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Effect of uncertainties in prediction models of shallow and deep foundations

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Abstract

The primary objective in foundation design is to ensure that the foundation exhibits satisfactory behavior under regulatory loading conditions while addressing critical safety concerns. This study aims to investigate the impact of parameter uncertainties in prediction models for bearing capacity and settlement of shallow and deep foundations on resulting failure probabilities using various approximation methods. Recognizing the complexity of reliability studies, this research explores the possibility of not probabilizing all variables and simplifying calculations for future applications. Additionally, it aims to assess the effectiveness of commonly used approximation methods in geotechnical engineering for estimating failure probabilities. The study's findings would enhance foundation reliability indices and decrease the probability of failure. The parametric study has enabled us to identify the variables that are most sensitive to changes in their coefficient of variation. The influence of model selection on foundation reliability is emphasized, and a critical evaluation of model bias is crucial for reliability analysis. It is evident that conservative models may shift subsequently calculated failure probabilities by inaccurately assigning situations from the safe domain to the failure domain due to inherent conservatism.

Keywords: Reliability, Failure probability, Sensitivity, Foundation, Settlement, Bearing capacity

1. Introduction

Uncertainty is a fact of life in geotechnical practice. The complexity of nature often presents soil profiles that are significantly different from those assumed in analysis and design. Similarly, loads and environmental conditions challenge even the most precise predictions. Limited site investigations, measurement errors, and imperfections in analysis procedures further complicate the engineer's task [1-6].

The consideration of bearing capacity and allowable settlement is crucial in foundation design [7-9]. Bearing capacity is determined by the shear strength of the soil and is estimated through theories proposed by [9-13], and others. However, the various equations for bearing capacity exhibit significant variations. Furthermore, the proposed theories for bearing capacity incorporate several assumptions aimed at simplifying the problems.

The primary objective of foundation design is to ensure satisfactory behavior under regulatory loading conditions, while keeping in mind the need to balance safety and economy [7]. It is evident that damaging behavior remains possible under certain conditions. Faced with potentially conflicting criteria of unfavorable behavior and economic constraints, the engineer establishes a reasonable level of safety based on empirical results and personal experience [7-10]. The question that arises systematically is how to assess the level of safety that can be considered acceptable. An experienced designer can intuitively appreciate the effect of various factors on foundation performance, such as variations in loads and material strengths, model errors, execution errors or misdirected work, and unforeseen site conditions. In traditional design practice, an overall assessment of these factors is typically made using a factor of safety. Such an assessment of safety heavily depends on the framework of the analysis and does not distinguish between model uncertainties and parameter uncertainties. The factor of safety is essentially based on experience gained from similar projects. The variation in safety level resulting from new conditions or advancements in the state of the art cannot be justified by traditional practice.

In the traditional method, an appreciation of risk can be obtained by incorporating various uncertainties into a mathematical formulation of risk. This approach forms the basis of probabilistic analysis, where uncertain quantities are modeled as random variables, and risk is evaluated in terms of a failure probability [14-20]. Stochastic modeling encompasses a set of methods that account for data uncertainties in computational models. In geotechnical engineering, reliability analysis aims to determine the reliability or failure probability of a structure, soil, or rock mass. The difficulty of calculating this probability has led to the development of various approximation methods [21-30].

We have good reason to be concerned about the reliability index's the accuracy when it is computed using the safety factors' anticipated values. The dependability index derived using this technique may in reality be deceptive if the traditional design utilizing safety factors is inaccurate as a result of numerous assumptions or simplifications. This is because the dependability index is determined using the presumptive safety factors, and the ensuing reliability rating may not be valid if the safety factors do not precisely reflect the

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foundation's real behavior. There is an increasing focus on probabilistic approaches, which more rigorously account for uncertainties, in contemporary geotechnical engineering procedures. Probabilistic techniques, as opposed to depending entirely on set safety factors.

The objective of this study is to examine the influence of parameter uncertainties in prediction models of bearing capacity and settlement of shallow and deep foundations on the failure probability obtained by each approximation method. Since reliability studies can involve heavy computations, this study explores the possibilities of reducing the number of random variables to consider in the analysis. The aim of this work is also to identify the advantages and limitations of certain approximation methods for failure probability used in geotechnical engineering.

2. Uncertainty concept in geotechnical engineering

The concept of uncertainty in geotechnical engineering refers to the consideration of the numerous sources of uncertainties that can affect analyses and predictions related to soils and foundations. Due to the complexity and inherent variability in soil properties, there are uncertainties regarding soil characterization, modeling of geotechnical behaviors, parameters used in calculations, loading conditions, etc. [25-35].

Uncertainty in geotechnical engineering can arise from various sources, such as measurement errors during site investigations, natural variations in soil properties, approximations in the models used, variations in environmental conditions, etc.

Soil is a discontinuous, heterogeneous, anisotropic, and non-conservative material that is defined by a set of physico-mechanical characteristics (bulk density, elastic modulus, strength parameters, etc.). Due to the unique nature of soil material, as well as the history of stresses and geological conditions, this set of physico-mechanical characteristics varies from one point to another within the soil mass. A complete characterization of a site, accounting for measurement errors and uncertainties, requires quantifying all the physico-mechanical characteristics at every point within the soil mass. The technical and practical impossibility of fully characterizing a site necessitates considering the values of all the physico-mechanical characteristics at any point within the soil mass as subject to debate [22, 34].

In geotechnical engineering, and as a first approximation, uncertainties arise from three sources:

- Randomness inherent in certain characteristics of the studied media or systems.
- Imperfections in the methods for estimating randomness and, more generally, the models used for analyzing the studied systems.
- Inability to know the social objective and social aversion to risk.

In most applications, the influence of this uncertainty is not considered [16, 34]. In the probabilistic approach, uncertainties regarding the load S and the resistance R are explicitly taken into account. The fundamental idea in the design of a system is to achieve a state that lies within the safety region on Figure 1 throughout the entire service life of the structure. The design criteria are located above the failure surface (R=S); the strength above the required strength represents the safety margin.

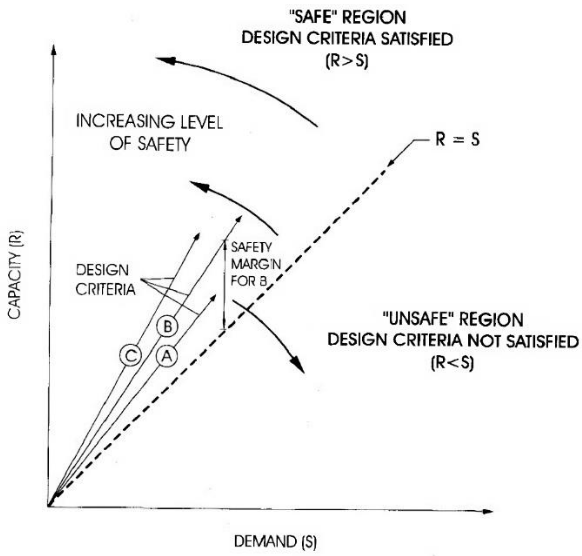

Figure 1 General design criterion for resistance as a function of solicitation [16].

The probability of the load exceeding the resistance is given by:

$$P[S > R] = P[R \leq S] = P[R - S \leq 0] = P[g(x) \leq 0] \quad (1)$$

where $x = [x_1, x_2, ..., x_n]$ denotes the set of random variables, and $g(x)$ represents the limit state function. When an expression is available to describe the limit state, such as bearing capacity or settlement, which allows for the expression of a safety margin, the failure probability is the probability that this margin is less than zero or a chosen value. A fundamental problem in reliability theory is related to the calculation of the multiple probability integral to estimate the failure probability P_f:

$$P_f = P[g(X) \leq 0] = \int_{g(X) \leq 0} f(x)dx \quad (2)$$

where $f(x)$ represents the joint probability density function of x. In the special case of simple functions, where an analytical expression for the joint probability density function of the limit state function $g(x)$ is given, analytical methods can be used. However, in practical cases where the joint probability densities are non-Gaussian and the basis function is nonlinear, determining the exact value of P_f through analytical methods is impossible. This difficulty has led to the development of various approximation methods.

The reliability index β is a measure of the reliability of a system that reflects the system's ability to perform a required function under given conditions for a specified duration while incorporating knowledge about parameter uncertainty. Calculation of the reliability index requires:

- A deterministic mechanical model (e.g., a calculation model for the bearing capacity of a foundation).
- A definition of the limit state $g(x)$.
- The characteristic statistical parameters of uncertainty (or variability) of random variables- mean value (μ), standard deviation (σ). The shapes of parameter distributions can be highly significant. Parameters can encompass not only the properties of geotechnical materials but also loads and system geometry.
- An approximation method to estimate the value of the reliability index.

The reliability index does not provide a direct answer to the posed problem. A more comprehensive or useful piece of information is the probability of failure. In most reliability analysis methods, the probability of failure is defined from the reliability index, except in simulation methods that allow for a direct estimation of the value.

