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The development of supplier comparison using lower process capability index for Weibull distribution model

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Abstract

Durability, one of the dimensions of quality, is the important criterion for comparing suppliers. In many industries, durability is widely measured by lifetime data modeled with Weibull distribution because of its flexible shape. The process capability indices (PCIs) is an appropriate tool for comparing suppliers regarding quality aspect. Many methods for comparing suppliers using the PCIs are developed to correspond to manufacturing conditions. However, the methods cannot apply with the lifetime data since they are developed under the normality assumption. The mentioned violation leads to probably misleading results in comparison. To consider the lifetime, this paper proposes the new supplier comparison using the PCIs for Weibull distribution model. Two parameters (scale and shape) of the Weibull distribution are studied. Since the shape parameter is sensitive to the PCIs estimation, this paper emphasizes on various shape parameters, e.g. symmetric, right, and left skewed. Regarding the lifetime, this paper studies single quality characteristic and the lower PCIs (C_{pl}). The producer's risk and the power of test obtained from Monte Carlo simulation are used to evaluate the performance of the proposed method. This method is compared with supplier comparison methods applying percentile PCIs and Box-Cox transformation for the PCIs estimation.

Keywords: Supplier comparison, Process capability indices, Weibull distribution, Durability, Lifetime

1. Introduction

The quality aspect is one of the important criteria for comparing suppliers. Process capability indices (PCIs) based on normal distribution are widely used to compare suppliers regarding the quality aspect because the quality of product can be explained by the PCIs straightforwardly. Several researchers apply the PCIs to compare suppliers with single quality characteristics such as Chou [1], Hubele et al. [2], Pearn et al. [3], Manomat and Sudasna-na-Ayudthya [4], Pearn et al. [5], Wu et al. [6], etc. Then, supplier comparison is extended to multiple quality characteristics such as Pearn and Wu [7], Pearn and Wu [8], Lan and Lin [9], etc.

In many industries, quality dimensions presented by Garvin [10] are widely employed to evaluate the quality aspect. Durability is an important quality dimension which manufacturers should focus on because it represents the product lifetime. The lifetime data commonly are modeled as Weibull distribution because the distribution has a variety of shapes providing a good model for small data set [11]. For supplier comparison using the PCIs, the existing methods are often based on normal distribution (symmetric shape). Thus, applying Weibull distribution in these methods may yield misleading results in comparison because Weibull distribution has asymmetric shape. As a result, an unqualified supplier may be selected. The manufacturers obtain the non-quality raw materials or parts leading to produce a non-quality product. The example of the misleading result also sees Thavorn and Sudasna-na-Ayudthya [12].

Therefore, this paper proposes the new supplier comparison method for lifetime data modeled with Weibull distribution and focuses on the single quality characteristic. The proposed method is modified from the method of Manomat and Sudasna-na-Ayudthya [4]. From the literature review, the method is appropriate for comparing suppliers regarding lifetime data. The product lifetime is focused on the lower bound since customers prefer to have their products last long. Therefore, this study will emphasize on the lower PCIs (C_{pl}) for assessing the process capability of the suppliers.

2. Material and methods

2.1 The proposed method

This paper proposes the new method for comparing suppliers using the C_{pl}. The development of this method is divided into two stages as the process capability estimation for Weibull distribution model and supplier comparison using the C_{pl}.

Since the quality characteristic of lifetime data is larger-the-better, the C_{pl} is the powerful tool for assessing capability process. The C_{pl} developed by Kane [13] is the index used to assess process capability by lower specification limits (LSL) as shown in the equation (1). The important assumption of C_{pl} calculation is the process that follows normal distribution. In practical, process mean (μ) and standard deviation (σ) are unknown, so they are replaced by an estimated sample mean ($\bar{x}$) and standard deviation (s) as the equation (2).

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$$C_{pl} = \frac{\mu - LSL}{3\sigma}, \quad (1)$$

$$\hat{C}_{pl} = \frac{\bar{X} - LSL}{3S} \quad (2)$$

For interpretation of the C_{pl}, Pearn *et al.* [3] present the quality conditions that commonly used in many industries as: if the $C_{pl} < 1.00$, process is inadequate; if $1.00 \leq C_{pl} < 1.33$, process is capable; if $1.33 \leq C_{pl} < 1.50$, process is satisfactory; if $1.50 \leq C_{pl} < 1.67$, process is good; if $1.67 \leq C_{pl} < 2.00$, process is excellent; if $C_{pl} \geq 2.00$, process is super.

However, the lower process capability assessment with Weibull distribution using the conventional C_{pl} obtained from equation (1) to (2) leads to the erroneous interpretation since the conventional C_{pl} is developed under the normality assumption. Although Weibull distribution is widely used to study in engineering and reliability due to its various shape, the shape is sensitive to measure the process capability.

To handle the Weibull distribution model, the C_L index is developed to assess lifetime performance based on Weibull distribution. This index is calculated by non-negative lifetime data and *LSL*. Many researches emphasize on applying the index in several situations [14, 15]. The drawback of this index is the interpretation depending on shape parameter that is different from the conventional C_{pl}. In practice, it is difficult to compare multiple suppliers when the process shape of each supplier is various.

For another method, applying Cumulative Distribution Function (CDF) of underlying distribution (i.e. Weibull, gamma, lognormal, etc.) for estimating process capability called "CDF PCIs method" is one of the popular methods. Several researchers [16-19] employ this method to estimate the process capability for non-normal distributed process. Moreover, the method is referenced by the Automotive Industry Action Group (AIAG) for non-normal PCIs estimation.

Regarding the CDF PCIs method, the proportion of non-conforming parts (*PNC*) are estimated by CDF of underlying distribution. Then, the *PNC* is converted to the C_{pl} by inverse CDF of standard normal distribution as shown in equation (3). The interpretation of process capability for the C_{pl} calculated from equation (3) is same to the conventional C_{pl}. Therefore, this method is applied in this study to assess the process capability for Weibull distribution model.

$$C_{pl} = \frac{-\Phi^{-1}(p)}{3} \quad (3)$$

Where Φ^{-1} is the inverse CDF of standard normal distribution, and p is the *PNC* defined as $P = P(X < LSL)$ corresponding to underlying distribution.

This study focuses on the two-parameter Weibull distribution $W(\delta, \gamma)$ with scale parameter δ and shape parameter γ. The cumulative distribution function is shown as:

$$F(x) = 1 - e^{-\left(\frac{x}{\delta}\right)^{\gamma}}, x > 0, \quad \delta > 0, \quad \gamma > 0 \quad (4)$$

Thus, the corresponding *PNC* for Weibull distribution is probability that the data of the process fall out of lower specification as shown the relation in equation (5).

$$p = P(X < LSL) = 1 - e^{-\left(\frac{LSL}{\delta}\right)^{\gamma}}, LSL > 0, \quad \delta > 0, \quad \gamma > 0 \quad (5)$$

From equation (3) and (5), the C_{pl} for Weibull distribution is calculated as equation (6):

$$C_{pl}(w) = \frac{-\Phi^{-1}(1 - e^{-\left(\frac{LSL}{\delta}\right)^{\gamma}})}{3}, LSL > 0, \quad \delta > 0, \quad \gamma > 0 \quad (6)$$

Where $C_{pl}(W)$ is the C_{pl} for Weibull distribution calculated from the CDF PCIs method

