonic patterns. Moreover, Swish supports self-gating, meaning the function adjusts its own output based on the input, providing adaptive flexibility during learning [27]. This leads to more precise curve fitting and better generalization when handling real-world variations in soil compaction behavior. Furthermore, the gradient flow in Swish remains active across both positive and negative input ranges, avoiding gradient vanishing or dead neuron problems that can hinder training with other functions [24, 26, 28]. As a result, networks trained with Swish converge more efficiently and reliably, especially in deeper architectures or more variable datasets [25, 27]. Finally, Swish inherently balances the benefits of both linear and nonlinear activation, allowing it to remain sensitive to small input changes while also promoting robust learning across broader input domains [27, 28]. These properties make Swish particularly well-suited for establishing standard lines in compaction testing, where accurate curve modeling under varying tolerances is critical.

Furthermore, tolerance-based evaluations were enhanced by applying tolerance levels to optimize performance, an approach inspired by [34]. This allowed for a balanced analysis that aligns with practical recommendations while addressing the shortcomings of overly strict or lenient tolerance levels, as previously identified by [31]. The study's emphasis on the 5% tolerance level, supported by geometric evaluations, ensures a reliable framework for compaction testing that surpasses traditional methods, as explored by [1] and [6].

In addition to contributing new methodological insights, this study expands on the application of computational models in soil mechanics. Unlike prior studies such as [10] and [3], which focused on basic ANN implementations, this research incorporates multiple activation functions and evaluates their impact on tolerance-based coverage. This approach closes the gap in understanding how activation functions influence the generalizability of standard lines, building on the work by [24] and [28].

Artificial Neural Networks (ANNs) are highly effective in predicting soil compaction parameters such as Maximum Dry Density (MDD) and Optimum Moisture Content (OMC) due to their ability to model complex, non-linear relationships, particularly in large datasets with varied input features [3, 10, 15]. However, ANNs can require more computational resources and longer training times compared to other machine learning techniques. Support Vector Machines (SVMs) tend to offer high accuracy, especially for smaller, well-structured datasets, but may struggle with larger or noisier datasets unless kernel adjustments are made [17]. Random Forests (RF), being an ensemble method, are robust to noisy data and computationally less demanding than ANNs, although they might not capture highly complex, non-linear relationships as effectively [17, 21]. k-Nearest Neighbors (k-NN), while simple and intuitive, may encounter challenges with large datasets due to high computational costs during prediction and performance degradation in high-dimensional spaces [11]. Gradient Boosting (GB) often excels in precision for non-linear relationships but is more computationally expensive and can be prone to overfitting without proper tuning [21, 22]. Ultimately, the selection of a model depends on the dataset size, complexity, and available computational resources.

In conclusion, this study closes several gaps in soil compaction research by introducing machine learning-driven methodologies, tolerance-based performance metrics, and geometric algorithms for coverage evaluation. It establishes Swish as the optimal activation function for generating standard lines, outperforming traditional statistical and computational methods. By addressing variability, operator-induced errors, and nonlinear data patterns, this research lays the groundwork for more accurate, adaptable, and reliable standards in soil compaction testing.

5. Conclusion

This study has significantly advanced the modeling of standard lines for soil compaction testing by introducing Artificial Neural Networks (ANNs) and tolerance-based evaluation metrics. By addressing key gaps in traditional methodologies, such as mean-based and polynomial fitting approaches, the research demonstrates the potential of machine learning to handle nonlinear relationships and dataset variability more effectively. The integration of Swish, ReLU, Tanh, and Sigmoid activation functions into ANN models provided a comprehensive comparison, with Swish emerging as the most effective activation function for generating accurate and reliable standard lines. The use of a robust geometric algorithm, the Ray-Casting Algorithm, allowed for precise evaluation of coverage areas, accounting for variability within the dataset. Tolerance levels of 2%, 5%, and 10% were analyzed, with the 5% tolerance level identified as the most balanced approach, offering reliable standard lines while minimizing overfitting or overgeneralization. This research contributes significantly to geotechnical engineering by introducing a machine learning-driven framework that enhances the accuracy, adaptability, and standardization of compaction testing.

