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An integrated model of preventive maintenance and safety stock for a used production unit under leasing

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Abstract

In this paper, we propose an integrated model of preventive maintenance (PM) and safety stock for an unreliable production unit, subject to random failures, acquired by leasing. Maintenance activities, both preventive and corrective, are performed by the lessor under the lease contract, incurring maintenance costs to the lessor. The leased production unit is used by the lessee to produce parts for the subsequent production stage. To prevent the subsequent production stage from being interrupted during breakdowns and PM, the lessee must produce a certain amount of safety stock, incurring inventory costs to the lessee. Unlike other models in the literature that implicitly assume that the leased production unit is a new item, in this paper, we investigate the case where the leased production unit is one that has already been used. In this case, the lessor can perform reconditioning, prior to leasing, on the production unit being leased incurring a reconditioning cost to the lessor. A mathematical model is developed to determine the optimal PM cycle duration, reconditioning level, and safety stock size to minimise the total expected cost to the lessor and the lessee combined over the leasing horizon. The numerical results show that calculating the optimal values of all three decision variables by considering the lessor and the lessee simultaneously yields a more satisfactory result, in terms of the total expected cost over the leasing horizon, than when the parties are considered individually. Some managerial insights useful to both the lessor and the lessee are provided in the sensitivity analysis section.

Keywords: Preventive maintenance, Safety stock, Leased equipment, Used equipment, Machine leasing

1. Introduction

Reliability is an important part of production planning and inventory control. This issue is important because most production systems are inherently unreliable and occasionally fail during production. Frequent disruptions due to failures can result in high corrective maintenance (CM) and shortage costs to a company. Traditionally, a firm can employ the following two strategies to reduce these costs:

1) Producing safety stock to cope with shortages that might occur during breakdowns, or

2) Executing preventive maintenance (PM) to improve the reliability of the production system and reduce the possibility of machine failure during production [1].

Originally, these two strategies were independently researched in the literature. The integrated model, considering both options for dealing with breakdowns, was first proposed by Cheung and Hausman [2]. The main objective of the integrated model was to simultaneously calculate the optimal PM cycle duration and safety stock size to minimise the total maintenance and inventory costs. This type of problem was later known in the literature as the joint determination of PM and safety stock problem. The work of Cheung and Hausman [2] was later modified by Dohi et al. [3]. A similar problem applied to an assembly line was proposed by Chelbi and Ait-Kadi [4]. That research was extended by Chelbi and Rezg [5], who added the availability level requirement to the problem. A practical model that considered failures that might occur during the accumulation of safety stock was proposed by Gharbi et al. [6]. Murino et al. [7] developed a model of this problem using condition-based maintenance (CBM) as a strategy for performing PM. The impact of maintenance, process deterioration, and safety stock on the economic production quantity (EPQ) model was studied by Chakraborty and Giri [8]. A model that considered both safety stock and spare parts inventory was proposed by Gan et al. [9]. Cheng et al. [10] applied the concept of imperfect maintenance to the problem. Zhou and Liu [11] incorporated the impact of quality loss due to system deterioration into the problem. In their work, based on the theory of constraints (TOC), different machines were maintained differently according to different PM policies. Nahas [12] proposed a joint PM and safety stock allocation model for a system consisting of n machines and $n-1$ stocking points. A model that considered a quality inspection policy was presented by Lopes [13]. A model that allowed the manufacturer to purchase some finished goods from an external supplier in addition to producing safety stock was proposed by Deiranlou et al. [14].

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One of the assumptions implicitly made in the above studies was that production facilities were purchased and maintained internally. In this case, the firm was responsible for all costs related to maintenance actions. However, in recent years, largely due to financial crises in many industrialized countries, there has been a trend towards leasing of production machinery in the industry, especially for young companies. Namely, instead of buying production machinery at a high cost, young companies prefer to rent it [15]. This trend has introduced many new issues to businesses, one of which is the issue of maintenance of leased machinery. Namely, in the case of leasing, the lessor is responsible for maintenance actions and all related costs without additional expenses incurred to the lessee. Terms of maintenance services and penalties that might occur to the lessor are fully specified in the lease contract. In other words, maintenance has become an issue for the lessor, but not for the lessee [16].

The first attempt at applying the concept of leased machinery maintenance to production/inventory systems was found in the work of Hajej et al. [17]. The authors studied a joint production/inventory and maintenance problem, assuming that the production machine was acquired by leasing with a warranty period. A mathematical model was developed to determine the optimal production planning, maintenance policy, and warranty period, minimising the total expected cost of both the lessor and the lessee over the leasing horizon. That research was later extended by Hajej et al. [18], who considered the possibility to extend the warranty period at an additional cost. Askri et al. [19] applied concepts similar to those of Hajej et al. [17] to a production system consisting of multiple leased parallel machines. Based on their previous work [18], Hajej et al. [20] proposed an analytical model to analyse the impact of production rates and warranty periods on the optimal maintenance policy. Askri et al. [21] developed a joint production/inventory and maintenance model for parallel leased machines. To reduce the impact of interruptions due to PM, all leased machines were maintained simultaneously by a group PM policy. An integrated model of capacity design, production planning, and maintenance policy for leased machines was proposed by Hajej et al. [22]. In their work, the optimal number of leased machines was determined, as well as the corresponding production planning and maintenance policies of those machines. Concerning the joint PM and safety stock problem, Niyamosoth and Pathumnakul [23] studied an unreliable production/inventory system similar to that proposed by Chelbi and Ait-Kadi [4], assuming that the production unit was acquired by leasing. A mathematical model was developed to simultaneously determine the optimal PM cycle duration and safety stock size to minimise the total expected cost of both the lessor and the lessee over the leasing horizon.

