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Published by the Faculty of Engineering, Khon Kaen University, Thailand

Mixed-Integer Linear programming for scheduling of radiotherapy patients

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Received 18 September 2023
Revised 4 November 2023
Accepted 7 December 2023

Abstract

This study presents an advanced mathematical model to optimize the scheduling of radiotherapy patients, thereby expediting solution discovery. The paper commences with an in-depth analysis of cancer treatment protocols and prior mathematical models. We then introduce some enhancements to an existing mathematical framework with the intent of expediting the derivation of solutions. The validity of the model is ensured through a meticulous evaluation of the constraints, leading to the removal of redundant constraints. This improved model is validated through the generation and assessment of five small-scale cases, and its efficacy is confirmed. The experimental results underscore the substantial time reduction achieved by the enhanced mathematical model in terms of finding solutions. To bolster its applicability to real-world scenarios, the model is enriched by incorporating additional constraints, for example related to surgical and radiotherapy processing times. The application of this comprehensive model to a real-world case demonstrates its ability to accurately determine the durations of simulation and radiotherapy while adhering to the specified constraints. It successfully allocates patients to specific rooms and technologies, and outlines the optimal frequency for radiotherapy sessions within each interval. The proposed model is expected to assume a pivotal role in facilitating informed decision-making among stakeholders. By substantially curtailing the treatment planning time and mitigating errors in radiotherapy patient scheduling, this model will be a valuable asset to healthcare practitioners and decision-makers alike.

Keywords: Mixed-Integer programming, Radiotherapy, Patient scheduling, Thai hospital case study

1. Introduction

Over the past decade, cancer has emerged as one of the most prevalent diseases globally, with a substantial impact on hospital management systems. The surge in numbers of cancer patients has significantly disrupted hospital operations, and an important challenge faced by numerous hospitals in Thailand pertains to the scheduling of radiotherapy (RT) patients for cancer treatment. This issue arises due to limitations on resources such as treatment rooms and the necessary technological infrastructure. At present, scheduling of RT patients is carried out manually by RT staff, who rely on their experience to devise schedules. However, this approach falls short of achieving an optimal schedule due to the intricate nature of the task and the multitude of constraints at play. Consequently, this method is prone to errors, potentially compromising the precision of cancer treatment planning.

Inadequacies in patient scheduling can profoundly impact the effectiveness of cancer treatment for patients, and the scheduling of RT patients therefore has a pivotal importance in ensuring timely and appropriate treatment [1-3]. With these challenges in mind, the focus of this research is directed toward investigating RT patient scheduling. Although several prior studies have proposed mathematical models to address the challenges associated with RT patient scheduling [1, 2], these models often overlook certain real-world conditions. Most existing research has focused on developing mathematical models for specific RT problems, without considering the broader integration of patient scheduling with related fields such as chemotherapy and surgery [4-7]. This holistic approach is notably lacking in many existing models. Furthermore, a common trend in the literature is the complexity of these models, which are characterized by a multitude of parameters and variables that can significantly impact the time needed to generate a solution. While several authors have proposed heuristic and metaheuristic algorithms to address this issue, these often provide local solutions. Hence, when an optimal solution is crucial, exact algorithms become indispensable. Boonmee et al. [1] recently introduced a mixed-integer linear programming (MILP) model for RT patient scheduling at a Thai hospital. Their primary objective was to minimize makespan, and various factors such as room availability, doctor schedules, treatment techniques, technology constraints, and treatment procedures were considered in the model. Notably, this model addressed RT patient scheduling in conjunction with related elements such as simulation and chemotherapy, which represented a significant aspect of this study. Nonetheless, the model neglected to account for surgical constraints, overlooking the inherent interplay between surgery and RT for certain patients. Typically, after undergoing surgery, patients require a prescribed recovery period before commencing their RT treatment. In addition, the maximum capacity constraint for the model was incomplete, as it relied solely on the number of treatable patients per day. This approach may lead to issues such as overtime and treatment delays due to the variability in individual treatment times, which could potentially cause some patients to miss treatment within the given time frame. Furthermore, upon reviewing the model presented in [1], it becomes apparent that the model may contain redundant constraints that adversely affect the computational time required to find the optimal solution.

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doi: 10.14456/easr.2024.12

In response to the shortcomings identified in the model presented in [1], this research has two primary objectives: (1) to significantly reduce the computational time for optimizing RT patient scheduling, and (2) to enhance the mathematical model by incorporating constraints related to RT processing time capacity and surgical considerations, to address the practical challenges of real-world scenarios.

These objectives give rise to two significant contributions of this research: (1) the development of an improved mathematical model that streamlines the data processing time compared to the model in [1], and (2) the creation of an advanced MILP model for RT patient scheduling that is capable of addressing real-world constraints, including RT processing time and surgical considerations. By bridging the gap between mathematical modeling and practical constraints, this research strives to facilitate more efficient and precise RT patient scheduling, which can benefit both patients and healthcare providers.

2. Literature review

Efficient patient scheduling is imperative for ensuring timely and precise RT treatment. However, this is a multifaceted challenge characterized by numerous objectives and constraints. Many studies, including those in [4, 6, 8-14] have used mathematical models and optimization techniques to address these challenges. Their objectives included minimizing patient waiting times, optimizing resource utilization, and accommodating constraints such as doctor schedules and machine availability. A summary of the papers covered in this review is presented in Table 1.