The use of Artificial Neural Networks (ANNs) for predicting Modified Proctor Test results faces several limitations. One of the main challenges is handling highly variable data, such as uneven moisture content in soil samples and environmental fluctuations, which can lead to decreased model accuracy. Additionally, the complexity of ANN models makes it difficult to interpret their results, which can reduce the reliability of the findings. The process of selecting appropriate model parameters, such as the number of layers or neurons, requires extensive testing and tuning, which may lead to instability in the results. These factors hinder the overall performance and reliability of ANN models in the context of soil compaction testing.

Future research should focus on addressing these limitations and enhancing the applicability of ANN models for Modified Proctor Test predictions. Studies could validate the framework across diverse soil types, incorporate environmental factors such as temperature and humidity, and develop hybrid models combining ANNs with advanced techniques like Support Vector Machines (SVM) or Random Forests to improve accuracy and reduce overfitting. Real-time applications using IoT devices for environmental monitoring, optimization of activation functions, and exploration of alternative geometric algorithms could further strengthen model reliability and precision. Additionally, integrating other geotechnical tests, such as permeability and consolidation, would expand the model's practical use. Importantly, applying explainable AI (XAI) techniques, such as SHAP or LIME, would improve the interpretability of ANN results, enhance model transparency, and support practical decision-making. Finally, developing performance indicators specifically tailored to compaction data analysis would improve the evaluation of standard lines and coverage accuracy.

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7. Appendix

Table 6 The raw data with Average (AVG), Standard Deviation (SD), Max and Min

| No. Test | Dry Density | | | | | Moisture Content | | | | | MDD | OMC |
|-------------|-------------|--------|--------|--------|--------|------------------|---------|---------|---------|---------|--------|---------|
| | 2% | 4% | 6% | 8% | 10% | 2% | 4% | 6% | 8% | 10% | | |
| 1 | 1.3011 | 1.5813 | 1.6745 | 1.6176 | 1.5805 | 9.2233 | 11.33 | 12.5 | 14.5078 | 15.544 | 1.6745 | 12.5 |
| 2 | 1.054 | 1.4075 | 1.7455 | 1.3915 | 1.2127 | 8.9591 | 11.7927 | 14.2919 | 20.6675 | 22.6269 | 175% | 14.2919 |