3. Approximate methods for determining the failure probability

The methods available to estimate the failure probability P_f, can be classified into two groups: moment methods and simulation methods.

The moments of the limit state function g(x) must be calculated at the proper point in the case of moment techniques. In the literature, a number of iterative strategies have been presented. First Order Second Moment (FOSM) reliability method, First Order Reliability Method (FORM), Second Order Reliability Method (SORM), and other specialized methods that employ third or fourth-order statistical moments, such as the First Order Third Moment (FOTM) method, are the main techniques.

The development of the FOSM (First-Order Second-Moment) method began in the 1970s, with Cornell's reliability index [34]. This method has firmly established itself in the field of geotechnical engineering [16, 17].

The FORM method proposed by Hasofer and Lind [35] is considered one of the most reliable computational methods for geotechnical applications. The FORM method was developed to address the non-invariance of the reliability index β with respect to the chosen limit state function and to overcome the mathematical limitations of the FOSM method.

Approximating the limit state at the design point by a straight line or plane introduces errors in the FORM analysis, the magnitude of which depends on the nonlinearity of the limit state equation. This is why higher-order moment methods, such as the SORM method, have been developed [16]. In this method, a quadratic or higher-order polynomial is used to describe the limit state surface at the design point.

Improving the FORM method by incorporating higher-order moments led to the First Order and Third Moment (FOTM) reliability method and the Higher-Order Moment Simulation Technique (HOMST). These two methods rely on the first-order Taylor series expansion of the basis function at the design point. The FOTM method is based on the third moment of the limit state function and assumes that the centered and standardized limit state function follows a lognormal distribution [30]. The HOMST method involves using the fourth-order moment of the limit state function to transform it into a standardized normal variable during the calculation of the reliability index [31]. Higher-order moment methods provide better approximations but are generally not considered necessary for the majority of applications.

Another commonly used method is the Point Estimate Method (PEM), which is an approximate method that estimates the first two moments of a limit state function based on the first two moments of the basic random variables. This method is robust and provides satisfactory accuracy for certain types of problems if the coefficient of variation is not very large [16]. The coefficient of variation (COV) is defined as = (Standard Deviation/Mean) × 100.

Simulation methods are based on generating a set of responses from which the failure probability can be estimated. The simplest technique is the Monte Carlo method (MCS), proposed by Von Neumann during World War II [16]. This method involves randomly sampling N numerical values for each basic variable, and by solving the problem N times in a deterministic manner, we can obtain a sample that represents the different responses of the limit state function, $g(x)$. If N_f is the number of responses that fall within the failure domain, then the failure probability can be estimated by:

$$P_f = \frac{N_f}{N} \tag{3}$$

The main drawback of the simulation method is that it requires very long computation times compared to moment methods. Specialized methods such as Conditional Simulation (CS), Importance Sampling (EST), and Directional Simulation (DS) have been proposed to reduce the number of simulations needed.

In this study, the moment methods FOSM, FORM, and SORM, as well as MCS, were used to analyze the reliability of shallow and deep foundations. Detailed descriptions of these methods can be found in Melchers [25] and Haldar and Mahadevan [36].

4. Design of shallow and deep foundations

Shallow foundations and deep foundations are the two primary foundation types utilized to support structures in foundation engineering. The design engineer for foundations must take into account two failure types while keeping safety and economy in mind:

a) Bearing Capacity Failure: This failure mode occurs when the soil beneath the foundation is unable to support the applied loads. It is important to ensure that the foundation can withstand the anticipated loads without excessive settlement or shear failure.
b) Settlement Failure: Excessive settlement of the foundation can lead to structural damage and loss of functionality. It is crucial to design the foundation in a way that minimizes settlement and provides adequate support to prevent significant deformations.

Reliability analysis techniques, such as the ones mentioned earlier (FOSM, FORM, SORM, MCS), can be employed to assess the failure probability for both shallow and deep foundations. These methods take into account uncertainties in material properties, soil characteristics, and applied loads to provide a quantitative measure of the foundation's reliability. By incorporating these probabilistic analyses into the design process, engineers can make informed decisions to ensure the safety and efficiency of foundation systems.

4.1 Design of shallow foundations

4.1.1 Bearing capacity of shallow foundations

The bearing capacity of a shallow foundation is typically determined based on the mechanical properties of the soil measured either in the laboratory or in-situ. The general equation for this bearing capacity is given by:

$$q_u = CN_cI_cS_cT_cd_c + \frac{1}{2}B'\gamma'N_\gamma I_\gamma S_\gamma T_\gamma d_\gamma + q_0N_qI_qS_qT_qd_q \tag{4}$$

where C is the cohesion of the soil, q_o is the effective stress of the soil at the foundation base, B' is the reduced width of the foundation, γ' is the unit weight of the soil below the base. N_c, N_γ, and N_q are the bearing capacity factors, which depend on the soil's friction angle φ. I_c, I_γ, I_q; S_c, S_γ, S_q; T_c, T_γ, T_q; and d_c, d_γ, d_q are correction factors.

This study considers two commonly used models in the calculation of bearing capacity for shallow foundations, namely the Hansen model [11] and the Vesic model [12].

4.1.2 Settlement of shallow foundations

The application of loads on soil causes displacements. The most significant displacements are vertical displacements or settlements. The total settlement S of a foundation, which is the response to the applied load on the soil, can be calculated as the sum of three components:

$$S = S_i + S_c + S_f \tag{5}$$

where S_i represents instantaneous settlement, S_c is consolidation settlement, and S_f is creep settlement.

4.2 Design of deep foundations

4.2.1 Bearing capacity of deep foundations

The ultimate vertical load carried by a pile is generally given by:

$$Q_u \approx q_{pu}A_P + \sum_{I=1}^{n} Q_{fui} - W_p \tag{6}$$

where Q_u is the ultimate bearing capacity of the pile, q_{pu} is the ultimate point load stress at the pile tip, A_p is the cross-sectional area of the pile, L is the length of the pile, Q_s is the ultimate vertical load due to lateral friction, and W is the weight of the pile.

Different methods have been proposed to solve Equation (6), with the most recommended ones being the Hansen method, the Vesic method, and the Generalized Ultimate Capacity Equation method by USACE [37].

4.2.2 Settlement of deep foundations

The settlement of an isolated pile under loads is generally low and is not a determining calculation parameter for most civil engineering structures. However, in some cases of pile groups, it may be imperative to consider settlement, and reliable estimation often requires an accurate assessment of the settlement of an individual pile.

In the case of linear elasticity, the settlement at the head of a pile with diameter D under the action of a vertical load Q_v at the pile head is given by:

$$S_{0p} = \frac{Qv.I_v}{E(L).D} \tag{7}$$

where I_v is the settlement factor at the pile head and *E(L)* is the Young's modulus of the soil at the base of the pile.

5. Reliability of shallow and deep foundations

In this study, the probabilistic methods FOSM, FORM, SORM, and MCS are used to account for and assess the influence of random uncertainties on the considered design model parameters. The variation of the reliability index with respect to the level of uncertainty in the parameters is examined for each of the probabilistic methods. The variation of the reliability index with respect to the variance of one of the random variables is obtained by keeping the mean of the variable under consideration constant and fixing the coefficient of variation of the other random variables at an arbitrarily set value of 5%. The distributions of the random variables are assumed to be normal or lognormal. The margin of safety is given by:

$$g(R, L) = R(x) - L(x) \tag{8}$$

where *R(x)* represents the bearing capacity or settlement of the foundation, and *L(x)* represents the applied load or allowable settlement of the foundation.

5.1 Reliability of shallow foundations

The parameters considered as random variables (normal or lognormal distributions) in the reliability analyses of shallow foundations are indicated in Table 1. The geometric parameters are assumed to be deterministic with the following average values: width B = 2 m, length L = 3 m, and depth d = 1.5 m. The allowable settlement is set to 5 cm.

Table 1 Parameters and average values considered in the probabilistic analysis of a shallow foundation.

Parameter	Symbol	Variable	Mean value
Specific parameters for bearing capacity prediction models			
Friction angle	$Ø$	Random	25°
Cohesion	C	Random	15 kN/m^2
Dry unit weight	γ	Random	18 kN/m^3
Vertical load	Q_v	Random	400 kN
Horizontal load	Q_h	Random	120 kN
Moment parallel to B	M_B	Random	100 $kN.m$
Moment parallel to L	M_L	Random	100 $kN.m$
Slope inclination	T	Random	10°
Base inclination	S	Random	25°
Cohesion between soil and base	Ca	Random	10 kN/m^2
Specific parameters for settlement prediction models			
Elastic modulus	E	Random	40 MPA
Preconsolidation stress	q_c	Random	100 kN/m^2
Coefficient of consolidation	C_v	Random	2,5 ×10^{-8} m^2/s
Poisson's ratio	ν	Random	0,3
Void index	e_i	Random	0,8
Compression index	C_c	Random	0,1
Recompression index	C_r	Random	0,02
Creep index	C_α	Random	0,004

In the problem of probabilistic analysis of the bearing capacity of shallow foundations, the bearing capacity $R(x)$ is given by Equation (4), and the load $L(x)$ is the vertical load divided by the foundation area. The diagonal values of the correlation matrix are equal to unity. The friction angle and cohesion are assumed to be negatively correlated with ρ = -0.5; the vertical and horizontal loads are assumed to be positively correlated with ρ = 0.5; the moments M_B and M_L are assumed to be positively correlated with ρ = 0.5.