Granja et al. [8] introduced an innovative approach that employed linear programming techniques to optimize patient admissions scheduling. Their mathematical model served a dual purpose: reducing patient waiting times, while simultaneously enhancing care quality and cost efficiency. Notably, their model tackled challenges such as slot overbooking and in-clinic patient wait times by sequentially allocating heterogeneous no-show patients to predefined time slots. Conforti et al. [2] presented novel integer programming formulations for optimizing RT patient scheduling. These models took into account both the quality and efficiency of healthcare while accommodating patient preferences. The effectiveness and robustness of these models in addressing the scheduling problem were demonstrated through computational experiments using real-world data. A subsequent work by Conforti et al. [15] addressed the challenge of optimizing patient scheduling for RT, given the long waiting times associated with cancer treatments. They introduced an innovative approach based on integer linear optimization to either minimize patient waiting times or maximize the scheduling of new patients. Numerical experiments based on real-world scenarios confirmed the effectiveness and reliability of this approach. Petrovic et al. [5] devised a sophisticated optimization model and associated algorithms for scheduling RT treatments for categorized cancer patients. This model is particularly noteworthy because it utilized real-life data from the Arden Cancer Centre in the UK, and considered practical constraints such as doctors' schedules, machine availability, and patient categories. The primary objectives of the study included minimizing average patient waiting times and addressing breaches of waiting time targets. Upon reviewing the literature, it is evident that many articles commonly frame the RT patient scheduling problem as a MILP or integer linear programming challenge. Furthermore, a recurring theme across various studies, including those in [4, 7, 10, 11, 13, 16], is an emphasis on minimizing patient waiting times. However, the research landscape also reveals a diverse array of optimization objectives, such as makespan [1], service level [3], completion time [8], maximum lateness [9], and cost [12].

Constraints play a pivotal role in resolving the RT patient scheduling problem, with various factors significantly impacting the quest for solutions. Constraints typically include capacity, treatment technology, RT techniques, treatment procedures, time limitations, room allocation, and resource management. These complex constraints have been addressed in multiple articles in the field. For instance, Castro and Petrovic [9] presented a real-world pre-treatment scheduling challenge in a UK hospital, framing it as a multi-objective optimization problem. Their proposed mathematical model took into account several constraints, including capacity, treatment duration, lead time, and the efficient management of resources. Similarly, Burke et al. [11] introduced an integer linear programming model tailored for practical RT treatment scheduling within daily hospital operations. This model considered constraints related to capacity and machine management while also considering the standard timeframes defined by: Joint Council for Clinical Oncology (JCCO). Vieira et al. [14] devised a MILP model focused on scheduling and sequencing RT sessions, which took into consideration patient preferences for time windows and a comprehensive set of constraints, including capacity, treatment sessions, and resource management, thus ensuring efficient and accurate scheduling.

The quest for solutions to the RT patient scheduling problem has given rise to a spectrum of algorithmic approaches, including exact, heuristic, and meta-heuristic algorithms. Jacquemin et al. [6] introduced an original RT scheduling model underpinned by the precision of integer linear optimization and a non-block scheduling strategy. Their elegant scheme considered the integration of treatment patterns across the entire patient care process, resulting in enhanced room utilization, increased patient treatment rates, and reduced waiting times. This particular research used an exact algorithm as the method of choice for solution-seeking. In a similar vein, Yoan et al. [16] developed an innovative RT scheduling model based on integer linear optimization, embracing a non-block scheduling paradigm. The introduction of treatment patterns within the patient treatment process significantly increased the room utilization, expanded patient treatment opportunities, and curtailed waiting times. In this study, an exact algorithm was adopted to tackle the problem. The utilization of exact algorithms, while advantageous in terms of delivering optimal solutions, does have limitations, especially when dealing with larger problem sizes. In such cases, exact algorithms may require prolonged processing times, which may make them less practical. Consequently, several articles have proposed heuristic or meta-heuristic algorithms for addressing the RT patient scheduling problem [5, 7, 8, 12, 13, 17]. However, for smaller problem instances, the exact algorithm remains a viable choice for achieving precise solutions. The choice of algorithmic approach depends on the size and complexity of the specific problem; exact algorithms offer accuracy at the cost of increased computational time, while heuristic and meta-heuristic algorithms provide expedited solutions for larger, more intricate problems.

As mentioned above, Boonmee et al. [1] introduced a MILP model tailored for RT patient scheduling with the primary objective of minimizing the makespan. This model took into account a diverse range of constraints, such as room availability, doctor schedules, treatment techniques, and others. What set this proposed model apart was its comprehensive approach: it not only focused on the RT process but also placed significant emphasis on related processes, such as chemotherapy and simulation. The recognition that patients often undergo chemotherapy prior to RT is a crucial insight, and the timing of the completion of chemotherapy and the commencement of RT is intricately linked. This aspect is a notable strength of this research, as the interplay between these two treatment modalities is highlighted. Nevertheless, this model disregarded surgical constraints, despite the close connection between surgery and RT for certain patients. Moreover, the maximum capacity constraint of the model remained incomplete, and it relied solely on treatable patient counts per day. This approach may lead to issues such as overtime and treatment delays due to the variability in individual treatment time, potentially resulting in some patients not receiving treatment within the provided period.

Given this research gap, this study aims to develop a comprehensive mathematical model for RT patient scheduling. Our objectives are twofold: firstly, to enhance the existing model from [1] to expedite solution discovery; and secondly, to enrich the augmented model with additional constraints, including surgical considerations and treatment processing time limitations.