One of the assumptions implicitly made in the above studies was that the leased production machines were new items. However, as suggested by Pongpech et al. [24], in practice, leased machinery can already be used. Given this research gap, in this paper, we broaden the work of Niyamosoth and Pathumnakul [23] by assuming that the leased production unit has already been used. In this case, the reconditioning level for the used production unit and the related cost are added to the proposed model. The aim of this research is to determine the optimal PM cycle duration, reconditioning level, and safety stock size to minimise the total expected cost to the lessor and the lessee combined over the leasing horizon.

This paper is structured as follows. Section 2 describes the concept of the proposed model. Section 3 is dedicated to the model formulation. In section 4, an algorithm to solve the proposed model is developed. In section 5, a numerical example and a sensitivity analysis on some model parameters are presented. Finally, the conclusion, including suggestions for future work, is given in section 6.

2. Concept of model

In our model, a production unit of age$A$, owned by the lessor, is leased to the lessee for a period$L$. According to the lease contract, the lessor will execute PM with a constant period, T. We assume that a PM action is a perfect maintenance action and that failures occurring during the lease period are repaired by the lessor. We also assume that a CM action is a minimal repair and that the repair time is much shorter than the mean time between failures (MTBF). Both PM and CM actions result in maintenance costs for the lessor. In addition, prior to leasing, the lessor can perform reconditioning on the production unit being leased. The reconditioning makes the leased production unit younger by reducing its current age from A to $A - x$. However, the reconditioning causes a reconditioning cost to the lessor, which depends on how much the age is reduced after the reconditioning.

The lessee will use the production unit to produce parts for the subsequent production stage. The demand rate for parts is α. To prevent the subsequent production stage from being interrupted, the lessee must produce safety stock of size S at rate ω after a PM action is completed. This safety stock will be withdrawn for use at rate α during breakdowns and PM. The production of such safety stock incurs a cost of the holding inventory to the lessee. We require the safety stock size to be at least greater than the demand for parts during CM actions. If the safety stock is insufficient for the demand during a PM action, shortages will occur, presenting a shortage cost to the lessee. We assume that this cost depends only on the number of shortages, regardless of their duration.

Given all the mentioned costs, one can see that the cost of the system consists of two parts: the cost to the lessor and the cost to the lessee. The decision variables that affect the system cost are (i) the duration of the PM cycle; (ii) the reconditioning level, which is the reduction in the age of the production unit following reconditioning; and (iii) the size of the safety stock. The purpose of our proposed model is to determine the optimal values of these three decision variables to minimise the total expected cost of the system over the leasing horizon.

3. Model formulation

3.1 Notation

The notation used in this research is as follows:

$F(t)$	Cumulative distribution function (CDF) of the time to failure
$f(t)$	Probability density function (PDF) of the time to failure, defined as $f(t) = \frac{dF(t)}{dt}$
$\lambda_0(t)$	Failure rate function in the absence of PM actions
$\lambda(t)$	Failure rate function with PM actions
$\Lambda_0(t)$	Cumulative failure rate function in the absence of PM actions, defined as $\Lambda_0(t) = \int_0^t \lambda_0(x)dx$
$\Lambda(t)$	Cumulative failure rate function with PM actions, defined as $\Lambda(t) = \int_0^t \lambda(x)dx$
$N(t)$	Number of failures in the interval $(0, t)$
$m(t_1, t_2)$	Number of failures in the interval (t_1, t_2)

Y	Repair time (time to repair a failure)
$G(y)$	CDF of the repair time
$g(y)$	PDF of the repair time, defined as $g(y) = \frac{dG(y)}{dy}$
τ	Repair time limit
T	Constant period to execute PM
Z	Duration of a PM action
$h(z)$	PDF of the duration of a PM action
C_f	Average cost of a CM action
a	Fixed cost of a PM action
b	Variable cost per unit of the reduction in the failure rate after a PM action
δ_j	Reduction in the failure rate after the jth PM action
C_t	Penalty cost per time unit if $Y > \tau$ (Penalty 1)
C_n	Penalty cost per failure (Penalty 2)
S	Size of safety stock
h	Inventory-holding cost per unit per time unit
ω, α	Production rates
π	Shortage cost per unit, regardless of the duration of the shortage
L	Lease contract duration (or the leasing horizon)
A	Age of the production unit being leased
x	Reconditioning level, which is the reduction in the age of the production unit following reconditioning
$C_u(x)$	Reconditioning cost, which increases as x increases

3.2 Failure rate function

An important function for maintenance studies is the failure rate function. In our work, we initially examine the failure rate function of a used production unit being leased without reconditioning and PM actions. Let $\lambda_0(t)$ denote the failure rate function of a new unit in the absence of PM actions, which is an increasing function of the time to failure, t. Given that the age of the unit being leased is A, the failure rate function of the used unit without reconditioning and PM actions is given by $\lambda(t) = \lambda_0(A + t), t > 0$. The age of the used unit can be reduced by executing reconditioning. Let x be the reconditioning level, which is the reduction in the age following reconditioning. Then, the failure rate function of the used unit with reconditioning only is given by

$$\lambda(t) = \lambda_0(A + t - x). \tag{1}$$

We assume a periodic PM policy, whereby the leased unit is preventively maintained at times $jT, j = 1, 2, \dots, k$. The jth PM action will reduce the failure rate by$\delta_j, j = 1, 2, \dots, k$. Note that, for perfect PM, $\delta_1 = \delta_2 = \cdots = \delta_k = \lambda_0(A - x + T) - \lambda_0(A - x)$. As a result, the failure rate function with both reconditioning and PM actions is given by

$$\lambda(t) = \lambda_0(A + t - x) - j\delta_j, jT < t \le (j + 1)T, \tag{2}$$

where $\delta_j = \lambda_0(A - x + T) - \lambda_0(A - x)$ for $j = 1, 2, \dots, k$. Substituting $\lambda_0(A - x + T) - \lambda_0(A - x)$ for $\delta_j, j = 1,2, \dots, k$, we can rewrite λ(t) in equation (2) as follows:

$$\lambda(t) = \lambda_0(A + t - x) - j[\lambda_0(A - x + T) - \lambda_0(A - x)] \text{ for } j = 1,2, \dots, k. \tag{3}$$