We examined the changes of the reliability index according to the covariance of each of the parameters of the model under consideration, as determined using the FORM Method. The results of the parametric reliability analysis of a shallow foundation subjected to an inclined and eccentric load near a sloping base are presented in Figures 2. These figures show the variations of the reliability index for each parameter of a limit state function. The main results obtained are as follows:

• For all the considered limit state functions, the reliability index strongly depends on the internal friction angle of the soil, the vertical load, and the horizontal load.

• For all the considered limit state functions, the reliability index is not significantly dependent on the cohesion between the base of the footing and the foundation soil; this insensitivity is more pronounced when the parameters are modeled using a lognormal distribution.

• When dealing with the same limit state function, the importance of parameter influences is closely tied to the nature of the distribution of the random variables. Nevertheless, the internal friction angle of the soil stands out as the most influential parameter, irrespective of the distribution type of the random variables and the considered correlation matrix. This result underscores the dominance of the internal friction angle over other factors in shaping the behavior of the system, reaffirming its critical role in various scenarios and distribution patterns.

5.2 Probabilistic analysis of settlement in shallow foundations

In the problem of probabilistic analysis of settlement in shallow foundations, the settlement $R(x)$ is given by Equation (5), and the allowable settlement $L(x)$ is fixed at a value of 5 cm. All random variables are considered independent.

Using the FORM Method, Figure 3 shows that the reliability index strongly depends on the preconsolidation pressure, unit weight, and vertical load, while it has very little dependence on the other variables, where the variation of the reliability index is practically constant. Furthermore, these figures demonstrate that the preconsolidation pressure is the random variable that governs the reliability for this mode of failure.

5.3 Variable reduction

Based on the results obtained in the previous sections, it is reasonable to question whether reliability analysis conducted with all random variables of a limit state function is only of theoretical interest. In other words, in practice, can we perform reliability analysis by considering a reduced number of random variables? To answer this question, a practical example is considered, where the measured variability in practice is taken into account.

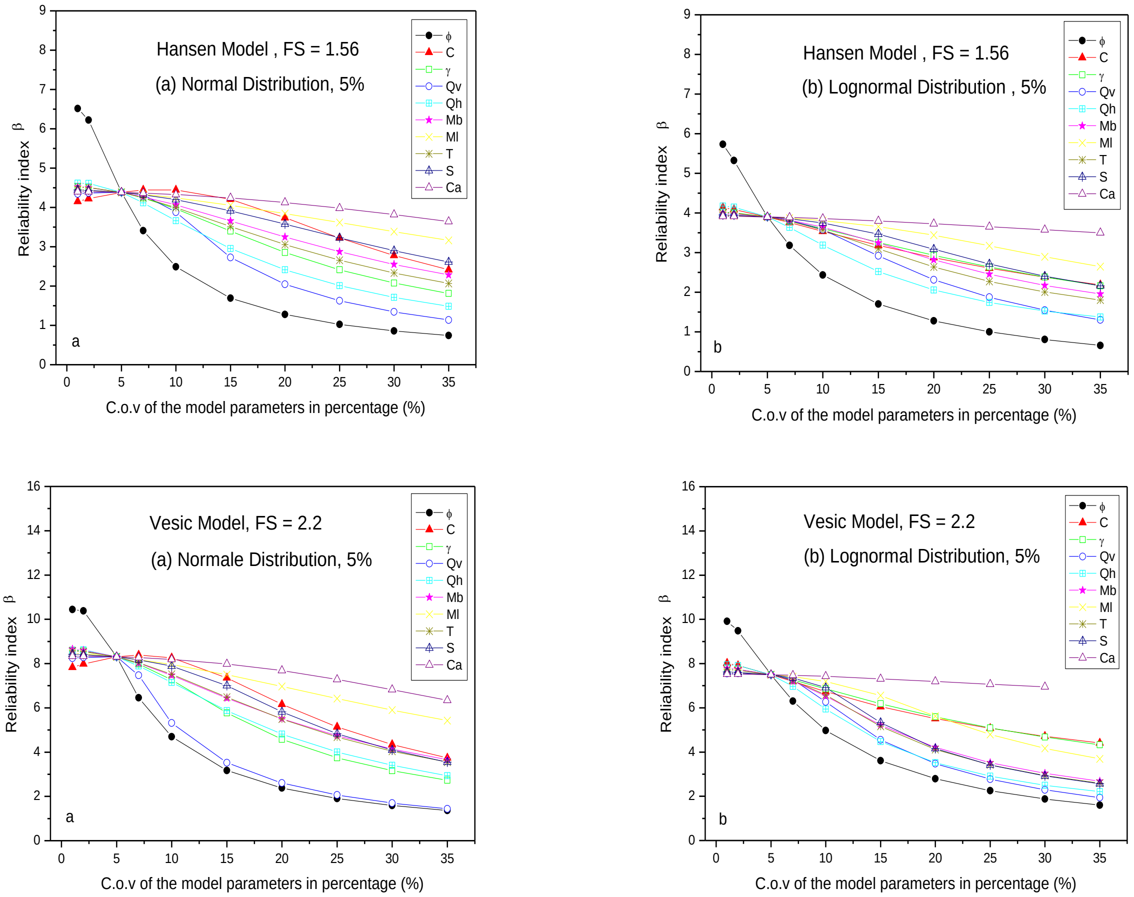

Figure 2 Variation of the reliability index with the coefficient of variation (C.o.v) of the capacity parameters of shallow foundations. (a) Normal distribution, (b) Lognormal distribution.

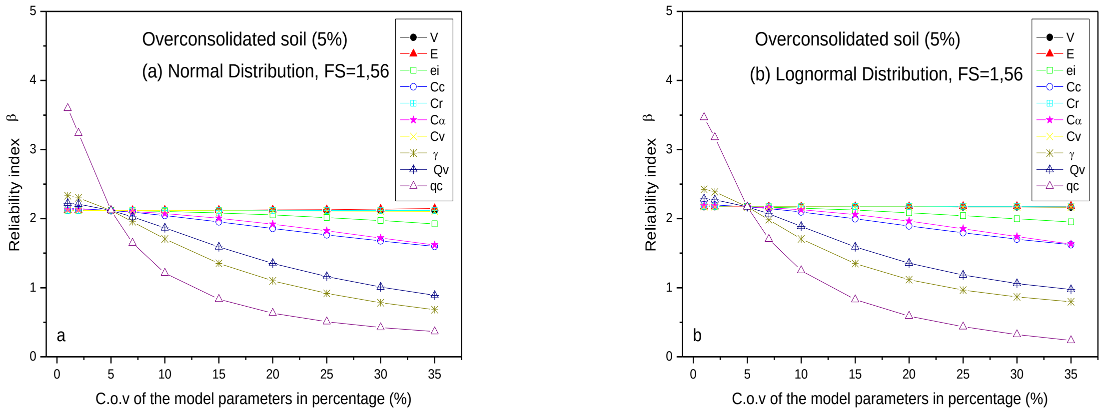

Figure 3 Variation of reliability index with coefficient of variation (C.o.v) of settlement parameters - overconsolidated soil. (a) Normal distribution, (b) Lognormal distribution

5.3.1 Variable reduction methodology

Studying the sensitivity of random variables in any limit state is crucial. This is justified by the fact that changing the coefficient of variation of a given random variable can lead to significant modifications in the reliability index values. When a random variable is replaced with a deterministic value, the omission coefficient is used to assess the effect of this change on the engineer's judgment. The omission coefficient for a random variable is expressed as follows:

$$\Omega_i = |\beta - \beta_i| \quad (9)$$

β_i corresponds to the reliability index calculation where all variables are random, except for x_i which is deterministic. β is the reliability index when all variables are probabilistic.

The guidelines provided by the U.S. Army Corps of Engineers [37], given in Table 2, can be used to determine whether a variable can be replaced with a deterministic value. In fact, to transition from a lower safety level to a more satisfactory safety level, an increase of 0.5 in the reliability index is required if the reliability index is less than 3. If the calculated reliability index is greater than 3, an increase of 1 is required to transition from an acceptable safety level to a higher safety level.

Table 2 Relationship between reliability index, failure probability, and safety level [37].