| 3 | 0.926 | 1.5975 | 1.8464 | 1.3074 | 1.1115 | 9.5206 | 12.8549 | 14.2381 | 18.1856 | 21.3229 | 185% | 14.2381 |
| 4 | 1.2771 | 1.6361 | 1.8737 | 1.644 | 1.5026 | 8.4026 | 10.5059 | 14.1662 | 18.7085 | 21.9352 | 187% | 14.1662 |
| 5 | 0.9761 | 1.3643 | 1.589 | 1.3585 | 1.2125 | 8.5294 | 12.7623 | 15.7447 | 18.6669 | 21.8957 | 159% | 15.7447 |
| 6 | 1.0035 | 1.3336 | 1.5505 | 1.3156 | 1.1618 | 8.8797 | 12.6699 | 14.8111 | 18.382 | 20.0406 | 155% | 14.8111 |
| 7 | 1.0641 | 1.5825 | 1.7615 | 1.5437 | 1.1561 | 8.501 | 12.3427 | 13.6881 | 16.6317 | 20.3101 | 1.7615 | 13.6881 |
| 8 | 0.9711 | 1.3548 | 1.5332 | 1.3774 | 1.1496 | 10.0675 | 13.8257 | 15.5464 | 20.1402 | 23.0404 | 1.5332 | 15.5464 |
| 9 | 1.2011 | 1.6059 | 1.8288 | 1.6315 | 1.506 | 8.8596 | 13.3877 | 16.4241 | 20.4728 | 22.4022 | 1.8288 | 16.4241 |
| 10 | 0.9234 | 1.5817 | 1.9552 | 1.578 | 1.1302 | 8.6005 | 11.2478 | 12.8403 | 16.466 | 18.2646 | 1.9552 | 12.8403 |
| 11 | 0.9995 | 1.4875 | 1.769 | 1.3768 | 1.2407 | 9.9891 | 13.9889 | 15.6165 | 19.4756 | 21.5359 | 1.769 | 15.6165 |
| 12 | 0.9819 | 1.481 | 1.7013 | 1.3449 | 1.1531 | 8.5964 | 13.8225 | 15.9992 | 20.41 | 22.7611 | 1.7013 | 15.9992 |
| 13 | 0.9974 | 1.3901 | 1.556 | 1.3017 | 1.1146 | 8.8404 | 13.853 | 15.3149 | 18.6418 | 23.1962 | 1.556 | 15.3149 |
| 14 | 0.9986 | 1.296 | 1.515 | 1.3229 | 1.1607 | 9.8257 | 13.2031 | 14.7705 | 20.0019 | 21.7362 | 1.515 | 14.7705 |
| 15 | 1.0228 | 1.37 | 1.7893 | 1.3125 | 1.1936 | 9.4803 | 12.5084 | 16.7434 | 20.2134 | 23.0257 | 1.7893 | 16.7434 |
| 16 | 0.9415 | 1.3357 | 1.5243 | 1.3239 | 1.1414 | 9.1705 | 13.0765 | 14.4694 | 17.6676 | 20.4885 | 1.5243 | 14.4694 |
| 17 | 1.0619 | 1.5983 | 1.9244 | 1.6144 | 1.1454 | 8.556 | 11.5368 | 13.4478 | 17.3758 | 20.8672 | 1.9244 | 13.4478 |
| 18 | 1.0243 | 1.4124 | 1.6787 | 1.334 | 1.1836 | 9.2154 | 12.1316 | 13.8321 | 16.8591 | 22.9208 | 1.6787 | 13.8321 |
| 19 | 1.1972 | 1.5352 | 1.7683 | 1.513 | 1.3227 | 8.5597 | 13.2828 | 15.2121 | 20.006 | 22.5374 | 1.7683 | 15.2121 |
| 20 | 1.1486 | 1.5363 | 1.9081 | 1.4911 | 1.2813 | 8.9654 | 11.1081 | 14.0432 | 16.443 | 21.5924 | 1.9081 | 14.0432 |
| 21 | 0.9869 | 1.4356 | 1.6719 | 1.3751 | 1.2397 | 9.1332 | 13.3317 | 15.6433 | 18.2374 | 23.2285 | 1.6719 | 15.6433 |
| 22 | 1.0331 | 1.6206 | 1.8824 | 1.5532 | 1.3205 | 8.4578 | 11.7475 | 13.1056 | 16.5407 | 17.8348 | 1.8824 | 13.1056 |
| 23 | 0.9856 | 1.4516 | 1.6946 | 1.4499 | 1.3273 | 9.8507 | 13.002 | 14.3603 | 16.7039 | 20.1158 | 1.6946 | 14.3603 |