3.3 Expected cost to the lessor in one PM cycle

In one PM cycle of duration T, four types of cost are incurred by the lessor: (i) the cost of a CM action, (ii) the cost of a PM action, (iii) the Penalty-1 cost, and (iv) the Penalty-2 cost. The expected cost to the lessor in one PM cycle is equal to the sum of the expected values of these four costs. (i) Cost of a CM action. For a new production unit, the expected number of failures in one PM cycle is given by

$$E[N(T)] = \Lambda_0(T) = \int_0^T \lambda_0(t)dt, \tag{4}$$

where $\lambda_0(t)$ is the failure rate function in the absence of PM actions. Let $m(t_1, t_2)$ denote the number of failures in the interval(t_1, t_2). The expected number of failures in one PM cycle for a used production unit is given by

$$E[m(A - x, A - x + T)] = E[N(A - x + T)] - E[N(A - x)]. \tag{5}$$

Using equation (4), we have

$$E[m(A - x, A - x + T)] = \Lambda_0(A - x + T) - \Lambda_0(A - x), \tag{6}$$

where

$$\Lambda_0(A - x + T) = \int_0^{A-x+T} \lambda_0(t)dt, \tag{7}$$

and

$$\Lambda_0(A-x)=\int_0^{A-x}\lambda_0(t)dt. \tag{8}$$

To calculate the expected cost of a CM action in one PM cycle, $E\left[TC_f(T,x)\right]$, we multiply $E[m(A-x,A-x+T)]$ by the average cost of a CM action, C_f. The result is as follows:

$$E\left[TC_f(T,x)\right]=C_fE[m(A-x,A-x+T)], \tag{9}$$

where $E[m(A-x,A-x+T)]$ is given by equation (6). As a result, we have

$$E\left[TC_f(T,x)\right]=C_f[\Lambda_0(A-x+T)-\Lambda_0(A-x)]. \tag{10}$$

(ii) Cost of a PM action. In general, PM costs consist of two elements: fixed costs and variable costs. Fixed costs are the same every time PM is executed. By contrast, variable costs depend on the reduction in the failure rate after each PM action, δ_j. Given that $a>0$ is the fixed cost of a PM action and $b\geq 0$ is the variable cost per unit of the reduction in the failure rate after a PM action, the cost of the j^{th} PM action can be expressed as $C_p(\delta_j)=a+b\delta_j, j=1,2,\dots,k$ [16]. Because $\delta_1=\delta_2=\cdots=\delta_k=\lambda_0(A-x+T)-\lambda_0(A-x)$, the cost of a PM action in one PM cycle, $C_p(T,x)$, can be expressed as

$$C_p(T,x)=a+b[\lambda_0(A-x+T)-\lambda_0(A-x)]. \tag{11}$$

(iii) Penalty-1 cost. Penalty 1 occurs if the time to repair a failure, Y, is longer than the specified value, $\tau\ (>0)$. Given that C_t is the penalty cost per unit time if $Y>\tau$, the expected Penalty-1 cost in one PM cycle, $E[\phi_1(T,x)]$, can be expressed as

$$E[\phi_1(T,x)]=C_tE[m(A-x,A-x+T)]\int_\tau^\infty(y-\tau)g(y)dy, \tag{12}$$

where $g(y)$ is the PDF of Y. Substituting $\Lambda_0(A-x+T)-\Lambda_0(A-x)$ for $E[m(A-x,A-x+T)]$, we have

$$E[\phi_1(T,x)]=C_t[\Lambda_0(A-x+T)-\Lambda_0(A-x)]\int_\tau^\infty(y-\tau)g(y)dy, \tag{13}$$

or

$$E[\phi_1(T,x)]=C_t[\Lambda_0(A-x+T)-\Lambda_0(A-x)]\int_\tau^\infty\left(1-G(y)\right)dy, \tag{14}$$

where $G(y)$ is the CDF of Y. (iv) Penalty-2 cost. For each failure occurring during the lease period, the lessor will be given Penalty 2. To calculate the expected Penalty-2 cost in one PM cycle, $E[\phi_2(T,x)]$, we multiply $E[m(A-x,A-x+T)]$ by the penalty cost per failure, C_n. The outcome is as follows:

$$E[\phi_2(T,x)]=C_nE[m(A-x,A-x+T)], \tag{15}$$

or

$$E[\phi_2(T,x)]=C_n[\Lambda_0(A-x+T)-\Lambda_0(A-x)]. \tag{16}$$

The expected cost to the lessor in one PM cycle, $E[CC_L(T,x)]$, is determined by adding the above expected costs together. The result is as follows:

$$\begin{aligned}E[CC_L(T,x)]=&\ C_f[\Lambda_0(A-x+T)-\Lambda_0(A-x)]+a+b[\lambda_0(A-x+T)-\lambda_0(A-x)]\\&+C_t[\Lambda_0(A-x+T)-\Lambda_0(A-x)]\int_\tau^\infty\left(1-G(y)\right)dy\\&+C_n[\Lambda_0(A-x+T)-\Lambda_0(A-x)].\end{aligned} \tag{17}$$

3.4 Expected cost to the lessee in one PM cycle

In one PM cycle, two types of cost are incurred by the lessor: (i) the inventory-holding cost and (ii) the shortage cost. The expected cost to the lessee in one PM cycle is equal to the sum of the expected values of these two costs. (i) Inventory-holding cost

At the start of a PM cycle, the leased production unit spends S/ω units of time to produce safety stock of size S (see Figure 1). This safety stock is collected at rate α for use during breakdowns and PM.