Reliability index β	Reliability	Failure probability	Level of performance
5	0,999 999 7	0,000 000 3	High
4	0,933 968 3	0,000 031 7	Good
3	0,998 650 0	0,001 350 0	Above average
2,5	0,993 790 3	0,006 209 7	Below average
2,0	0,977 249 9	0,022 750 1	Poor
1,5	0,933 192 8	0,066 807 2	Unsatisfactory
1,0	0,841 344 7	0,158 655 3	Hazardous

In our approach, calculating the omission coefficient will allow us to verify if the reduction of one or more variables will compromise the engineer's judgment. A value of the omission coefficient less than 0.5 for a low safety level or less than 1 for a high safety level indicates that the probabilization of that variable has no effect on the engineer's decision. On the other hand, a result greater than 0.5 for a low safety level or greater than 1 for a high safety level highlights an overestimated or underestimated safety level. In both situations, the variable should be included as a random variable. Since reliability studies can involve heavy calculations, this verification examines the possibility of reducing the number of variables in the problem and ensuring simplified calculations for practical applications.

5.3.2 Variable reduction for bearing capacity of a sloping shallow foundation

To assess the sensitivity of the random variables in the bearing capacity of a sloping shallow foundation subjected to an eccentric and inclined load near a slope, two sets of calculations were performed. Table 3 summarizes the mean values and covariances of the parameters considered in the sensitivity analysis.

Table 3 Coefficients of variation and mean values considered in the sensitivity study of the variables for the bearing capacity of a sloping shallow foundation.

Parameters		Case 1		Case 2	
		Mean value	C.o.v (%)	Mean value	C.o.v (%)
Friction Angle	$\varnothing$	25°	2-50	30°	2-50
Cohesion	C	15 kN/m^2	2-50	18 kN/m^2	2-50
Unit weight	γ	18 kN/m^3	5	18 kN/m^3	10
Vertical load	Q_v	400 kN	10	400 kN	15
Horizontal load	Q_h	120 kN	10	120 kN	15
Moment parallel to B	M_B	100 $kN.m$	10	100 $kN.m$	10
Moment parallel to L	M_L	100 $kN.m$	10	100 $kN.m$	10
Slope inclination	S	10°	2	10°	2
Base inclination	T	25°	2	25°	2
Base adhesion to soil	Ca	10 kN/m^2	10	10 kN/m^2	10

For the two cases considered in Table 3, the covariances of the Mohr-Coulomb resistance parameters are assumed to vary jointly from 2% to 50%, while the covariances of the other random variables are kept constant. Two formulations of the SORM method, namely SORMCai [18] and SORMZhao [31], were used. Using the limit state function given by Equation (8) and the Monte Carlo simulation technique, the reliability index is calculated in two different ways: first, by considering the statistics of the limit state function (MCSS), and second, by counting failures - $g(x) < 0$ in different simulation cycles (MCSC). This parametric study led to the following conclusions (Figures 4 and 5):

• For both cases considered, the FORM, SORM, and CMCSC methods provide very similar results, even when the limit state function is nonlinear or when the covariance of a parameter exceeds 15%. This result is not in line with the findings of the Committee on Reliability Methods for Geotechnical Engineering [20], which states that for a covariance exceeding 15% of a parameter, the reliability methods mentioned above yield different results. This conclusion is closely related to the nature of the limit state function used. The results obtained by the FORM, SORM, and MCSC methods, which do not impose any restriction on the variability of the parameters, show that in certain cases, a high coefficient of variation can be assigned to the parameters in a reliability analysis with FORM/SORM without significant error.

• The second-order reliability method (SORM) uses second-degree terms of the Taylor series. The computational difficulty is greater, and the improvement in accuracy is not always commensurate with the additional computational effort. The numerical examples presented demonstrate that the precision provided by the first-order reliability method (FORM) is sufficient for practical applications.

• The Monte Carlo simulation technique has the advantage of conceptual simplicity, but it requires a large number of samples - 500,000 samples in our case - of the limit state function to achieve satisfactory accuracy. The numerical examples show that the reliability indices obtained from the statistics of the limit state function and the failure counting methods are quite different. In many situations, the MCSC method is more accurate than the MCSS method.

• In the case of a nonlinear limit state function, which is common in geotechnical problems, the FORM method produces significant errors.

• For low covariance values of the parameters, the FORM and MCSS methods yield very similar results.

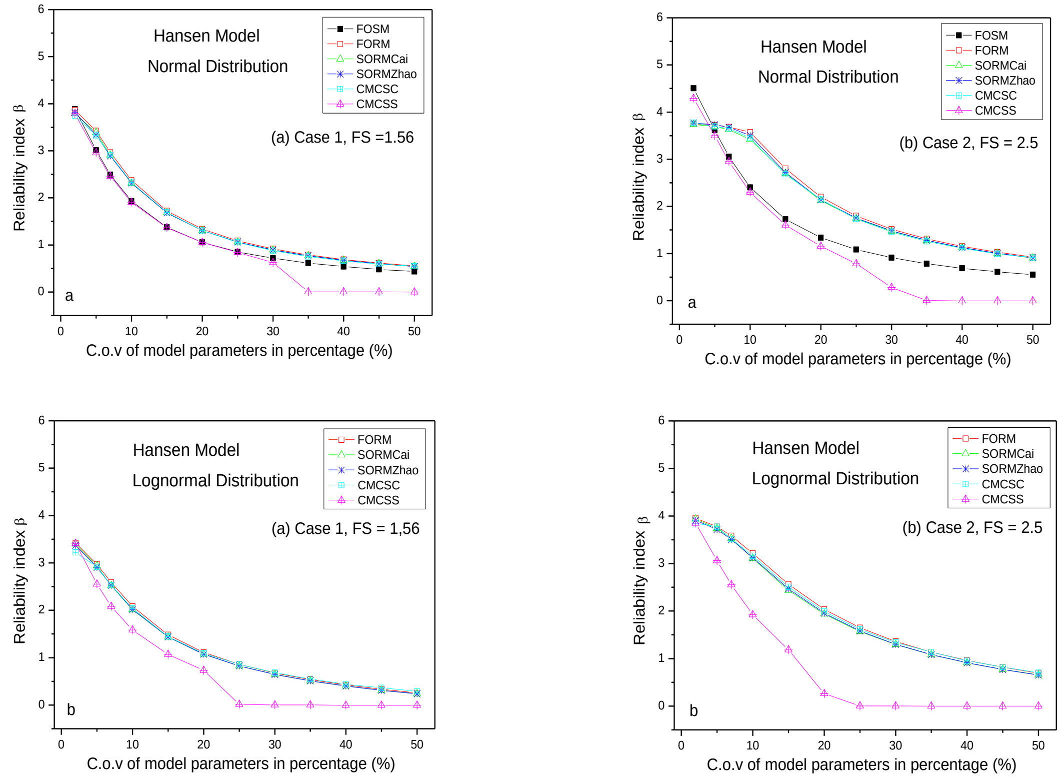

Figure 4 Comparison of reliability indices calculated with FOSM, FORM, SORM, and MCS - Hansen model. (a) Case 1, (b) Case 2.

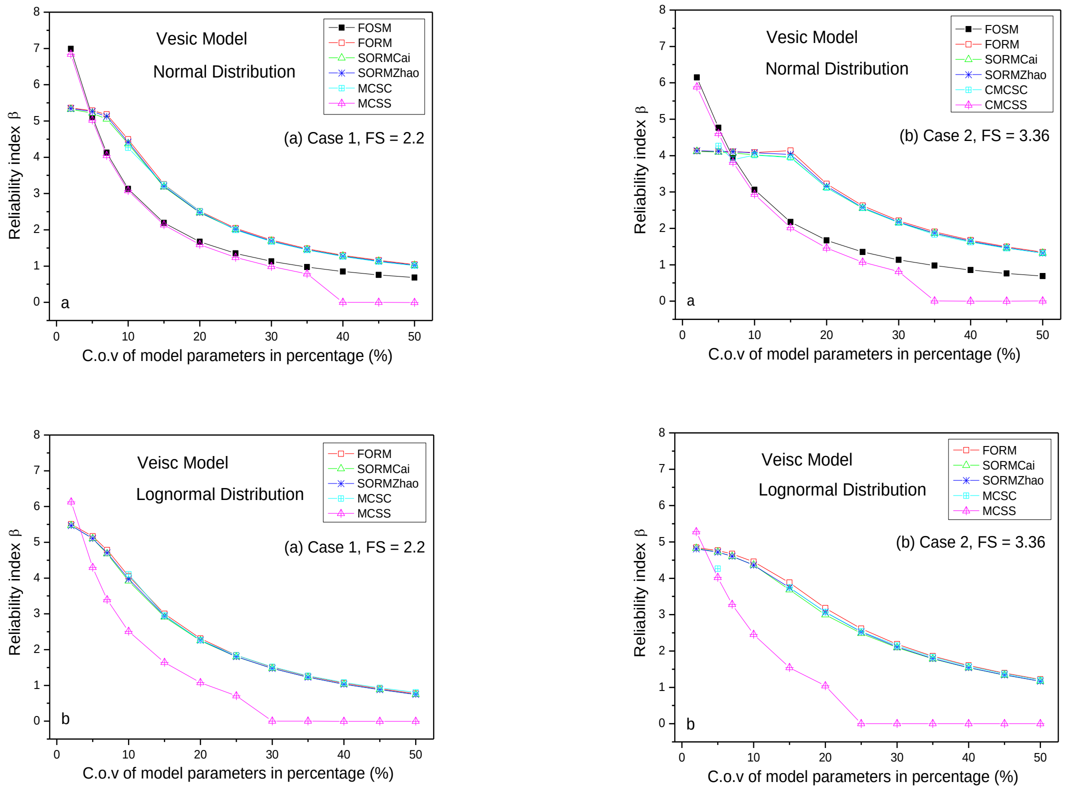

Figure 5 Comparison of reliability indices calculated with FOSM, FORM, SORM, and MCS - Vesic model. (a) Case 1, (b) Case 2.