In practice, scale parameter $\hat{\delta}$ and shape parameter γ are unknown, so theses parameters are replaced by the estimated scale parameter $\hat{\delta}$ and shape parameter $\hat{\gamma}$ obtained from the maximum likelihood method. Therefore, the estimated $C_{pl}(W)$ is calculated as equation (7):

$$\hat{C}_{pl}(w) = \frac{-\Phi^{-1}\left(1 - e^{-\left(\frac{LSL}{\hat{\delta}}\right)^{\hat{\gamma}}}\right)}{3}, LSL > 0, \quad \hat{\delta} > 0, \quad \hat{\gamma} > 0 \quad (7)$$

For comparing suppliers, this paper applies the equality test of the PCIs. The test is presented by Manomat and Sudasna-na-Ayudthya [4] developed from Hubele et al. [2]. The procedures of supplier comparison are shown as:

1. Rearrange $\hat{C}_{pl}(W)$ in order from the smallest to greatest. Let $\hat{C}_{pl}(W)_{,l}$ is the supplier with the smallest $\hat{C}_{pl}(W)$ (the 1st supplier), and $\hat{C}_{pl}(W)_{,k}$ is the supplier with the greatest $\hat{C}_{pl}(W)$ (the kth supplier); where *i* (the *i*th supplier) =1,2,…,*k*;

2. Define d vector as the vector of the difference of $C_{pl}(W)$ between the 1st supplier and other suppliers as shown in equation (8) as:

$$\mathbf{d} = \begin{bmatrix} C_{PL}(w)_{,1} - C_{PL}(w)_{,2} \\ C_{PL}(w)_{,1} - C_{PL}(w)_{,3} \\ \vdots \\ C_{PL}(w)_{,1} - C_{PL}(w)_{,k} \end{bmatrix}; \quad (8)$$

3. Define the test hypothesis and significance level (α) as:

$$H_0: d = 0 \text{ และ } H_1: d \neq 0; \quad (9)$$

4. Calculate the Wald's statistics;

4.1 Construct the estimated d vector ($\hat{\mathrm{d}}$) as equation (10) because d vector is unknown in practice:

$$\hat{\mathbf{d}} = \begin{bmatrix} \hat{C}_{PL}(w)_{,1} - \hat{C}_{PL}(w)_{,2} \\ \hat{C}_{PL}(w)_{,1} - \hat{C}_{PL}(w)_{,3} \\ \vdots \\ \hat{C}_{PL}(w)_{,1} - \hat{C}_{PL}(w)_{,k} \end{bmatrix}; \quad (10)$$

4.2 Construct H and V matrix as:

$$\mathbf{H} = \left[\frac{\partial \mathbf{d}}{\partial C_{PL}(w)}\right] = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 1 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & -1 \end{bmatrix}; \quad (11)$$

Table 1 The summarized data of the example

Summarized statistic	Supplier A	Supplier B	Supplier C
1. Descriptive Statistics			
- Sample mean ($\bar{X}$)	0.9596	0.8658	1.1091
- Sample standard deviation (S)	0.3527	0.3527	0.9419
- Sample size (n)	25	25	25
- Skewness, Kurtosis	0.3561, 2.5924	-0.1571, 1.9560	1.4852, 5.4518
2. Goodness of fit test for Weibull distribution	P-value>0.250	P-value>0.250	P-value>0.250
3. Parameter estimation W($\hat{\delta}$, $\hat{\gamma}$)	W(1.0704,3.1313)	W(0.9741,2.7893)	W(1.1954,1.2525)

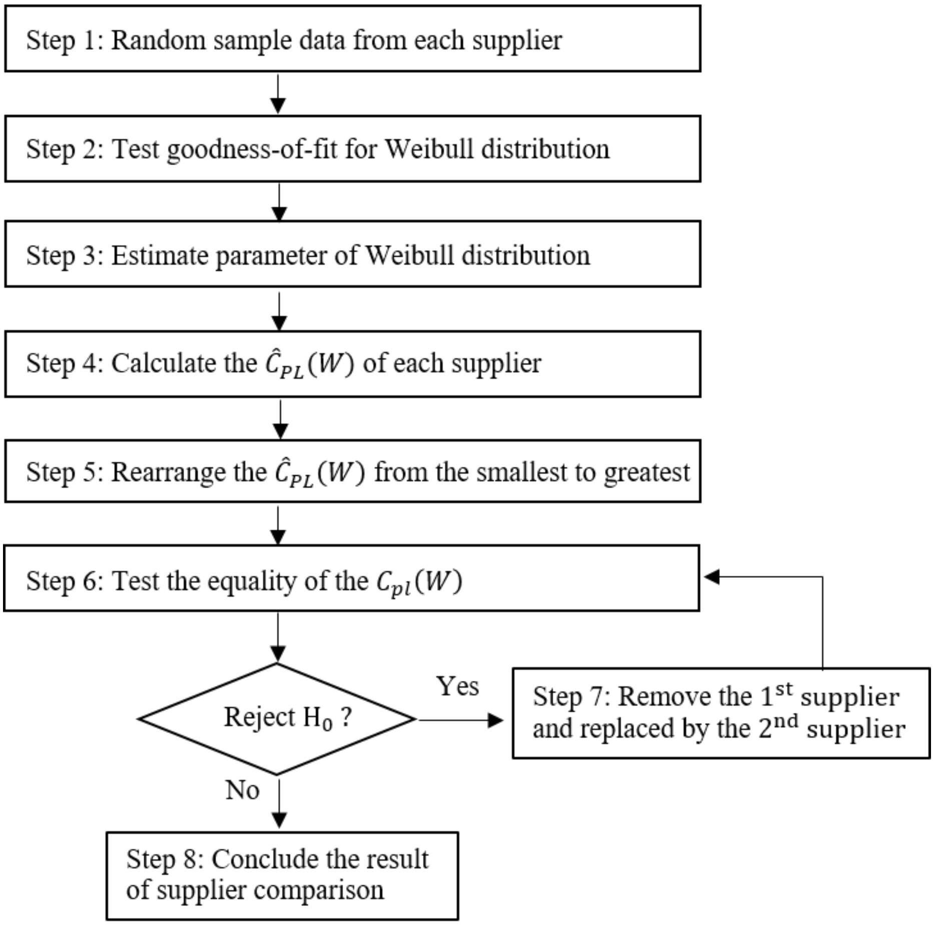

Figure 1 The procedure of calculation for the proposed method

$$\mathbf{V} = diag\left[\frac{1}{n_1}\left(\frac{1}{9}+\frac{1}{2}\hat{C}^2_{PL}(w)_{,1}\right),\frac{1}{n_2}\left(\frac{1}{9}+\frac{1}{2}\hat{C}^2_{PL}(w)_{,2}\right),\ldots,\frac{1}{n_k}\left(\frac{1}{9}+\frac{1}{2}\hat{C}^2_{PL}(w)_{,k}\right)\right]; \quad (12)$$

Where n_i is the sample size of the i^{th} supplier;

4.3 Hence, Wald's statistic is calculated as:

$$W = \hat{\mathbf{d}}^T\left[\mathbf{HVH}^T\right]^{-1}\hat{\mathbf{d}}; \quad (13)$$

5. Decide whether the first supplier is different from at least one other supplier (reject the null hypothesis (H0)) in case of $W > X^2_{\alpha,k-1}$

6. Remove the 1st supplier and replaced by the 2nd supplier when null hypothesis is rejected and repeat steps 2 to 5. If the testing results do not reject the null hypothesis, there is no difference among all suppliers.