Table 1 Summary of radiotherapy patient scheduling schemes

Article	Objective	Constraint									Model Type	Method
		Capacity	Technology	Technique	Procedure	Simulation	Time	Room	Resource	Other		
[1]	Max makespan	✓	✓	✓	✓	✓		✓	✓	Patient type, time gap, chemotherapy	MILP	Exact
[2]	Min waiting time	✓	✓		✓	✓	✓			Priority	MILP	Exact
[3]	Min service level	✓				✓	✓	✓	✓	Time gap (CT scan only)	DP	Simu
[4]	Min Waiting time	✓			✓	✓	✓		✓	Priority	MILP	Exact
[5]	Min waiting time, Min length of breaches of waiting time targets	✓	✓		✓		✓	✓	✓	Priority	-	GA, KB-GA, Weighted-GA
[6]	Max penalty	✓			✓	✓	✓		✓	Priority	ILP	Exact
[7]	Min waiting time	✓					✓			Machines, cancer site, time slot, penalty, patient type	LP	Column Generation
[8]	Min total completion and total waiting	✓					✓		✓	Time slot	-	Simu and SA
[9]	Min waiting time, Min the maximum lateness, Min the sum of weighted lateness	✓			✓		✓	✓	✓	Priority, time gap, lead time	MILP	Exact
[10]	Min Waiting time, Min tardiness	✓	✓			✓		✓	✓	Machines, facilities	-	GA
[11]	Min number of patients who miss the standard time, Min waiting time	✓					✓		✓	Machines, Standard time of JCCO	MILP	Exact
[12]	Min cost	✓					✓		✓	Machines, cost, priority,	MILP	GA
[13]	Min waiting time	✓			✓		✓		✓	Patient type	-	GA
[14]	Min overall deviation	✓			✓		✓		✓	Treatment sessions	MILP	Exact
[15]	Min waiting time	✓	✓				✓			Time slot, Machines, priority	MIP	Exact
[16]	Min waiting time	✓					✓		✓	Machines, time window, priority, penalty	MILP	Exact
[17]	Min weighted lateness	✓	✓				✓		✓	Machines, priority, time slot	-	Heuristics, Hill climbing
[18]	Min waiting time	✓					✓			Time slot, priority	MIP	Exact

Remark: MILP: Mixed-Integer Linear Programming, ILP: Integer Linear Programming, LP: Linear Programming, DP: Dynamic Programming, Sim: Simulation, SA: Simulated Annealing, GA: Genetic Algorithm, KB-GA: Knowledge-Based Genetic Algorithm, CT: Computer Tomography Scans, JCCO: Joint Council for Clinical Oncology.

3. Research methodology

A scheduling model for RT patients is developed and improved in this paper. In this section, we explain the specifics of our research methodology.

3.1 Data collection

This study represents an in-depth analysis and compilation of the RT procedures pertinent to cancer patients. This analysis extends beyond the mere enumeration of treatment fractions, encompassing variables such as cancer classifications, availability of simulation rooms, treatment room capacities, physician resources, technique classifications, and patient categorizations. Furthermore, an examination of existing mathematical models for the scheduling of radiation therapy patients is undertaken, with the specific aim of identifying constraints that might have been overlooked.

3.2 Improvements to existing mathematical model

In view of the gaps identified within the mathematical model presented in [1], the primary objective of this research is to enhance the efficiency and effectiveness of this model by minimizing the data processing time. A comprehensive review and refinement of all of the existing constraints are undertaken to ensure a more robust and optimized mathematical framework.

3.3 Model verification and validation

In this section, we describe the rigorous process of validation and verification applied to the enhanced mathematical model to ascertain its accuracy prior to the incorporation of additional constraints. A comparative analysis is conducted between the refined mathematical model and that introduced in [1], with the aim of assessing the performance enhancements. To comprehensively explore the robustness of our model, five distinct test cases are generated, each of which is executed five times to demonstrate consistency in the processing times. A performance evaluation of the improved mathematical model is then carried out using a T-test, as demonstrated in Equation (1).

$$\begin{aligned} &H_0: \mu_1 - \mu_2 = 0 \\ &H_1: \mu_1 - \mu_2 > 0 \end{aligned} \tag{1}$$

where
μ_1: Processing period for the mathematical model from [1]
μ_2: Processing period for the improved mathematical model.

3.4 Development of the mathematical model

To enhance the practicality of the mathematical model for real-world applications, a set of additional variables and constraints is introduced, such as surgery-related constraints and constraints related to the duration of treatment.

3.5 Case study application

Following the formulation of the mathematical model, numerical data are generated based on information from [1]. This numerical dataset serves as the foundation for assessing the efficacy of the proposed mathematical model in Section 3.4.

3.6 Conclusion and discussion

The results are elaborated upon and summarized. This section also suggests avenues for further research.

4. Results

The aim of this research was to enhance a mathematical model for the scheduling of RT patients, with a dual focus on reducing the problem-solving time and crafting a more practical mathematical framework. This study represents an extension of the work presented in [1].

4.1 Data collection

Initially, the RT processes for cancer patients were thoroughly examined. As described in [1], RT entails a sequential progression through five essential steps: a preliminary consultation, simulation, planning and design of the treatment, verification, and delivery of the treatment (as shown in Figure 1). Once patients have received approval for RT from a multidisciplinary team, the scheduling of these patients becomes a significant aspect of managing RT services. The time duration for each step varies depending on the treatment technique, and whether it is two-dimensional (2D) or three-dimensional (3D). Following a conference, X-ray simulation for 2D techniques or computed tomography (CT) simulation for 3D techniques is performed to acquire patient images. Radiation oncologists then define treatment fields (2D) or the boundaries of tumors and normal organs (3D). The next step involves generating the treatment plan according to the prescribed dose. Upon approval of the plan, delivery of the treatment commences. Notably, verification occurs only during the initial RT session (the first fraction). However, the verification procedure must be repeated if the treatment site changes.