| 24 | 1.0427 | 1.3608 | 1.6623 | 1.3782 | 1.2274 | 10.632 | 14.0546 | 15.6691 | 18.3465 | 23.2735 | 1.6623 | 15.6691 |
| 25 | 1.037 | 1.4203 | 1.5922 | 1.4031 | 1.2634 | 8.546 | 11.4004 | 13.4481 | 19.4 | 21.6688 | 1.5922 | 13.4481 |
| 26 | 1.0053 | 1.3055 | 1.5491 | 1.3098 | 1.1485 | 11.7184 | 14.6646 | 16.3502 | 20.0742 | 22.2173 | 1.5491 | 16.3502 |
| 27 | 0.9212 | 1.5677 | 1.9524 | 1.5802 | 1.4079 | 9.2254 | 11.3502 | 13.1772 | 17.9842 | 23.2562 | 1.9524 | 13.1772 |
| 28 | 0.965 | 1.395 | 1.73 | 1.3387 | 1.1193 | 8.562 | 12.8707 | 14.4333 | 17.1185 | 20.2722 | 1.73 | 14.4333 |
| 29 | 1.0742 | 1.5272 | 1.9006 | 1.2957 | 1.1838 | 9.0579 | 12.0531 | 13.6581 | 16.1222 | 20.4444 | 1.9006 | 13.6581 |
| 30 | 1.0455 | 1.3698 | 1.5372 | 1.3975 | 1.1248 | 8.7735 | 10.9522 | 17.2134 | 20.7495 | 22.8132 | 1.5372 | 17.2134 |
| 31 | 0.984 | 1.6492 | 1.9043 | 1.4777 | 1.2786 | 8.4688 | 13.5041 | 15.4242 | 19.4018 | 21.1716 | 1.9043 | 15.4242 |
| 32 | 1.1049 | 1.45 | 1.66 | 1.4544 | 1.3189 | 8.5848 | 12.4521 | 14.0204 | 20.7095 | 22.9726 | 1.66 | 14.0204 |
| 33 | 1.0484 | 1.3615 | 1.5302 | 1.2972 | 1.1662 | 10.0752 | 13.2882 | 16.1056 | 19.4355 | 23.0631 | 1.5302 | 16.1056 |
| 34 | 1.1901 | 1.6228 | 1.8284 | 1.4374 | 1.2516 | 8.5293 | 11.8257 | 14.8456 | 18.3337 | 21.8292 | 1.8284 | 14.8456 |
| 35 | 1.0047 | 1.6799 | 1.9467 | 1.3009 | 1.1337 | 9.3862 | 12.9364 | 14.8127 | 18.2001 | 21.6997 | 1.9467 | 14.8127 |
| 36 | 0.9154 | 1.5395 | 1.739 | 1.4554 | 1.2604 | 9.4516 | 11.648 | 13.0863 | 15.3496 | 22.6829 | 1.739 | 13.0863 |
| 37 | 0.9368 | 1.3227 | 1.6263 | 1.3213 | 1.14 | 8.6958 | 11.6355 | 15.5311 | 18.3203 | 23.0127 | 1.6263 | 15.5311 |
| 38 | 0.9716 | 1.3392 | 1.8706 | 1.3546 | 1.1749 | 11.0421 | 13.7116 | 15.6621 | 19.3535 | 21.5418 | 1.8706 | 15.6621 |
| 39 | 1.2924 | 1.6549 | 1.8737 | 1.4185 | 1.1307 | 8.671 | 13.2565 | 15.9838 | 20.214 | 23.2365 | 1.8737 | 15.9838 |
| 40 | 0.9571 | 1.4339 | 1.6124 | 1.3543 | 1.2364 | 10.6833 | 14.3711 | 15.9815 | 20.1553 | 23.0351 | 1.6124 | 15.9815 |
| 41 | 0.9575 | 1.3793 | 1.561 | 1.2959 | 1.1163 | 8.6285 | 11.2066 | 13.4549 | 16.6139 | 18.7585 | 1.561 | 13.4549 |
| 42 | 0.987 | 1.767 | 1.615 | 1.505 | 1.394 | 4.881 | 10.714 | 18.627 | 21.827 | 27.604 | 1.767 | 10.714 |
| 43 | 0.7996 | 1.5728 | 1.6835 | 1.2946 | 1.0696 | 4.7412 | 11.1515 | 21.2972 | 31.0944 | 40.1821 | 1.6835 | 21.2972 |
| 44 | 0.7024 | 1.7852 | 1.7807 | 1.2163 | 0.9804 | 5.0383 | 12.1559 | 21.217 | 27.3603 | 37.8665 | 1.7852 | 12.1559 |