To understand and apply the proposed method effectively, this paper summarizes the procedure of calculation as in Figure 1.

2.2 The example

This study presents the calculating method via the example for comparing three supplier's processes using the proposed method. This example assumes that the processes are failure time (in hour) obtained from lifetime testing process with LSL = 0.001. The calculating methods are divided into 8 main steps as:

Step 1: Random sample data from each supplier process: The descriptive statistic of the processes summarized in Table 1 shows that processes of A and C are right-skewed because the skewness values of A (0.3561) and C (1.4852) are positive. In contract, the process of B (-0.1571) is left-skewed due to the negative skewness.

Step 2: Test goodness-of-fit for Weibull distribution: From Table 1, All supplier processes are Weibull distributed because the p-value obtained from Anderson-Darling test of supplier A (>0.25), B (>0.25), and C (>0.25) are greater than 0.05.

Step 3: Estimate parameter of Weibull distribution: This paper employs the maximum likelihood method for estimating the parameters. According to Table 1, the estimated scale and shape parameter of supplier A are 1.07 and 3.13, B's are 0.97 and 2.7, and C's are 1.19 and 1.25 respectively.

Step 4: Calculate $\hat{C}_{pl}(W)$ of each supplier from equation (7): $\hat{C}_{pl}(W)$ of supplier A, B, and C are 2.0596, 1.9148, and 1.2112 respectively.

Step 5: Rearrange $\hat{C}_{pl}(W)$ from the smallest to greatest: $\hat{C}_{PL}(w)_{,1}$ = 1.2112 (supplier C), $\hat{C}_{PL}(w)_{,2}$ = 1.9148 (supplier B), and $\hat{C}_{PL}(w)_{,3}$ = 2.0596 (supplier A).

Step 6: Test the equality of the $C_{pl}(W)$:

6.1 Define d vector as $\mathbf{d} = \begin{bmatrix} C_{PL}(w)_{,1} - C_{PL}(w)_{,2} \\ C_{PL}(w)_{,1} - C_{PL}(w)_{,3} \end{bmatrix}$;

6.2 Define the hypothesis testing and significance level as: H_0 = d vs. $H_1 \neq$ d and α =0.05;

6.3 Construct $\hat{d}$ vector, H and V matrix as:

$$\hat{\mathbf{d}} = \begin{bmatrix} \hat{C}_{PL}(w)_{,1} - \hat{C}_{PL}(w)_{,2} \\ \hat{C}_{PL}(w)_{,1} - \hat{C}_{PL}(w)_{,3} \end{bmatrix} = \begin{bmatrix} 1.2112 - 1.9148 \\ 1.2112 - 2.0596 \end{bmatrix} = \begin{bmatrix} -0.7036 \\ -0.8484 \end{bmatrix};$$

$$\mathbf{H} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \text{ and } \mathbf{V} = \begin{bmatrix} 0.0338 & 0 & 0 \\ 0 & 0.0778 & 0 \\ 0 & 0 & 0.0893 \end{bmatrix};$$

6.4 Calculate Wald's statistic as: W = 8.0148;

6.5 Calculate $X^2_{\alpha,k-1}$ as: $X^2_{0.05,3-1}$=5.9915;

6.6 Decide the testing result: reject H_0 at α = 0.05 because the $W > X^2_c$.The 1st supplier (C) is different from other suppliers (B and C) at 5% significance levels.

Step 7: Remove the 1st supplier and replaced by the 2nd supplier: Since the H_0 is rejected, the 1st supplier (C) is removed and replaced by the 2nd supplier (B) and repeat steps 6 to 7.

Step 6 (the repeated step). Test the equality of the $C_{pl}(W)$:
d vector is defined as: d = $[C_{PL}(W)_{,1} - C_{PL}(W)_{,2}]$;
Hence, $\hat{d} = [\hat{C}_{PL}(W)_{,1} - \hat{C}_{PL}(W)_{,2}]$
$= [1.9148 - 2.0596]$
$= -0.1448$;
H, V matrix, W statistic, and X^2_c are calculated as:
H = $[1 \;\; -1]$, V = $\begin{bmatrix} 0.0778 & 0 \\ 0 & 0.0893 \end{bmatrix}$, and W = 0.1255;
$X^2_{\alpha,k-1}$=$X^2_{0.05,2-1}$= 3.8415;

From W statistic and X^2_c, the H_0 is not rejected at α = 0.05 because $W < X^2_{0.05,2-1}$.The 1st supplier (B) and 2nd supplier (A) are not different at 5% significance levels. So, the procedure can jump to step 8.

Step 8: Conclude the result of supplier comparison: The appropriate supplier is both supplier A and B.

2.3 Simulation experiment

2.3.1 Parameter setting

In this study, lifetime data from each supplier process are modeled by Weibull distribution with scale (δ) and shape (γ) parameters. Since shape of distribution measured by skewness values affects the PCIs estimation, this paper emphasizes on various shape of distribution, e.g. symmetry, right and left skewness. For Weibull distribution, the γ represents shape of distribution. Thus, this study uses γ = 3.6 as the criterion for explaining process shape because this process shape is nearly symmetric (skewness=0). If γ <3.6, the process shape is right-skewed (skewness>0). If γ >3.6, the process shape is left-skewed (skewness<0). In this study, the δ is specified to 1 because the scale parameter is no effect on shape of distribution [20]. The parameters in this study including details are illustrated in Table 2 and Figure 2.

This paper studies C_{pl} =1.00 (capable process), 1.50 (good process), and 2.00 (super process) since the supplier comparison in industries emphasizes on the capable process (C_{pl} >1.00).In normal distribution, the C_{pl} can be determined by the relationship between *PNC* and the C_{pl} as:

$$PNC = 1 - \Phi(3C_{pl}); \tag{14}$$

Where C_{pl} =1.00; *PNC* = 1,350 *ppm*, C_{pl} =1.50; *PNC* = 3.40 *ppm*, C_{pl} =2.00; *PNC* = 0.001 *ppm*. (*ppm* or non-conforming part per million). For Weibull distribution study, the studied C_{pl} can be determined from the corresponding *LSL* of Weibull distribution with the same *PNC*.

2.3.2 The performance measures

This paper applies the producer's risk and power of test to evaluate the performance of supplier comparison method. In quality engineering, producer's risk (α) is the probability that a good lot will be rejected, and consumer's risk (α) is the probability of accepting a lot of poor quality (a bad lot). To convenient work, consumer's risk is sometimes presented as the power of test $(1-\beta)$ [21]. The power implies that the probability of making a correct decision (reject the bad lot).

The 5% significance level (α = 0.05) is employed to study in this paper because this value is applied in the industries popularly. For appropriate method, producer's risk should be controlled under the α = 0.05, and power of test should be closed to 1.00 (maximum value) when sample size is larger.

The performance measures are calculated by Monte Carlo simulation. Iteration of 10,000 times are employed in this simulation because it is the accuracy varying numbers of iterations for calculating the producer's risk and the power of test.