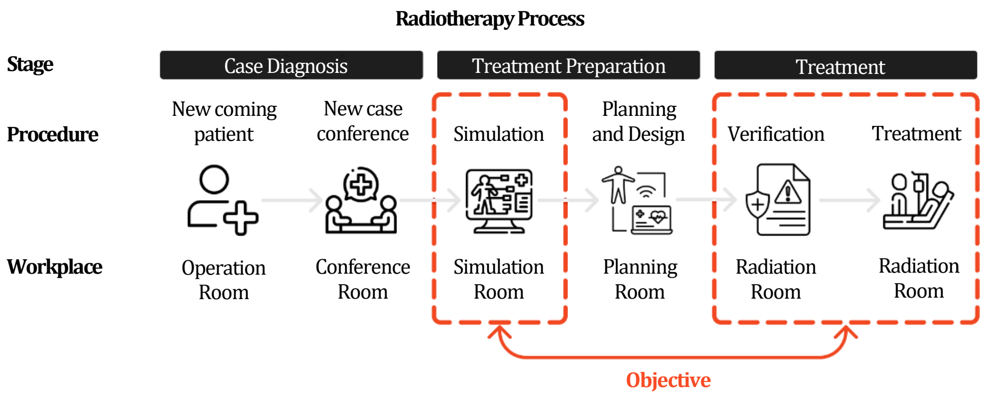

Figure 1 Case study: radiotherapy procedure at the Thai Cancer Center

More specific aspects include the number of treatment fractions, classification of cancer types, numbers of simulation and radiation rooms, medical personnel, technique categories, and patient profiles, which are meticulously gathered and scrutinized.

At the end of this phase, a thorough investigation into the constraints governing radiotherapy patient scheduling was undertaken, and essential information needed to facilitate the enhancement and development of the mathematical model was collated.

4.2 Improvements to the mathematical model

The mathematical model introduced in [1] was identified as a candidate for refinement. The structure of this model is that of a MILP problem, underpinned by several fundamental assumptions, as follows:

1. Individual patients undergo irradiation using distinct techniques and varying numbers of fractions aligned with their treatment sites;
2. Certain techniques are confined to specific treatment rooms;
3. Scheduling is designed without preemptive rescheduling;
4. There is no prioritization of patients;
5. Only the interrelationships between the initial radiotherapy, simulation, and the culmination of chemotherapy are established;
6. The relationship between the surgery process and radiotherapy is not determined;
7. All of the parameters remain constant, known, and deterministic.

Notations:
P: Set of patients with cancer (*p* = 1, 2, 3, …, *P*)
F: Set of treatment fractions (*f* = 1, 2, 3, …, *F*)
R: Set of radiation rooms (*r* = 1, 2, 3, …, *R*)
T: Set of time periods (*t* = 1, 2, 3, …, *T*)
A: Set of radiation sites (*a* = 1, 2, 3, …, A)
D: Set of physicians (*d* = 1, 2, 3, …, *D*)
M: Set of radiation techniques (*m* = 1, 2, 3, …, *M*)
S: Set of simulation rooms (*s* = 1, 2, 3, …, *S*)
C: Set of patient categories by radiation period (*c* = 1, 2, 3, …, *C*)

Parameters:
$TGsim_a$: Time interval between the simulated period and the initial fraction of radiation for the radiation site *a*
$Pcat_{pc}$: 1, if patient *p* is the patient category *c*, 0, otherwise.
$Ppat_{af}$: 1, Radiation should be delivered to the f^{th} fraction of treatment site *a*, 0, otherwise.
$TGfrac_a$: Time interval between each fraction of radiation site *a*
$Pdoc_{pd}$: 1, if patient *p* is treated by physician *d*, 0, otherwise.
Dav_{dt}: 1, if physician *d* is available at period *t*, 0, otherwise.
$Pdiag_{pa}$: 1, if patient *p* is determined to be a radiation site *a*
$Ftech_{af}^{m}$: 1, if technology type *m* is used to irradiate the f^{th} fraction of the radiation site *a*, 0, otherwise.
$Rtech_{rm}$: 1, if technology *m* is implemented in the radiation room *r*, 0, otherwise.
$Rcap_{cr}^{t}$: Number of patients type *c* that can fit in each irradiation room *r* at period *t*
$Scap_{cs}^{t}$: Number of patients type *c* that can fit in each simulation room *s* at period *t*
$Pchemo_p$: 1, If patient *p* is instructed to undergo chemotherapy before receiving radiation treatment, 0, otherwise.
$Fchemo_p$: Finished chemotherapy period for patient *p*
$TGchemo_a$: Time interval between the completion of the chemotherapy period and the initiation of the initial fraction of radiation site *a*
$Time_t$: Amount of periods *t*
M: Large number

Decision variables
Z: Total time of completion
x_{ps}^{t}: 1, if patient *p* is allocated to the simulation room *s* at period *t* for simulation, 0, otherwise.
y_{pf}^{rt}: 1, if patient *p* undergoes the f^{th} fraction of radiation treatments in radiation room *r* at period *t*, 0, otherwise.
z_{pr}^{m}: 1, if patient *p* is determined to receive radiation in radiation room *r* by technology *m*, 0 , otherwise
$CTime_{pf}$: Completion treatment time of patient *p* for the f^{th} fraction
$CMax_p$: Maximum completion time of patient *p*

Mathematical Model

Objective function:
$$Min\ \sum_{p\in P} CMax_p \tag{2}$$

Subject to
$$CMax_p \geq CTime_{pf}\ ;\forall p,f \tag{3}$$

$$\sum_{r\in R}\sum_{t\in T} y_{pf}^{rt} = \sum_{a\in A} Pdiag_{pa}Ppat_{af}\ ;\forall p,f \tag{4}$$

$$\sum_{a\in A} Ctime_{pf}Pdiag_{pa}Ppat_{af} = \sum_{r\in R}\sum_{t\in T} y_{pf}^{rt}Time_t\ ;\forall p,f \tag{5}$$

$$-M\left(\left(Pdiag_{pa}Ppat_{af}\right)-1\right)+\ CTime_{pf}-CTime_{pf'}\geq TGfrac_{a};\forall p,f,f',a \tag{6}$$

$$\sum_{p\in P}\sum_{f\in F}y_{pf}^{rt}Pcat_{pc}\leq Rcap_{cr}^{t}\ ;\forall r,t,c \tag{7}$$

$$\sum_{r\in R}z_{pr}^{m}\leq 1;\forall p,m \tag{8}$$

$$(\sum_{f\in F}\sum_{t\in T}\sum_{a\in A}y_{pf}^{rt}\,Pdiag_{pa}\,Ftech_{af}^{m})\leq(\sum_{f\in F}\sum_{a\in A}Pdiag_{pa}\,Ftech_{af}^{m}Rtech_{rm}z_{pr}^{m})\ ;\forall p,r,m \tag{9}$$