| 45 | 0.9688 | 1.8283 | 1.807 | 1.5295 | 1.3253 | 4.4467 | 9.9347 | 21.1099 | 28.147 | 38.9537 | 1.8283 | 9.9347 |
| 46 | 0.7404 | 1.5246 | 1.5325 | 1.2639 | 1.0695 | 4.5138 | 12.0684 | 23.4621 | 28.0844 | 38.8836 | 1.5325 | 23.4621 |
| 47 | 0.7613 | 1.4903 | 1.4953 | 1.224 | 1.0247 | 4.6992 | 11.981 | 22.0709 | 27.6558 | 35.5892 | 1.4953 | 22.0709 |
| 48 | 0.8073 | 1.7684 | 1.6988 | 1.4362 | 1.0197 | 4.4988 | 11.6716 | 20.3975 | 25.0224 | 36.0678 | 1.7684 | 11.6716 |
| 49 | 0.7367 | 1.514 | 1.4787 | 1.2815 | 1.014 | 5.3277 | 13.074 | 23.1666 | 30.3011 | 40.9165 | 1.514 | 13.074 |
| 50 | 0.9112 | 1.7946 | 1.7637 | 1.5179 | 1.3283 | 4.6885 | 12.6598 | 24.4746 | 30.8014 | 39.7831 | 1.7946 | 12.6598 |

Table 6 (continued) The raw data with Average (AVG), Standard Deviation (SD), Max and Min

| No. Test | Dry Density | | | | | Moisture Content | | | | | MDD | OMC |
|----------|-------------|--------|--------|--------|--------|------------------|---------|---------|---------|---------|--------|---------|
| | 2% | 4% | 6% | 8% | 10% | 2% | 4% | 6% | 8% | 10% | | |
| 51 | 0.7005 | 1.7675 | 1.8857 | 1.4681 | 0.9968 | 4.5514 | 10.6363 | 19.1342 | 24.7732 | 32.4354 | 1.8857 | 19.1342 |
| 52 | 0.7582 | 1.6622 | 1.7061 | 1.2809 | 1.0943 | 5.2863 | 13.2283 | 23.271 | 29.3011 | 38.2447 | 1.7061 | 23.271 |
| 53 | 0.7449 | 1.655 | 1.6408 | 1.2513 | 1.0171 | 4.5492 | 13.0709 | 23.8414 | 30.7069 | 40.4205 | 1.655 | 13.0709 |
| 54 | 0.7566 | 1.5534 | 1.5007 | 1.2111 | 0.983 | 4.6784 | 13.0998 | 22.8217 | 28.0467 | 41.1931 | 1.5534 | 13.0998 |
| 55 | 0.7575 | 1.4482 | 1.4611 | 1.2308 | 1.0238 | 5.1998 | 12.4852 | 22.0104 | 30.0929 | 38.6003 | 1.4611 | 22.0104 |
| 56 | 0.7759 | 1.5309 | 1.7256 | 1.2212 | 1.0527 | 5.017 | 11.8283 | 24.9504 | 30.4112 | 40.8903 | 1.7256 | 24.9504 |
| 57 | 0.7142 | 1.4926 | 1.4701 | 1.2317 | 1.0067 | 4.8531 | 12.3655 | 21.5617 | 26.581 | 36.3847 | 1.4926 | 12.3655 |
| 58 | 0.8056 | 1.7861 | 1.856 | 1.502 | 1.0102 | 4.5278 | 10.9095 | 20.0394 | 26.142 | 37.0572 | 1.856 | 20.0394 |
| 59 | 0.7771 | 1.5783 | 1.619 | 1.2412 | 1.044 | 4.8768 | 11.472 | 20.612 | 25.3646 | 40.704 | 1.619 | 20.612 |
| 60 | 0.9082 | 1.7156 | 1.7054 | 1.4076 | 1.1666 | 4.5298 | 12.5606 | 22.6684 | 30.0992 | 40.0231 | 1.7156 | 12.5606 |
| 61 | 0.8713 | 1.7168 | 1.8403 | 1.3873 | 1.1301 | 4.7445 | 10.5041 | 20.9266 | 24.7385 | 38.345 | 1.8403 | 20.9266 |
| 62 | 0.7486 | 1.6043 | 1.6124 | 1.2794 | 1.0935 | 4.8333 | 12.6068 | 23.3111 | 27.4383 | 41.2506 | 1.6124 | 23.3111 |