As can be seen in Figure 1, to calculate the inventory-holding cost in one PM cycle, we first need to determine the MTBF and mean time to repair (MTTR) values. Note that, for minimal repairs [25],

$$MTBF=\frac{T}{E[m(A-x,A-x+T)]}=\frac{T}{\Lambda_0(A-x+T)-\Lambda_0(A-x)}. \tag{18}$$

MTTR is simply the expected value of Y, given by

$$MTTR=E[Y]=\int_0^\infty yg(y)dy=\int_0^\infty\left(1-G(y)\right)dy. \tag{19}$$

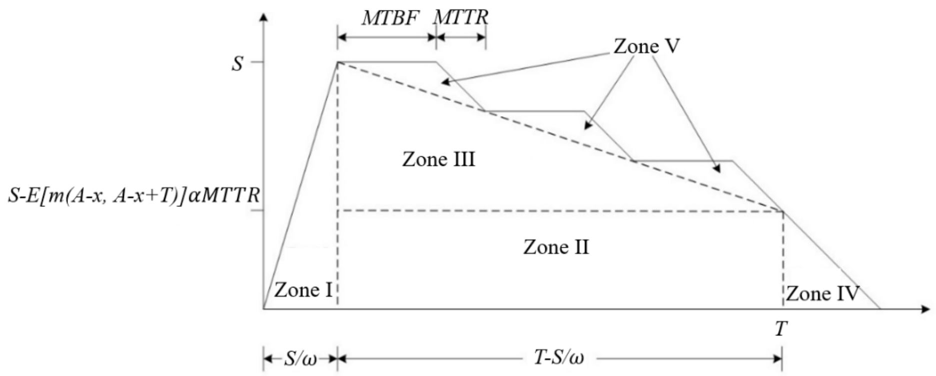

Figure 1 Changes in safety-stock levels during one cycle of PM

In addition to the MTBF and MTTR, we need to know the level of remaining safety stock, R, at the time when the PM action is commenced. The value of R is calculated using the following equation:

$$R = S - E[m(A-x, A-x+T)]\alpha MTTR. \tag{20}$$

Substituting $\Lambda_0(A-x+T) - \Lambda_0(A-x)$ for $E[m(A-x, A-x+T)]$, we have

$$R = S - [\Lambda_0(A-x+T) - \Lambda_0(A-x)]\alpha MTTR. \tag{21}$$

To facilitate the calculation of the expected inventory quantity in one PM cycle, we divide Figure 1 into five zones. The expected inventory quantity in one PM cycle is equal to the sum of the areas from Zone I to Zone V. Given that h is the inventory-holding cost per unit per time unit, the expected inventory-holding cost in one PM cycle, $E[IHC(T,x,S)]$, is given by

$$\begin{aligned} E[IHC(T,x,S)] = {} & \frac{hS^2}{2\omega} + h(S - E[m(A-x, A-x+T)]\alpha MTTR)(T - S/\omega) \\ & + \frac{h}{2}(T - S/\omega)E[m(A-x, A-x+T)]\alpha MTTR \\ & + \frac{h}{2\alpha}(S - E[m(A-x, A-x+T)]\alpha MTTR)^2 \\ & + \frac{h}{2}E[m(A-x+s/\omega, A-x+T)]\alpha MTTR.MTBF. \end{aligned} \tag{22}$$

The term $E[m(A-x+S/\omega, A-x+T)] = \Lambda_0(A-x+T) - \Lambda_0(A-x+S/\omega)$ represents the expected number of times that failure will occur in the interval $(S/\omega, T)$. For simplicity, we assume that no failures happen during the accumulation of the safety stock. Therefore, we have

$$E[m(A-x+S/\omega, A-x+T)] \approx \Lambda_0(A-x+T) - \Lambda_0(A-x). \tag{23}$$

Replacing $E[m(A-x, A-x+T)]$ and $E[m(A-x+S/\omega, A-x+T)]$ by $\Lambda_0(A-x+T) - \Lambda_0(A-x)$, we can rewrite $E[IHC(T,x,S)]$ in equation (22) as follows:

$$\begin{aligned} E[IHC(T,x,S)] = {} & \frac{hS^2}{2\omega} + h(S - [\Lambda_0(A-x+T) - \Lambda_0(A-x)]\alpha MTTR)(T - S/\omega) \\ & + \frac{h}{2}(T - S/\omega)[\Lambda_0(A-x+T) - \Lambda_0(A-x)]\alpha MTTR \\ & + \frac{h}{2\alpha}(S - [\Lambda_0(A-x+T) - \Lambda_0(A-x)]\alpha MTTR)^2 \\ & + \frac{h}{2}[\Lambda_0(A-x+T) - \Lambda_0(A-x)]\alpha MTTR.MTBF. \end{aligned} \tag{24}$$

(ii) Shortage cost. In each PM cycle, shortages occur only when the duration of the PM action, Z, is longer than the time taken to deplete the remaining safety stock, R. Let $\eta(z, R)$ be a random variable that represents the number of shortages occurring in one PM cycle. Then,

$$\eta(z,R) = \begin{cases} 0 & if \quad z < R/\alpha, \\ \alpha(z - R/\alpha) & if \quad z \geq R/\alpha. \end{cases} \tag{25}$$

Therefore, the expected number of shortages in one PM cycle can be expressed as follows:

$$E[\eta(z,R)] = \alpha \int_{R/\alpha}^{\infty} (z - R/\alpha)h(z)dz, \tag{26}$$

where $h(z)$ is the PDF of Z. Given that π is the shortage cost per unit, regardless of the duration of the shortage, the expected shortage cost in one PM cycle, $E[SC(T,x,S)]$, is given by

$$E[SC(T,x,S)] = \pi\alpha \int_{R/\alpha}^{\infty} (z - R/\alpha)h(z)dz. \tag{27}$$