This paper study two and five suppliers (k=2 and 5) with sample size n =15, 30, 50, 100, 200, and 300 respectively. The producer's risk can be calculated as:

1. Define the C_{pl} of each supplier: All the C_{pl} of supplier are equal ($C_{pl,1} = C_{pl,2}$) where $C_{pl,1} = C_{pl,2}$ = 1.00, 1.50, and 2.00 respectively; (for five suppliers, define $C_{pl,1} = C_{pl,2} = C_{pl,3} = C_{pl,4} = C_{pl,5}$);
2. Take random samples (n) from each supplier;
3. Compare suppliers using the proposed method with α = 0.05 and record the number of rejecting the null hypothesis;
4. Simulate using Monte Carlo method with iteration of 10,000 times;
5. Calculate producer's risk from equation (15);

$$\text{Producer's risk} = \frac{\text{Number of reject H}_0}{\text{10,000 times}}. \tag{15}$$

The power of test can be calculated as:

1. Define the C_{pl} of each supplier: The C_{pl} of the 1st supplier is different from other suppliers ($C_{pl,1} \neq C_{pl,2}$) where $C_{pl,1}$ = 1.00 vs. $C_{pl,2}$ = 1.50, $C_{pl,1}$ = 1.00 vs. $C_{pl,2}$ = 2.00 and $C_{pl,1}$ = 1.50 vs. $C_{pl,2}$ = 2.00; (for five suppliers, define $C_{pl,1} \neq C_{pl,2} = C_{pl,3} = C_{pl,4} = C_{pl,5}$);
2. Take random samples from each supplier;
3. Compare suppliers using the proposed method with α = 0.05 and record the number of rejecting the null hypothesis;
4. Simulate using Monte Carlo method with iteration of 10,000 times;
5. Calculate power of test from equation (16);

Table 2 Parameters including details in the simulation study

Parameters W (δ,γ),	Skewness	Kurtosis
W(1,1.0)	1.9971	8.9626
W(1,1.5)	1.0714	4.3839
W(1,3.6)	0.0004	2.7167
W(1,8.0)	-0.5335	3.3274

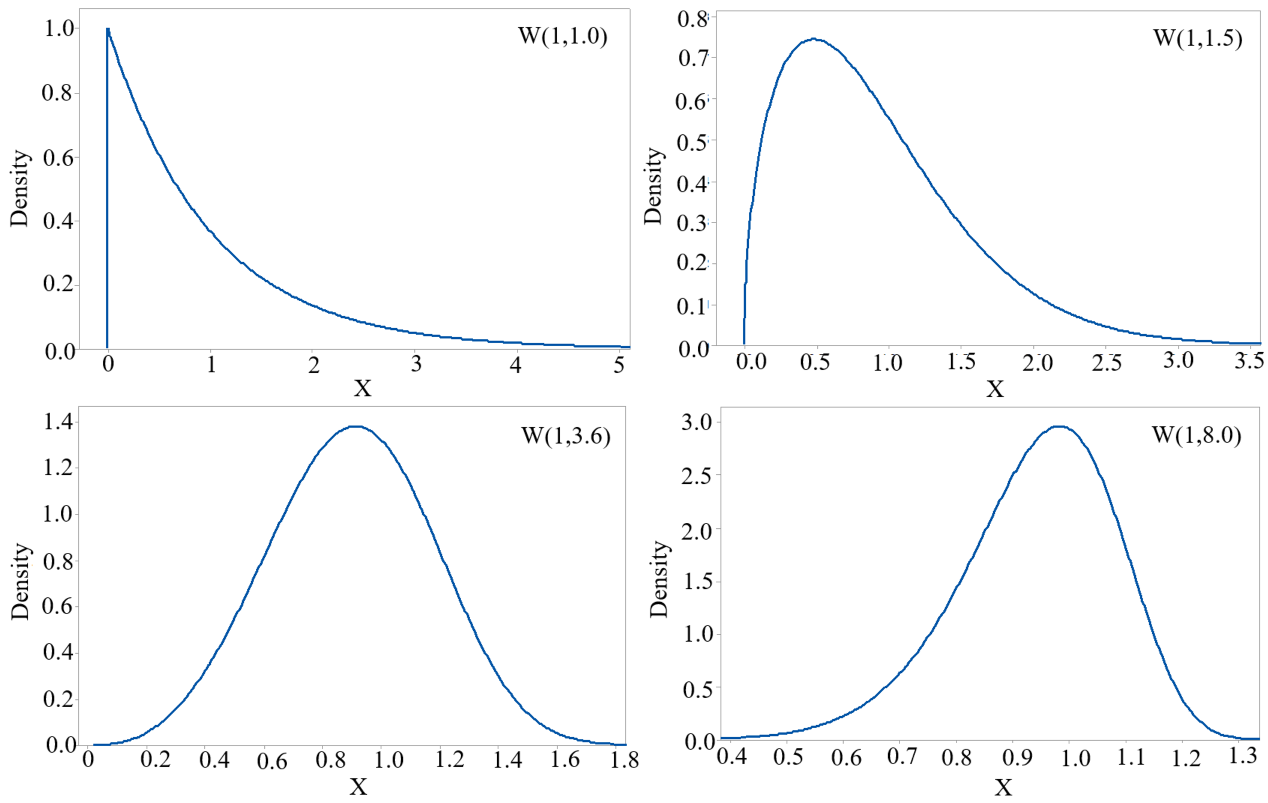

Figure 2 The various shape parameters of Weibull distribution in this study

$$\text{The power of test} = \frac{\text{Number of reject } H_0}{10{,}000 \text{ times}} . \qquad (16)$$

To verify the performance, the proposed method is compared with two supplier comparison methods applying the popular method for handling non-normal PCIs (i.e. percentile PCIs and Box-Cox transformation) for C_{pl} estimation. The details of two comparison methods are illustrated as:

1. Appling the percentile PCIs method: the supplier's processes are evaluated the process capability by the percentile PCIs method sometimes called "ISO method" and then compared by Manomat and Sudasna-na-Ayudthya method [4] respectively. The percentile method is widely used to evaluate non-normal process capability when the underlying distribution is discovered. This method can be calculated comfortably via the statistical software (e.g. Minitab). The method estimates PCIs using percentiles (e.g. 0.135, 50.0, and 99.865) of the underlying distribution. The C_{pl} of this method can be calculated from equation (17). For interpretation of process capability, the $C_{pl}(q)$ is same to the conventional C_{pl}. (e.g. $C_{pl}(q) > 1.00$ implies that process is capable)

$$C_{pl}(q) = \frac{X_{0.5} - LSL}{X_{0.5} - X_{0.00135}} \qquad (17)$$

Where $X_{0.05}$ and $X_{0.00135}$ is the 50th and 0.135th percentile for Weibull distribution respectively.

2. Appling Box-Cox transformation method: the supplier's processes are compared by Manomat and Sudasna-na-Ayudthya method [4] using the transformed data via Box-Cox transformation method. Box-Cox transformation [22] is popularly used for transforming non-normal into normal distributed data. In addition, this method easily calculates the PCIs by statistical software (e.g. Minitab). The process characteristic data are converted to normal distributed data (Y) as:

$$Y = \begin{cases} \dfrac{X^{\lambda} - 1}{\lambda} & ;\lambda \neq 0; \\ \ln X & ;\lambda = 0 \end{cases} \qquad (18)$$

Where λ is an unknown parameter estimated by the maximum likelihood method (see the detail from Box and Cox [22]).