| 63 | 0.7837 | 1.811 | 1.8154 | 1.4451 | 1.1647 | 4.4759 | 11.1088 | 19.5294 | 24.8855 | 31.6721 | 1.8154 | 19.5294 |
| 64 | 0.7477 | 1.6221 | 1.6344 | 1.3489 | 1.1707 | 5.213 | 12.2951 | 21.3992 | 25.1311 | 35.7229 | 1.6344 | 21.3992 |
| 65 | 0.791 | 1.5207 | 1.6032 | 1.2823 | 1.0826 | 5.6265 | 13.2905 | 23.3494 | 27.6024 | 41.3305 | 1.6032 | 23.3494 |
| 66 | 0.7866 | 1.5871 | 1.5355 | 1.3054 | 1.1144 | 4.5226 | 10.7806 | 20.0398 | 29.1874 | 38.4806 | 1.5871 | 10.7806 |
| 67 | 0.7627 | 1.4588 | 1.494 | 1.2186 | 1.013 | 6.2014 | 13.8672 | 24.3643 | 30.2017 | 39.4547 | 1.494 | 24.3643 |
| 68 | 0.6988 | 1.7518 | 1.8829 | 1.4702 | 1.2418 | 4.8821 | 10.733 | 19.6361 | 27.0573 | 41.2998 | 1.8829 | 19.6361 |
| 69 | 0.7321 | 1.5588 | 1.6685 | 1.2455 | 0.9872 | 4.531 | 12.1709 | 21.5079 | 25.7549 | 36.0005 | 1.6685 | 21.5079 |
| 70 | 0.8149 | 1.7066 | 1.833 | 1.2055 | 1.0441 | 4.7935 | 11.3977 | 20.3527 | 24.2559 | 36.3064 | 1.833 | 20.3527 |
| 71 | 0.7932 | 1.5308 | 1.4826 | 1.3002 | 0.9921 | 4.643 | 10.3567 | 25.6507 | 31.2177 | 40.513 | 1.5308 | 10.3567 |
| 72 | 0.7465 | 1.843 | 1.8366 | 1.3748 | 1.1277 | 4.4817 | 12.7698 | 22.9845 | 29.19 | 37.5977 | 1.843 | 12.7698 |
| 73 | 0.8382 | 1.6203 | 1.6009 | 1.3531 | 1.1633 | 4.5431 | 11.7751 | 20.8927 | 31.1576 | 40.7961 | 1.6203 | 11.7751 |
| 74 | 0.7953 | 1.5214 | 1.4758 | 1.2069 | 1.0286 | 5.3318 | 12.5657 | 23.9999 | 29.2408 | 40.9568 | 1.5214 | 12.5657 |
| 75 | 0.9028 | 1.8135 | 1.7634 | 1.3373 | 1.1039 | 4.5137 | 11.1827 | 22.1223 | 27.5831 | 38.7655 | 1.8135 | 11.1827 |
| 76 | 0.7622 | 1.8773 | 1.8774 | 1.2103 | 0.9999 | 4.9672 | 12.233 | 22.0733 | 27.3821 | 38.5356 | 1.8774 | 22.0733 |
| 77 | 0.6945 | 1.7204 | 1.6772 | 1.354 | 1.1117 | 5.0018 | 11.0147 | 19.5006 | 23.0936 | 40.2816 | 1.7204 | 11.0147 |
| 78 | 0.7106 | 1.4781 | 1.5685 | 1.2293 | 1.0055 | 4.6018 | 11.0029 | 23.1438 | 27.563 | 40.8673 | 1.5685 | 23.1438 |
| 79 | 0.7371 | 1.4965 | 1.8041 | 1.2603 | 1.0363 | 5.8435 | 12.9661 | 23.3391 | 29.1175 | 38.2551 | 1.8041 | 23.3391 |
| 80 | 0.9804 | 1.8493 | 1.807 | 1.3197 | 0.9973 | 4.5887 | 12.5357 | 23.8184 | 30.412 | 41.2648 | 1.8493 | 12.5357 |
| 81 | 0.726 | 1.6023 | 1.5551 | 1.26 | 1.0905 | 5.6536 | 13.5897 | 23.8151 | 30.3237 | 40.907 | 1.6023 | 13.5897 |
| 82 | 0.7263 | 1.5414 | 1.5055 | 1.2057 | 0.9846 | 4.5662 | 10.5972 | 20.0499 | 24.9956 | 33.3124 | 1.5414 | 10.5972 |
| 83 | 1.2422 | 1.4728 | 1.6 | 1.5196 | 1.4361 | 14.2857 | 16.2791 | 18.3432 | 21.8935 | 27.1605 | 1.6 | 18.3432 |