The expected cost to the lessee in one PM cycle, $\mathrm{E}[\mathrm{CC}_l(\mathrm{T}, \mathrm{x}, \mathrm{S})]$, is acquired by adding the above expected costs together. The result is as follows:

$$\begin{aligned} E[CC_l(T,x,S)] = & \frac{hS^2}{2\omega} + h(S - [\Lambda_0(A-x+T) - \Lambda_0(A-x)]\alpha MTTR)(T - S/\omega) \\ & + \frac{h}{2\alpha}(S - [\Lambda_0(A-x+T) - \Lambda_0(A-x)]\alpha MTTR)^2 \\ & + \frac{h}{2}[\Lambda_0(A-x+T) - \Lambda_0(A-x)]\alpha MTTR.MTBF \\ & + \pi\alpha \int_{R/\alpha}^{\infty} (z - R/\alpha) h(z) dz. \end{aligned} \tag{28}$$

3.5 Total expected cost in one PM cycle

To obtain the total expected cost per cycle, $\mathrm{E}[\mathrm{TCC}(\mathrm{T}, \mathrm{x}, \mathrm{S})]$, we combine the expected costs to the lessor, $\mathrm{E}[\mathrm{CC}_L(\mathrm{T}, \mathrm{x})]$, and to the lessee, $\mathrm{E}[\mathrm{CC}_l(\mathrm{T}, \mathrm{x}, \mathrm{S})]$. Therefore, using equations (17) and (28), the total expected cost in one PM cycle can be expressed as follows:

$$\begin{aligned} E[TCC(T,x,S)] = & C_f[\Lambda_0(A-x+T) - \Lambda_0(A-x)] + a + b[\lambda_0(A-x+T) - \lambda_0(A-x)] \\ & + C_t[\Lambda_0(A-x+T) - \Lambda_0(A-x)] \int_{\tau}^{\infty} (1 - G(y)) dy \\ & + C_n[\Lambda_0(A-x+T) - \Lambda_0(A-x)] + \frac{hS^2}{2\omega} \\ & + h(S - [\Lambda_0(A-x+T) - \Lambda_0(A-x)]\alpha MTTR)(T - S/\omega) \\ & + \frac{h}{2}(T - S/\omega)[\Lambda_0(A-x+T) - \Lambda_0(A-x)]\alpha MTTR \\ & + \frac{h}{2\alpha}(S - [\Lambda_0(A-x+T) - \Lambda_0(A-x)]\alpha MTTR)^2 \\ & + \frac{h}{2}[\Lambda_0(A-x+T) - \Lambda_0(A-x)]\alpha MTTR.MTBF \\ & + \pi\alpha \int_{R/\alpha}^{\infty} (z - R/\alpha) h(z) dz. \end{aligned} \tag{29}$$

3.6 Expected duration of one PM cycle

The duration of a PM cycle is the period from the end of one PM action to the end of the subsequent PM action (see Figure 1). This period is composed of a constant cycle of duration T and a PM action of duration Z. The expected duration of this period, $E[CL(T)]$, is given by

$$E[CL(T)] = T + E[Z] = T + \int_0^{\infty} z h(z) dz. \tag{30}$$

3.7 Expected number of PM cycles

To calculate the expected number of PM cycles, $E[NC(T)]$, over the leasing horizon, we simply divide the lease contract duration, L, by the expected duration of one PM cycle, $E[CL(T)]$, as follows:

$$E[NC(T)] = \frac{L}{E[CL(T)]}. \tag{31}$$

3.8 Total expected cost over the leasing horizon

The total expected cost over the leasing horizon consists of two main parts: (i) the total expected operating cost and (ii) the reconditioning cost.

(i) Total expected operating cost. The total operating cost over the leasing horizon, $TOC(T,x,S)$, includes the operating costs incurred by both the lessor, $TOC_L(T,x)$, and the lessee, $TOC_l(T,x,S)$. To calculate its expected value, $E[TOC(T,x,S)]$, we multiply $E[TCC(T,x,S)]$ by $E[NC(T)]$, as follows:

$$E[TOC(T,x,S)] = E[NC(T)]E[TCC(T,x,S)], \tag{32}$$

or

$$E[TOC(T,x,S)] = \frac{L.E[TCC(T,x,S)]}{E[CL(T)]}. \tag{33}$$

We can then write $E[TOC(T,x,S)]$ as the sum of the expected values of $TOC_L(T,x)$ and $TOC_l(T,x,S)$, as follows:

$$E[TOC(T,x,S)] = E[TOC_L(T,x)] + E[TOC_l(T,x,S)], \tag{34}$$

where

$$E[TOC_L(T,x)] = \frac{L.E[CC_L(T,x)]}{E[CL(T)]}, \tag{35}$$

$$E[TOC_l(T,x,S)] = \frac{L.E[CC_l(T,x,S)]}{E[CL(T)]}. \tag{36}$$

(ii) Reconditioning cost. In our work, we adopt the concept of reconditioning used equipment from Pongpech et al. [24]. In their paper, the reconditioning cost, $C_u(x)$, depends on the reconditioning level, x, and is defined by the following two-parameter increasing function:

$$C_u(x) = \frac{\psi x}{1-e^{-\varphi(A-x)}}, 0 \le x < A, \tag{37}$$

where $\psi > 0, \varphi > 0$ are the parameters of the function.
Note that if x is equal to zero, there is no reconditioning. As $x \to A$, $C_u(x) \to \infty$, which means that it is impractical to recondition the used production unit to a new condition [24].