$$y_{pf}^{rt}Pdiag_{pa}Ftech_{af}^{m}\leq Rtech_{rm};\forall p,f,r,t,a,m \tag{10}$$

$$\sum_{r\in R}y_{pf}^{rt}\,Pdoc_{pd}\leq Dav_{dt}\ ;\forall p,d,t,f=1 \tag{11}$$

$$\sum_{r\in R}\sum_{t\in T}y_{pf}^{rt}Time_{t}>\sum_{a\in A}\left(TGchemo_{a}Pchemo_{p}Pdiag_{pa}\right)+Fchemo_{p}\ ;\forall p,f=1 \tag{12}$$

$$[(\sum_{r\in R}\sum_{t\in T}y_{pf}^{rt}Time_{t}-\sum_{s\in S}\sum_{t\in T}x_{ps}^{t}\,Time_{t})(Pdiag_{pa})]=Pdiag_{pa}(TGsim_{a}+1);\forall p,a,f=1 \tag{13}$$

$$\sum_{t\in T}\sum_{s\in S}x_{ps}^{t}=1\ ;\forall p \tag{14}$$

$$\sum_{p\in P}x_{ps}^{t}Pcat_{pc}\leq Scap_{cs}^{t}\ ;\forall s,t,c \tag{15}$$

$$x_{ps}^{t}\in\{1,0\};\forall p,s,t \tag{16}$$

$$y_{pf}^{rt}\in\{1,0\};\forall p,r,t,f \tag{17}$$

$$z_{pr}^{m}\in\{1,0\};\forall p,r,m \tag{18}$$

$$CTime_{pf}\in\{Integer\};\forall p,f \tag{19}$$

$$CMax_{p}\in\{Integer\}\ ;\forall p \tag{20}$$

The objective function in Equation (2) aims to reduce the total completion time, as shown in Equation (3). Equation (4) represents the mandate for each patient to undergo radiation treatment according to a prescribed pattern for their specific treatment type during each interval. Equations (5) and (6) stipulate that the completion time for each radiation session must exceed or equal a specified time interval. Equation (7) represents the capacity requirement, in which a specific number of patients must be accommodated within the radiation room concurrently. This capacity extends to each patient category. Equations (8), (9), and (10) enforce the condition that patients must be exposed to designated technologies within designated radiation rooms. Equation (11) mandates the continuous presence of a doctor during the first treatment session. Equation (12) ensures that the initial irradiation session can only commence following the completion of chemotherapy and a specified recovery interval. Simulation-related stipulations are encapsulated in Equations (13) and (14), while Equation (15) defines the capacity requirements within the simulation room for patients of various categories. The non-negativity and binary constraints associated with the decision variables are covered by Equations (16) to (20).

Drawing on the constraints analysis in [1], we identify the hard constraint in Equation (10) as redundant, as its essence is already represented by Equation (9), which mandates technology-specific irradiation within the radiation rooms. Consequently, Equation (10) was omitted from the refined model, thereby decreasing the complexity by reducing the number of constraints. This streamlined the optimization process, improved the alignment of the model with real-world scenarios, and enhanced the flexibility of the scheduling model.

4.3 Model verification and validation

This section describes the validation of the enhanced mathematical model based on five distinct case scenarios, with variations in patient profiles, maximum treatment fractions, availability of radiotherapy rooms, treatment duration, treatment sites, medical personnel, treatment techniques, simulated treatment rooms, and patient types. We note that this experiment assumes the absence of predetermined chemotherapy or surgery processes. Comprehensive details of these treatments are summarized in Table 2.

Table 2 Numbers of variables used in the five case studies

Variables	1	2	3	4	5
Patient (*p*)	3	4	7	8	10
Fraction (*f*)	5	8	20	25	33
Irradiation room (*r*)	2	2	3	3	4
Day (*t*)	10	20	40	50	72
Treatment site types (*a*)	3	4	5	6	7
Doctor (*d*)	2	2	3	4	5
Technique (*m*)	2	2	3	3	4
Simulation room (*s*)	2	2	2	2	2
Patient type (*c*)	2	2	2	2	2

Throughout this experimental phase, all of the mathematical models were precisely solved by employing the Gurobi Optimizer version 9.1, software for solving mathematical programming problems, which was implemented using Python 3.7. The computational tests were executed on a personal computer equipped with an Intel(R) Core (TM) i7-1065G7 CPU clocked at 1.30 GHz and 16.00 GB.

In order to assess the performance of the enhanced model in terms of computational time in comparison to the initial model, we examined five case studies: two small-scale cases, two medium-scale cases, and one large-scale case (referred to as the "real-world case"). Each case was run five times to explore the consistency of the processing times. The performance of the improved mathematical model was demonstrated using a T-test, and the results are shown in Table 3.

Table 3 Statistics for computation time

Case	Variables	Initial Model [1]			Improved Model			Comparison	
		No. of constraints	Mean (Second)	SD	No. of constraints	Mean (Second)	SD	Mean difference	P-value (CL=95%)
1	195	2,198	0.18	0.02	398	0.12	0.00	0.06	0.002
2	372	11,600	0.61	0.02	1,360	0.29	0.02	0.32	0.000
3	1,068	267,093	11.54	1.20	15,093	3.37	0.06	8.19	0.000
4	1,489	571,660	24.10	2.07	31,660	6.74	0.03	17.46	0.000
5	2,648	2,740,784	113.17	1.90	79,664	27.33	0.87	85.83	0.000

Upon analyzing the results in Table 3, it became evident that all five case studies yielded p-values of below 0.05, leading to the rejection of the null hypothesis outlined in Equation (1). This unequivocally confirmed a substantial reduction in computational time for the improved mathematical model compared to the model presented in [1]. The primary reason for this reduction in computation time can be attributed to the diminished number of constraints. In addition, identical values of the objective function were generated across all cases. A graphical depiction of the processing time comparison is provided in Figure 2. It is noteworthy that although there appeared to be minimal differences when assessing the small cases, substantial disparities emerged in the testing of the medium and large cases. The processing times for the enhanced mathematical model exhibited notable improvements, especially in comparison to the model in [1], which required lengthier solution times.