3. Results and discussion

The simulation results for comparing two suppliers (k=2) are illustrated in Appendix Table 1 and 2. According to the proposed method, the producer's risks are not sensitive to the process's shape while they are decreased when sample size is larger. Since the higher C_{pl} influences on the decreasing of producer's risk, this method is suitable to compare the capable supplier (C_{pl} >1.00). However, the risks are controlled under the specified value (α = 0.05) because the risks are not over 0.01 in all cases. The example of producer's risk in case of $C_{pl,1}$ = $C_{pl,2}$ =1.00 are illustrated in Figure 3A. For the power of test, it is close to maximum value (1.00) in all process's shapes when sample size is larger. For example, the power of tests in case of $C_{pl,1}$ =1.00 and $C_{pl,2}$ =1.50 tend to increase continuously and are maximum when sample size is greater than 200 as shown in Figure 4A.

For applying the percentile PCIs method, although the producer's risks are lower than α = 0.05, they depend on the process's shape obviously as the example shown in Figure 3B. These risks are lower than the proposed method's risks at symmetrical and right-skewed process (shape parameter≤3.6). Then, the risks are greater than the proposed method's risks when process is left-skewed (shape parameter>3.6). The risks are decreased in case of the higher C_{pl} and larger sample size. In addition, the power of tests are sensitive to the process's shape. They close to 0 (minimum value) at right-skewed process but tend to increase at left-skewed process as example shown in Figure 4B. From the results, although the producer's risks are lowest at right-skewed process, the power of tests are no performance.

From supplier comparison using the transformed data via Box-Cox transformation, the simulated results show that the process's shape does not influence on the producer's risks and power of tests. The producer's risks are decreased by larger sample size and increased by higher C_{pl}. Moreover, the risks are greater than 0.20 in all cases as example in Figure 3C. As a result, they are uncontrollable under α = 0.05. The mentioned results show that the risks of this method are greater than the proposed method's obviously in all cases. Although the power of tests are greater than the proposed method's at small sample size (n<30), they are not over 0.80 in all cases as examples shown in Figure 4C.

In this study, the results of two performance measures (i.e. producer's risk and power of test) show that the proposed method is more appropriate for supplier comparison regarding Weibull distributed model than the two methods. Since the process capability of each supplier is straightforwardly estimated by the *PNC* value corresponding to Weibull distribution, the C_{pl} indices of two suppliers are estimated accurately and precisely. This result agrees with Hosseinifard and Abbasi [19], who mention that the CDF PCIs method is the best estimation for the PCIs if the underlying distribution can be known. Moreover, the performance of proposed method does not depend on the shape of supplier's process because the CDF PCIs method is not sensitive to distribution of process [18].

Although, percentile PCIs method and Box-Cox transformation are widely used in many industries to handle non-normal PCIs, the performance measures show that two methods are not appropriate for applying in supplier comparison. The percentile PCIs method is suitable with the slightly skewed process [23], so the heavily skewed process lead to the erroneous interpretation of process capability. Since the right-skewed process in this study is quite heavy (skewness>1), applying the $C_{pl}(q)$ to compare suppliers results in the lack of performance in the comparison as shown in the simulated results. Practically, lifetime data are commonly the right-skewed process. Therefore, the supplier comparison using the $C_{pl}(q)$ leads to a possibly misleading result in comparison. Manufacturer should be aware.

Since the supplier's processes are transformed to normal distribution, the C_{pl} estimation is not sensitive to process's shape. This result agrees with Swamy et. al. [24], who conclude that the accuracy of process capability estimation is robust to departure from normal distribution. However, the results of two performance measures demonstrate that supplier comparison along with Box-Cox transformation is no performance in case of Weibull distribution model although the sample size n>100 is employed.

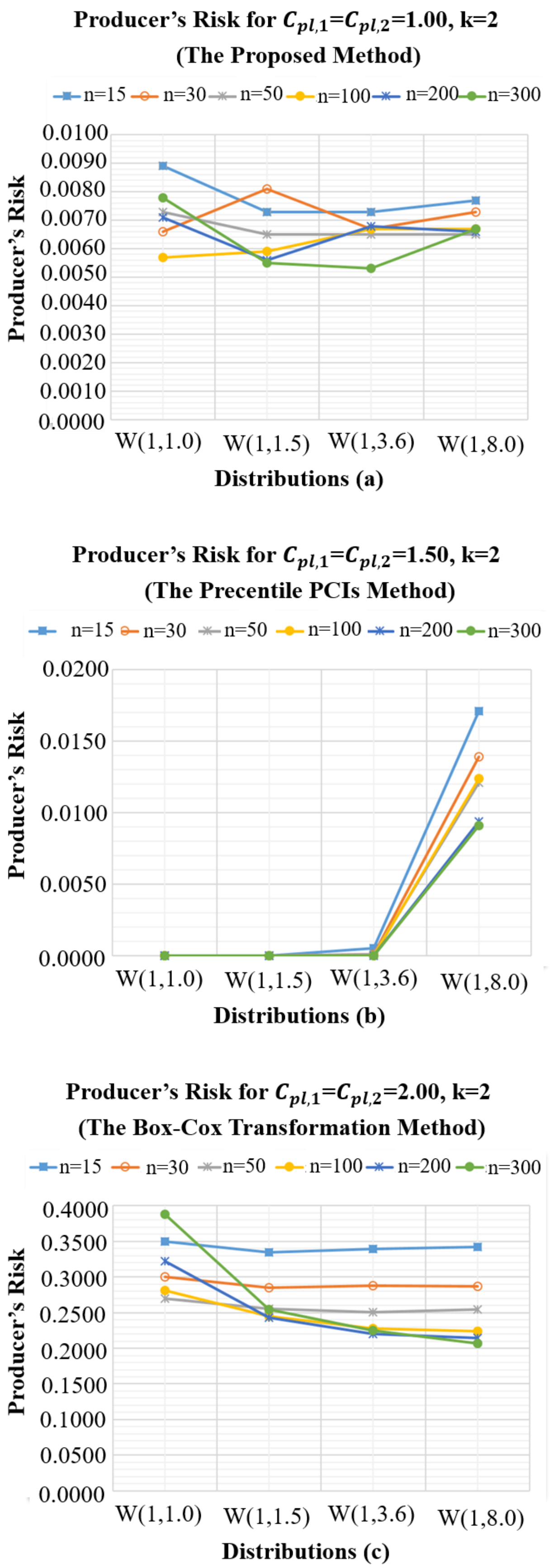

Figure 3 The producer's risks for two suppliers (k=2) in case of $C_{pl,1}$ = $C_{pl,2}$ =1.00; A: the proposed method, B: applying the percentile PCIs method; C: applying Box-Cox transformation method