The total expected cost over the leasing horizon, $E[TC(T,x,S)]$, which is the objective function of this research, can be acquired by combining $E[TOC(T,x,S)]$ and $C_u(x)$, as follows:

$$E[TC(T,x,S)] = E[TOC(T,x,S)] + C_u(x). \tag{38}$$

3.9 Constraints on S, x, and T

As previously mentioned, our proposed model assumes that no failures occur during the accumulation of the safety stock. As a result, the time needed to accumulate safety stock, S/ω, is very short and clearly shorter than T (see Figure 1). Therefore, a first constraint on S is given by

$$\frac{S}{\omega} < T, \tag{39}$$

or

$$S < \omega T. \tag{40}$$

In addition, safety stock must be large enough to be used during CM actions without causing shortages. Therefore, a second constraint on S is given by

$$S \ge E[m(A-x, A-x+T)]\alpha MTTR, \tag{41}$$

or

$$S \ge [\Lambda_0(A-x+T) - \Lambda_0(A-x)]\alpha MTTR. \tag{42}$$

Given equation (37), a constraint on the reconditioning level, x, is $0 \le x < A$. Moreover, a constraint on the constant period to execute PM, T, is $0 < T < L$.

3.10 Optimisation model

Based on the objective function and the constraints obtained in the previous sections, the optimisation model for our work is as follows:

$$\text{Minimise } Z = E[TC(T,x,S)], \tag{43}$$

subject to

$$[\Lambda_0(A-x+T) - \Lambda_0(A-x)]\alpha MTTR \le S < \omega T, \tag{44}$$

$$0 \le x < A, \tag{45}$$

$$0 < T < L. \tag{46}$$

4. Algorithm

The algorithm used to solve the proposed problem is as follows:
Step 1: Fix $x = x_0 = 0$.
Step 2: Fix $T = T_0 \in (0, L)$.
Step 3: Solve fo $\bar{S}^*(T,x)r$, the value of S that minimises $E[TC(T,x,S)]$. Note that, in this case, $E[TC(T,x,S)]$ is a function of a single variable. Therefore, it can be solved quickly using commercial software.
Step 4: Retain the value of $\bar{S}^*(T,x)$. Then, go to step 5.
Step 5: Set $T = T + \Delta T$, where $\Delta T > 0$ is a sufficiently small value.
Step 6: Check if $T < L$. If so, go to step 3. If not, keep $\bar{S}^*(\bar{T}^*, x)$, the minimum value of $\bar{S}^*(T,x)$ retained in step 4, then go to step 7.
Step 7: Set $x = x + \Delta x$, where $\Delta x > 0$ as a sufficiently small value.
Step 8: Check if $x < A$. If so, go to step 3. If not, keep $\bar{S}^*(\bar{T}^*, \bar{x}^*)$, the minimum value of $\bar{S}^*(\bar{T}^*, x)$ retained in step 6, then stop. The optimal solution, (T^*, x^*, S^*), has been obtained.

5. Numerical example and sensitivity analysis

In this section, we present a numerical example and a sensitivity analysis on some model parameters.

5.1 Functions and test values

In maintenance studies, exponential distributions are often used in modelling systems with constant failure and repair rates [26]. For time-dependent failure and repair models, various types of probability distributions, such as Weibull, normal, lognormal, and gamma distributions, are generally applied for both failure and repair times [1, 25]. The assumptions on the probabilistic functions and the values of constants used in testing the proposed model are given as follows:

(i) Failure rate function: a two-parameter Weibull failure rate function given by

$$\lambda_0(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}, t \geq 0, \tag{47}$$

where $\beta > 0$ is the shape parameter and $\theta > 0$ is the scale parameter. We set $\beta = 2$ so that the increase in the failure rate is linear. With no loss of generality, we set $\theta = 1$.
(ii) CDF of the repair time: a two-parameter Weibull distribution function given by

$$G(y) = 1 - exp\left[-\left(\frac{y}{\mu}\right)^{m}\right], y \geq 0, \tag{48}$$

where $m > 0$ is the shape parameter and $\mu > 0$ is the scale parameter. We set $m = 0.5$ and $\mu = 0.5$ so that the MTTR is 1 day.
(iii) PDF of the duration of PM action: a gamma density function given by

$$h(z) = \frac{z^{\gamma-1}e^{-z/\rho}}{\rho^{\gamma}\Gamma(\gamma)}, z \geq 0, \tag{49}$$

where $\gamma > 0$ is the shape parameter and $\rho > 0$ is the scale parameter. We set $\gamma = 2.5$ and $\rho = 0.02$ so that E[Z] is 0.05 months.
(iv) Reconditioning cost: a two-parameter increasing function given by equation (37) with $\psi = 500$ and $\varphi = 0.01$.
(v) Other constants used in this research are as follows:
$L = 5$ years, $A = 5$ years, $C_f = \$2{,}500$, $a = \$100$, $b = \$50$, $C_t = \$300$, $C_n = \$200$, $\pi = 2$ *days*, $h = \$6$ /unit/year, $\omega = 86{,}400$ units/year, $\alpha = 345{,}600$ units/year, and $\pi = \$2$/unit.

5.2 Numerical results

Table 1 presents the optimal solutions to the proposed model and two special cases. The optimal solution to the proposed model is the set of values of T, x, and S that minimises the total expected cost over the leasing horizon, $E[TC(T,x,S)]$. The optimal solution to special case 1 is the set of values that minimises the total expected cost to the lessor over the leasing horizon, $E[TOC_L(T,x)] + C_u(x)$. The optimal solution to special case 2 is the set of values that minimises the total expected cost to the lessee over the leasing horizon, $E[TOC_l(T,x,S)]$. Note that $E[TC(T,x,S)] = E[TOC_L(T,x)] + C_u(x) + E[TOC_l(T,x,S)]$, using equations (34) and (38).