| 84 | 1.0063 | 1.311 | 1.6679 | 1.3072 | 1.1019 | 13.8764 | 16.9438 | 20.9727 | 31.1891 | 39.5365 | 1.6679 | 20.9727 |
| 85 | 0.8841 | 1.488 | 1.7642 | 1.2281 | 1.01 | 14.7461 | 18.4699 | 20.8937 | 27.4436 | 37.2581 | 1.7642 | 20.8937 |
| 86 | 1.2194 | 1.5239 | 1.7903 | 1.5444 | 1.3653 | 13.0145 | 15.0949 | 20.7883 | 28.2328 | 38.3278 | 1.7903 | 20.7883 |
| 87 | 0.9319 | 1.2708 | 1.5183 | 1.2762 | 1.1018 | 13.2109 | 18.337 | 23.1046 | 28.17 | 38.2589 | 1.5183 | 23.1046 |
| 88 | 0.9581 | 1.2422 | 1.4815 | 1.2359 | 1.0557 | 13.7535 | 18.2042 | 21.7346 | 27.74 | 35.0174 | 1.4815 | 21.7346 |
| 89 | 1.016 | 1.474 | 1.6831 | 1.4502 | 1.0505 | 13.167 | 17.7341 | 20.0868 | 25.0986 | 35.4883 | 1.6831 | 20.0868 |
| 90 | 0.9272 | 1.2619 | 1.465 | 1.2939 | 1.0446 | 15.5932 | 19.8649 | 22.8136 | 30.3934 | 40.2591 | 1.465 | 22.8136 |
| 91 | 1.1468 | 1.4958 | 1.7474 | 1.5327 | 1.3685 | 13.7224 | 19.2355 | 24.1017 | 30.8952 | 39.1439 | 1.7474 | 24.1017 |
| 92 | 0.8816 | 1.4733 | 1.8682 | 1.4824 | 1.027 | 13.3212 | 16.1609 | 18.8426 | 24.8486 | 31.9142 | 1.8682 | 18.8426 |
| 93 | 0.9543 | 1.3855 | 1.6903 | 1.2933 | 1.1273 | 15.4719 | 20.0993 | 22.9165 | 29.3904 | 37.6303 | 1.6903 | 22.9165 |
| 94 | 0.9375 | 1.3794 | 1.6256 | 1.2634 | 1.0478 | 13.3147 | 19.8602 | 23.4782 | 30.8005 | 39.771 | 1.6256 | 23.4782 |
| 95 | 0.9523 | 1.2948 | 1.4867 | 1.2228 | 1.0127 | 13.6927 | 19.9041 | 22.474 | 28.1321 | 40.5313 | 1.4867 | 22.474 |
| 96 | 0.9534 | 1.2071 | 1.4475 | 1.2427 | 1.0547 | 15.2187 | 18.9703 | 21.6751 | 30.1846 | 37.9801 | 1.4475 | 21.6751 |
| 97 | 0.9765 | 1.2761 | 1.7096 | 1.233 | 1.0845 | 14.6838 | 17.9722 | 24.5702 | 30.5038 | 40.2333 | 1.7096 | 24.5702 |
| 98 | 0.8989 | 1.2441 | 1.4565 | 1.2436 | 1.0371 | 14.204 | 18.7883 | 21.2332 | 26.6619 | 35.8001 | 1.4565 | 21.2332 |
| 99 | 1.0139 | 1.4887 | 1.8388 | 1.5166 | 1.0407 | 13.2521 | 16.5761 | 19.7341 | 26.2216 | 36.4618 | 1.8388 | 19.7341 |
| 100 | 0.978 | 1.3155 | 1.604 | 1.2532 | 1.0755 | 14.2735 | 17.4308 | 20.298 | 25.4418 | 40.05 | 1.604 | 20.298 |
| 101 | 1.1431 | 1.43 | 1.6896 | 1.4213 | 1.2019 | 13.2578 | 19.0848 | 22.323 | 30.1908 | 39.3801 | 1.6896 | 22.323 |
| 102 | 1.0966 | 1.431 | 1.8232 | 1.4008 | 1.1642 | 13.8863 | 15.9602 | 20.6077 | 24.8139 | 37.729 | 1.8232 | 20.6077 |
| 103 | 0.9422 | 1.3372 | 1.5975 | 1.2918 | 1.1265 | 14.1461 | 19.1551 | 22.9559 | 27.5219 | 40.5878 | 1.5975 | 22.9559 |
| 104 | 0.9863 | 1.5095 | 1.7986 | 1.4591 | 1.1999 | 13.1 | 16.8789 | 19.2319 | 24.9613 | 31.1632 | 1.7986 | 19.2319 |