To reduce the size of the probabilistic analysis problem for the bearing capacity of shallow foundations, the variations of the calculated reliability indices considering each of the ten variables as random are compared to the variations of the reliability indices calculated using a limited but chosen number of random variables (Figure 6 and 7). A good agreement can be observed between the variations of the reliability indices calculated with ten random variables and the variations with a reduced number of random variables.

It is observed that the reliability index for this mode of failure of shallow foundations can be accurately estimated using only the Mohr-Coulomb resistance parameters (c, φ) if their covariances exceed 15%. For lower covariances, uncertainties in the vertical and horizontal loads must also be considered.

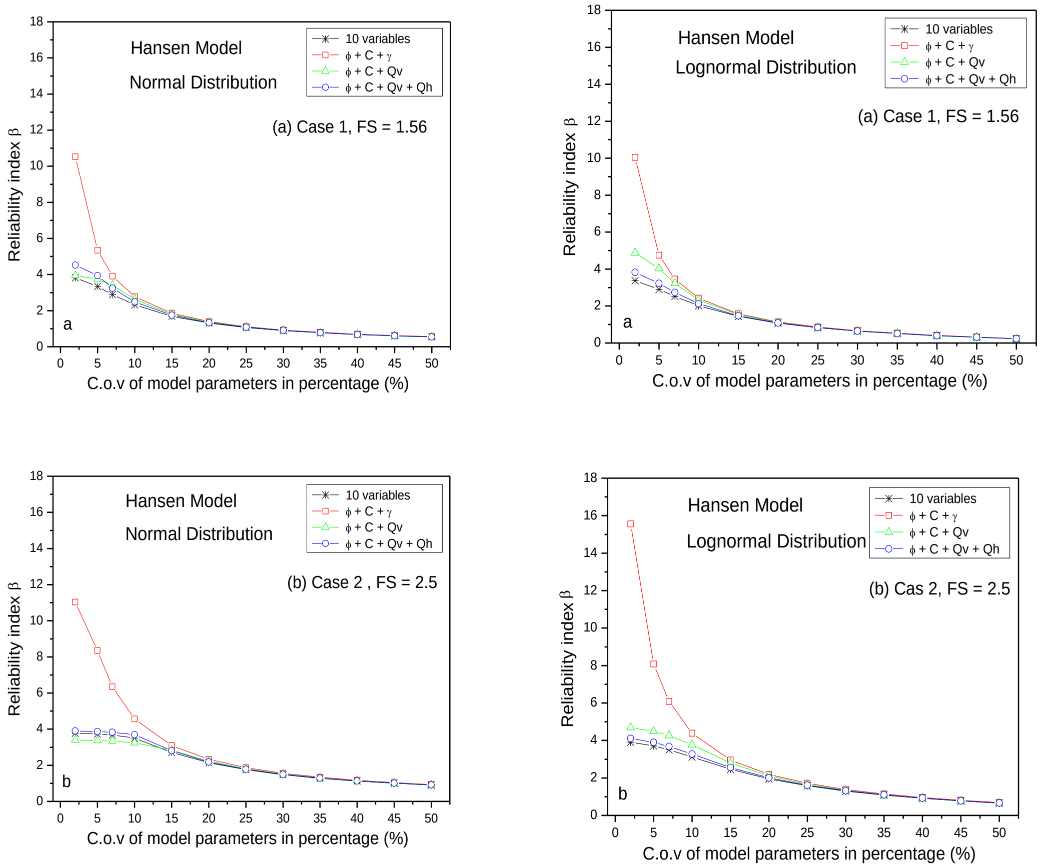

Figure 6 Reduction of random parameters - Hansen model. (a) Case 1, (b) Case 2.

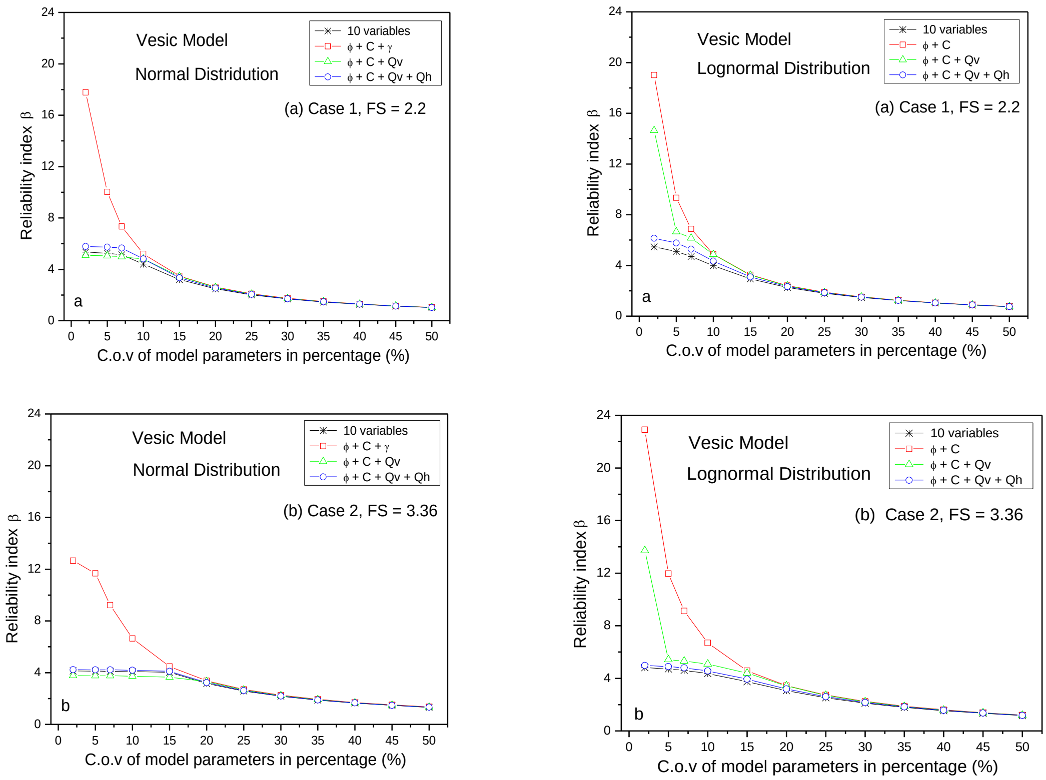

Figure 7 Reduction of random parameters - Vesic model. (a) Case 1, (b) Case 2.

The results of the sensitivity analysis on the variables predicting the bearing capacity are presented in Table 4. The analysis shows that to achieve a reduction in the number of random variables, it is necessary to consider the angle of friction, the cohesion term, the vertical load, and the horizontal load as random variables. These parameters are considered crucial in reliability assessment due to their significant impact, with omission factors less than 0.5. On the other hand, the remaining variables can be treated as deterministic, as their influence on reliability is relatively smaller.

Table 4 Reduction of variables for the bearing capacity of a shallow foundation.

Random combination of parameters used	Distribution	Influence of reduction		Appropriate random parameter combination
		Case 1	Case 2	
		Hansen Model		
$Ø + C + \gamma$	Normal	▲	▲	*Friction angle* + *Cohesion* + *Vertical load* + *Horizontal load*
	Lognormal	▲	▲	
$Ø + C + Q_v$	Normal	●	●	
	Lognormal	▲	●	
$Ø + C + Q_v + Q_h$	Normal	●	●	
	Lognormal	●	●	
		Vesic Model		
$Ø + C + \gamma$	Normal	▲	▲	*Friction angle* + *Cohesion* + *Vertical load* + *Horizontal load*
	Lognormal	▲	▲	
$Ø + C + Q_v$	Normal	●	●	
	Lognormal	▲	▲	
$Ø + C + Q_v + Q_h$	Normal	●	●	
	Lognormal	●	●	
▲ : Risk	**● : Without Risk**			

5.3.3 Variable reduction for settlement of a shallow foundation

The number of parameters involved in the probabilistic analysis of settlement in a shallow foundation on overconsolidated soil is quite large (10 parameters). However, it is possible to identify among these parameters those that have a negligible influence in the reliability analysis. In order to assess the sensitivity of the random variables in settlement, two sets of calculations were performed. Additionally, two other calculations were conducted using a different mean value for the allowable settlement compared to the first case. Table 5 summarizes the mean values and coefficients of variation used in each case for the sensitivity analysis of the variables.