Table 1 Optimal values of T^*, x^*, and S^*

Model	T^* (years)	x^* (years)	S^* (units)	Cost to the lessor ($) $(E[TOC_L(T^*,x^*)] + C_u(x^*))$	Cost to the lessee ($) $(E[TOC_l(T^*,x^*,S^*)])$	Total cost ($) $(E[TC(T^*,x^*,S^*)])$
Proposed model	0.04	2	3,456	122,788	77,918.50	200,707
Special case 1	0.10	2	2,532.39	121,738	89,799.90	211,538
Special case 2	0.04	4.5	3,456	476,189	77,346.80	553,536

As shown in Table 1, the optimal solution to the proposed model is $T^* = 0.04$ years, $x^* = 2$ years, and $S^* = 3{,}456$ units. When implementing this policy, the total expected cost is \$200,707. The optimal solution to special case 1 ($T^* = 0.10$ years, $x^* = 2$ years, and $S^* = 2{,}532.39$ units) limits the total expected cost to the lessor to \$121,738. However, applying this policy increases the cost to the lessee, which then raises the total expected cost to \$211,538. Implementing the optimal solution to special case 2 ($T^* = 0.04$ years, $x^* = 4.5$ years, and $S^* = 3{,}456$ units) limits the total expected cost to the lessee to \$77,346.80. However, for this policy, the value of x^* is close to that of A. Therefore, the reconditioning cost to the lessor is very high, which greatly increases the total expected cost to \$553,536.

Based on these results, the proposed model provides a better solution, in terms of the total expected cost over the leasing horizon, than do both special cases, in which the lessor and the lessee are individually considered. Note that the optimal solution to the proposed model is not the best answer for both parties. However, the lessor and the lessee should be able to agree on the solution because the costs to both parties are considered. In other words, the solution can limit potential conflicts between the parties over the choice of optimal policy.

5.3 Sensitivity analysis

In this section, we conduct a sensitivity analysis on some important model parameters to determine how changes in those parameters affect the optimal solution of the proposed model.

(i) Effect of changes in A Table 2 presents the impact of changes in the age of the production unit being leased, A, on the optimal solution.

Table 2 Effect of changes in A on the optimal solution

Optimal solution	A		
	3	**5**	**7**
$E[TC(T^*, x^*, S^*)]$ ($)	166,271	200,707	230,712
T^* (years)	0.04	0.04	0.04
x^* (years)	1	2	3.5
S^* (units)	3,456	3,456	3,456

As shown in Table 2, the optimal reduction in the age following reconditioning, x^*, increases as A increases. This result implies that the lessor is more likely to raise the reconditioning level when the leased production unit is older. The total expected cost over the leasing horizon, $E[TC(T^*, x^*, S^*)]$, also increases as A increases. However, the optimal period to execute PM actions, T^*, and the optimal size of safety stock, S^*, do not change.

(ii) Effect of changes in ψ Table 3 presents the impact of changes in the reconditioning cost parameter, ψ, on the optimal solution.

Table 3 Effect of changes in ψ on the optimal solution

Optimal solution	ψ		
	250	**500**	**1,000**
$E[TC(T^*, x^*, S^*)]$ ($)	178,897	200,707	218,312
T^* (years)	0.04	0.04	0.04
x^* (years)	3	2	1
S^* (units)	3,456	3,456	3,456

As shown in Table 3, $E[TC(T^*, x^*, S^*)]$ increases and x^* decreases as ψ increases. However, T^* and S^* do not change.

(iii) Effect of changes in h Table 4 presents the impact of changes in the inventory-holding cost per unit per unit time, h, on the optimal solution.

Table 4 Effect of changes in h on the optimal solution

Optimal solution	h		
	2	**6**	**12**
$E[TC(T^*, x^*, S^*)]$ ($)	153,022	200,707	261,978
T^* (years)	0.05	0.04	0.03
x^* (years)	2	2	2
S^* (units)	4,320	3,456	2,592

As shown in Table 4, S^* decreases as h increases, which implies that the lessee tends to hold less safety stock if h is high. Because $T > S/\omega$, T^* also decreases as h increases, which implies that the lessor tends to perform PM actions more frequently if h is high. Additionally, $E[TC(T^*, x^*, S^*)]$ increases as h increases. However, x^* does not change.

(iv) Effect of changes in π Table 5 presents the impact of changes in the shortage cost per unit, π, on the optimal solution.

Table 5 Effect of changes in π on the optimal solution

Optimal solution	π		
	2	**10**	**30**
$E[TC(T^*, x^*, S^*)]$ ($)	200,707	218,411	231,762
T^* (years)	0.04	0.05	0.06
x^* (years)	2	2	2
S^* (units)	3,456	4,320	5,184

As shown in Table 5, S^* increases as π increases, which implies that the lessee tends to hold large quantities of safety stock if shortages create high costs. Because $T > S/\omega$, T^* also increases as π increases, which implies that the lessor tends to perform PM actions less frequently if the shortage cost per unit is high. Additionally, $E[TC(T^*, x^*, S^*)]$ increases as π increases. However, x^* does not change.

6. Conclusion

In this work, we propose an integrated PM and safety stock model for a used production unit under lease. The lessee uses this production unit to produce parts for the subsequent production stage. In addition to those parts, safety stock must be produced as a buffer during stoppages due to CM and PM actions, incurring inventory costs to the lessee. Both CM and PM actions, during the period of the lease agreement, are performed by the lessor, incurring maintenance costs to the lessor. In addition to maintenance actions, the lessor can perform reconditioning, prior to leasing, to improve the reliability of the leased production unit, incurring a reconditioning cost to the lessor. The purpose of the proposed model is to determine the optimal PM cycle duration, reconditioning level and safety stock size to minimise the total expected cost to the lessor and the lessee combined over the leasing horizon.