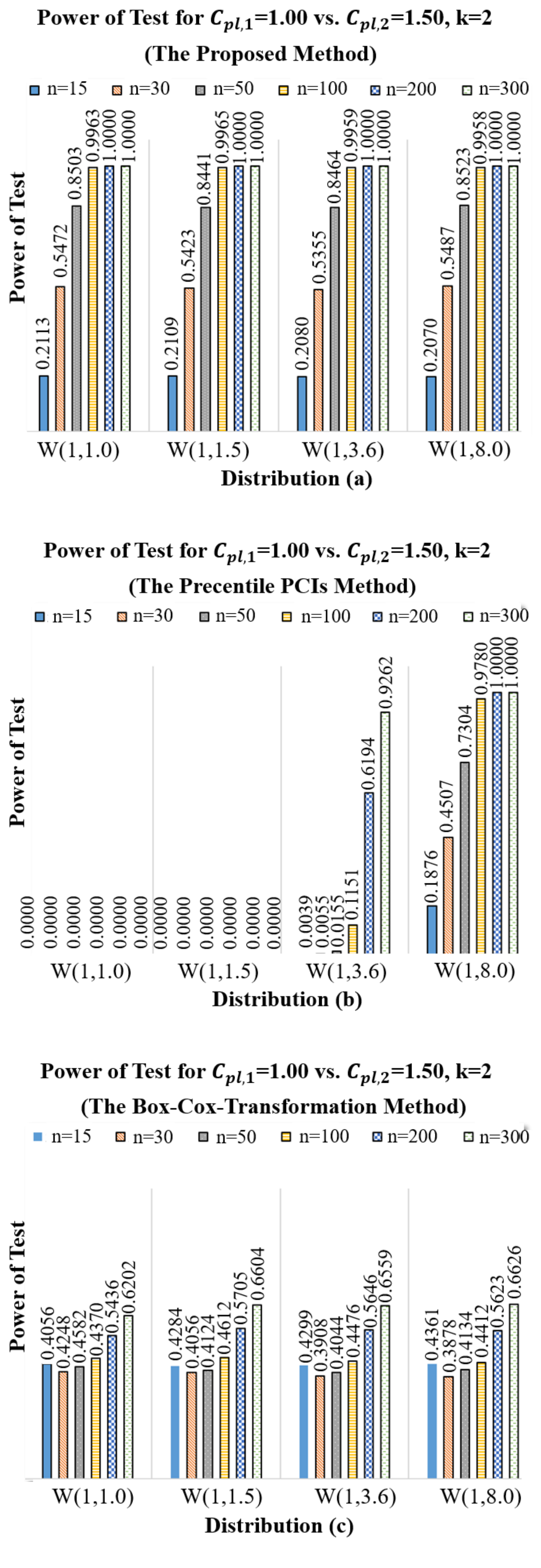

Figure 4 The power of tests for two suppliers (k=2) in case of $C_{pl,1}$ =1.00 and $C_{pl,2}$ =1.50; A: the proposed method, B: applying the percentile PCIs method and C: applying Box-Cox transformation method

To understand the proposed method, this paper presents the calculating method via the example. The example shows that the proposed method is appropriate to apply in the industries because it is able to handle the multiple suppliers having the different process shape.

Moreover, the method is not complex for calculating. The steps 1 to 3 can be comfortably calculated by the statistical software. For applying the proposed method, the data and *LSL* should be greater than zero. Since the result of supplier comparison is sensitive to sample size, manufacturers should select an appropriate sample size.
To verify the performance of the proposed method, five suppliers (k=5) are studied. From the simulated results shown in Appendix Table 3 and 4, the results indicate that the proposed method is appropriate to compare multiple suppliers and also more effective than the two method.

4. Conclusion

Durability is one of the important criteria that manufacturers should focus on in supplier comparison. The durable material obtained from supplier yields that the products are satisfactorily used over a long period of time. Lifetime is widely used to measure the durability and commonly modeled with Weibull distribution. In many industries, PCIs are widely employed in supplier comparison. However, the existing methods cannot apply with Weibull distribution model because it is developed under normality assumption. Therefore, this paper proposes the new method developed from Manomat and Sudasna-na-Ayudthya [4] to compare Weibull distribution models. Regarding the lifetime process, this study focuses on single quality characteristic and the C_{pl}. The performance of the method is measured by the producer's risk and power of test obtained from Monte Carlo simulation.

According to the simulated results, the two performance measures point out that the proposed method is more appropriate to compare suppliers regarding Weibull distribution model than the comparison methods applying percentile PCIs and Box-Cox transformation for C_{pl} estimation. In the proposed method, two performance measures are not sensitive to process's shape. Manufacturer can compare supplier's processes with any shape. Moreover, the producer's risk is controlled under the specified significance level, and the power of test is the best among two methods. Qualify supplier is selected. However, there are some drawbacks of the proposed method as: 1) the data of supplier's process (i.e. quality characteristic data, *LSL*) should be greater than zero and 2) this method cannot be adapted to apply with upper PCIs (C_{pu}).

In manufacturing industry, the existing method for supplier comparison using C_{pl} cannot conduct with lifetime process due to limitation of normality assumption. Although manufacturers apply the popular methods for handling non-normal C_{pl} such as percentile method and Box-Cox transformation to compare suppliers, the comparing result is possibly erroneous as shown in the simulated results. Therefore, the proposed method is an appropriate method that assists the manufacturers to compare supplier's processes regarding the product life or lifetime data effectively. The raw materials and parts of products received from qualified suppliers are operated efficiently under the specification of lifetime. As a result, customers are satisfied with the quality products acquired from manufacturers. Moreover, the proposed method can apply to other non-normal distributed data, which are discovered in other processes such as exponential and lognormal because the distributions come from the Gamma family as Weibull distribution but the scale and shape parameters are different.

The further study may develop the method for comparing multiple suppliers based on non-normal distribution for two-sided specification.

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Appendix

Table 1 The simulated results of the producer's risk for two suppliers (k=2)

The Proposed Method

	$C_{pl,1}$ = $C_{pl,2}$=1.00 (*PNC*=1349.890 ppm)				$C_{pl,1}$ = $C_{pl,2}$=1.50 (*PNC*=3.398 ppm)				$C_{pl,1}$ = $C_{pl,2}$=2.00 (*PNC*=0.001 ppm)			
n	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**
15	0.0089	0.0073	0.0073	0.0077	0.0027	0.0033	0.0030	0.0033	0.0019	0.0028	0.0022	0.0024
30	0.0066	0.0081	0.0067	0.0073	0.0032	0.0029	0.0027	0.0030	0.0017	0.0023	0.0019	0.0020
50	0.0073	0.0065	0.0065	0.0065	0.0030	0.0023	0.0023	0.0025	0.0018	0.0019	0.0018	0.0018
100	0.0057	0.0059	0.0067	0.0067	0.0030	0.0021	0.0028	0.0029	0.0023	0.0018	0.0020	0.0017
200	0.0071	0.0056	0.0068	0.0066	0.0035	0.0024	0.0025	0.0033	0.0018	0.0017	0.0020	0.0014
300	0.0078	0.0055	0.0053	0.0067	0.0032	0.0027	0.0028	0.0025	0.0017	0.0016	0.0016	0.0015