Table 5 Coefficients of variation and mean values considered in the sensitivity analysis of the variables for settlement of a shallow foundation

Parameters		Case 1		Case 2	
		Mean value	**C.o.v (%)**	**Mean value**	**C.o.v (%)**
Vertical load	Q_v	400 *kN*	10	400 *kN*	10
Dry unit weight	γ	18 kN/m^3	5	18 kN/m^3	5
Elastic modulus	E	40 *MPA*	10	40 *MPA*	10
Preconsolidation stress	q_c	100 KN/m^2	2 - 50	100 kN/m^2	2 - 50
Consolidation coefficient	C_v	2,5 ×10^{-8} m^2/s	25	2,5 ×10^{-8} m^2/s	50
Poisson's ratio	v	0 ,3	10	0,3	20
Void ratio	e_i	0,8	10	0,8	20
Compression index	C_c	0,1	10	0,1	20
Recompression index	C_r	0,02	10	0,02	20
Creep index	C_α	0,004	10	0,004	20
Allowable settlement	S_{admi}	5 *cm*	0	6 *cm*	0

The reduction of variables for settlement prediction of a shallow foundation is presented in Table 6. From this table, it can be observed that a valid reduction of random variables for all cases can only be achieved if the preconsolidation pressure, vertical load, dry unit weight of soil, and creep index are considered as random variables. These parameters are deemed essential in reliability evaluation due to omission factors less than 0.5. The other variables can be considered as deterministic.

Table 6 Reduction of variables for settlement of a shallow foundation

Random combination of parameters used	Distribution	Influence of the reduction		Appropriate random combination of parameters
		Case 1	Case 2	
		Overconsolidated soil		
$q_c + Q_v$	Normal	▲	▲	*Preconsolidation stress* + *Vertical load* + *Soil weight* + *Creep index*
	Lognormal	▲	▲	
$q_c + Q_v + \gamma$	Normal	●	▲	
	Lognormal	●	▲	
$q_c + Q_v + \gamma + C_\alpha$	Normal	●	●	
	Lognormal	●	●	
▲ : Risk	**● : Without Risk**			

5.4 Reliability of deep foundations

The assessment of the reliability of a deep foundation involves verifying its suitability for service. This process is of great importance as it determines whether the foundation is inacceptable conditions of safety and functionality. Geotechnical investigation is the first step in this process, as it allows for the best possible consideration of site-specific data.

However, the available information may remain incomplete or even inaccurate due to the limited depth of investigation. Moreover, the structure is located in deep layers where the actions it experiences are not always precisely known. This leads to a large number of uncertainties that need to be understood and considered in the evaluation process. For this reason, the analysis of the performance of this type of structure must be developed based on probabilistic foundations, using the principles and methods of reliability theory.

For the reliability analysis of deep foundations, the example of an isolated pile under a vertical compressive load is considered. Table 7 provides an overview of the variables required in the prediction models for bearing capacity and settlement, as defined by Equation 6 and 7.

Table 7 Parameters and mean values considered in the probabilistic analysis of an isolated pile.

Parameter	Symbol	Variable	Mean value
Parameters common to prediction models			
Vertical load	Q_v	Random	400 *kN*
Pile diameter	D	Deterministic	0,9 *m*
Height of sand clay	L_a	Deterministic	6 *m*
Height of sand layer	Ls	Deterministic	10 *m*
Pile weight	$\gamma_{béton}$	Deterministic	25 kN/m^3
Specific parameters for bearing capacity prediction models			
Friction angle	$\bar{\phi}_s$	Random	30°
Undrained cohesion	C_u	Random	15 *kN*
Dry unit weight of sand	γ_s	Random	10 kN/m^3
Dry unit weight of clay	γ_a	Random	12 kN/m^3
Specific parameters for settlement prediction model			
Elastic modulus of the sand layer	E_s	Random	30 *MPA*
Elastic modulus of the clay layer	E_a	Random	15 *MPA*
Elastic modulus of the pile	E_p	Random	30 *MPA*
Poisson's ratio	ν	Random	0,35
Allowable settlement	$Spadmi$	Deterministic	4 *cm*

5.4.1 Probabilistic analysis of the bearing capacity of deep foundations

While many geotechnical calculation models are "simple," reasonable predictions of the complex behavior of soil-structure interaction can still be achieved through empirical calibration. Due to the unique nature of geotechnics where empiricism is unavoidable, model uncertainties can be significant. A simple assessment of the bias (or mean) of a model is crucial for reliability analysis. If the model is conservative, it is evident that calculated failure probabilities will be biased, as situations belonging to the safe domain will be incorrectly assigned to the failure domain due to built-in conservatism.

For the analysis of bearing capacity, two prediction models for tip resistance are considered: the Vesic model and the Generalized Bearing Capacity Equation [37].

In the probabilistic analysis of the bearing capacity of deep foundations, the bearing capacity $R(x)$ is given by Equation (6), and the load $L(x)$ represents the vertical load divided by the foundation area. All random variables are considered independent.

The parametric study allowed us to identify the variables most sensitive to variations in their standard deviation (or coefficient of variation). This study showed that the friction angle is the most dominant variable, and the order of influence of the other variables depends on the choice of distribution law (Figure 8).

5.4.2 Probabilistic analysis of settlement of deep foundations

In the probabilistic analysis of settlement of deep foundations, the settlement $R(x)$ is given by Equation (7), and the allowable settlement $L(x)$ is set to a value of 4 cm. All random variables are considered independent.

The parametric study allowed us to identify the variables most sensitive to variations in their standard deviation (Figure 9). The following observations can be made:

• The reliability index is highly dependent on the coefficient of variation of the vertical load, the modulus of elasticity of the pile, and has little dependence on the coefficient of Poisson.

• The limit state function is highly sensitive to the choice of distribution law used to model the random variables - the order of influence of the parameters and the values of the reliability index depend on it.

5.4.3 Variable reduction

This section is adopted to study the sensitivity of variables and verify if it is possible to identify, among these variables, those that have a negligible influence in the probabilistic analysis. Table 8 presents the coefficients of variation and biases (means) that are used in the sensitivity analysis of random parameters.

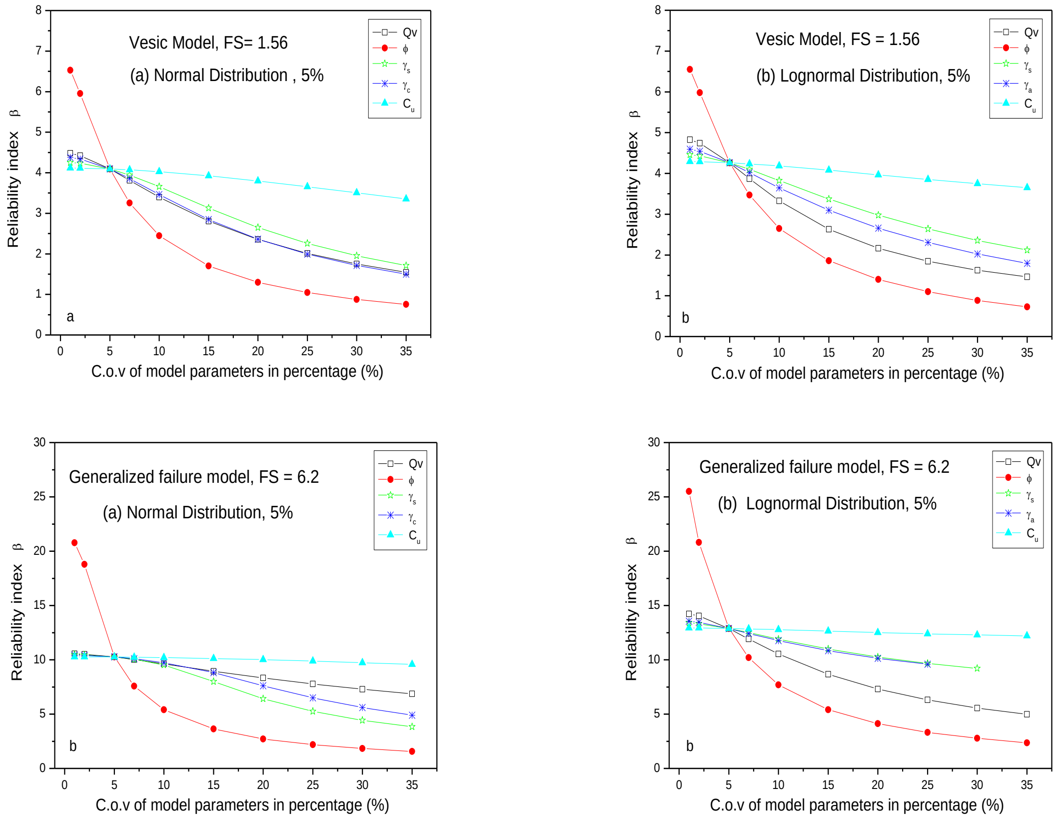

Figure 8 Variation of the reliability index with the coefficient of variation (C.o.v) of the parameters for the bearing capacity of an isolated pile. (a) Normal distribution, (b) Lognormal distribution.