Based on our experimental results, calculating the optimal values of all three decision variables by considering the lessor and the lessee simultaneously yields a more satisfactory result, in terms of the total expected cost over the leasing horizon, than when the parties are considered individually. Because the optimal solution considers the costs to both parties, it can limit potential conflicts between the parties over the choice of optimal policy. Furthermore, the results obtained from the sensitivity analysis show several managerial perspectives. First, the lessor tends to raise the reconditioning level when the leased production unit is relatively old. Second, when the inventory-holding cost per unit per time unit is high, the lessee tends to hold less safety stock and the lessor tends to perform PM actions more frequently. Lastly, if the shortage cost per unit is high, the lessee tends to hold large quantities of safety stock and the lessor tends to perform PM actions less frequently.

Imperfect maintenance is a common practice within industry. Therefore, our future work will consider the case in which imperfect maintenance is included in the problem. We will also adjust Penalty 2 so that it is more realistic. Namely, instead of requiring Penalty 2 to occur every time a failure occurs, the penalty will happen only when the number of failures exceeds a predetermined value.

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8. References

[1] Blischke WR, Murthy DNP. Reliability. New York: John Wiley & Sons; 2000.
[2] Cheung KL, Hausman WH. Joint determination of preventive maintenance and safety stocks in an unreliable production environment. Nav Res Logist. 1997;44:257-72.
[3] Dohi T, Okamura H, Osaki S. Optimal control of preventive maintenance schedule and safety stocks in an unreliable manufacturing environment. Int J Prod Econ. 2001;74:147-55.
[4] Chelbi A, Ait-Kadi D. Analysis of a production/inventory system with randomly failing production unit submitted to regular preventive maintenance. Eur J Oper Res. 2004;156:712-8.
[5] Chelbi A, Rezg N. Analysis of a production/inventory system with randomly failing production unit subjected to a minimum required availability level. Int J Prod Econ. 2006;99:131-43.
[6] Gharbi A, Kenne JP, Beit M. Optimal safety stocks and preventive maintenance periods in unreliable manufacturing systems. Int J Prod Econ. 2007;107:422-34.
[7] Murino T, Romano E, Zoppoli P. Maintenance policies and buffer sizing: an optimization model. WSEAS Trans Bus Econ. 2009;6:21-30.
[8] Chakraborty T, Giri BC. Joint determination of optimal safety stocks and production policy for an imperfect production system. Appl Math Model. 2012;36:712-22.
[9] Gan S, Zhang Z, Zhou Y, Shi J. Joint optimization of maintenance, buffer, and spare parts for a production system. Appl Math Model. 2015;39:6032-42.
[10] Cheng GQ, Zhou BH, Li L. Joint optimisation of buffer size and preventive maintenance for a deteriorating upstream machine. Int J Syst Sci Oper Logist. 2015;2:199-210.
[11] Zhou B, Liu Z. Optimizing preventive maintenance: a deteriorating system with buffers. Ind Manag Data Syst. 2016;116: 1719-40.
[12] Nahas N. Buffer allocation and preventive maintenance optimization in unreliable production lines. J Intell Manuf. 2017;28: 85-93.
[13] Lopes R. Integrated model of quality inspection, preventive maintenance and buffer stock in an imperfect production system. Comput Ind Eng. 2018;126:650-6.
[14] Deiranlou M, Dehghanian F, Pirayesh MA. The simultaneous effect of holding safety stock and purchasing policies on the economic production quantity model subject to random machine breakdown. Int J Eng. 2019;32:1643-55.
[15] Schutz J, Rezg N. Maintenance strategy for leased equipment. Comput Ind Eng. 2013;66:593-600.
[16] Jaturonnatee J, Murthy DNP, Boondiskulchok R. Optimal preventive maintenance of leased equipment with corrective minimal repairs. Eur J Oper Res. 2006;174:201-15.
[17] Hajej Z, Rezg N, Gharbi A. Integrated maintenance policy optimization under lease/warranty contract. Proceedings of the 13th International Conference on Industrial Engineering; 2013 Oct 21-23; Dubai, UAE.
[18] Hajej Z, Rezg N, Gharbi A. A decision optimization model for leased manufacturing equipment with warranty under forecasting production/maintenance problem. Math Probl Eng. 2015:1-14.
[19] Askri T, Hajej Z, Rezg N. Optimal integrated maintenance strategy for parallel leased machines with warranty periods. Proceedings of the 3rd Conference on Control and Fault-Tolerant Systems; 2016 Sep 7-9; Barcelona, Spain.
[20] Hajej Z, Rezg N, Ali G. An optimal production/maintenance strategy under lease contract with warranty periods. J Qual Mainten Eng. 2016;22:35-50.
[21] Askri T, Hajej Z, Rezg N. Jointly production and correlated maintenance optimization for parallel leased machines. Appl Sci. 2017;7(5):1-20.
[22] Hajej Z, Rezg N, Askri T. Joint optimization of capacity, production and maintenance planning of leased machines. J Intell Manuf. 2020;31:351-74.
[23] Niyamosoth T, Pathumnakul S. Joint determination of preventive maintenance and buffer stock for a production unit under lease. J Ind Eng Manag. 2018;11:497-512.
[24] Pongpech J, Murthy DNP, Boondiskulchock R. Maintenance strategies for used equipment under lease. J Qual Mainten. 2006;12:52-67.
[25] Ebeling CE. An introduction to reliability and maintainability engineering. 2nd ed. Illinois: Waveland Press; 2010.
[26] Nandal N, Malik SC. On use of gamma distribution for evaluation of reliability and availability of a single unit system subject to arrival time of the server. J Reliab Stat Stud. 2019;12:93-102.