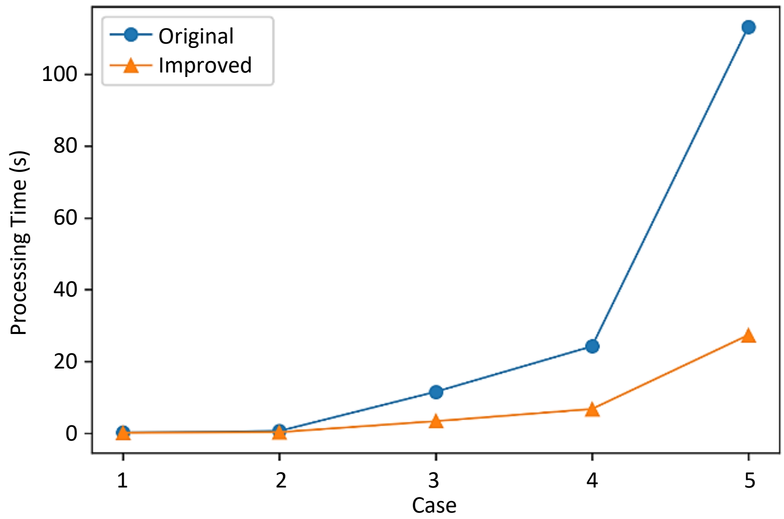

Figure 2 Comparison of processing times

4.4 Development of the mathematical model

With a view to real-world applications, this study seeks to enhance an existing mathematical model of RT patient scheduling. This enhancement involves the inclusion of pertinent constraints, and in particular surgical considerations and RT processing times. The introduction of surgical conditions involves scenarios where certain patients require surgery before undergoing radiation therapy, followed by a recovery period. Three additional parameters and one constraint were incorporated into the model to accommodate this surgical complexity. The next section provides a detailed exposition of these supplementary parameters and constraints.

Additional parameters:

$Psur_p$: Parameter with a value of one when patient *p* is given surgery before radiotherapy; otherwise zero.

$Fsur_p$: Date of completion of surgery for patient *p*.

$TGsur_a$: Interval time between the end of surgery and the first radiation treatment, position *a*

Additional constraint:

$$\sum_{r\in R}\sum_{t\in T} y_{pf}^{rt}\, Time_t > \sum_{a\in A}\left(TGsur_a Psur_p Pdiag_{pa}\right) + Fsur_p\ ; \forall p, f = 1 \qquad (21)$$

A supplementary constraint is outlined in Equation (21), which represents the date of the patient's initial RT session. This date is contingent upon the completion of surgery, followed by the stipulated recovery period.

A constraint on the RT processing time was also included within this model. The framework presented in [1] imposes a maximum capacity constraint per day for both simulation and irradiation rooms, which determines the limit on the number of patients that can be treated daily. However, relying solely on the number of treatable patients per day might not accurately capture the dynamics of real-world scenarios, due to the varying processing times for each patient's RT. Hence, in this study, we construct a capacity constraint that aligns with the actual processing time for radiation. The parameters $Rcap_{cr}^{t}$ and $Scap_{cs}^{t}$ and Equations (7) and (15) were therefore removed from the enhanced model. The radiation processing time constraint was then clearly defined by the addition of new parameters and constraints, as explained in the next section.

Additional parameters:
$Stime_a$: Processing time for simulation at radiation site type a
$Rtime_{am}$: Processing time for RT at radiation site type a using technology m
$Tcap_c$: Hours of availability for patient type c

Additional constraint:
$$\left(\sum_{p\in P}\sum_{a\in A} x_{ps}^{t} Pdiag_{pa} Stime_a Pcat_{pc}\right) + \left(\sum_{p\in P}\sum_{f\in F}\sum_{a\in A}\sum_{m\in M} y_{pf}^{rt}\, Ftech_{af}^{m} Rtime_{am} Pcat_{pc}\right) \le Tcap_c \text{ ; } \forall s,r,t,c \tag{22}$$

Equation (22) states that the cumulative simulation processing time within each room for each patient category should not exceed the daily working hours, and the combined radiation therapy processing time within each room for each patient category must also stay within the confines of the daily working hours.

The final mathematical model is shown below:

Objective function:
$$Min\ \sum_{p\in P} CMax_p \tag{23}$$

subject to Equations (3)–(6), (8)–(14) and (16)–(22).

In the improved mathematical model, assumption 6 from the existing model is removed, and assumption 5 is revised to state, "The established interrelationships are limited to the initial radiotherapy, simulation, culmination of chemotherapy, and completion of surgery."

4.5 Case study application

To evaluate the efficacy of the proposed mathematical model, a numerical example dataset was generated using information from [1]. This dataset contained information on 13 patients, four distinct treatment techniques (image-guided RT (IGRT), 2D, 3D, and intensity-modulated RT (IMRT)), two simulation rooms, four radiation rooms, seven radiation sites, and two patient types. The technological resources available for RT in each room are detailed in Table 4. For the purposes of this illustration, we note that patient type 1 was given an eight-hour workday (office hours clinic), whereas patient type 2 was given a four-hour day (after-hours clinic). The specific numerical data are given in Table 5.

After inputting the data and executing the optimization solver, the outcomes revealed a total treatment completion time of 750 days, with a solution-finding time of 225.66 s. The RT patient schedule for this case study is visually represented in Figure 3. The scheduling output demonstrates the model’s ability to adhere to the prescribed constraints and offer a viable patient scheduling strategy in accordance with the proposed mathematical model.