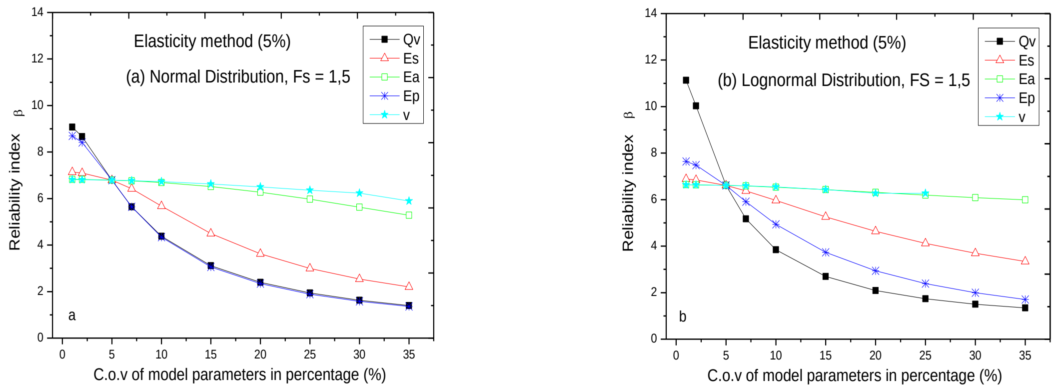

Figure 9 Variation of reliability index with coefficient of variation of settlement parameters for an isolated pile. (a) Normal distribution, (b) Lognormal distribution.

Table 8 Coefficients of variation and mean values considered in the sensitivity analysis of the variables for the bearing capacity and settlement of an isolated pile.

Parameters	Notation	Means		Cov (%)	
		Cas 1	Cas 2	Cas 1	Cas 2
Common parameters in prediction models					
Vertical load	Q_v	400 *kN*	360 *kN*	10	15
Please refer to Table 7 for the geometric variables.					
Specific parameters for bearing capacity prediction models					
Friction Angle	$\bar{\phi}_s$	30°	32°	2 - 50	2 - 50
Undrained cohesion	C_u	15 kN/m^2	15 kN/m^2	2 - 50	2 - 50
Dry unit weight of sand	γ_s	10 kN/m^3	10 kN/m^3	5	10
Dry unit weight of clay	γ_a	12 kN/m^3	12 kN/m^3	5	10
Specific parameters for settlement prediction model					
Elastic modulus of the sand layer	E_s	30 *MPA*	30 *MPA*	2 - 50	2 - 50
Elastic modulus of the clay layer	E_a	15 *MPA*	15 *MPA*	2 - 50	2 - 50
Elastic modulus of the pile	E_p	30 *MPA*	35 *MPA*	15	10
Poisson's ratio	v	0,35	0,35	10	20
Allowable settlement	*Spadmi*	4 *cm*	4 *cm*	0	0

Figures 10 to 12 present the result of the variations of reliability indices using different models for predicting the bearing capacity and settlement of a pile, different probability density functions, different means and coefficients of variation, and different proposed combinations to assess the sensitivity of the random variables. The following observations can be made:

• In all models for predicting the bearing capacity and settlement of a pile, the vertical load appears to have a pronounced influence on the behavior of the reliability index. The gap between the reference curves (5 random variables) and the curves that consider the vertical load as a deterministic variable is evident.

• After analyzing the four proposed combinations for reducing the variables involved in predicting the bearing capacity of a pile using the Hansen model and the general failure model (Figure 10 and 11), it is observed that for low coefficients of variation of the unit weights of the soil layers, the standard deviation seems to have a pronounced influence on the behavior of the reliability index. However, the unit weights of the two considered layers can be treated as deterministic. The rationale for this decision lies in the significant increase in the reliability index, which is generally greater than 5 for low coefficients of variation, and thus the engineer's judgment will not be affected by any increase resulting from the reduction of these two variables.

• The results obtained in the sensitivity analysis of the settlement parameters reaffirmed that the vertical load and the pile's modulus of elasticity are the two random variables that govern reliability for this failure mode (Figure 12).

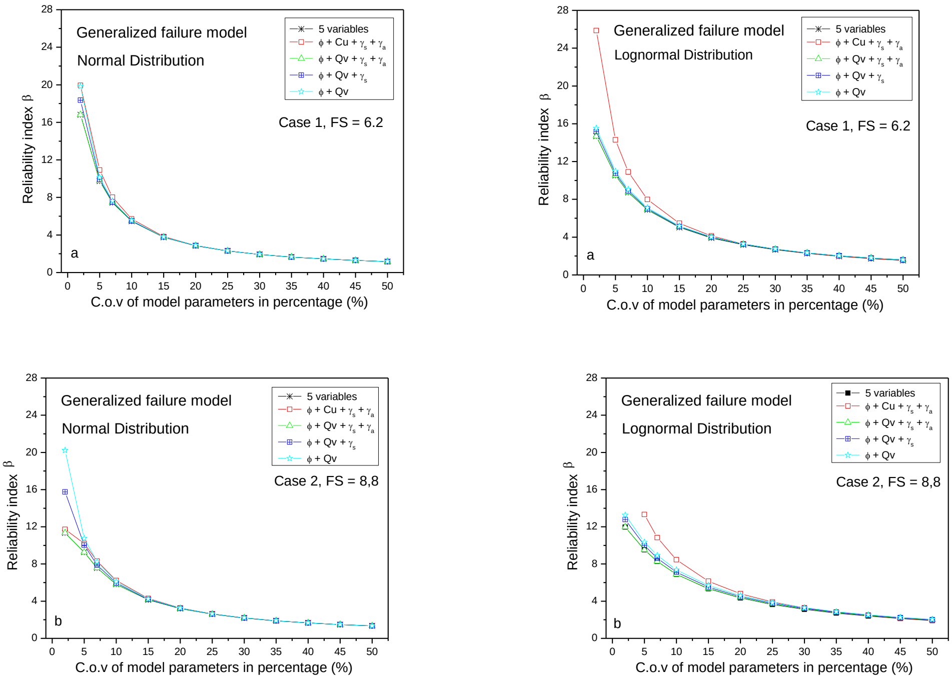

Figure 10 Reduction of random parameters - generalized failure model.

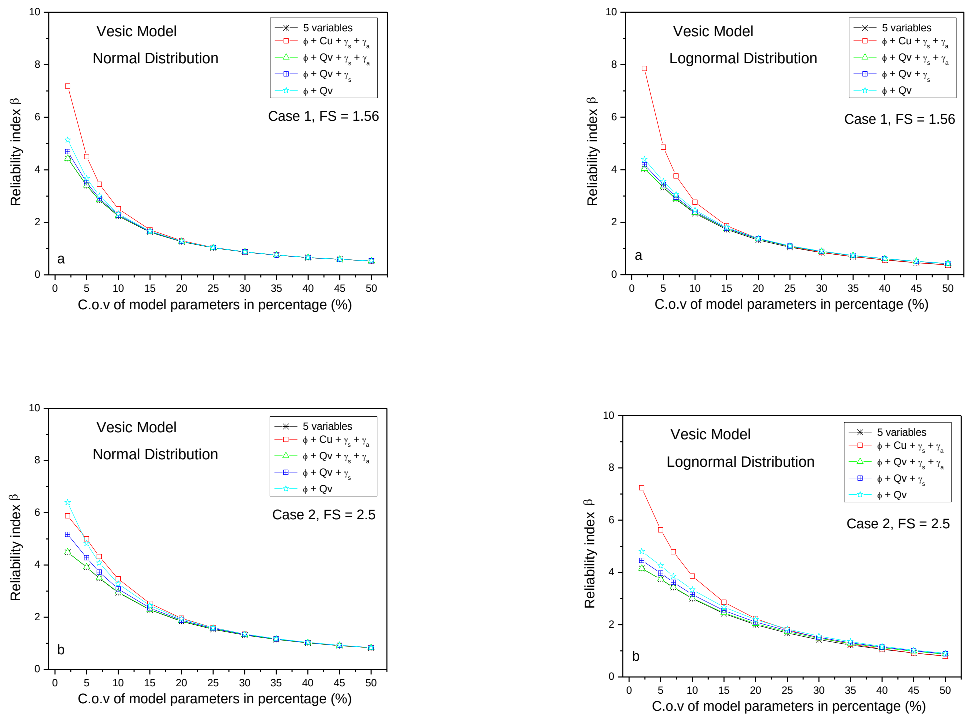

Figure 11 Reduction of random parameters - Vesic model.