The Percentile PCIs Method

	$C_{pl,1}$ = $C_{pl,2}$=1.00 (*PNC*=1349.890 ppm)				$C_{pl,1}$ = $C_{pl,2}$=1.50 (*PNC*=3.398 ppm)				$C_{pl,1}$ = $C_{pl,2}$=2.00 (*PNC*=0.001 ppm)			
n	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**
15	0.0000	0.0000	0.0005	0.0171	0.0000	0.0000	0.0001	0.0168	0.0000	0.0000	0.0007	0.0126
30	0.0000	0.0000	0.0001	0.0139	0.0000	0.0000	0.0001	0.0117	0.0000	0.0000	0.0002	0.0097
50	0.0000	0.0000	0.0001	0.0121	0.0000	0.0000	0.0000	0.0092	0.0000	0.0000	0.0001	0.0075
100	0.0000	0.0000	0.0000	0.0124	0.0000	0.0000	0.0000	0.0070	0.0000	0.0000	0.0000	0.0053
200	0.0000	0.0000	0.0000	0.0094	0.0000	0.0000	0.0000	0.0059	0.0000	0.0000	0.0000	0.0063
300	0.0000	0.0000	0.0000	0.0091	0.0000	0.0000	0.0000	0.0079	0.0000	0.0000	0.0000	0.0072

Box-Cox Transformation Method

	$C_{pl,1}$ = $C_{pl,2}$=1.00 (*PNC*=1349.890 ppm)				$C_{pl,1}$ = $C_{pl,2}$=1.50 (*PNC*=3.398 ppm)				$C_{pl,1}$ = $C_{pl,2}$=2.00 (*PNC*=0.001 ppm)			
n	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**
15	0.3498	0.3348	0.3391	0.3421	0.4945	0.4909	0.4809	0.4867	0.5520	0.5496	0.5410	0.5362
30	0.2998	0.2853	0.2875	0.2872	0.4497	0.4498	0.4463	0.4395	0.5032	0.5016	0.4978	0.4987
50	0.2694	0.2550	0.2503	0.2541	0.4443	0.4279	0.4189	0.4206	0.4961	0.4805	0.4889	0.4789
100	0.2812	0.2448	0.2279	0.2238	0.4624	0.4127	0.3957	0.3938	0.5118	0.4719	0.4540	0.4516
200	0.3220	0.2427	0.2198	0.2139	0.4878	0.4333	0.3985	0.3817	0.4984	0.4965	0.4566	0.4439
300	0.3883	0.2547	0.2251	0.2063	0.4711	0.4164	0.3828	0.3695	0.4649	0.4595	0.4470	0.4438

Table 2 The simulated results of the power of test for two suppliers (k=2)

The Proposed Method

n	$C_{pl,1}$ = 1.00 vs. $C_{pl,2}$ = 1.50				$C_{pl,1}$ = 1.00 vs. $C_{pl,2}$ = 2.00				$C_{pl,1}$ = 1.50 vs. $C_{pl,2}$ = 2.00			
	W(1,1.0)	W(1,1.5)	W(1,3.6)	W(1,8.0)	W(1,1.0)	W(1,1.5)	W(1,3.6)	W(1,8.0)	W(1,1.0)	W(1,1.5)	W(1,3.6)	W(1,8.0)
15	0.2113	0.2109	0.2080	0.2070	0.7378	0.7468	0.7484	0.7478	0.0642	0.0658	0.0611	0.0660
30	0.5472	0.5423	0.5355	0.5487	0.9886	0.9885	0.9883	0.9883	0.2114	0.2147	0.2138	0.2142
50	0.8503	0.8441	0.8464	0.8523	1.0000	1.0000	1.0000	1.0000	0.4659	0.4563	0.4607	0.4571
100	0.9963	0.9965	0.9959	0.9958	1.0000	1.0000	1.0000	1.0000	0.8828	0.8681	0.8776	0.8771
200	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9986	0.9989	0.9985	0.9985
300	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

The Percentile PCIs Method

n	$C_{pl,1}$ = 1.00 vs. $C_{pl,2}$ = 1.50				$C_{pl,1}$ = 1.00 vs. $C_{pl,2}$ = 2.00				$C_{pl,1}$ = 1.50 vs. $C_{pl,2}$ = 2.00			
	W(1,1.0)	W(1,1.5)	W(1,3.6)	W(1,8.0)	W(1,1.0)	W(1,1.5)	W(1,3.6)	W(1,8.0)	W(1,1.0)	W(1,1.5)	W(1,3.6)	W(1,8.0)
15	0.0000	0.0000	0.0039	0.1876	0.0000	0.0000	0.0046	0.4460	0.0000	0.0000	0.0004	0.0338
30	0.0000	0.0000	0.0055	0.4507	0.0000	0.0000	0.0119	0.8306	0.0000	0.0000	0.0002	0.0585
50	0.0000	0.0000	0.0155	0.7340	0.0000	0.0000	0.0378	0.9792	0.0000	0.0000	0.0000	0.1098
100	0.0000	0.0000	0.1151	0.9780	0.0000	0.0000	0.2958	0.9999	0.0000	0.0000	0.0000	0.2675
200	0.0000	0.0000	0.6194	1.0000	0.0000	0.0000	0.8893	1.0000	0.0000	0.0000	0.0000	0.6006
300	0.0000	0.0000	0.9262	1.0000	0.0000	0.0000	0.9959	1.0000	0.0000	0.0000	0.0001	0.8258

Box-Cox Transformation Method

n	$C_{pl,1}$ = 1.00 vs. $C_{pl,2}$ = 1.50				$C_{pl,1}$ = 1.00 vs. $C_{pl,2}$ = 2.00				$C_{pl,1}$ = 1.50 vs. $C_{pl,2}$ = 2.00			
	W(1,1.0)	W(1,1.5)	W(1,3.6)	W(1,8.0)	W(1,1.0)	W(1,1.5)	W(1,3.6)	W(1,8.0)	W(1,1.0)	W(1,1.5)	W(1,3.6)	W(1,8.0)
15	0.4370	0.4284	0.4299	0.4361	0.4556	0.4637	0.4593	0.4539	0.5146	0.5158	0.5173	0.5182
30	0.4056	0.4035	0.3908	0.3878	0.4476	0.4397	0.4351	0.435	0.4903	0.4707	0.4749	0.4745
50	0.4248	0.4124	0.4044	0.4134	0.461	0.4547	0.4445	0.4491	0.4646	0.4557	0.4433	0.4465
100	0.4582	0.4612	0.4476	0.4412	0.5278	0.5111	0.5141	0.5167	0.4931	0.4557	0.4321	0.4262
200	0.5436	0.5705	0.5646	0.5623	0.596	0.6477	0.6479	0.6419	0.5109	0.4547	0.4332	0.421
300	0.6202	0.6604	0.6559	0.6626	0.675	0.749	0.7488	0.7524	0.4933	0.4416	0.4251	0.4089

Table 3 The simulated results of the producer's risk for five suppliers (k=5)