| 105 | 0.941 | 1.3521 | 1.6192 | 1.362 | 1.206 | 15.2574 | 18.6814 | 21.0731 | 25.2077 | 35.1489 | 1.6192 | 21.0731 |
| 106 | 0.9955 | 1.2675 | 1.5884 | 1.2947 | 1.1153 | 16.4677 | 20.1938 | 22.9937 | 27.6865 | 40.6664 | 1.5884 | 22.9937 |
| 107 | 0.99 | 1.3229 | 1.5213 | 1.3181 | 1.148 | 13.2367 | 16.3802 | 19.7344 | 29.2764 | 37.8624 | 1.5213 | 19.7344 |
| 108 | 0.9599 | 1.216 | 1.4802 | 1.2304 | 1.0436 | 18.1503 | 21.0702 | 23.9931 | 30.2938 | 38.8208 | 1.4802 | 23.9931 |
| 109 | 1.2103 | 1.3869 | 1.6005 | 1.2742 | 1.061 | 13.468 | 16.7179 | 22.791 | 27.6464 | 40.2099 | 1.6005 | 22.791 |
| 110 | 1.2554 | 1.4042 | 1.841 | 1.3063 | 1.0935 | 17.1019 | 19.7009 | 22.9833 | 29.2055 | 37.6398 | 1.841 | 22.9833 |
| 111 | 1.6697 | 1.7353 | 1.8439 | 1.368 | 1.0524 | 13.4295 | 19.0469 | 23.4552 | 30.504 | 40.6011 | 1.8439 | 23.4552 |
| 112 | 1.2365 | 1.5035 | 1.5869 | 1.306 | 1.1508 | 16.5462 | 20.6484 | 23.452 | 30.4154 | 40.249 | 1.5869 | 23.452 |
| 113 | 1.237 | 1.4463 | 1.5363 | 1.2497 | 1.039 | 13.3637 | 16.1016 | 19.7442 | 25.0712 | 32.7765 | 1.5363 | 19.7442 |
| 114 | 1.1464 | 1.5155 | 1.7315 | 1.2998 | 1.0756 | 10.9666 | 19.4141 | 23.2634 | 28.4801 | 44.5978 | 1.7315 | 23.2634 |
| 115 | 0.985 | 1.4162 | 1.5171 | 1.1987 | 1.0407 | 11.1718 | 23.3004 | 25.9142 | 31.5882 | 47.9771 | 1.5171 | 25.9142 |
| 116 | 1.0311 | 1.5987 | 1.7081 | 1.354 | 1.1085 | 10.3456 | 20.5316 | 21.7103 | 28.6493 | 36.8367 | 1.7081 | 21.7103 |
| 117 | 0.9837 | 1.4319 | 1.5377 | 1.2639 | 1.1142 | 12.0494 | 22.7242 | 23.7888 | 28.932 | 41.548 | 1.5377 | 23.7888 |
| 118 | 1.0407 | 1.3424 | 1.5084 | 1.2014 | 1.0303 | 13.0052 | 24.5639 | 25.9568 | 31.7771 | 48.0701 | 1.5084 | 25.9568 |
| 119 | 1.035 | 1.401 | 1.4447 | 1.2231 | 1.0606 | 10.4536 | 19.925 | 22.2776 | 33.6019 | 44.7555 | 1.4447 | 22.2776 |
| 120 | 1.0034 | 1.2878 | 1.4057 | 1.1417 | 0.9641 | 14.3341 | 25.6299 | 27.0851 | 34.7696 | 45.8884 | 1.4057 | 27.0851 |
| AVG | 0.9547 | 1.5065 | 1.6745 | 1.3497 | 1.1381 | 9.1910 | 14.3878 | 19.5967 | 24.8372 | 32.7768 | 1.6812 | 17.8943 |
| SD | 0.1644 | 0.1586 | 0.1425 | 0.1127 | 0.1214 | 3.7954 | 3.5089 | 3.8480 | 5.1238 | 8.6169 | 0.1409 | 4.5020 |
| Max | 1.6697 | 1.8773 | 1.9552 | 1.644 | 1.5805 | 18.1503 | 25.6299 | 27.0851 | 34.7696 | 48.0701 | 1.9552 | 27.0851 |
| Min | 0.6945 | 1.2071 | 1.4057 | 1.1417 | 0.9641 | 4.4467 | 9.9347 | 12.5 | 14.5078 | 15.544 | 1.4057 | 9.9347 |

Remark: The percentages of 2%, 4%, 6%, 8%, and 10% represent the amounts of moisture content added, based on the guidelines of ASTM D1557.