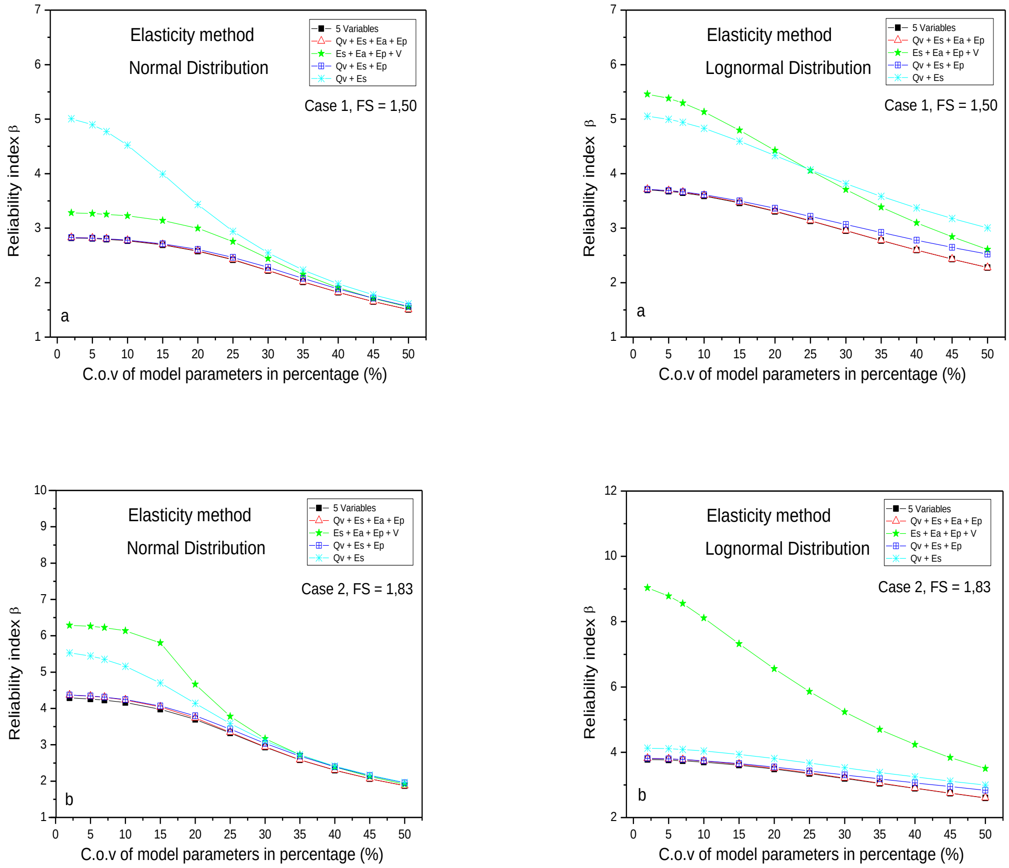

Figure 12 Reduction of random parameters - elasticity method.

Table 9 summarizes the results obtained from the sensitivity analysis of the variables involved in the probabilistic analysis of an isolated pile. It can be observed from this table that most variables can be treated as deterministic. This is explained by the fact that, for the selected performance function (ultimate limit state of bearing capacity, ultimate limit state of settlement), the dispersion of certain variables has little influence on the calculations. On the contrary, the deterministic value to be considered in the calculations will have a considerable influence for certain variables.

Table 9 Reduction of variables for the bearing capacity and settlement of an isolated pile

Random combination of parameters used	Distribution	Influence of reduction		Appropriate random parameter combination
		Cas 1	**Cas 2**	
		Prediction models for bearing capacity		
		Vesic Model		
$\bar{\phi}_s + C_u + \gamma_s + \gamma_a$	Normal	▲	▲	*Friction angle + Vertical load + Dry unit weight of soil(γ_s ou γ_a)*
	Lognormal	▲	▲	
$\bar{\phi}_s + Q_v + \gamma_s + \gamma_a$	Normal	●	●	
	Lognormal	●	●	
$\bar{\phi}_s + Q_v + \gamma_s$	Normal	●	●	
	Lognormal	●	●	
$\bar{\phi}_s + Q_v$	Normal	▲	▲	
	Lognormal	●	●	
		General shear failure model		
$\bar{\phi}_s + C_u + \gamma_s + \gamma_a$	Normal	●	●	*Friction angle + Vertical load*
	Lognormal	●	●	
$\bar{\phi}_s + Q_v + \gamma_s + \gamma_a$	Normal	●	●	
	Lognormal	●	●	
$\bar{\phi}_s + Qv + \gamma_s$	Normal	●	●	
	Lognormal	●	●	
$\bar{\phi}_s + Q_v$	Normal	●	●	
	Lognormal	●	●	

Table 9 (continued) Reduction of variables for the bearing capacity and settlement of an isolated pile

Random combination of parameters used	Distribution	Influence of reduction Cas 1	Cas 2	Appropriate random parameter combination
		Prediction models for bearing capacity		
		Hansen Model		
$\bar{\phi}_s + C_u + \gamma_s + \gamma_a$	Normal	●	●	*Friction angle + Vertical load*
	Lognormal	●	●	
$\bar{\phi}_s + Q_v + \gamma_s + \gamma_a$	Normal	●	●	
	Lognormal	●	●	
$\bar{\phi}_s + Q_v + \gamma_s$	Normal	●	●	
	Lognormal	●	●	
$\bar{\phi}_s + Q_v$	Normal	●	●	
	Lognormal	●	●	
		Settlement prediction model		
		Elasticity Method		
$Qv + E_s + E_a + E_p$	Normal	●	●	*Vertical load + Soil elasticity modulus (E_s ou E_a) + Pile's elasticity modulus*
	Lognormal	●	●	
$E_s + E_a + E_p + v$	Normal	▲	▲	
	Lognormal	▲	▲	
$Q_v + E_s + E_p$	Normal	●	●	
	Lognormal	●	●	
$Q_v + E_s$	Normal	▲	▲	
	Lognormal	▲	▲	

▲ : Risk ● : Without Risk

6. Conclusion

This parametric study of reliability analysis for shallow and deep foundations has allowed us to assess the effects of design parameter uncertainties on the reliability index and draw the following conclusions:

• The traditional practice of foundation design using safety factors can be misleading. Foundations designed with large safety factors are not exempt from the risk of failure, especially if there are significant uncertainties in the design parameters.

• Reliability analysis provides a logical framework for incorporating uncertainties into the design process and enhances the risk perception in traditional design methods.

• For all considered limit state functions in probabilistic analysis of shallow foundations, the reliability index depends strongly on the internal friction angle of the soil, the vertical load, and the horizontal load, and is less dependent on the cohesion between the foundation base and the soil. This independence is observed especially when the parameters are modeled by a lognormal distribution. Sensitivity analysis of the parameters shows that the reliability index of shallow foundations, in the general case considered, can be accurately estimated by considering only the Mohr-Coulomb resistance parameters (c, φ), the vertical load, and the horizontal load as random variables.

• The parametric analysis of settlement in shallow foundations shows that the reliability index is strongly influenced by the preconsolidation pressure, unit weight, and vertical load, while being less dependent on other variables.

• In all models for predicting the bearing capacity and settlement of deep foundations, the vertical load appears to have a pronounced influence on the reliability index. The gap between the reference curves and the curves considering the vertical load as a deterministic variable is evident.

• Sensitivity analysis has shown that a reduction of random parameters is still possible even when the number of random variables involved in the probabilistic analysis is relatively small (5 variables). The parametric study has allowed us to identify the variables most sensitive to variations in their standard deviation (or coefficient of variation). This study has shown that the internal friction angle of the soil is the most dominant variable, and the order of influence of other variables depends on the choice of the distribution law.

• The second-order reliability method (SORM) uses second-degree terms of the Taylor series. The calculation difficulty is greater, but the improvement in precision is not always commensurate with the additional computational effort.

• The first-order-second-moment method (FOSM) leads to significant errors that can compromise the engineer's judgment and jeopardize the safety of the structure. This method should be abandoned in favor of more robust methods.

• The numerical examples presented demonstrate that the precision provided by the first-order reliability method (FORM) is sufficient for practical geotechnical applications.

• Monte Carlo simulation has the advantage of conceptual simplicity but requires a large number of simulations. With advancements in computer technology, the time required to perform the necessary simulations may no longer be a problem.

• The probabilistic distribution type of parameters has a significant influence on the calculated reliability index. Table 2, which served as our guide, appears to be incomplete as it does not specify the type of probability distribution used for the considered parameters in the probabilistic analysis.

7. Acknowledgements

This work is dedicated to the memory of Smain Belkacemi, our mentor, supervisor and teacher who inspired us and many other polytechnical engineers (1957–2020).

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