The Proposed Method												
	$C_{pl,1} = C_{pl,2} = \ldots = C_{pl,5}$ = 1.00 (*PNC*=1349.890 ppm)				**$C_{pl,1} = C_{pl,2} = \ldots = C_{pl,5}$ = 1.50 (*PNC*=3.398 ppm)**				**$C_{pl,1} = C_{pl,2} = \ldots = C_{pl,5}$ = 2.00 (*PNC*=0.001 ppm)**			
n	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**
15	0.0016	0.0027	0.0016	0.0016	0.0003	0.0002	0.0001	0.0005	0.0003	0.0001	0.0002	0.0001
30	0.0014	0.0017	0.0014	0.0013	0.0005	0.0002	0.0003	0.0001	0.0001	0.0001	0.0001	0.0002
50	0.0021	0.0009	0.0015	0.0016	0.0001	0.0002	0.0001	0.0002	0.0001	0.0001	0.0002	0.0003
100	0.0015	0.0013	0.0015	0.0007	0.0002	0.0001	0.0004	0.0002	0.0002	0.0002	0.0001	0.0001
200	0.0007	0.0009	0.0011	0.0008	0.0005	0.0003	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002
300	0.0018	0.0011	0.0015	0.0013	0.0002	0.0002	0.0001	0.0001	0.0002	0.0001	0.0001	0.0001
The Percentile PCIs Method												
	$C_{pl,1} = C_{pl,2} = \ldots = C_{pl,5}$ = 1.00 (*PNC*=1349.890 ppm)				**$C_{pl,1} = C_{pl,2} = \ldots = C_{pl,5}$ = 1.50 (*PNC*=3.398 ppm)**				**$C_{pl,1} = C_{pl,2} = \ldots = C_{pl,5}$ = 2.00 (*PNC*=0.001 ppm)**			
n	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**
15	0.0000	0.0000	0.0000	0.0062	0.0000	0.0000	0.0000	0.0041	0.0000	0.0000	0.0001	0.0028
30	0.0000	0.0000	0.0000	0.0044	0.0000	0.0000	0.0000	0.0025	0.0000	0.0000	0.0000	0.0023
50	0.0000	0.0000	0.0000	0.0037	0.0000	0.0000	0.0000	0.0014	0.0000	0.0000	0.0000	0.0020
100	0.0000	0.0000	0.0000	0.0025	0.0000	0.0000	0.0000	0.0012	0.0000	0.0000	0.0000	0.0010
200	0.0000	0.0000	0.0000	0.0027	0.0000	0.0000	0.0000	0.0016	0.0000	0.0000	0.0000	0.0010
300	0.0000	0.0000	0.0000	0.0018	0.0000	0.0000	0.0000	0.0014	0.0000	0.0000	0.0000	0.0003
Box-Cox Transformation Method												
	$C_{pl,1} = C_{pl,2} = \ldots = C_{pl,5}$ = 1.00 (*PNC*=1349.890 ppm)				**$C_{pl,1} = C_{pl,2} = \ldots = C_{pl,5}$ = 1.50 (*PNC*=3.398 ppm)**				**$C_{pl,1} = C_{pl,2} = \ldots = C_{pl,5}$ = 2.00 (*PNC*=0.001 ppm)**			
n	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**
15	0.6166	0.6133	0.6215	0.6185	0.8272	0.8313	0.8224	0.8216	N/A	N/A	N/A	0.8722
30	0.5596	0.5445	0.5296	0.5397	0.8042	0.8013	0.7959	0.7950	0.8605	0.8540	0.8506	0.8519
50	0.5244	0.5083	0.4836	0.4875	0.8024	0.7934	0.7777	0.7776	0.8696	0.8488	0.8330	0.8399
100	0.5426	0.4785	0.4452	0.4485	0.8551	0.7893	0.7603	0.7591	0.9177	0.8529	0.8391	0.8225
200	0.6758	0.5030	0.4333	0.4110	0.8923	0.8039	0.7611	0.7525	0.8935	0.8563	0.8268	0.8253
300	0.7693	0.5101	0.4251	0.4060	0.8574	0.7846	0.7615	0.7549	0.8638	0.8208	0.8292	0.8176

Remark N/A means that the value is not calculable due to the limitation of the simulation program.

Table 4 The simulated results of the power of test for five suppliers (k=5)

The Proposed Method

	$C_{pl,1}$ = 1.00 vs. $C_{pl,2}$ =…= $C_{pl,5}$ = 1.50				$C_{pl,1}$ = 1.00 vs. $C_{pl,2}$ =…= $C_{pl,5}$ = 2.00				$C_{pl,1}$ = 1.50 vs. $C_{pl,2}$ =…= $C_{pl,5}$ = 2.00			
n	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**
15	0.1527	0.1553	0.1519	0.1568	0.7695	0.7785	0.7709	0.7752	0.0267	0.0284	0.0286	0.0249
30	0.4889	0.4881	0.4999	0.4965	0.9931	0.9927	0.9926	0.9934	0.1168	0.1159	0.1201	0.1175
50	0.8359	0.8319	0.8322	0.8381	1.0000	1.0000	1.0000	1.0000	0.3364	0.3449	0.3371	0.3363
100	0.9978	0.9971	0.9980	0.9983	1.0000	1.0000	1.0000	1.0000	0.8526	0.8518	0.8592	0.8514
200	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9986	0.9986	0.9994	0.9991
300	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

The Percentile PCIs Method

n	$C_{pl,1}$ = 1.00 vs. $C_{pl,2}$ =…= $C_{pl,5}$ = 1.50				$C_{pl,1}$ = 1.00 vs. $C_{pl,2}$ =…= $C_{pl,5}$ = 2.00				$C_{pl,1}$ = 1.50 vs. $C_{pl,2}$ =…= $C_{pl,5}$ = 2.00			
	W(1,1.0)	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**
15	0.0000	0.0000	0.0000	0.1487	0.0000	0.0000	0.0001	0.4441	0.0000	0.0000	0.0000	0.0121
30	0.0000	0.0000	0.0000	0.4051	0.0000	0.0000	0.0004	0.8464	0.0000	0.0000	0.0000	0.0227
50	0.0000	0.0000	0.0003	0.7193	0.0000	0.0000	0.0019	0.9859	0.0000	0.0000	0.0000	0.0442
100	0.0000	0.0000	0.0094	0.9849	0.0000	0.0000	0.0715	1.0000	0.0000	0.0000	0.0000	0.1578
200	0.0000	0.0000	0.2946	1.0000	0.0000	0.0000	0.7297	1.0000	0.0000	0.0000	0.0000	0.4917
300	0.0000	0.0000	0.7862	1.0000	0.0000	0.0000	0.9832	1.0000	0.0000	0.0000	0.0000	0.7892

Box-Cox Transformation Method

n	$C_{pl,1}$ = 1.00 vs. $C_{pl,2}$ =…= $C_{pl,5}$ = 1.50				$C_{pl,1}$ = 1.00 vs. $C_{pl,2}$ =…= $C_{pl,5}$ = 2.00				$C_{pl,1}$ = 1.50 vs. $C_{pl,2}$ =…= $C_{pl,5}$ = 2.00			
	W(1,1.0)	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**	**W(1,1.0)**	**W(1,1.5)**	**W(1,3.6)**	**W(1,8.0)**
15	0.8106	0.8017	0.7936	0.7934	N/A	N/A	N/A	0.8485	N/A	N/A	N/A	0.8624
30	0.7860	0.7757	0.7747	0.7703	0.8341	0.8270	0.8229	0.8334	0.8499	0.8467	0.8391	0.8471
50	0.8025	0.7824	0.7599	0.7693	0.8550	0.8329	0.8217	0.8233	0.8618	0.8317	0.8323	0.8214
100	0.8471	0.8187	0.7892	0.7920	0.8954	0.8761	0.8511	0.8468	0.9008	0.8442	0.8149	0.8168
200	0.8811	0.8677	0.8504	0.8496	0.9008	0.9195	0.9082	0.9139	0.8941	0.8503	0.8189	0.8156
300	0.8810	0.9105	0.9077	0.9031	0.9004	0.9428	0.9484	0.9499	0.8627	0.8100	0.8202	0.8157

Remark N/A means that the value is not calculable due to the limitation of the simulation program.