Table 4 Technology allocation in each radiation room, taking into consideration surgery

Radiation Room	Technology
Room 1	M1
Room 2	M2, M3, M4
Room 3	M1, M2, M3, M4
Room 4	M3, M4

Table 5 Patient treatment information, taking into consideration surgery

Patient	Patient category	Radiation Site	No. of Fractions	Technology	CT[1] before RT[2] (the last cycle of CT received)	RT after the last cycle of chemo	Surgery before RT (the last cycle of Surgery received)	RT after the last cycle of surgery	Sim. before RT	Doctor
1	In-hour clinic	A1	33	M2	Yes (Day 2)	28 days	No	None	28 days	D1
2	In-hour clinic	A2	33	M2	No	None	No	None	28 days	D2
3	In-hour clinic	A3	33	M2	No	None	No	None	28 days	D3
4	In-hour clinic	A4	23	M4	No	None	No	None	7 days	D4
5	Off-hour clinic	A5	21	M2	Yes (Day 1)	42 days	No	None	28 days	D5
6	In-hour clinic	A6	24	M3	Yes (Day 3)	42 days	No	None	28 days	D1

[1]CT refers to chemotherapy.
[2]RT refers to radiotherapy.

Table 5 (continued) Patient treatment information, taking into consideration surgery

Patient	Patient category	Radiation Site	No. of Fractions	Technology	CT[1] before RT[2] (the last cycle of CT received)	RT after the last cycle of chemo	Surgery before RT (the last cycle of Surgery received)	RT after the last cycle of surgery	Sim. before RT	Doctor
7	In-hour clinic	A7	22	M1	No	None	No	None	7 days	D2
8	In-hour clinic	A4	23	M4	No	None	No	None	7 days	D3
9	Off-hour clinic	A6	24	M3	Yes (Day 5)	42 days	No	None	28 days	D4
10	Off-hour clinic	A7	22	M1	No	None	No	None	7 days	D5
11	In-hour clinic	A5	21	M2	Yes (Day 1)	42 days	No	None	28 days	D1
12	In-hour clinic	A6	24	M3	Yes (Day 4)	42 days	Yes (Day 48)	14 days	28 days	D2
13	Off-hour clinic	A3	33	M2	No	None	Yes (Day 3)	28 days	28 days	D3

[1]CT refers to chemotherapy.
[2]RT refers to radiotherapy.

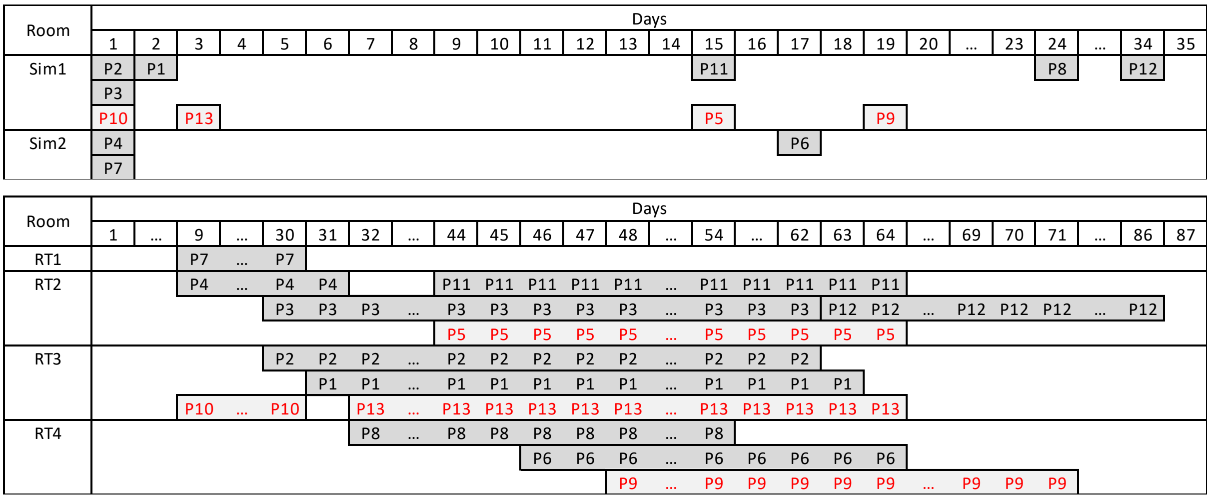

Figure 3 Appointment schedule for radiotherapy patients

5. Discussion

The outcomes of the proposed mathematical model show that it effectively generates a comprehensive set of results, including the creation of detailed RT schedules for each treatment period, patient assignments to specific rooms and technologies, and the determination of timeframes for simulation and RT. These results are achieved under varying conditions such as treatment processing times, chemotherapy, and surgical aspects. This robust set of outcomes reaffirms our confidence in the model's practical applicability.

Based on the refined model and the inclusion of practical constraints, we successfully achieved the two objectives of the study, as follows:

1. The research findings demonstrate that the removal of redundant constraints from the model gave a significant reduction in the processing time needed to find the optimal solution. The goal of significantly reducing the computation time was therefore met.
2. The previous lack of incorporation of surgical restrictions and treatment processing time constraints was addressed in the improved mathematical model. This was done to enhance the model's ability to tackle the practical challenges that are encountered in real-world scenarios. The results indicated that the enhanced model could provide more comprehensive treatment plans that included factors such as doctor availability, treatment techniques, and treatment procedures, thereby demonstrating the successful achievement of this objective.

The findings were successfully connected with the initial research goals, and the ways in which the insights gained from the study were directly aligned with and met these objectives were discussed. In light of these two primary goals, we can identify some managerial insights as follows:

- Optimization of RT patient scheduling: A valuable mathematical model has been presented that significantly reduces the computation time for optimizing RT patient scheduling. This will be important for healthcare managers and administrators, as it offers a more efficient and automated way to manage patient appointments and resources, ultimately improving the overall efficiency of RT services.
- Enhanced treatment planning: By incorporating constraints related to RT processing time capacity and surgical considerations, this research contributes to the development of more comprehensive treatment plans. This will be valuable for medical professionals and managers, as it allows for more tailored and efficient treatment planning, and considers factors such as doctor availability, treatment techniques, and treatment procedures.

- Interdisciplinary collaboration: This research emphasized the importance of collaboration between medical professionals, operations researchers, and decision makers. The proposed interdisciplinary approach enables the development of patient-centric scheduling strategies that consider both medical knowledge and optimization techniques. The need for collaboration among different stakeholders in healthcare management is therefore highlighted.
- Future development: The insights gained from this research pave the way for further advancements in patient scheduling methodologies. Our results suggest that ongoing developments could include refining the model, incorporating more practical constraints, integrating real-time data, and exploring the integration of emerging technologies. This will be valuable for managers and researchers, encouraging them to continue improving patient care and resource allocation in radiotherapy scheduling.

The proposed mathematical model, despite being promising, still has limitations that may hinder its direct application in real-world scenarios. These limitations stem from certain assumptions and simplifications within the model. The key constraints and areas for improvement are as follows. (1) The model assumes that patients are assigned to doctors based on team considerations, and that a single doctor oversees a patient's entire RT program. In practice, patients may require medical personnel with different levels of skill at various stages of treatment. (2) The model does not incorporate uncertainty or variability in patient treatment times or other parameters. Real-world situations are subject to change, which can impact scheduling and resource allocation. (3) Although room availability is considered, machine availability is not explicitly addressed. In practice, machine availability and maintenance schedules can affect treatment scheduling. (4) The model does not consider patient preferences or constraints related to specific treatment times. In practice, patients may have preferences or constraints on when they can receive treatment. (5) The proposed model assumes static scheduling and does not adapt to real-time changes, such as emergency cases or unexpected delays. (6) The model is dependent on the quality and accuracy of the input data, and any inaccuracies in the data may affect the model's performance. (7) Although the proposed model has a reduced computation time from the minimization of hard constraints, it may still face scalability issues when dealing with a large number of patients and complex scheduling scenarios. Addressing these limitations would require a more complex and sophisticated model, perhaps incorporating stochastic elements and dynamic scheduling features to better reflect real-world complexities.

In future work, it would be prudent to develop the proposed mathematical model into a more streamlined program, due to the difficulty of comprehending or translating the outputs from the optimization solver, particularly for those not well-versed in the field. The development of dedicated software solutions would facilitate the resolution of this issue. In addition, as the scale of the problem grows, the challenge of finding a solution may intensify. Hence, the incorporation of heuristic or metaheuristic algorithms may become necessary to find optimal solutions.

From an examination of the scheduling results in Figure 3, it is evident that certain patients are assigned to the same room on the same day. To effectively manage the patient queue within a given day, the staff must manually arrange the treatment slots. As the system evolves, this factor should be included in the mathematical model to ensure a comprehensive solution that considers the dynamic patient flow throughout the day.

6. Conclusions

The aim of this research was to improve a mathematical model for RT patient scheduling, to address a gap in extant research. Building upon the foundation established in [1], our study extends the existing mathematical model by refining it. Notable progress was achieved through the removal of redundant constraints, which gave a remarkable 75.85% decrease in processing time compared to the model in [1]. We also addressed the omission of surgical restrictions and treatment processing time constraints in the prior model, thereby enhancing it. To validate its effectiveness, we generated a comprehensive numerical example dataset. The results demonstrated the model's capacity to determine optimal solutions, including patient assignments to specific rooms and technologies, and calculating the required durations for simulation and RT, while adhering to defined constraints such as those related to chemotherapy and surgical restrictions. Our model also considers capacity based on RT processing time constraints, and offers advantages over the consideration of capacity based on maximum capacity constraints per day.

In future research, it will be highly advisable to incorporate fuzzy parameters into the model, as this will be crucial to address the variability and uncertainty in treatment processing times among patients. In a medical setting, each patient's treatment timeline can exhibit variations due to individual factors, and these variances can significantly impact the scheduling process. The integration of fuzzy parameters into the model will allow it to more effectively accommodate and adapt to these uncertainties, thereby bolstering the scheduling resilience and flexibility. This approach not only enhances the model's real-world applicability but also contributes to more precise and robust scheduling. Consequently, it ensures that patients receive their treatments in a timely and efficient manner, even in the face of potential variations in treatment durations.

Furthermore, to create a more comprehensive model that accurately mirrors real-world complexities, it is essential to introduce additional considerations such as emergency cases and priority scenarios. Including these elements will allow the model to effectively address challenges related to unforeseen disruptions and varying levels of patient urgency. Emergency cases often demand immediate treatment, and can disrupt standard scheduling practices; incorporating these scenarios will equip the model to efficiently adapt to such urgent cases, allowing for the optimal allocation of resources to ensure timely patient care. The model should also account for cases where patients have varying levels of priority based on the nature of their medical condition. The inclusion of priority scenarios enables the model to consider these cases judiciously, and to ensure that patients with greater medical needs are scheduled appropriately, thereby minimizing treatment delays and conflicts.

Proactively tackling these challenges will mean that the model becomes more adaptable and responsive to the dynamic nature of healthcare, ultimately advancing the quality of patient care and optimizing resource allocation in RT scheduling.

7. Acknowledgements

The Cancer Patient Center is to be thanked for supplying information on patient treatment and cancer treatment for the benefit of this study, as well as for giving the researchers with information on cancer treatment. In addition, this study appreciates the assistance of Miss Khwantira Tanak and Miss Sirikwan Jongjityothin in conducting the study. This research work was partially supported by Faculty of Engineering, Chiang Mai University.

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